$g\ g \rightarrow H$: small-x perspective

F Hautmann

- *Factorization*
- *High-energy resummation*
- *Large transverse momenta: the vector boson case*

Workshop on kT-dependent PDFs

Antwerp, June 2014

Motivation

Short-distance factorization:

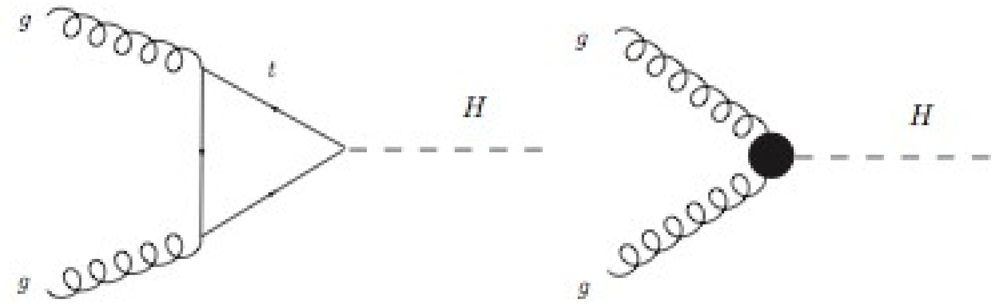
$$m_H^2 \ \sigma = \sum_{a,b} \int dx_1 \ dx_2 \ f_a(x_1, \mu) \ C(\alpha_s(\mu_R), x/(x_1 x_2), m_H/\mu) \ f_b(x_2, \mu)$$

- High-energy logarithms in hard-scattering function...

$$\begin{aligned} C(\alpha_s, x, m_H/\mu) \ &= \ c^{(0)}(x) + \frac{\alpha_s}{\pi} \left[c^{(1)}(x) + \overline{c}^{(1)}(x) \ L \right] \\ &+ \ \left(\frac{\alpha_s}{\pi} \right)^2 \left[c^{(2)}(x) + \overline{c}^{(2)}(x) \ L + \overline{\overline{c}}^{(2)}(x) \ L^2 \right] + \cdots \end{aligned}$$

$$L = \ln(m_H^2/\mu^2), \ x = m_H^2/s$$

- ...as well as in the evolution of parton density functions



heavy top effective theory

Moments of hard-scattering function:

$$C_N(\alpha_s, m_H/\mu) = \int_0^1 dx x^{N-1} C(\alpha_s, x, m_H/\mu)$$

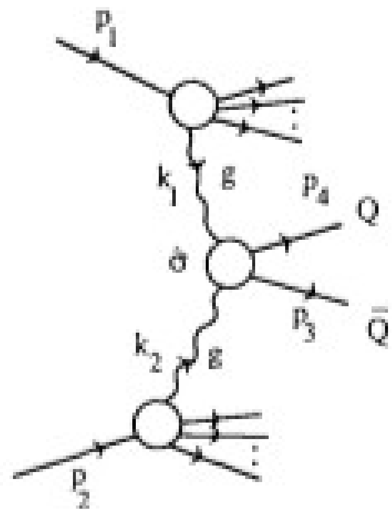
- $N \to 0$ (small-x) behavior to all orders in α_s

$$C_N(\alpha_s, m_H/\mu) = \underbrace{R_N^2(\gamma_N)\ (m_H^2/\mu^2)^{2\gamma_N}}_{RG\ \ factors}\ \underbrace{h_N(\gamma_N, \gamma_N)}_{finite\ \ part}$$

where

$$\gamma_N = \overline{\alpha}_s/N + 2\,\zeta(3)\ (\overline{\alpha}_s/N)^4\ +\ \ldots \qquad \text{(BFKL)}$$

Off-shell cross section



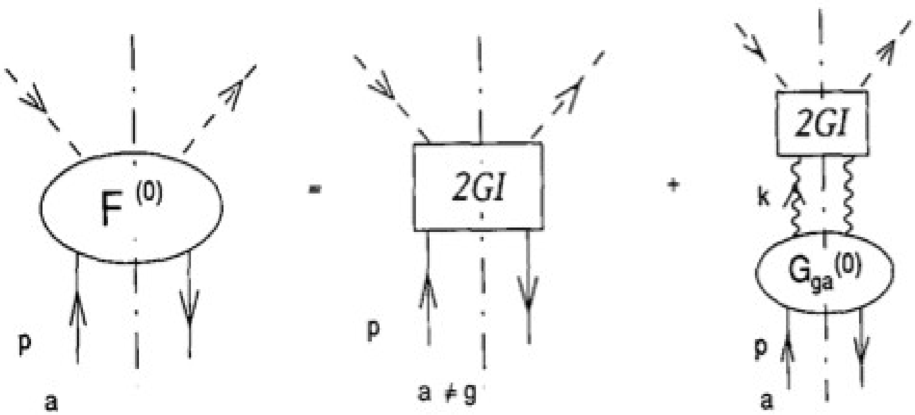
$$h_N(\gamma_1, \gamma_2) \quad = \quad \gamma_1 \ \gamma_2 \int \frac{d^2\mathbf{k}_1}{\pi \mathbf{k}_1^2} \left(\frac{\mathbf{k}_1^2}{m_H^2} \right)^{\gamma_1} \int \frac{d^2\mathbf{k}_2}{\pi \mathbf{k}_2^2} \left(\frac{\mathbf{k}_2^2}{m_H^2} \right)^{\gamma_2}$$

