gg→Higgs from the TMD perspective

Daniël Boer

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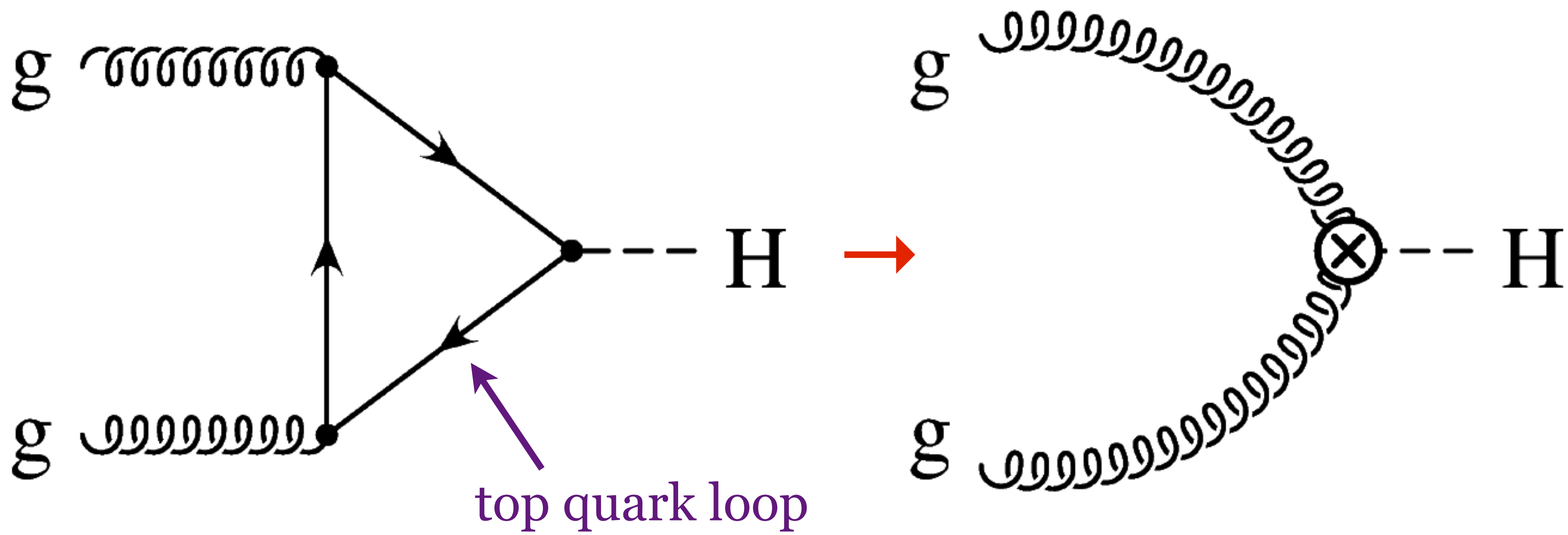
university of groningen

Higgs production in gluon fusion

Higgs production in gg → H happens via a top quark loop:



The inclusive Higgs production cross section at LHC can be described well because the collinear gluon distribution inside protons is known well

It becomes a different matter for the transverse momentum distribution
At large Q_T one can again use collinear factorization, but at small Q_T there are large logs of Q_T/Q (resummation) & nonperturbative contributions

Here: TMD perspective (TMD factorized expressions and TMD evolution)
(gauge links and process dependence will be discussed by Piet Mulders)

TMD factorization expressions

$$\frac{d\sigma}{dx_A dx_B d\Omega d^2\boldsymbol{q}_T} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T}\,\tilde{W}(\boldsymbol{b},Q;x_A,x_B) + \mathcal{O}\left(\frac{Q_T^2}{Q^2}\right)$$

$$\tilde{W}(\boldsymbol{b},Q;x_A,x_B) = \tilde{f}_1^g(x_A,\boldsymbol{b}^2;\zeta_A,\mu)\,\tilde{f}_1^g(x_B,\boldsymbol{b}^2;\zeta_B,\mu)H\left(Q;\mu\right)$$

TMD factorization expressions

Y term

$$\frac{d\sigma}{dx_A dx_B d\Omega d^2\boldsymbol{q}_T} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T}\tilde{W}(\boldsymbol{b}, Q; x_A, x_B) + \mathcal{O}\left(\frac{Q_T^2}{Q^2}\right)$$

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This is a naive expression, since gluons can be polarized inside unpolarized protons [Mulders, Rodrigues '01]

$$\Phi_g^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{n_\rho\, n_\sigma}{(p\cdot n)^2} \int \frac{d(\xi\cdot P)\, d^2\xi_T}{(2\pi)^3}\; e^{ip\cdot\xi}\, \langle P|\, \mathrm{Tr}\big[\, F^{\mu\rho}(0)\, F^{\nu\sigma}(\xi)\, \big]\, |P\rangle\, \big]_{\mathrm{LF}}$$

$$= -\frac{1}{2x}\left\{ g_T^{\mu\nu}\, f_1^g - \left(\frac{p_T^\mu p_T^\nu}{M^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M^2}\right) h_1^{\perp\, g} \right\}$$

Second term requires nonzero k_T, but is k_T even, chiral even and T even

$$\tilde{\Phi}_g^{ij}(x, \boldsymbol{b}) = \frac{1}{2x}\left\{ \delta^{ij}\, \tilde{f}_1^g(x, b^2) - \left(\frac{2b^i b^j}{b^2} - \delta^{ij}\right) \tilde{h}_1^{\perp\, g}(x, b^2) \right\}$$

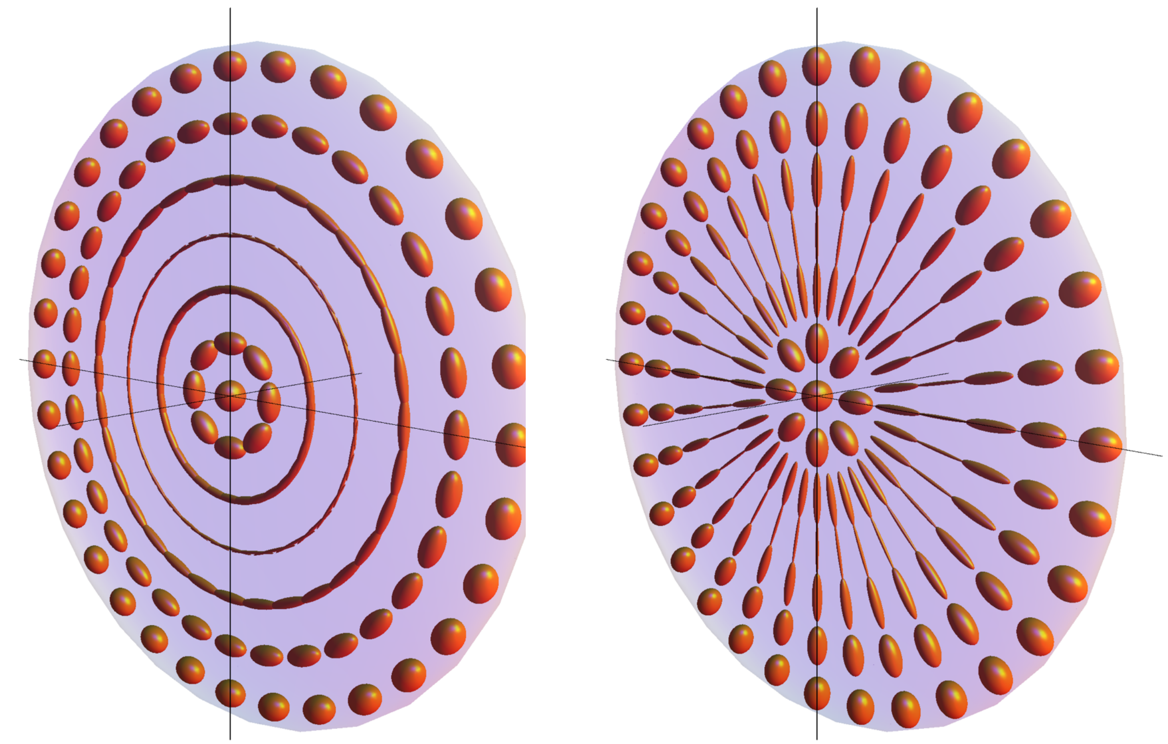
Gluon polarization inside unpolarized protons

Polarization densities:

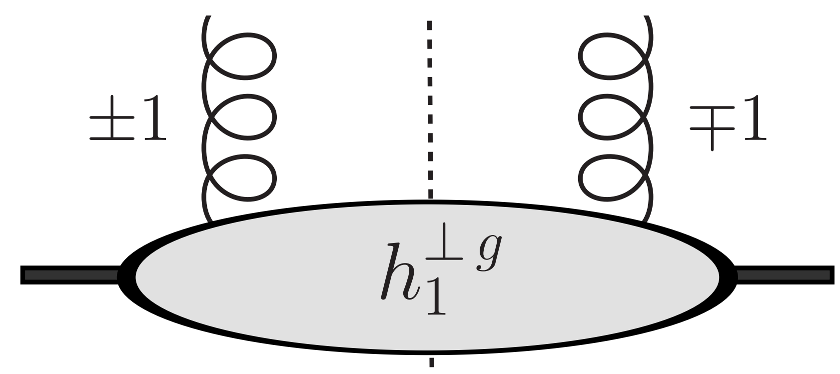
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ unpolarized gluons

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$ circularly polarized gluons

$$\begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix}$$ linearly polarized gluons



$$\Phi_g^{\mu\nu}(x, \boldsymbol{p}_T)_{\text{max pol}} = \frac{2}{x}\,\frac{p_T^\mu p_T^\nu}{\boldsymbol{p}_T^2}\,f_1^g$$



an interference between ±1 helicity gluon states

It means that gluons prefers to be polarized along k_T,
with a $\cos 2\phi$ distribution of linear polarization around it, where $\phi = \angle(k_T, \varepsilon_T)$

