Higgs q_T-distribution

Effective Field Theory approach

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Introduction / Motivation

- In the Effective Field Theory (EFT) approach we **DO NOT PROVE** factorization!! We assume the theory contains the right modes and can only **establish** factorization at any given order in perturbation theory.

- We perform a **matching** between two effective theories at each hard scale (>Lqcd).

- Resummation of logarithms is done by **running** between different scales.

- I will show, step by step, the derivation of the factorization theorem for the Higgs transverse momentum distribution by using the EFT approach

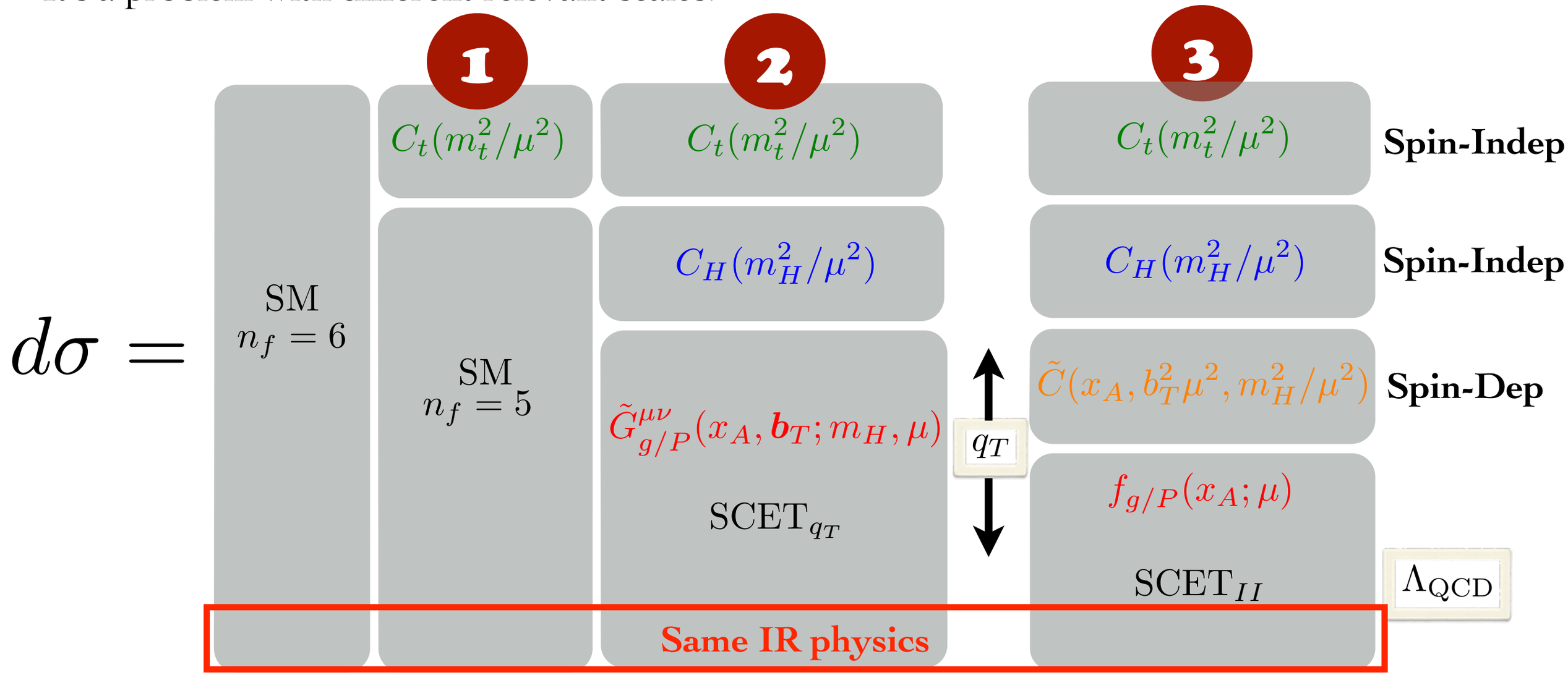
- Goal: understand the TMD factorization and discuss the possible connection with the small-x approach and uPDFs