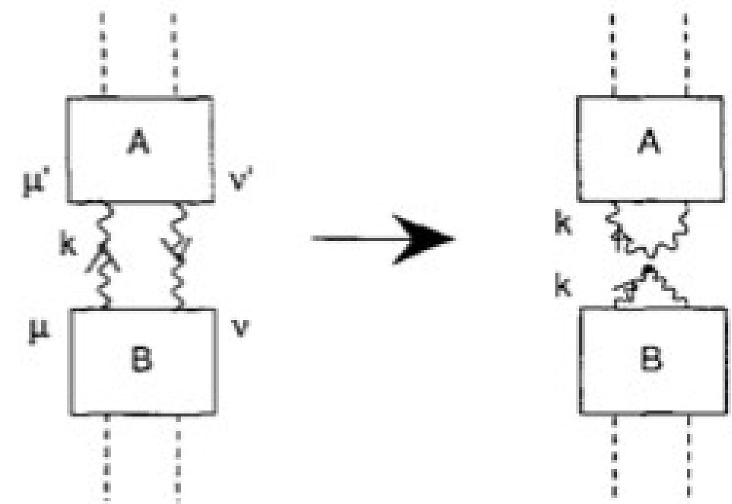
$$\times \quad \int_0^1 \frac{dx}{x} \, x^N \, \hat{\sigma} \left(x, \frac{\mathbf{k}_1}{m_H}, \frac{\mathbf{k}_2}{m_H} \right)$$

$\hat{\sigma}$ computed by coupling $gg \to H$ to eikonal gluon polarizations

$$\mathcal{M}^{(eik)}(k_1, k_2, p) = \frac{2 k_{1\perp}^{\mu_1} k_{2\perp}^{\mu_2}}{\sqrt{\mathbf{k}_1^2 \mathbf{k}_2^2}} \mathcal{M}_{\mu_1 \mu_2}(k_1, k_2, p)$$

MSbar transverse momentum dependent factorization

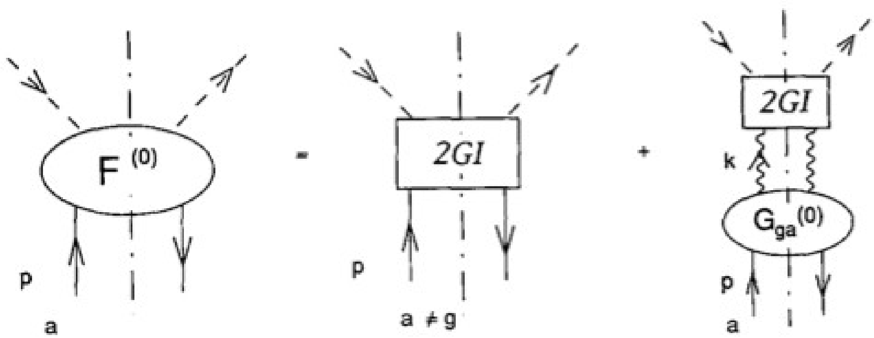




$$m_H^2 \; \sigma_N \;=\; \int d^{2+2\varepsilon}\mathbf{k}_1 \; d^{2+2\varepsilon}\mathbf{k}_2 \; \mathcal{F}_N(\mathbf{k}_1, \mu, \varepsilon)$$
$$\times \;\; \hat{\sigma}_N \left(\mathbf{k}_1/m_H, \mathbf{k}_2/m_H, \alpha_s(m_H/\mu)^{\varepsilon}, \varepsilon\right) \; \mathcal{F}_N(\mathbf{k}_2, \mu, \varepsilon)$$

$$\text{where} \;\; \mathcal{F}(z, \mathbf{k}, \mu, \varepsilon) = \int \frac{dk^2}{2(2\pi)^{4+2\varepsilon}} \; P_{\mu\nu}^{(H)} \; G^{\mu\nu}(k, p)$$

The renormalization group R factor



$$\int^{\mu} dk_{\perp} \, \mathcal{F}(k_{\perp}, \mu) \; = \; R \otimes f^{\overline{\rm MS}}(\mu)$$

- Perturbative expansion ($x \to 0$):

$$R(x) - \delta(1-x) \simeq 1.40 \; \alpha_s^3 \; \ln^2 x - 0.11 \; \alpha_s^4 \; \ln^3(1/x) + \ldots$$

- Relationship with RG evolution:

$$\mathcal{F}_N(k_{\perp}, \mu) = \underbrace{R_N \; [\gamma_N \; \frac{1}{k_{\perp}^2} \; \left(\frac{k_{\perp}^2}{\mu_F^2} \right)_N^{\gamma}]}_{\varepsilon \to 0} \; \underbrace{\Gamma_N (\frac{\mu_F^2}{\mu^2})}_{\text{series of poles } 1/\varepsilon}$$

γ_N = anomalous dimension
μ_F = factorization scale

Perturbative resummation of small-x logarithms

[H, PLB 535 (2002) 159]

♠ gg channel: $(m_{\rm top} \to \infty)$

$$C_{gg}(\alpha_s, x, m_H^2/\mu^2) = \frac{\alpha_s^2 m_H^2 G_F}{288\pi\sqrt{2}} \Bigg[\delta(1-x) + \frac{\alpha_s}{\pi}\, C_A \underbrace{(-2\ln x + 2\ L)}_{NLO}$$

$$+ \left(\frac{\alpha_s}{\pi}\right)^2 C_A^2 \underbrace{\left(-\frac{2}{3}\ln^3 x + 2\ln^2 x\ L - 2\ln x\ L^2\right)}_{NNLO} + \cdots \Bigg]$$

♠ quark channels: $(m_{\rm top} \to \infty)$

$$C_{qg}(\alpha_s, x, m_H^2/\mu^2) = \frac{\alpha_s^2 m_H^2 G_F}{288\pi\sqrt{2}} \Bigg[\frac{\alpha_s}{\pi}\, C_F\, (-\ln x + L) + \left(\frac{\alpha_s}{\pi}\right)^2 C_F C_A$$

$$\times \left(-\frac{1}{2}\ln^3 x + \frac{3}{2}\ln^2 x\ L - \frac{3}{2}\ln x\ L^2\right) + \cdots \Bigg] ,$$

$$C_{q\bar{q}}(\alpha_s, x, m_H^2/\mu^2) = \frac{\alpha_s^2 m_H^2 G_F}{288\pi\sqrt{2}} \left(\frac{\alpha_s}{\pi}\right)^2 C_F^2 \left(-\frac{1}{3}\ln^3 x + \ln^2 x\ L - \ln x\ L^2\right) + \cdots$$