Gluon polarization inside unpolarized protons

Linearly polarized gluons are generated perturbatively
[Nadolsky, Balazs, Berger, Yuan, '07; Catani, Grazzini, '10]

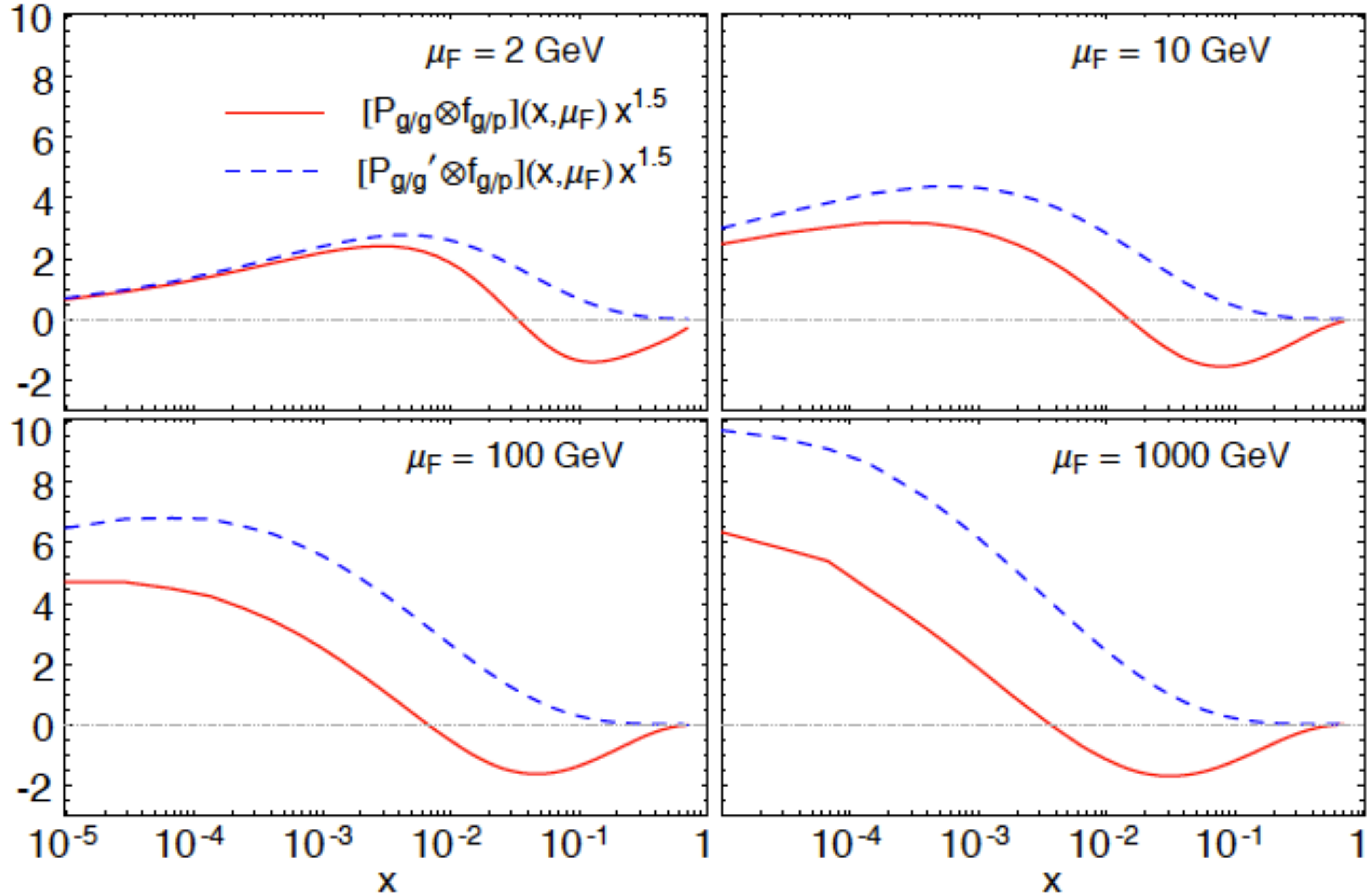


Figure 4: Comparison of $[P_{g/g} \otimes f_{g/p}](x, \mu_F)$ and $[P'_{g/g} \otimes f_{g/p}](x, \mu_F)$ for the gluon PDF $f_{g/p}(x, \mu_F)$ in the proton (multiplied by $x^{1.5}$ to better illustrate the small-x region) at several values of the factorization scale μ_F.

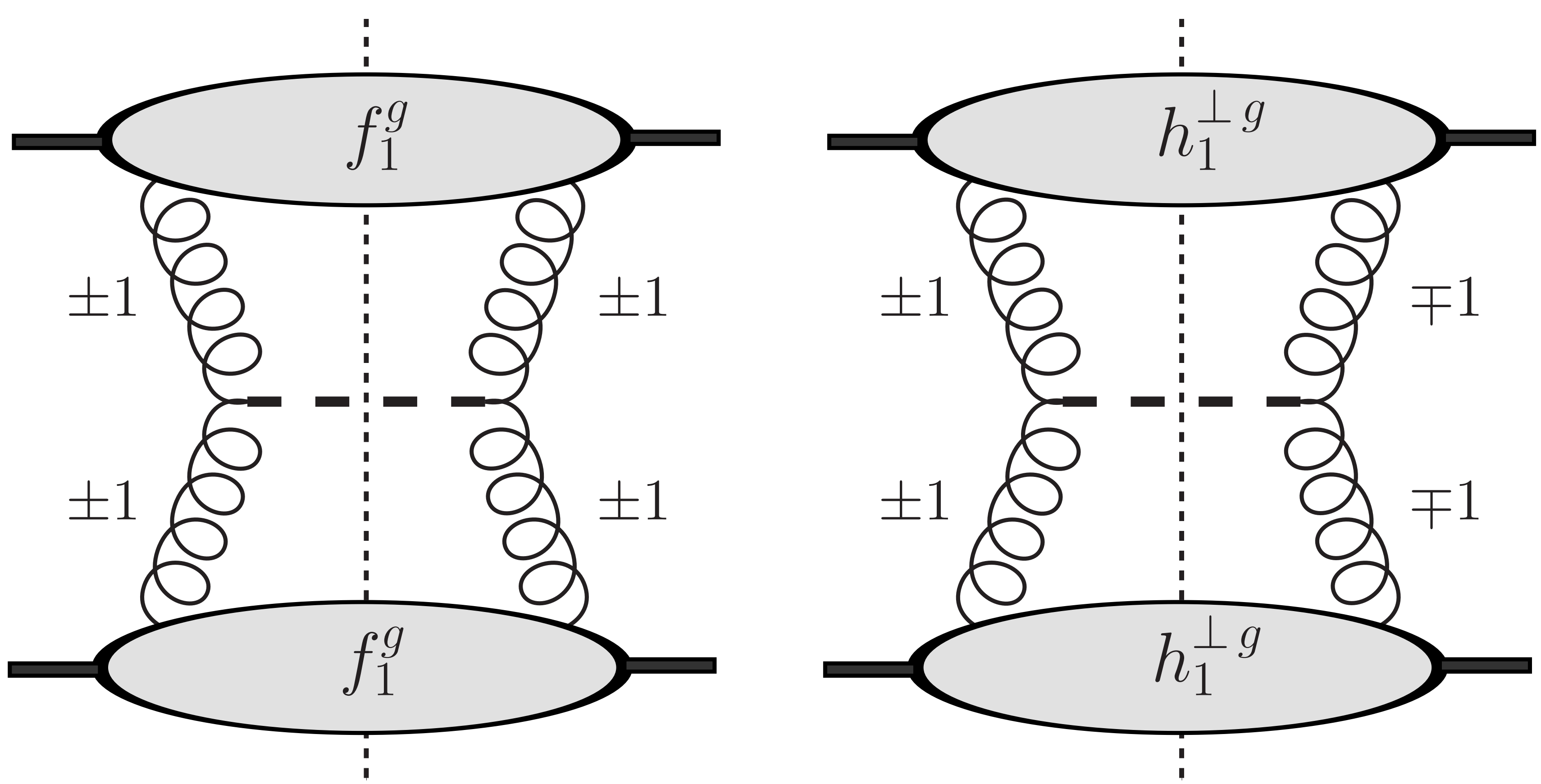
A nonperturbative distribution ($h_1^{\perp g}$) can be present too [Mulders, Rodrigues '01]

$h_1^{\perp g}$ in $p\,p \to H\ X$

It affects the transverse momentum distribution in $pp \to HX$ (Higgs production)

Linearly polarized gluons enter Higgs production ($\sigma(Q_T)$) at NNLO pQCD
[Catani & Grazzini, '10]

The nonperturbative distribution can be present at tree level and would affect Higgs production at low Q_T
[DB, Den Dunnen, Pisano, Schlegel, Vogelsang, '12]



Tree level expression

$$\left.\frac{E\, d\sigma^{pp\to HX}}{d^3\vec{q}}\right|_{q_T\ll m_H} = \frac{\pi\sqrt{2}G_F}{128 m_H^2 s}\left(\frac{\alpha_s}{4\pi}\right)^2 \left|\mathcal{A}_H(\tau)\right|^2$$
$$\times\left(\mathcal{C}\left[f_1^g\, f_1^g\right] + \mathcal{C}\left[w_H\, h_1^{\perp g}\, h_1^{\perp g}\right]\right) + \mathcal{O}\left(\frac{q_T}{m_H}\right)$$