Factorization in EFT approach: overview

- We want to factorize this process:

$$h_A(P, S_A) + h_B(\bar{P}, S_B) \to H(q_T) + X$$

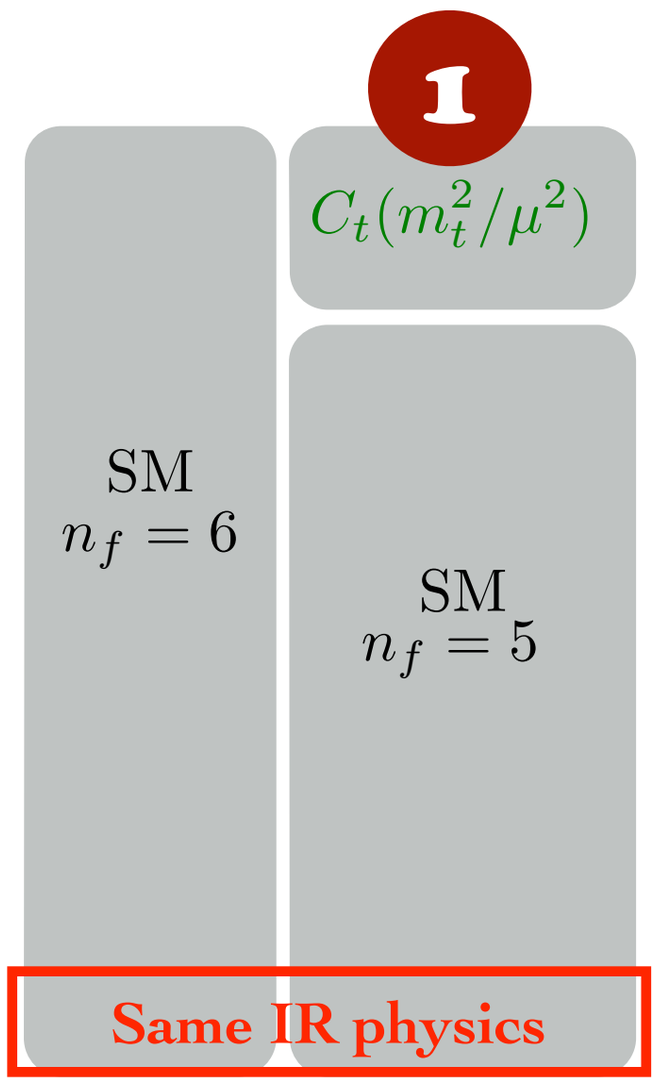
- It's a problem with different relevant scales:



Factorization Theorem = Multistep Matching Procedure

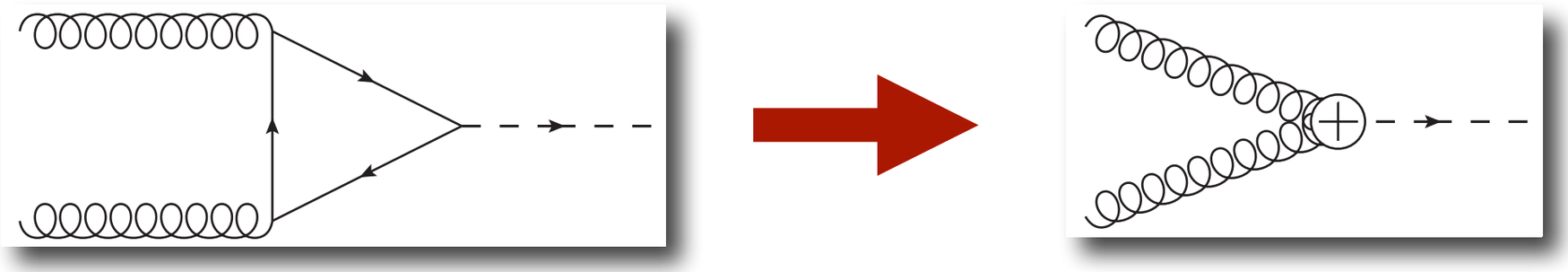
Step 1:
Factorization of the top mass

$$d\sigma =$$



Factorization of the top quark

- The glue-glue fusion process is well approximated by an effective lagrangian:



Top quark dominates!!

$$\mathcal{L}_{\rm eff} = \frac{\alpha_s(\mu^2)}{12\pi} C_t(m_t^2,\mu^2)\,\frac{H}{v}\,F^{\mu\nu,a}\,F^a_{\mu\nu}$$

- The coefficient (and thus its anomalous dimension) is known up to 3-loops:

$$C_t(m_t^2,\mu^2) = 1 + \frac{\alpha_s(\mu^2)}{4\pi}\,(5C_A - 3C_F)$$
$$+ \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^2 \left[\frac{27}{2}\,C_F^2 + \left(11\ln\frac{m_t^2}{\mu^2} - \frac{100}{3}\right) C_F C_A - \left(7\ln\frac{m_t^2}{\mu^2} - \frac{1063}{36}\right) C_A^2 \right.$$
$$\left. - \frac{4}{3}\,C_F T_F - \frac{5}{6}\,C_A T_F - \left(8\ln\frac{m_t^2}{\mu^2} + 5\right) C_F T_F n_f - \frac{47}{9}\,C_A T_F n_f \right]$$