Beyond x ⟶ 0: CCFM exclusive evolution

$$x\mathcal{A}(x,k_t,q) = x\mathcal{A}(x,k_t,q_0)\Delta_s(q) + \int dz \int \frac{dq'}{q'} \cdot \frac{\Delta_s(q)}{\Delta_s(q')} \tilde{P}(z,k_t,q') \frac{x}{z} \mathcal{A}\left(\frac{x}{z}, q'\right)$$

- solve integral equation via iteration:

$$x\mathcal{A}_0(x,k_t,q) \quad = \quad x\mathcal{A}(x,k_t,q_0)\Delta(q)$$

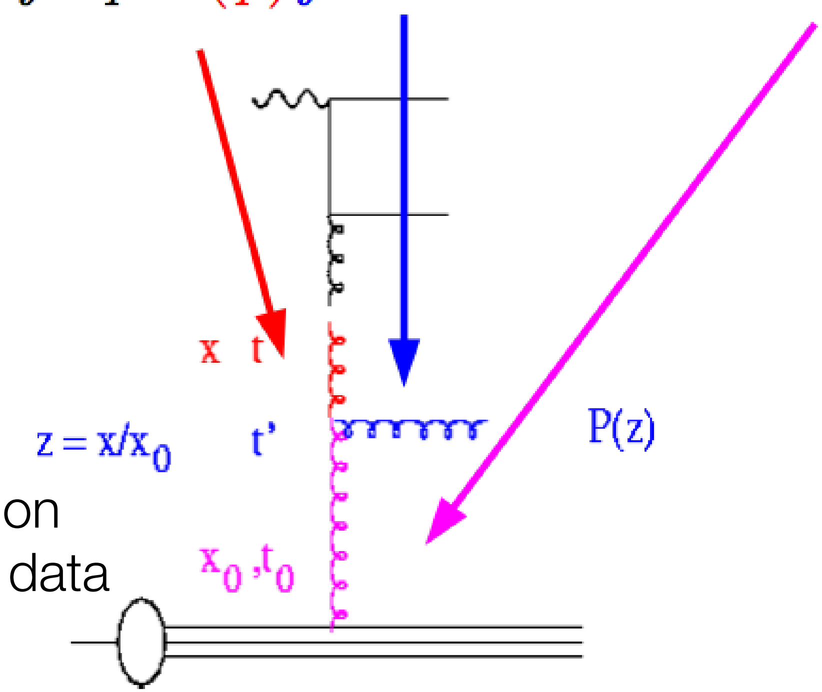
from q' to q w/o branching

branching at q'

from q_0 to q' w/o branching

$$x\mathcal{A}_1(x,k_t,q) \quad = \quad x\mathcal{A}(x,k_t,q_0)\Delta(q) + \int \frac{dq'}{q'} \frac{\Delta(q)}{\Delta(q')} \int dz \tilde{P}(z) \frac{x}{z} \mathcal{A}(x/z, k_t', q_0)\Delta(q')$$

- Note: evolution equation formulated with Sudakov form factor is equivalent to “plus” prescription, but better suited for numerical solution for treatment of kinematics



- kt dependent shower by CCFM evolution
- new parton density from DIS precision data

kT-dependent gluon density from precision DIS data

[Jung & H, Nucl. Phys. B 883 (2014) 1]