The gluon TMDs enter in convolutions:

$$\mathcal{C}[w\, f\, f] \equiv \int d^2\boldsymbol{p}_T \int d^2\boldsymbol{k}_T\, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f(x_A, \boldsymbol{p}_T^2)\, f(x_B, \boldsymbol{k}_T^2)$$

$$w_H = \frac{(\boldsymbol{p}_T\cdot\boldsymbol{k}_T)^2 - \frac{1}{2}\boldsymbol{p}_T^2\boldsymbol{k}_T^2}{2M^4} \qquad\qquad \tau = m_H^2/(4m_t^2)$$

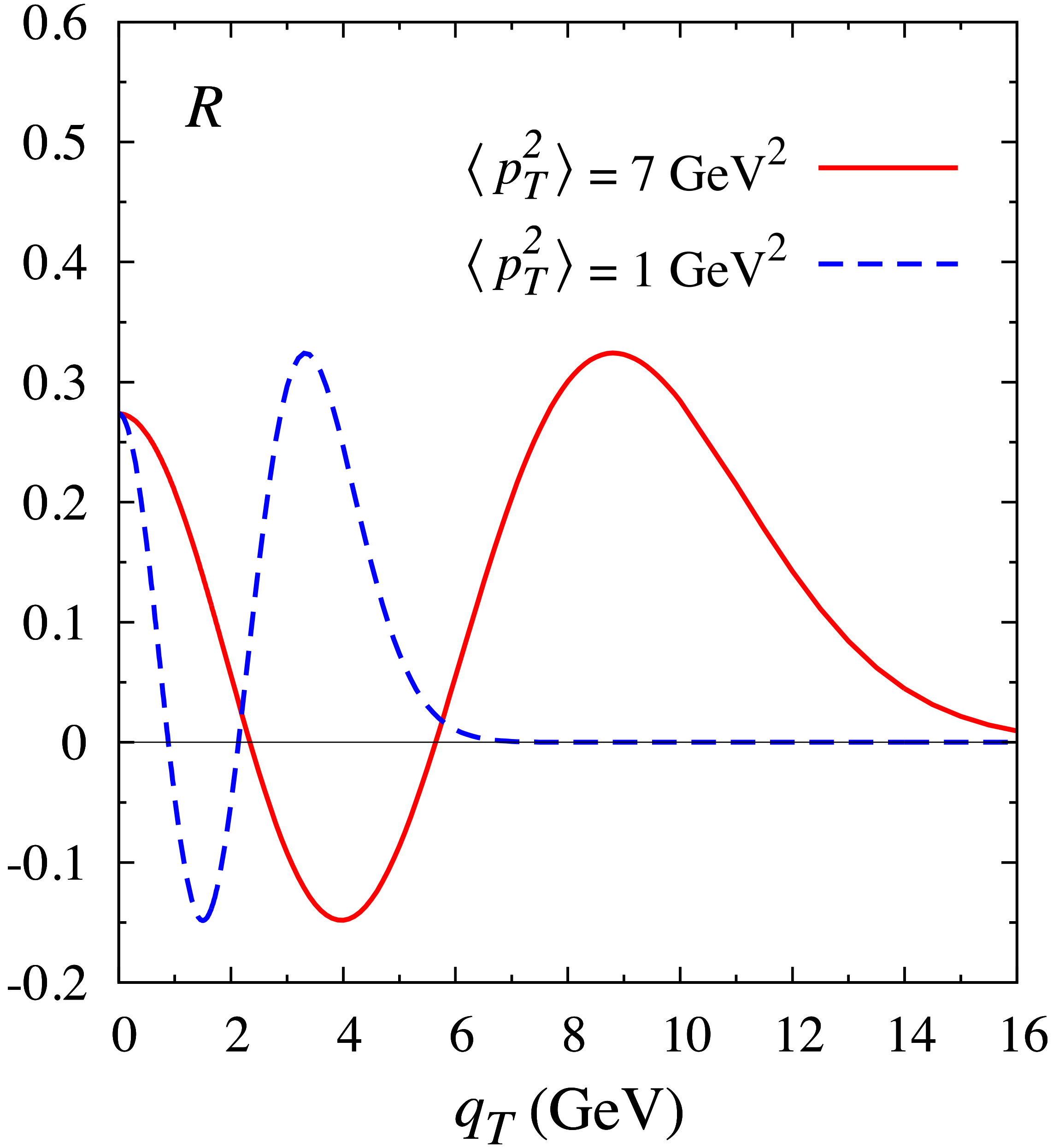
The relative effect of linearly polarized gluons:

$$\mathcal{R}(Q_T) \equiv \frac{\mathcal{C}[w_H\, h_1^{\perp g}\, h_1^{\perp g}]}{\mathcal{C}[f_1^g\, f_1^g]}$$

Angular independent cross section is of the form:

$$\frac{d\sigma}{dq_T} \propto [1 \pm R(q_T)] \qquad (+ \text{ for } H^0; - \text{ for } A^0)$$

$$R = \frac{\mathcal{C}[w_H h_1^{\perp g} h_1^{\perp g}]}{\mathcal{C}[f_1^g f_1^g]}$$



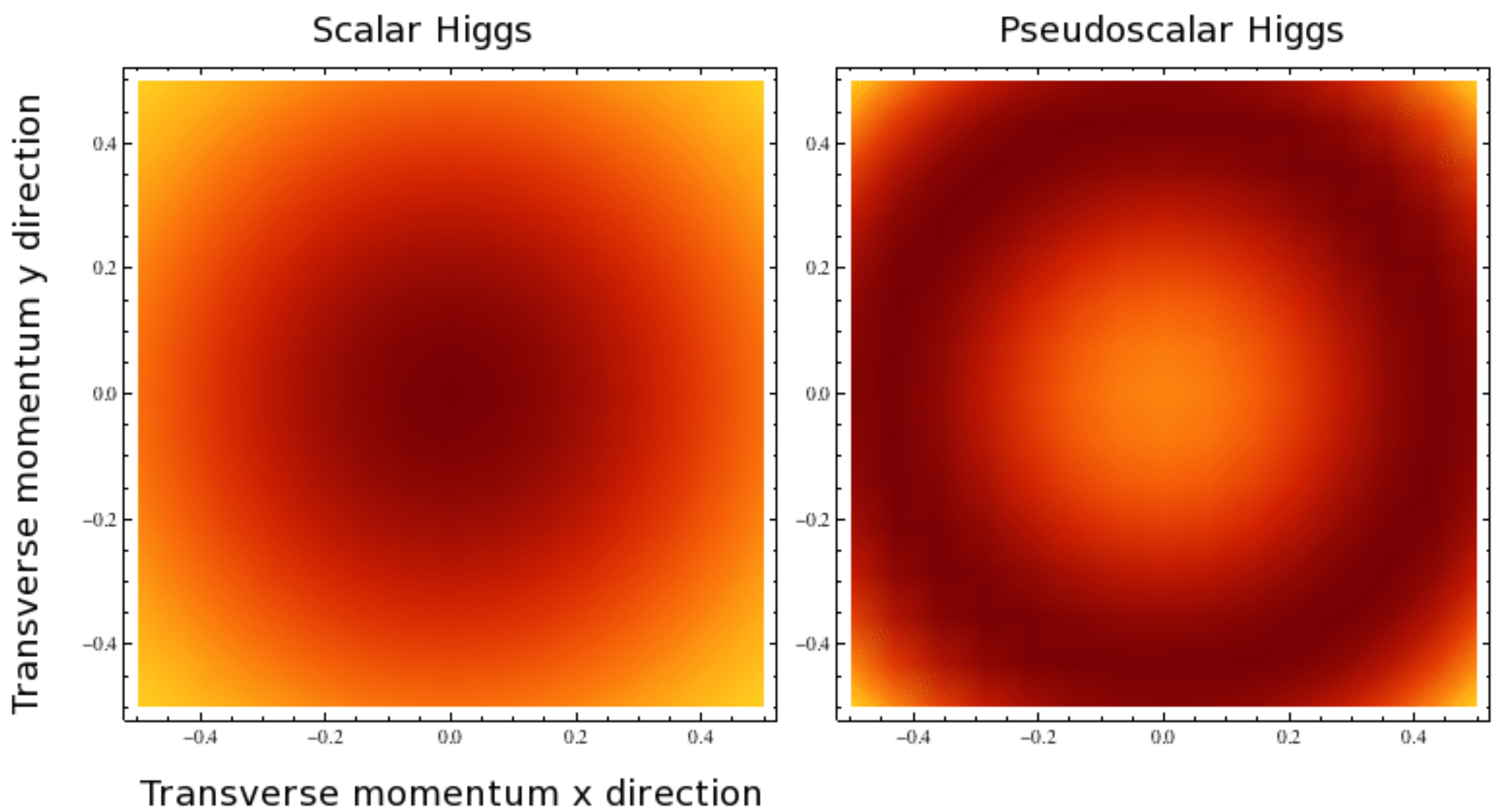
Characteristic modulation (double node)

Overall sign determined by the parity of the Higgs

[DB, Den Dunnen, Pisano, Schlegel, Vogelsang, '12]

Here a model function for $h_1^{\perp g}$ is used that is close to its bound for larger q_T