NNLO

Kramer, Laenen, Spira NPB'98
Chetyrkin, Kniehl, Steinhauser PRL'97

NNNLO

Schroder, Steinhauser JHEP'06
Chetyrkin, Kuhn, Sturm NPB'06

$$\frac{d\ln C_t(m_t^2,\mu^2)}{d\ln\mu} = \gamma^t\left(\alpha_s(\mu)\right)$$

$$\gamma^t\left(\alpha_s(\mu)\right) = \alpha_s^2\,\frac{d}{d\alpha_s}\,\frac{\beta(\alpha_s)}{\alpha_s^2}$$

Not surprising, since the effective lagrangian is pure Yang-Mills piece…

Factorization of the top quark

- Using the effective lagrangian, the cross-section can be written as:

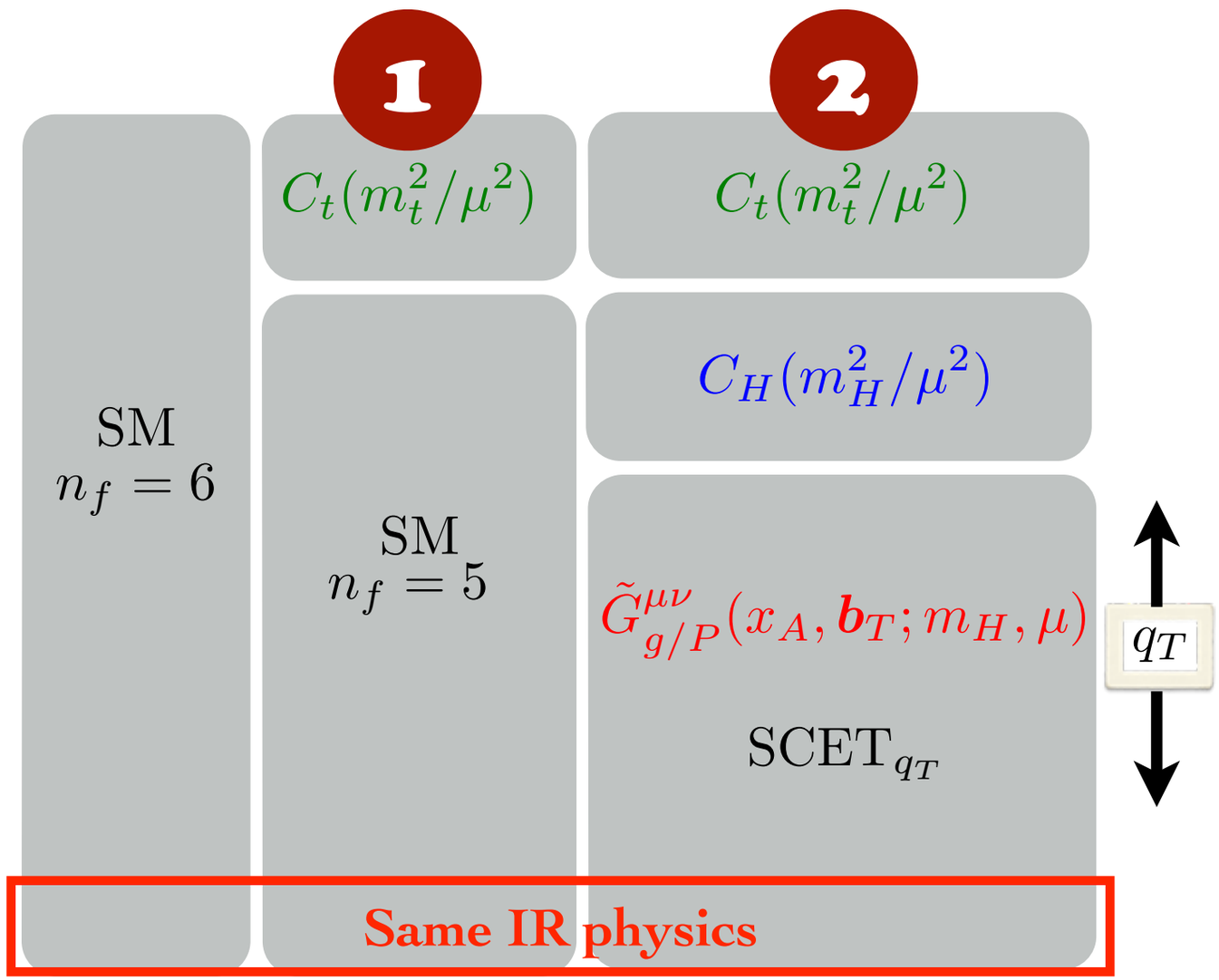
$$d\sigma = \frac{1}{2s}\left(\frac{\alpha_s(\mu)}{12\pi v}\right)^2 C_t^2(m_t^2,\mu^2)\frac{d^3q}{(2\pi)^2 2E_q}\int d^4y e^{-iq\cdot y}$$
$$\times \sum_X \langle PS_A, \bar{P}S_B | F^a_{\mu\nu}F^{\mu\nu,a}(y) | X\rangle \langle X | F^b_{\alpha\beta}F^{\alpha\beta,b}(0) | PS_A, \bar{P}S_B\rangle$$