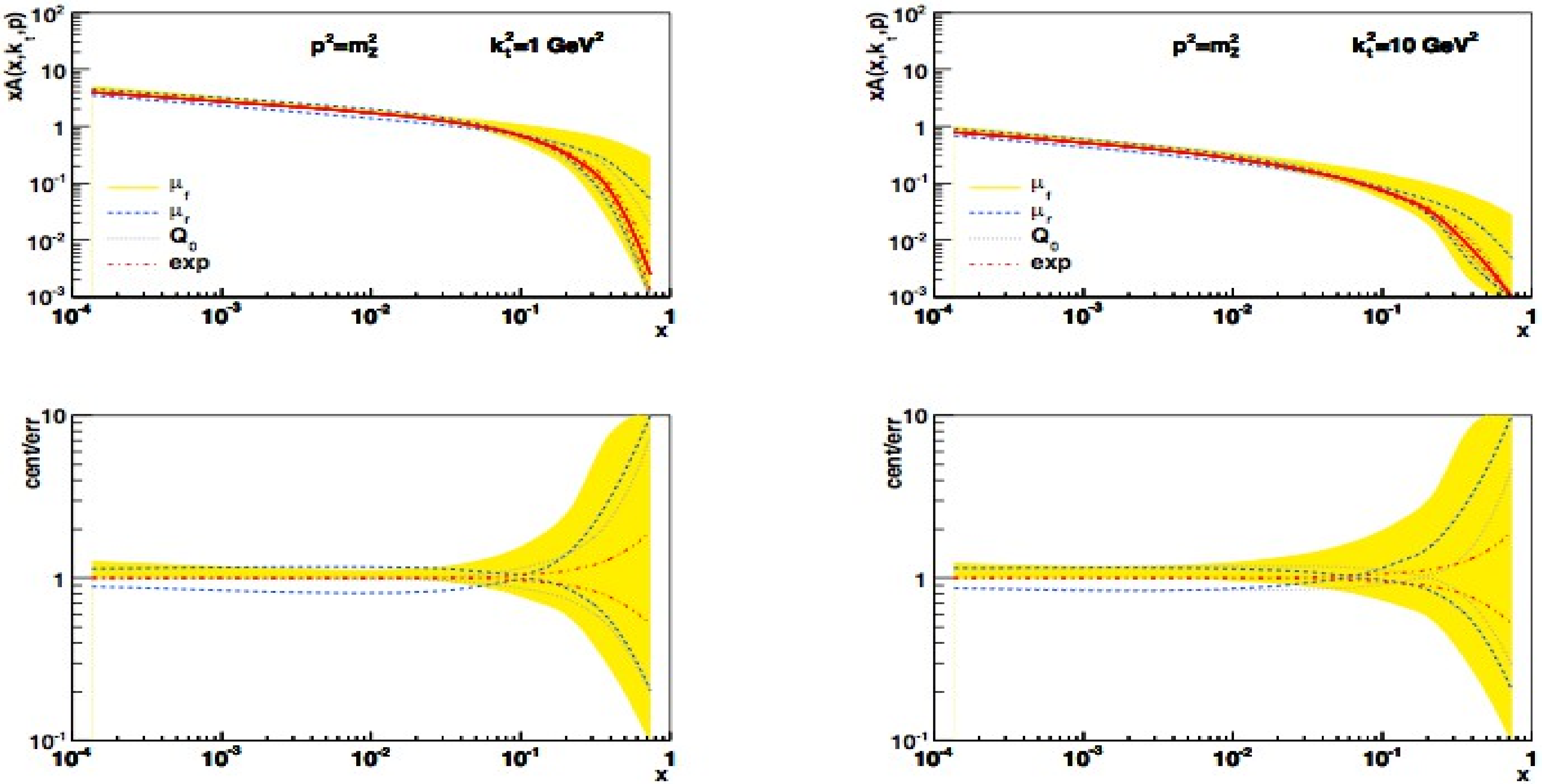
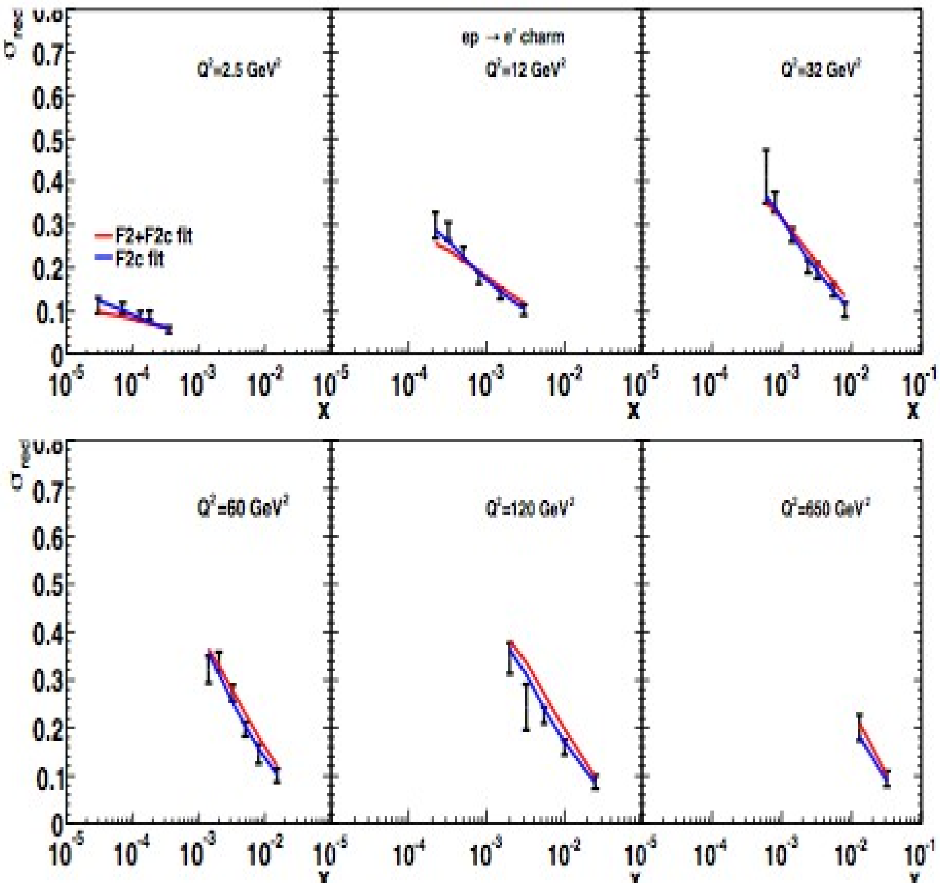


FIG. 6. *Experimental and theoretical uncertainties of the unintegrated TMD gluon density versus x for different values of transverse momentum at* $p^2 = m_Z^2$.