On-shell Higgs production



In reality the Higgs boson decays

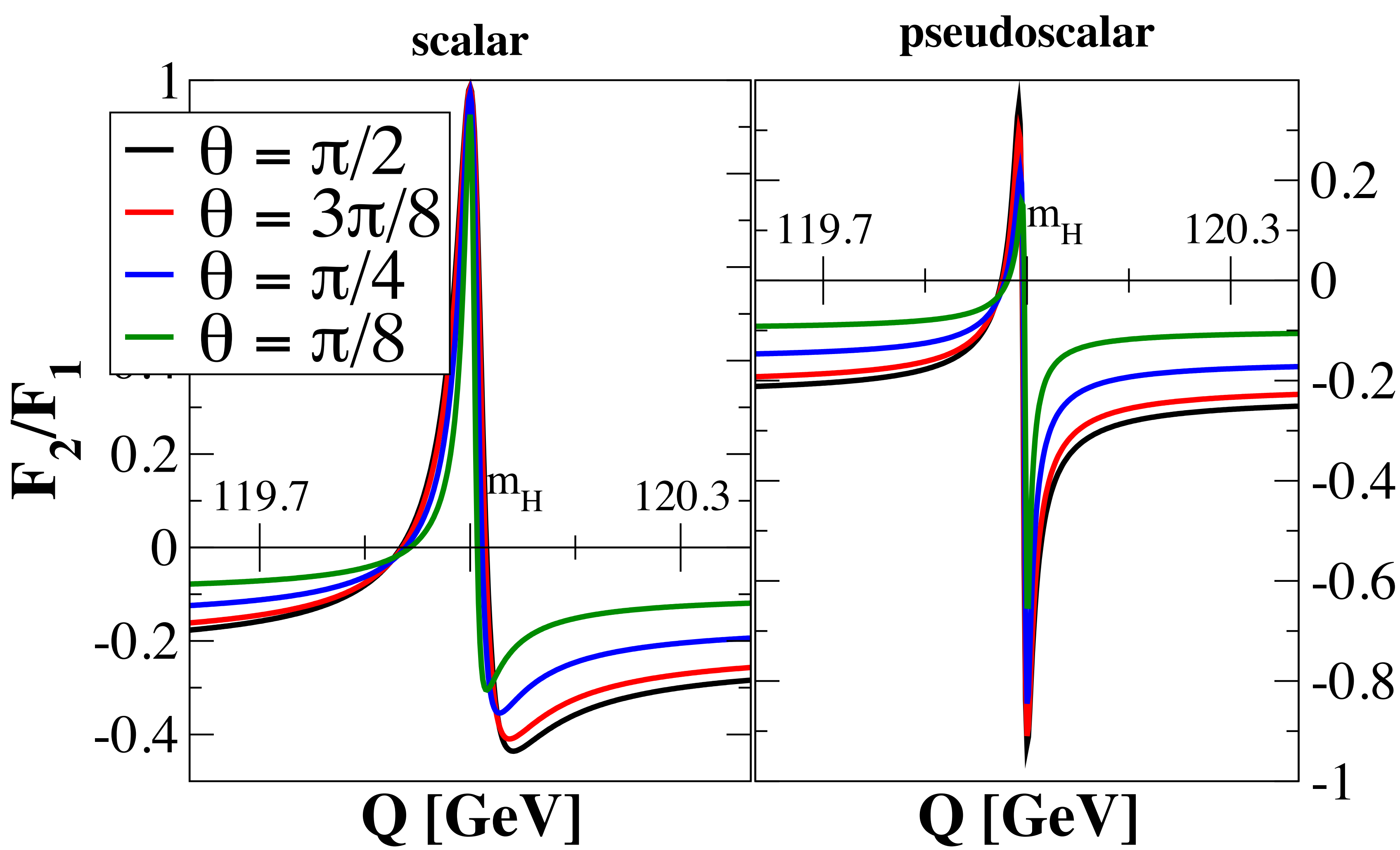
There will be background processes to deal with, which may dilute the modulation

Linearly polarized gluons also enter in the process gg → γγ without Higgs

[Nadolsky, Balazs, Berger, Yuan, '07; ,Qiu, Schlegel,Vogelsang '10]

$gg \to \gamma\,\gamma$

$$\int d\phi \frac{d\sigma}{d^4q\, d\Omega} \propto \left[1 + \frac{F_2}{F_1} R(q_T)\right]$$

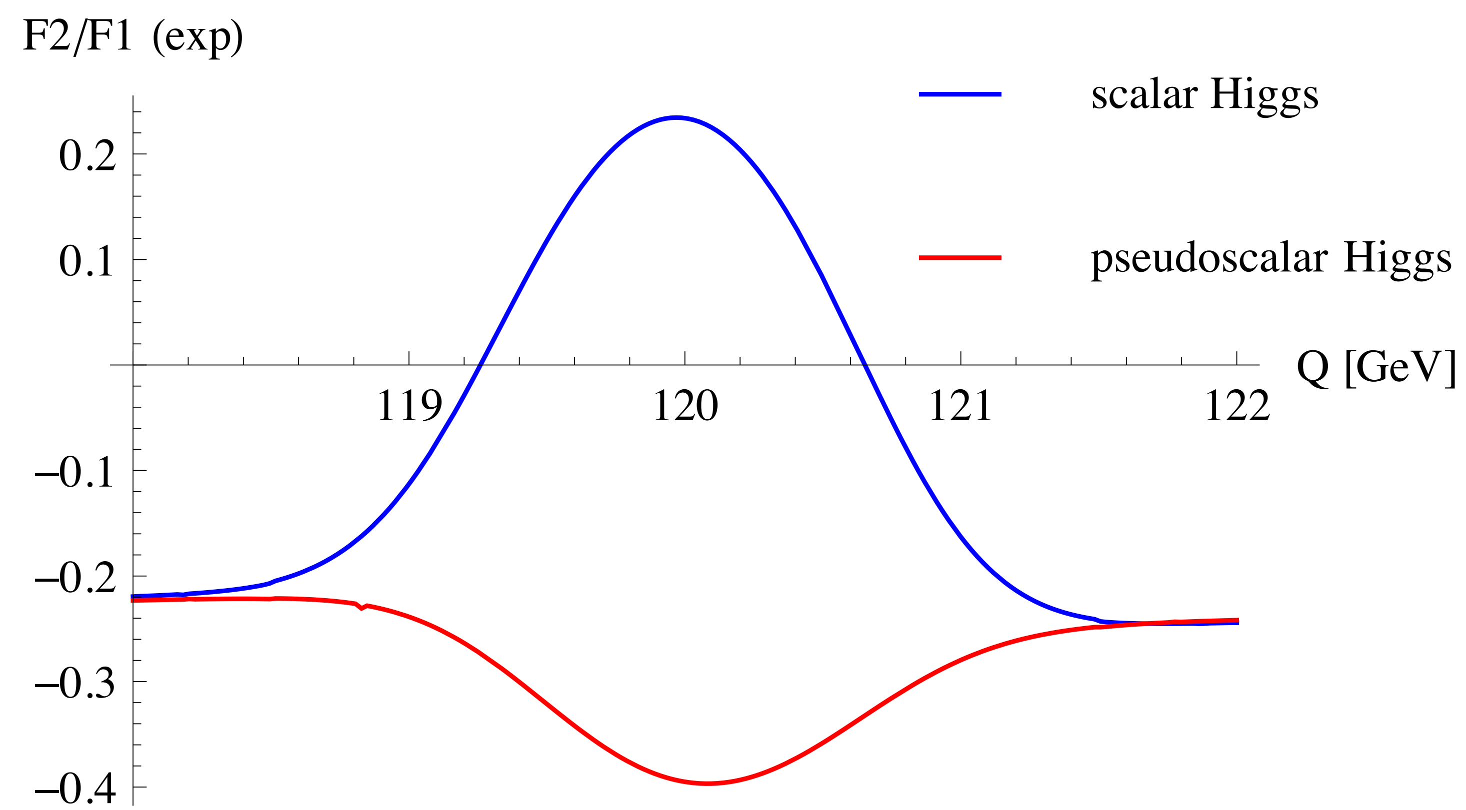


Discernable only in a narrow region around the Higgs mass (here: m_H=120 GeV)

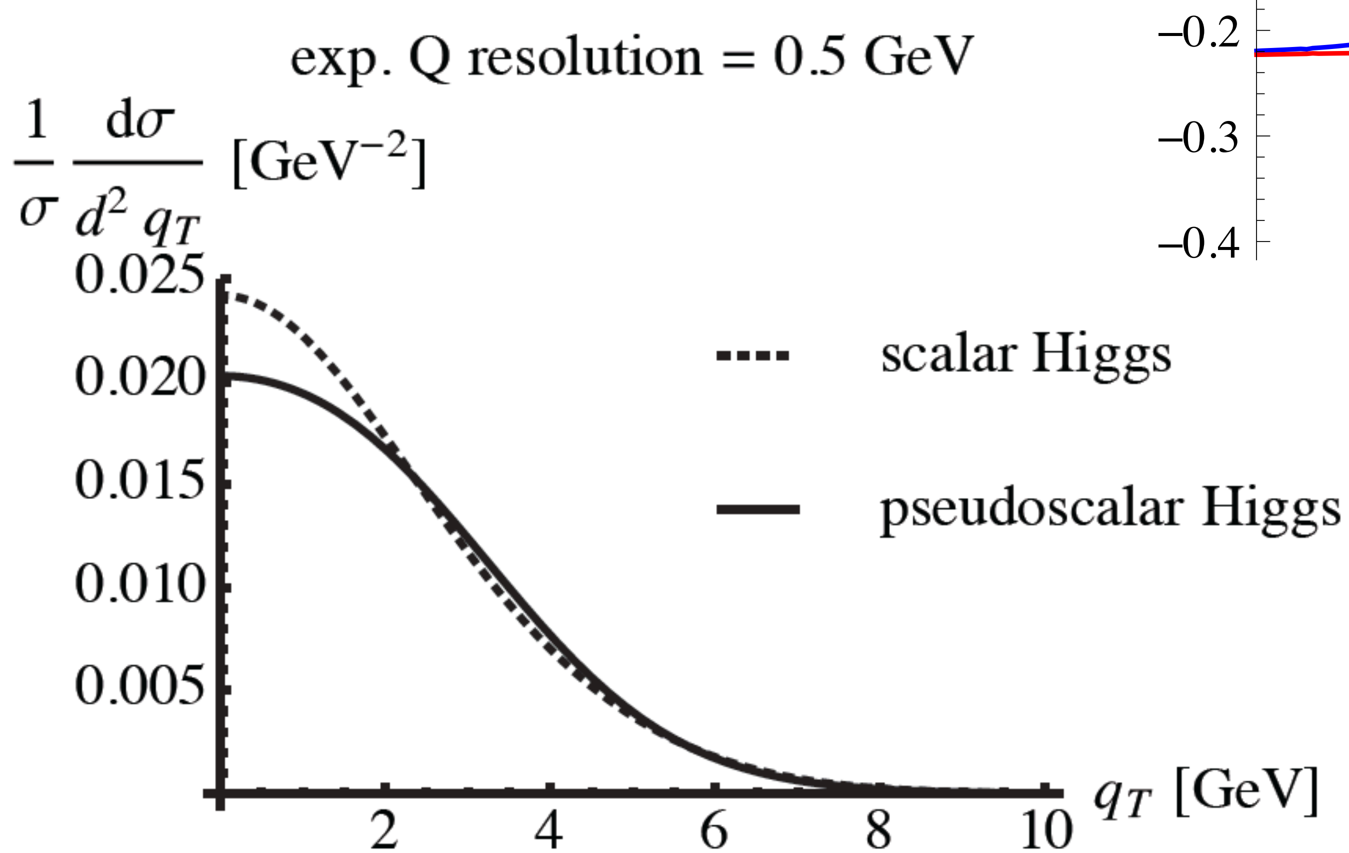
[DB, Den Dunnen, Pisano, Schlegel, Vogelsang, '12]