- We didn't talk about SCET yet!
- The factorization now follows similar steps as for DY or SIDIS

MGE, Idilbi, Scimemi JHEP'12
MGE, Idilbi, Scimemi PRD'14

Step 2:
Factorization of the Higgs mass

$$d\sigma =$$



SCET-I in a nutshell

Bauer, Fleming, Pirjol, Stewart '01
BPS '02

- Soft-Collinear Effective Theory is an effective theory of QCD
- SCET describes interactions between low energy (u)soft and collinear fields (very energetic in one light-cone direction)
- Expand the lagrangian in powers of a small parameter (lambda)
- **Assumption:** SCET captures all the IR physics of QCD (no Glaubers...): matching is possible
- SCET is useful to establish factorization theorems and resum large logs

$$p_n^\mu = Q(1, \lambda^2, \lambda)$$ *n-collinear*

$$p_{\bar{n}}^\mu = Q(\lambda^2, 1, \lambda)$$ *$\bar{n}$-collinear*

$$p_{us}^\mu = Q(\lambda^2, \lambda^2, \lambda^2)$$ *ultrasoft (SCET-I)*

$$p_s^\mu = Q(\lambda, \lambda, \lambda)$$ *soft (SCET-II)*

$$n^2 = \bar{n}^2 = 0\,, \quad n \cdot \bar{n} = 2$$
$$n^\mu = (1, 0, 0, 1)$$
$$\bar{n}^\mu = (1, 0, 0, -1)$$

$$p^\mu = \bar{n} \cdot p \frac{n^\mu}{2} + n \cdot p \frac{\bar{n}^\mu}{2} + p_\perp^\mu$$
$$\equiv p_+^\mu + p_-^\mu + p_\perp^\mu \equiv (p^+, p^-, p_\perp)$$

SCET-I in a nutshell

- In SCET we build the effective lagrangian starting from QCD…

$$\mathcal{L}_{\rm QCD} = \bar{\psi}\, i\not{D}\psi - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} \qquad D_\mu = \partial_\mu - igT^a A^a_\mu$$

e.o.m.

$$i\bar{n}\cdot D\xi_{\bar{n}} + i\frac{\not{\bar{n}}}{2}\not{D}_\perp \xi_n = 0$$

Splitting

$$\psi = \xi_n + \xi_{\bar{n}}$$

$$\xi_n = \frac{\not{n}\not{\bar{n}}}{4}\psi\,, \quad \not{n}\xi_n = 0$$

$$\xi_{\bar{n}} = \frac{\not{\bar{n}}\not{n}}{4}\psi\,, \quad \not{\bar{n}}\xi_{\bar{n}} = 0$$

$$\mathcal{L} = \bar{\xi}_n \left(in\cdot D + i\not{D}_\perp \frac{1}{i\bar{n}\cdot D} i\not{D}_\perp \right) \frac{\not{\bar{n}}}{2}\xi_n$$

QCD lagrangian still…

Power counting…

$$iD_\perp = i\partial_{n\perp} + gA_{n\perp} + i\partial_{us\perp} + gA_{us\perp}$$
$$\approx i\partial_{n\perp} + gA_{n\perp} \sim \lambda$$

$$\mathcal{L}_n^{(0)} = \bar{\xi}_n \left(in\cdot D_{us} + gn\cdot A_n + i\not{D}_{n\perp} \frac{1}{i\bar{n}\cdot D_n} i\not{D}_{n\perp} \right) \frac{\not{\bar{n}}}{2}\xi_n$$