kT-dependent gluon density from precision DIS data

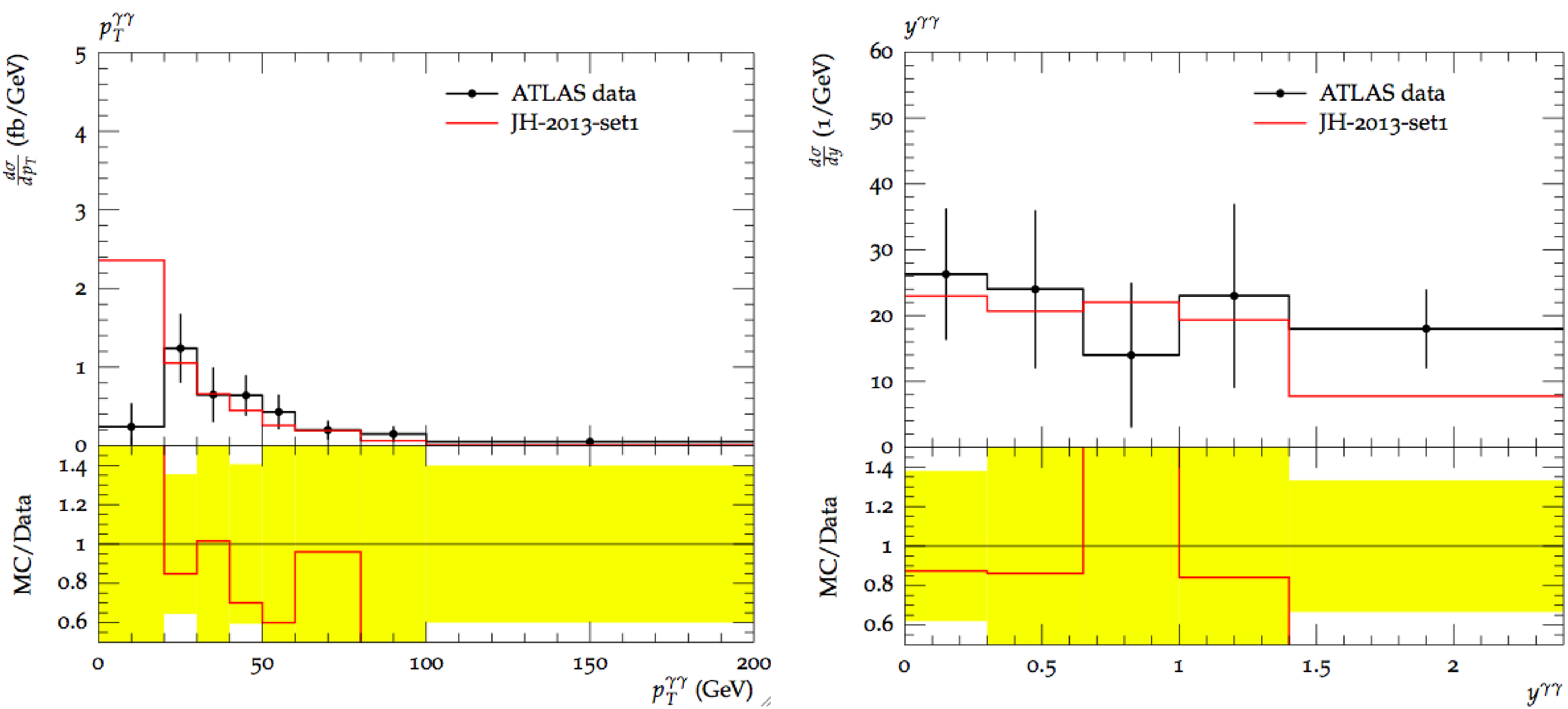


[Jung & H, Nucl. Phys. B 883 (2014) 1]

- Very good description of inclusive DIS data with TMD gluon
- Sea quark yet to be included at TMD level

	$\chi^2/ndf\,(F_2^{(\mathrm{charm})})$	$\chi^2/ndf\,(F_2)$	$\chi^2/ndf\,\left(F_2 \text{ and } F_2^{(\mathrm{charm})}\right)$
3-parameter	0.63	1.18	1.43
5-parameter	0.65	1.16	1.41

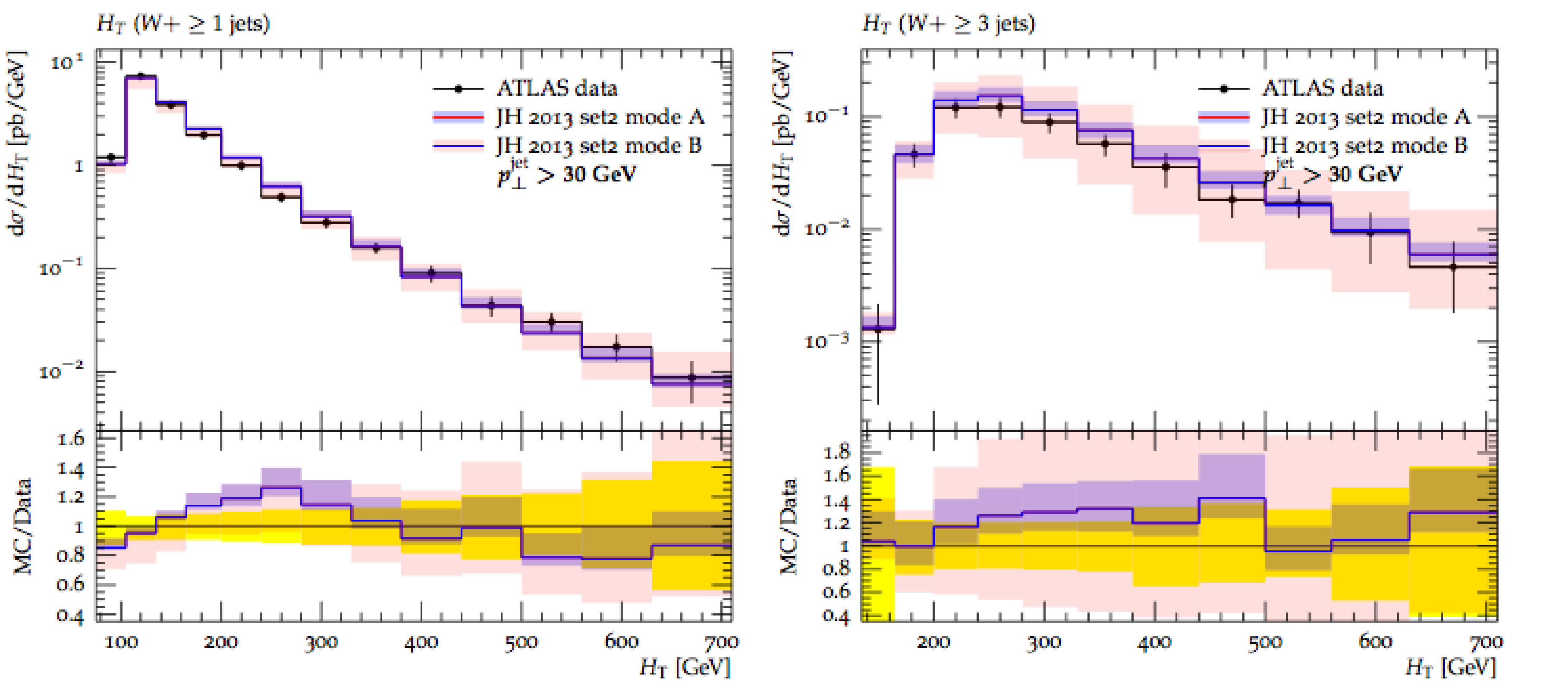
ATLAS pT and rapidity spectra
Higgs → gamma gamma