$gg \to \gamma\gamma$

$$\int d\phi \frac{d\sigma}{d^4q\,d\Omega} \propto \left[1 + \frac{F_2}{F_1} R(q_T)\right]$$



Energy resolution becomes important
Assume ΔQ=0.5 GeV



What do we know about the polarization?

At small x the WW (or CGC) gluon field and the dipole distribution have been studied:

$$h_{1,WW}^{\perp g} \ll f_{1,WW}^{\perp g} \quad \text{for } k_\perp \ll Q_s, \qquad h_{1,WW}^{\perp g} = 2 f_{1,WW}^{\perp g} \quad \text{for } k_\perp \gg Q_s$$

$$x h_{1,DP}^{\perp g}(x, k_\perp) = 2 x f_{1,DP}^{g}(x, k_\perp)$$

Metz, Zhou '11

At small x the kT factorization approach yields maximum polarization too:

$$\Phi_g^{\mu\nu}(x, \boldsymbol{p}_T)_{\text{max pol}} = \frac{2}{x} \frac{p_T^\mu p_T^\nu}{\boldsymbol{p}_T^2} f_1^g$$

Catani, Ciafaloni, Hautmann, 1991

One can also consider the perturbative tail, which is calculable

$$\tilde{f}_{g/P}(x, b^2; \mu, \zeta) = \sum_{i=g,q} \int_x^1 \frac{d\hat{x}}{\hat{x}} C_{i/g}(x/\hat{x}, b^2; g(\mu), \mu, \zeta) f_{i/P}(\hat{x}; \mu) + \mathcal{O}((\Lambda_{\text{QCD}} b)^a)$$

There is no theoretical reason why it should be small, especially at small x

$$\mathcal{R}(Q_T) \equiv \frac{\mathcal{C}[w_H\, h_1^{\perp g}\, h_1^{\perp g}]}{\mathcal{C}[f_1^g\, f_1^g]}$$

$$\begin{aligned}
\mathcal{C}\,[f_1^g\, f_1^g] &= \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T}\, \tilde{f}_1^g(x_A, b^2; \zeta_A, \mu)\, \tilde{f}_1^g(x_B, b^2; \zeta_B, \mu) \\
&= \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T}\, e^{-S_A(b,Q)}\, \tilde{f}_1^g(x_A, b^2; \mu_b^2, \mu_b)\, \tilde{f}_1^g(x_B, b^2; \mu_b^2, \mu_b)
\end{aligned}$$

$$S_A(b,Q) = \frac{C_A}{\pi} \int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \alpha_s(\mu) \left[\ln\left(\frac{Q^2}{\mu^2}\right) - \frac{11 - 2n_f/C_A}{6}\right] + \mathcal{O}(\alpha_s^2)$$

$$\begin{aligned}
S_A(b,Q) = &- \frac{36}{33 - 2n_f} \left[\ln\left(\frac{Q^2}{\mu_b^2}\right) + \ln\left(\frac{Q^2}{\Lambda^2}\right)\, \ln\left(1 - \frac{\ln\left(Q^2/\mu_b^2\right)}{\ln\left(Q^2/\Lambda^2\right)}\right)\right. \\
&\left. + \frac{11 - 2n_f/C_A}{6} \ln\left(\frac{\ln\left(Q^2/\Lambda^2\right)}{\ln\left(\mu_b^2/\Lambda^2\right)}\right)\right]
\end{aligned}$$

$$\frac{d\sigma}{dx_A dx_B d\Omega d^2\boldsymbol{q}_T} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T}\tilde{W}(\boldsymbol{b}, Q; x_A, x_B) + \mathcal{O}\left(\frac{Q_T^2}{Q^2}\right)$$

The integral is over all b, including nonperturbatively large b

$$\tilde{W}(b) \equiv \tilde{W}(b_*)\, e^{-S_{NP}(b)}$$

$$b_* = b/\sqrt{1 + b^2/b_{\rm max}^2} \leq b_{\rm max}$$

$$b_{\rm max} = 1.5\ {\rm GeV}^{-1} \Rightarrow \alpha_s(b_0/b_{\rm max}) = 0.62$$

Modified Aybat-Rogers S_{NP}

$$S_{NP}(b, Q, Q_0) = \frac{C_A}{C_F}\left[0.184 \ln\frac{Q}{2Q_0} + 0.332\right] b^2$$

$$\mathcal{R}(Q_T) = \frac{\int d^2\boldsymbol{b}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T} e^{-S_A(b_*,Q)-S_{NP}(b,Q)}\; \tilde{h}_1^{\perp g}(x_A, b_*^2; \mu_{b_*}^2, \mu_{b_*})\; \tilde{h}_1^{\perp g}(x_B, b_*^2; \mu_{b_*}^2, \mu_{b_*})}{\int d^2\boldsymbol{b}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T}\, e^{-S_A(b_*,Q)-S_{NP}(b,Q)} \tilde{f}_1^g(x_A, b_*^2; \mu_{b_*}^2, \mu_{b_*})\, \tilde{f}_1^g(x_B, b_*^2; \mu_{b_*}^2, \mu_{b_*})}$$

$$\tilde{h}_1^{\perp\, g}(x, b^2) = \int d^2\boldsymbol{p}_T\; \frac{(\boldsymbol{b}\cdot\boldsymbol{p}_T)^2 - \frac{1}{2}\boldsymbol{b}^2\boldsymbol{p}_T^2}{b^2 M^2}\; e^{-i\boldsymbol{b}\cdot\boldsymbol{p}_T}\; h_1^{\perp g}(x, p_T^2)$$

$$= -\pi \int dp_T^2 \frac{p_T^2}{2M^2} J_2(b p_T) h_1^{\perp g}(x, p_T^2)$$

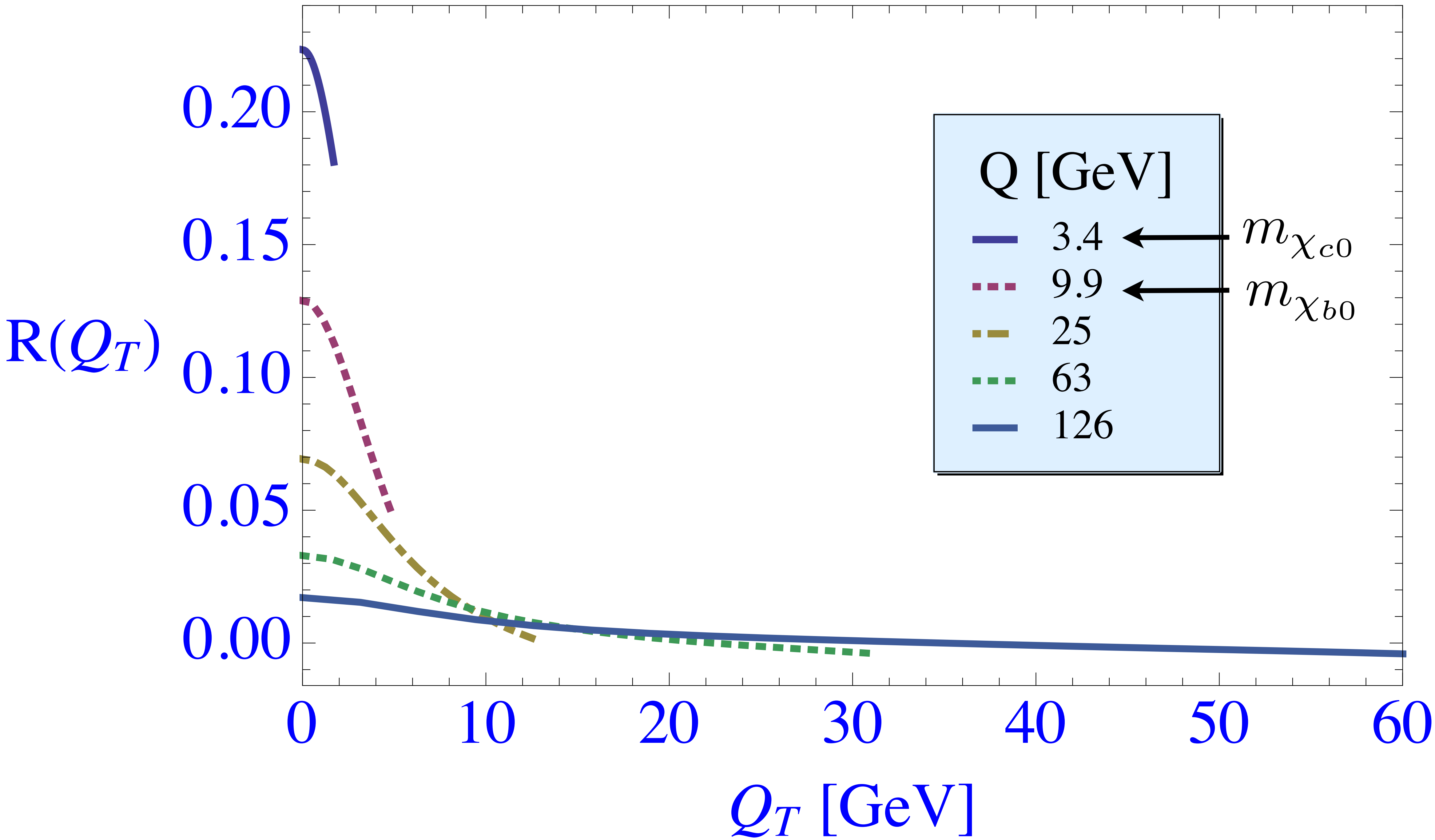
Consider now only the perturbative tail:

$$\tilde{f}_1^g(x, b^2; \mu_b^2, \mu_b) = f_{g/P}(x; \mu_b) + \mathcal{O}(\alpha_s)$$

$$\tilde{h}_1^{\perp g}(x, b^2; \mu_b^2, \mu_b) = \frac{\alpha_s(\mu_b) C_A}{2\pi} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1\right) f_{g/P}(\hat{x}; \mu_b) + \mathcal{O}(\alpha_s^2)$$

This coincides with the CSS approach

TMD evolution effects



$$x_A = x_B = Q/(8\text{TeV})$$

MSTW08 LO gluon distribution

D.B. & den Dunnen, 1404.6753

PHYSICAL REVIEW D **86,** 094026 (2012)

Improved resummation prediction on Higgs boson production at hadron colliders

Jian Wang,[1] Chong Sheng Li,[1,2,*] Hai Tao Li,[1] Zhao Li,[3,†] and C.-P. Yuan[2,3,‡]

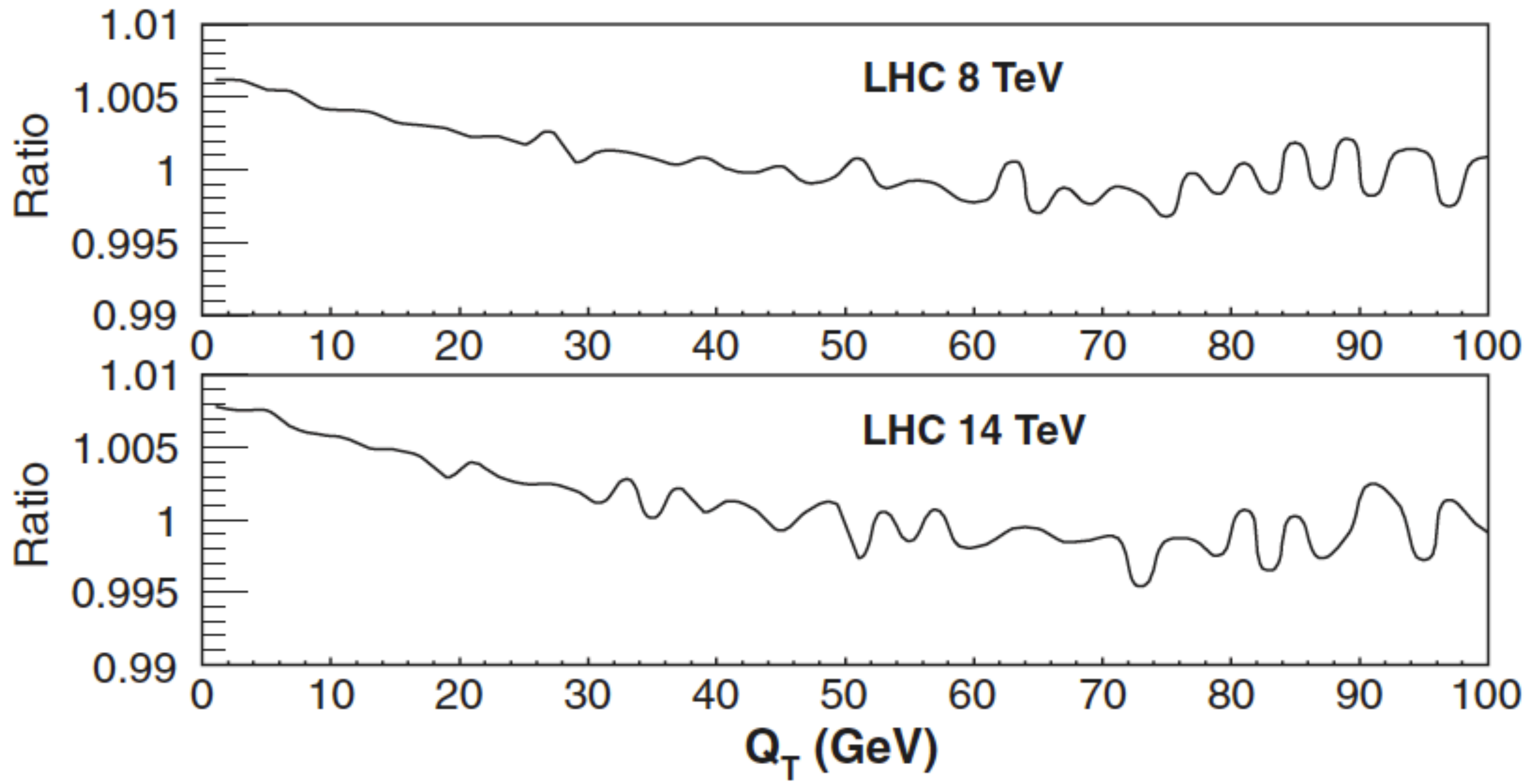


FIG. 3. The ratios between the transverse momentum distributions with and without G functions at the Tevatron (1.96 TeV) and the LHC (7, 8, and 14 TeV). The oscillations of the ratio curves in the figure are due to numerical uncertainties.

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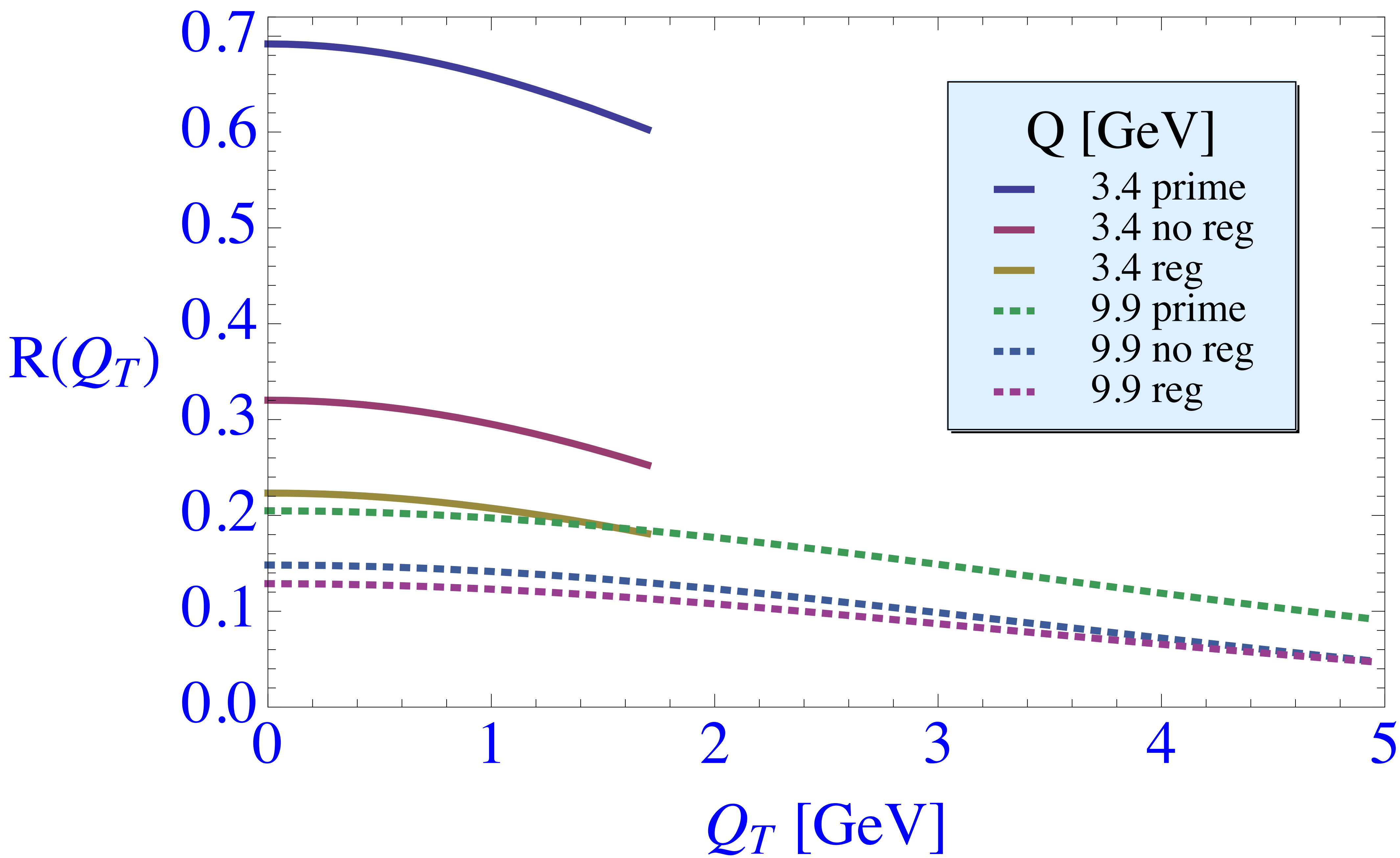
$$\frac{G^{(1)}G^{(1)}\alpha_s^2 + 2G^{(1)}G^{(2)}\alpha_s^3}{C^{(0)}C^{(0)} + 2C^{(0)}C^{(1)}\alpha_s + (C^{(1)}C^{(1)} + 2C^{(0)}C^{(2)})\alpha_s^2} \approx$$