JH-2013-set1: kT dependent gluon density from fits to precision DIS data

Can we go to large transverse momenta? Total H_T distribution in W + n jets final states at the LHC

Dooling, Jung & H, arXiv:1406.2994



mode A: uncertainties from renorm. scale, starting evol. scale, expt. errors

mode B: include factorization scale uncertainties

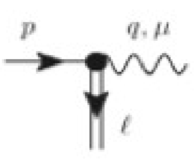
Application to vector bosons + jets at high energy

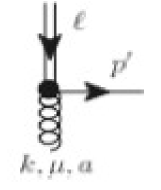
→ Use exclusive CCFM evolution

→ Determine TMD pdf from high-precision DIS data

→ Obtain predictions for final states associated with Drell-Yan using high-energy off-shell matrix elements

- High-energy effective theory → effective vertices

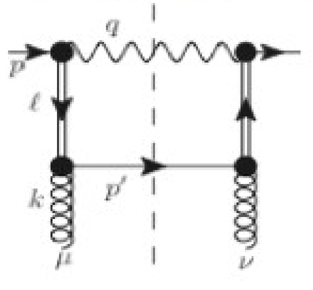




[Bogdan & Fadin, NPB740 (2006) 36]

[Lipatov & Vyazovsky, NPB597 (2001) 399]

- Parton matrix elements (gauge-invariant, despite off-shell parton)

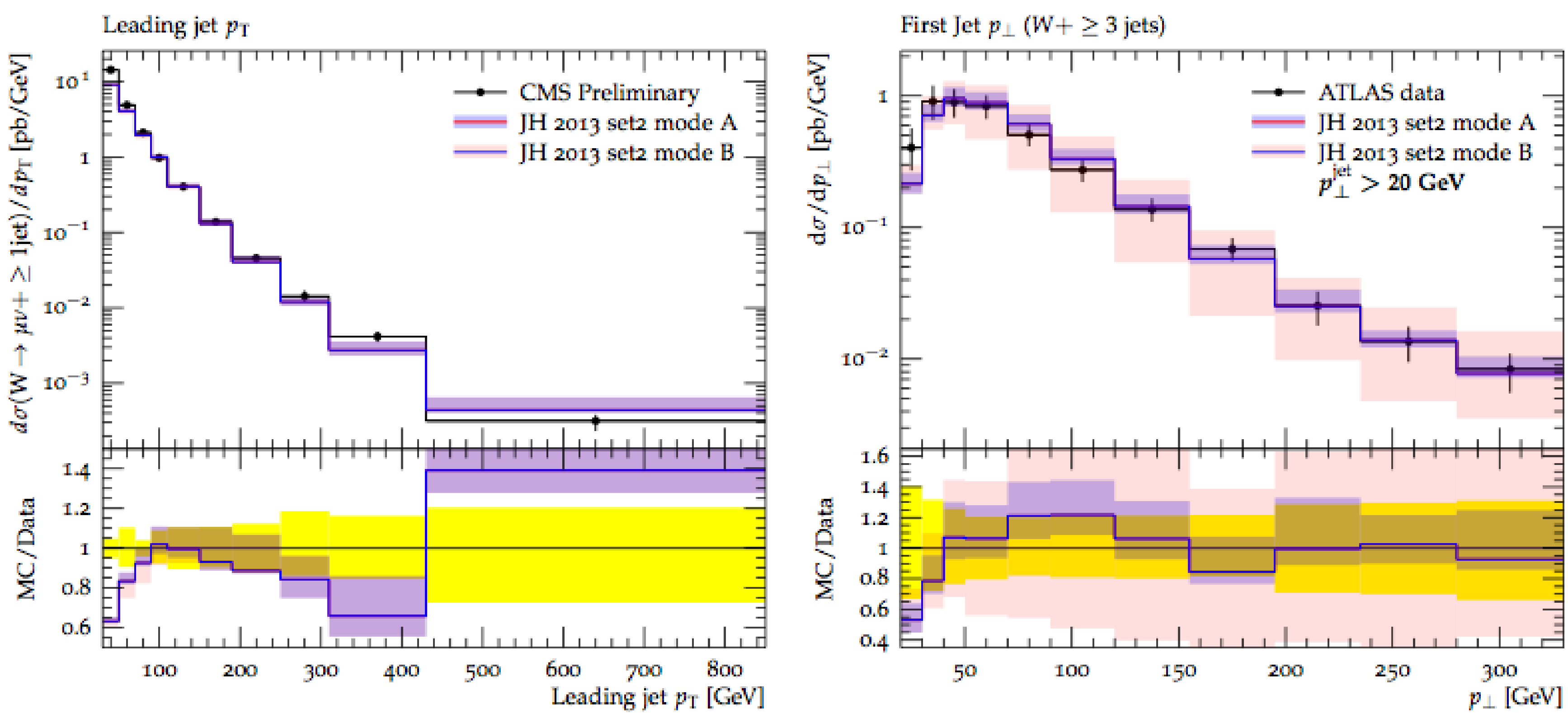


[Ball & Marzani, NPB814 (2009) 246]