$$\frac{G^{(1)}G^{(1)}\alpha_s^2}{C^{(0)}C^{(0)}} \left(1 + \frac{2G^{(1)}G^{(2)}}{G^{(1)}G^{(1)}}\alpha_s + \mathcal{O}(\alpha_s^2)\right) \left(1 - \frac{2C^{(0)}C^{(1)}}{C^{(0)}C^{(0)}}\alpha_s + \mathcal{O}(\alpha_s^2)\right)$$

They include third factor, but not second
No reason why not of same size, but not expected to be large effect on end result
Can reduce the suppression somewhat

Wang et al. use also different S_{NP}

At low Q there is quite some uncertainty from the very small b region (b << 1/Q) where the perturbative expressions for S_A are all incorrect (don't satisfy S(0)=0)



Standard regularization:

$$Q^2/\mu_b^2 = b^2 Q^2/b_0^2 \rightarrow Q^2/\mu_b'^2 \equiv (bQ/b_0 + 1)^2$$

Parisi, Petronzio, 1985

Very small b region

For very small b region (b << 1/Q) the perturbative expressions for S_A are all incorrect

$$S_A(b,Q) = \frac{C_A}{\pi}\int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\alpha_s(\mu)\,[\ldots] \stackrel{b\ll 1/Q}{\longrightarrow} -\frac{C_A}{\pi}\int_{Q^2}^{\mu_b^2} \frac{d\mu^2}{\mu^2}\alpha_s(\mu)\,[\ldots]$$

Sudakov suppression ($e^{-\#}$) becomes (fake) Sudakov enhancement ($e^{+\#}$)

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Sudakov suppression ($e^{-\#}$) becomes (fake) Sudakov enhancement ($e^{+\#}$)

$$\frac{d\sigma}{dq_T^2} = Y(q_T^2) + \int \frac{d^2\mathbf{b}}{4\pi} e^{-i\mathbf{q}_T\cdot\mathbf{b}}\sigma_0(1+A)\exp S(b)$$

where

$$S(b) = \int_0^{A_T^2}\frac{dk^2}{k^2}(J_0(b\,k)-1)\left(B\ln\frac{Q^2}{k^2}+C\right).$$

$$A_T^2 = A_T^2(y) = \frac{(S+Q^2)^2}{4S\cosh^2 y} - Q^2.$$

$$\exp S = \exp\int_0^{A_T^2} \approx \left(1+\int_{Q^2}^{A_T^2}\right)\exp\int_0^{Q^2}$$

Altarelli, Ellis, Martinelli, 1985

Does satisfy S(0)=0

Not yet 100% clear what is the exact expression to take in CS/TMD factorization (2011)

Gaussian+tail model

In the TMD factorized expression there may be nonperturbative contributions from small p_T which mainly affect large b

The perturbative tail holds for small b which is dominated by large p_T, but there is an intermediate region

CSS only allows NP contribution via S_{NP} and does not allow all possibilities of the TMD approach

To illustrate this we consider a model which is approximately Gaussian at low p_T and has the correct tail at high p_T or small b:

$$f_1^g(x, p_T^2) = f_1^g(x) \frac{R^2}{2\pi} \frac{1}{1 + p_T^2 R^2} \qquad R = 2\,\mathrm{GeV}^{-1}$$

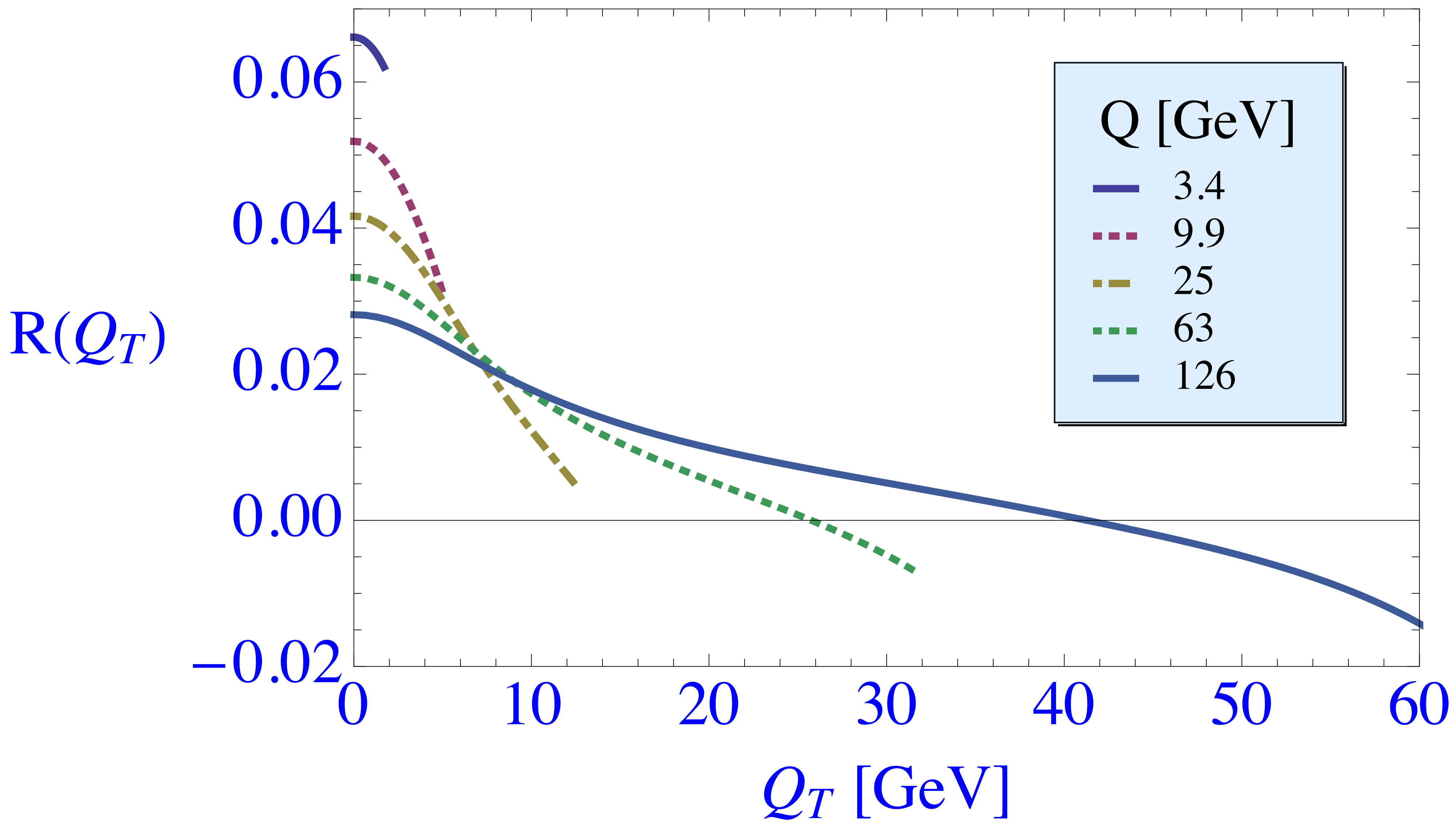
$$h_1^{\perp g}(x, p_T^2) = c f_1^g(x) \frac{M^2 R_h^4}{2\pi} \frac{1}{(1 + p_T^2 R_h^2)^2}$$