[Hentschinski, Jung & H, NPB865 (2012) 54]

W + n jets final states at the LHC

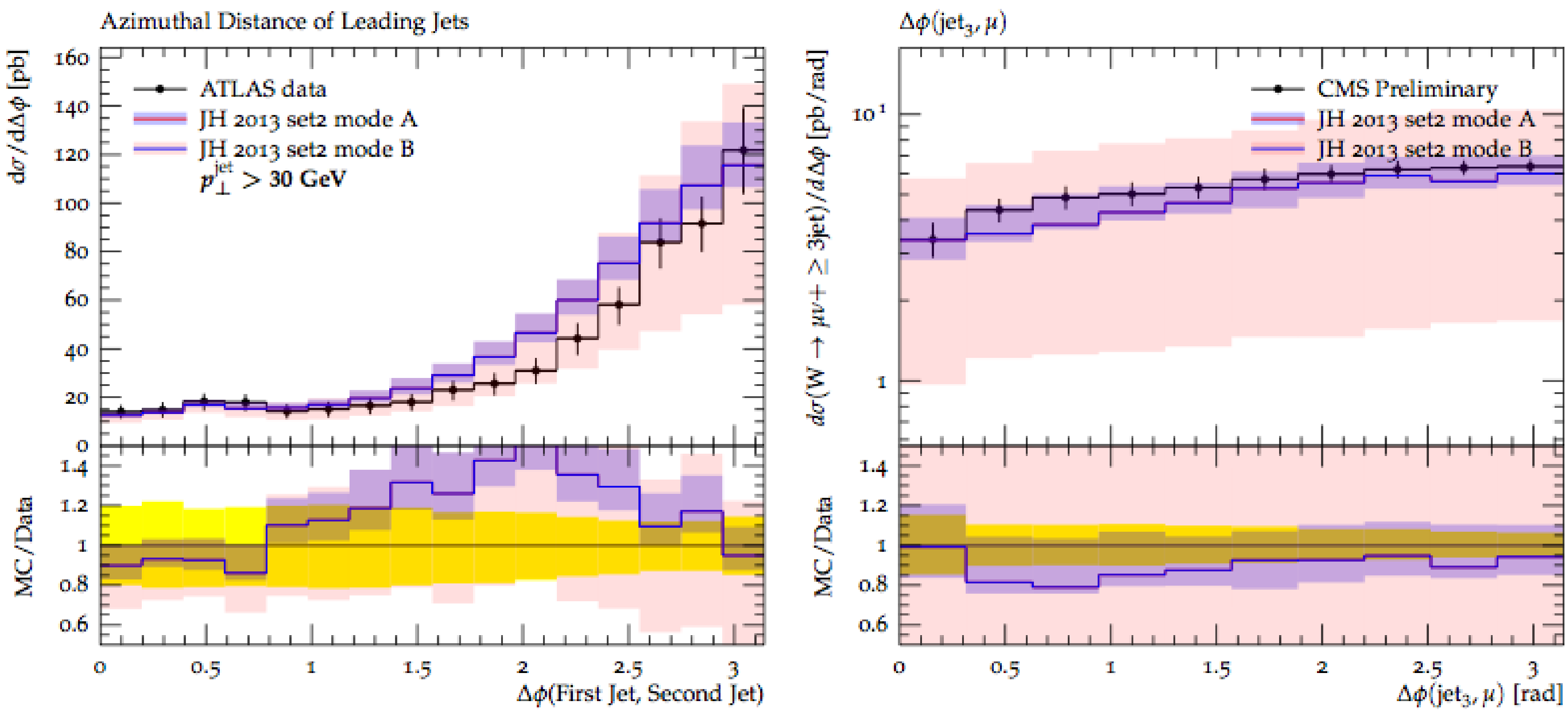
Dooling, Jung & H, arXiv:1406.2994



Leading jet pT: (left) inclusive; (right) n>=3

Angular correlations in W + n jets final states

Dooling, Jung & H, arXiv:1406.2994



(left) Delta-phi between two hardest jets; (right) vector boson - third jet correlation