To satisfy Soffer-like bound: $R_h^2 = 3R^2/2 \qquad c = 2$

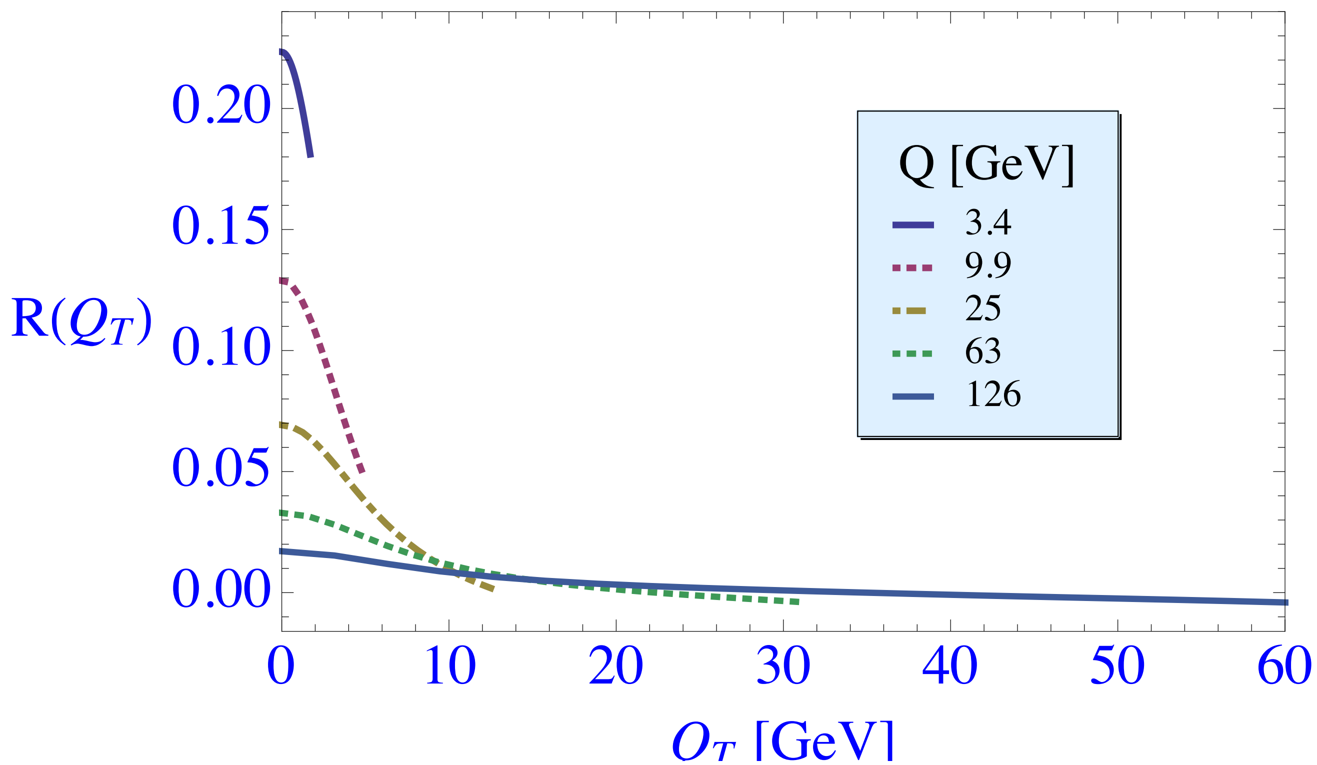
Gaussian+tail model

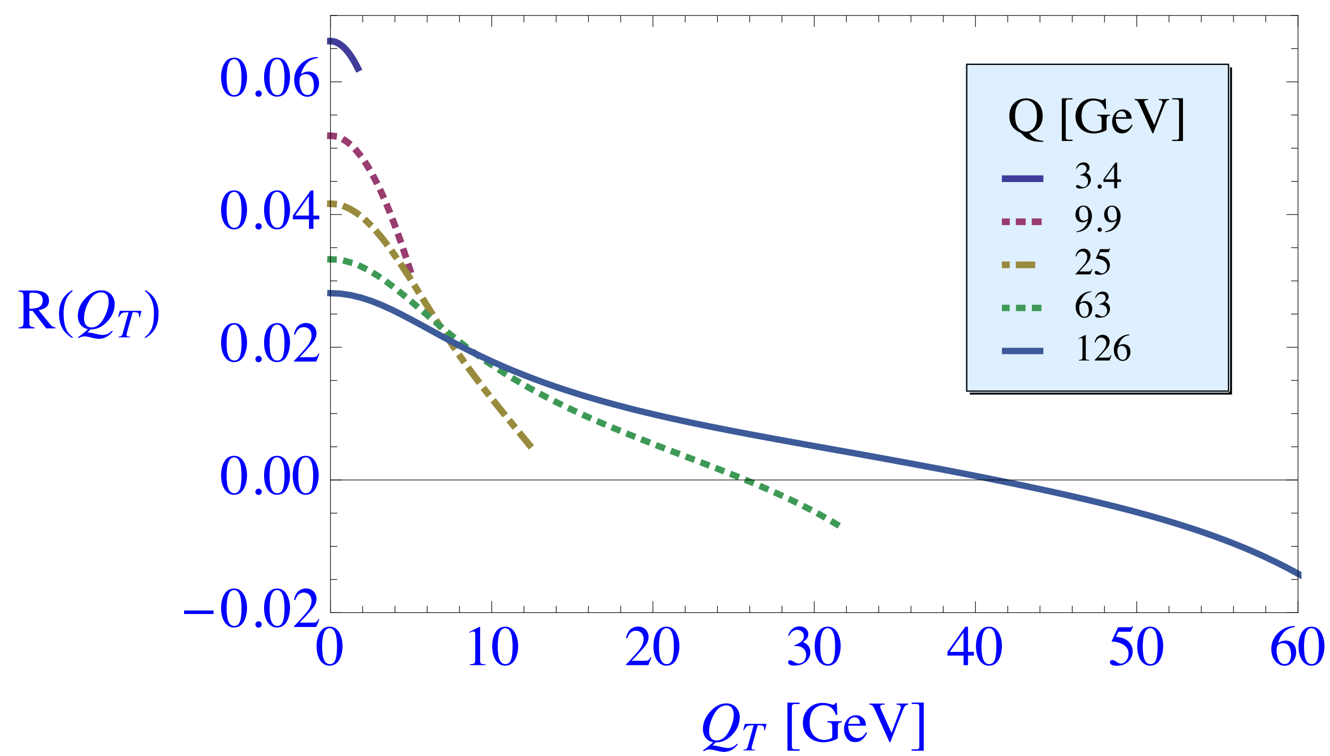
$$\tilde{f}_1^g(x, b^2; \mu_b^2, \mu_b) = f_1^g(x; \mu_b) K_0(b/R) / \ln(R b_0/b + 1)$$

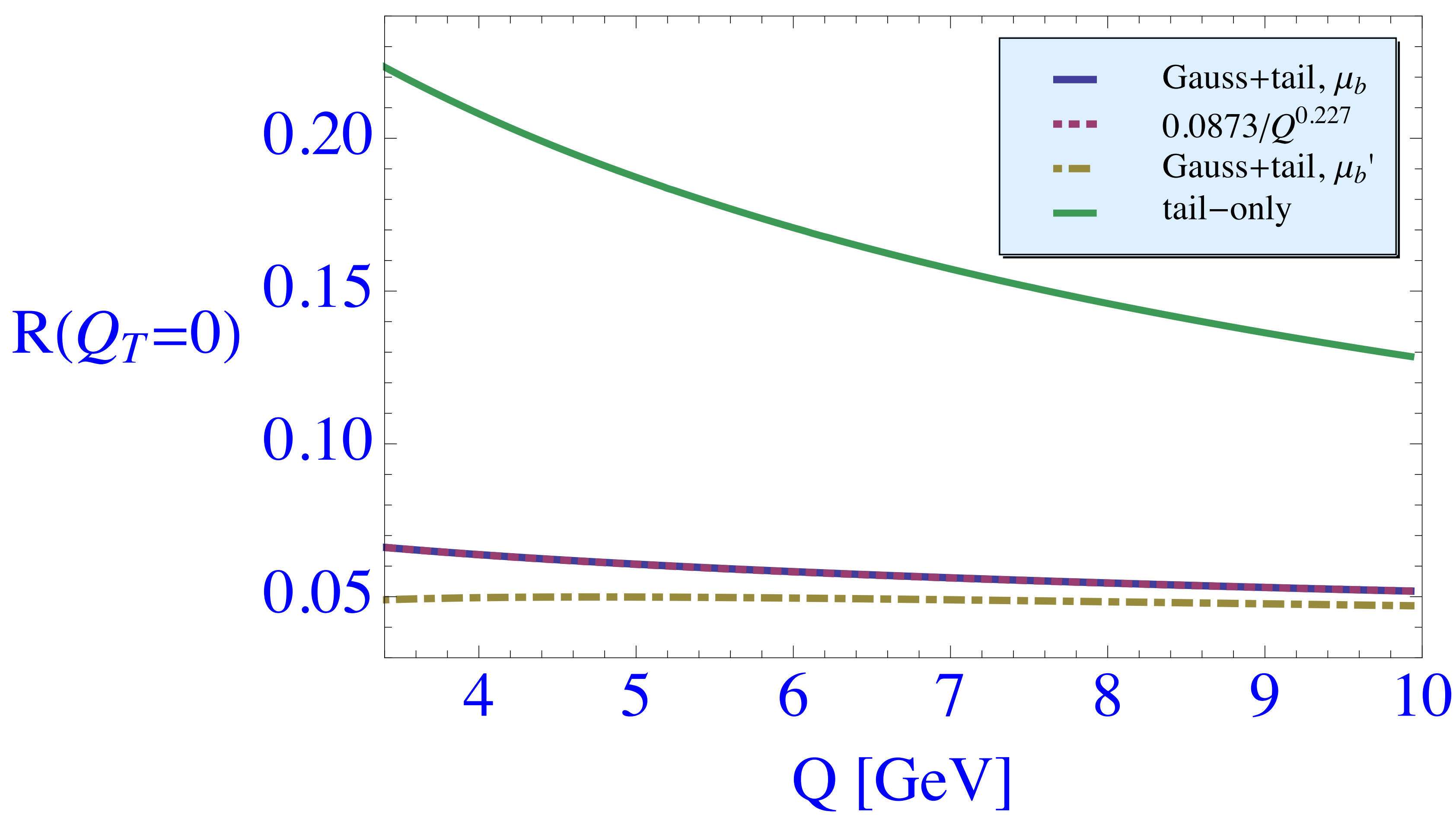
$$\tilde{h}_1^{\perp g}(x, b^2; \mu_b^2, \mu_b) = \frac{c}{4} f_1^g(x; \mu_b) \frac{b}{R_h} K_1(b/R_h) / \ln(R_h b_0/b + 1)$$



Comparison







Gaussian+tail evolves much more slowly than tail-only (CSS) expression