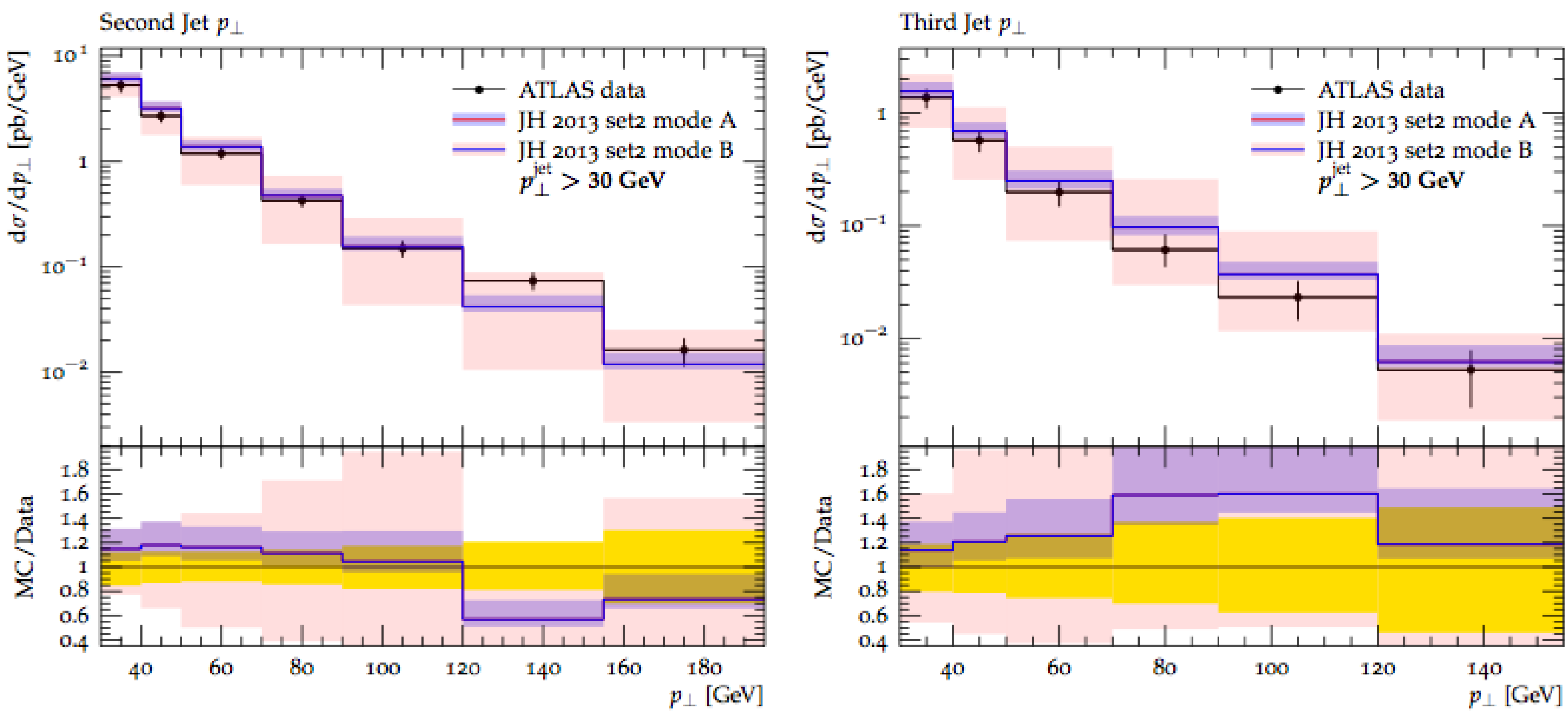
Summary

The small-x perspective on
transverse momentum dependent parton distributions
provides

- well-defined QCD evolution in high energy limit (BFKL)
- resummation of $N \to 0$ poles in hard scattering functions
- extendable to next-to-leading-logarithmic accuracy
- exclusive parton-shower formulation via CCFM

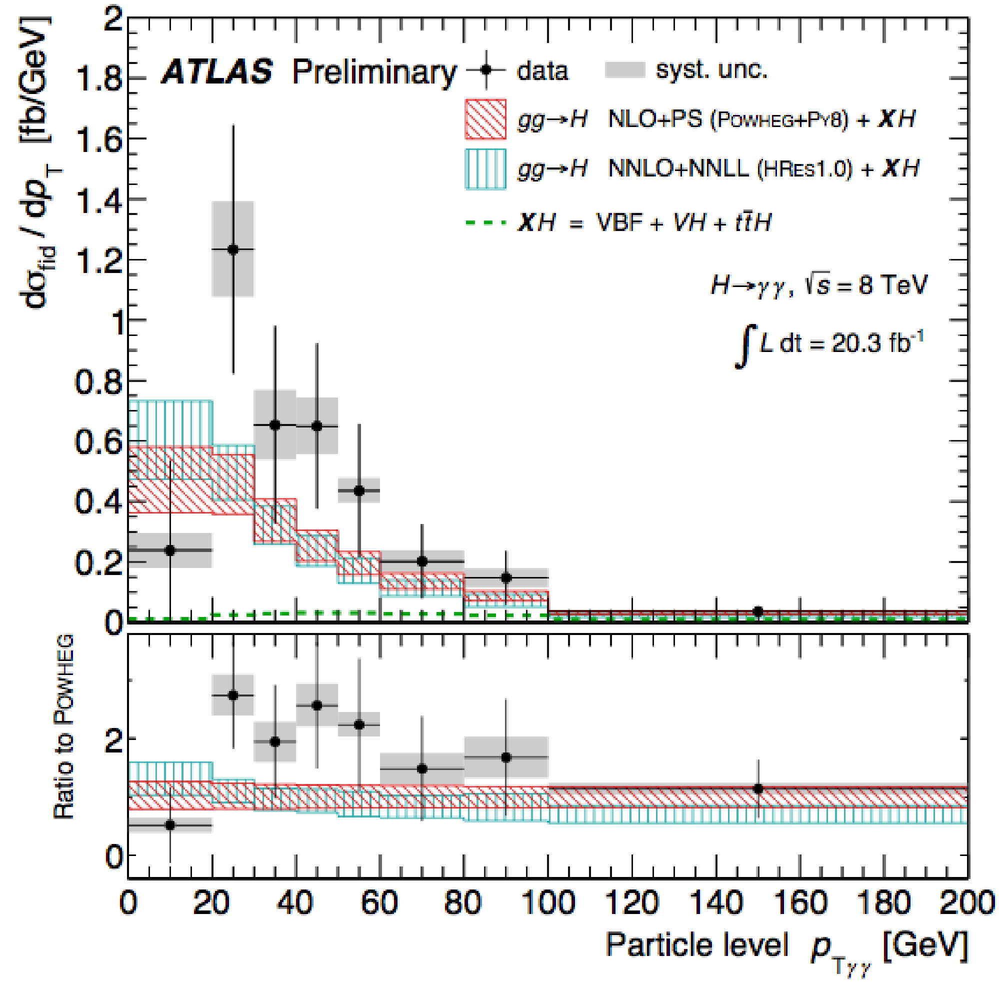
W + n jets final states at the LHC

Dooling, Jung & H, arXiv:1406.2994



Subleading jets: (left) second jet pT; (right) third jet pT

ATLAS preliminary Conf-2013-072



(a) $p_{\mathrm{T}}^{\gamma\gamma}$