LO SCET lagrangian for fermions

- We have **separate lagrangians** for (u)soft quarks/gluons and (anti)collinear quarks/gluons

Hard, soft and collinear modes…

- Now we integrate out the Higgs mass:

$$F^{\mu\nu,a}\,F^{a}_{\mu\nu} \longrightarrow -2q^2 C_H(-q^2,\mu^2)\, g^{\perp}_{\mu\nu} B^{\mu,a}_{n\perp}\left(\mathcal{S}^\dagger_n \mathcal{S}_{\bar n}\right)^{ab} B^{\nu,b}_{\bar n\perp}$$

Known at 3-loops!! Its anomalous dimension as well

$$B^{\mu}_{n\perp} = \frac{1}{g}[\bar n\cdot \mathcal{P} W_n^\dagger i D_n^{\perp\mu} W_n]$$

$$W_n(x) = \bar P \exp\left[\int_{-\infty}^{0} ds\, \bar n\cdot A^a_n(x+\bar n s) t^a\right]$$

$$S_n(x) = P \exp\left[\int_{-\infty}^{0} ds\, n\cdot A^a_s(x+ns) t^a\right]$$

Calligraphic Wilson line means adjoint representation: $(t^a)^{bc} = -i f^{abc}$

- And the cross section is given by:

$$d\sigma = \sigma_0(\mu)\, C_t^2(m_t^2,\mu)\, H(m_H,\mu)\, m_H^2\, \frac{dy\, d^2q_\perp}{(2\pi)^4} \int d^4y\, e^{-iq\cdot y} \times J^{\mu\nu}_n(y)\, J_{\bar n\,\mu\nu}(y)\, S(y)$$

$$\sigma_0(\mu) = \frac{m_H^2\,\alpha_s^2(\mu)}{72\pi(N_c^2-1)sv^2}$$

$$\tau = (m_H^2+q_T^2)/s \qquad x_{A,B} = \sqrt{\tau}\, e^{\pm y}$$

$$H = |C_H|^2$$

- Collinear and soft matrix elements are defined by:

$$J^{\mu\nu}_n(y) = \sum_{X_n} \langle PS_A|\, B^{\mu,a}_{n\perp}(y)\,|X_n\rangle\, \langle X_n|\, B^{\nu,a}_{n\perp}(0)\,|PS_A\rangle$$

$$S(y) = \frac{1}{N_c^2-1}\sum_{X_s} \langle 0|\left(\mathcal{S}^\dagger_n \mathcal{S}_{\bar n}\right)^{ab}(y)\,|X_s\rangle\, \langle X_s|\left(\mathcal{S}^\dagger_{\bar n} \mathcal{S}_n\right)^{ba}(0)\,|0\rangle$$

Power counting (Taylor expansion)

- We need to be consistent with the power counting and Taylor expand the previous expression:

$$q \sim Q(1,1,\lambda)$$
$$y \sim \frac{1}{Q}(1,1,\frac{1}{\lambda})$$

$$\left(\frac{\partial}{\partial y^-},\frac{\partial}{\partial y^+},\frac{\partial}{\partial y_\perp}\right) J_n^{\mu\nu}(y) \sim Q(1,\lambda^2,\lambda)$$
$$\left(\frac{\partial}{\partial y^-},\frac{\partial}{\partial y^+},\frac{\partial}{\partial y_\perp}\right) J_{\bar{n}}^{\mu\nu}(y) \sim Q(\lambda^2,1,\lambda)$$
$$\left(\frac{\partial}{\partial y^-},\frac{\partial}{\partial y^+},\frac{\partial}{\partial y_\perp}\right) S(y) \sim Q(\lambda,\lambda,\lambda)$$

$$\sigma_0(\mu) = \frac{m_H^2\,\alpha_s^2(\mu)}{72\pi(N_c^2-1)sv^2}$$

$$d\sigma = \sigma_0(\mu)\, C_t^2(m_t^2,\mu)\, H(m_H,\mu)\, \frac{m_H^2}{\tau s}\, dy\, \frac{d^2q_\perp}{(2\pi)^2} \int d^2y_\perp\, e^{-iq_\perp\cdot y_\perp}$$
$$\times\, 2J_n^{\mu\nu}(x_A,y_\perp,\mu)\, J_{\bar{n}\,\mu\nu}(x_B,y_\perp,\mu)\, S(y_\perp,\mu)$$

- Collinear and soft matrix elements are now (I multiply the collinear by -xP…):

$$J_n^{\mu\nu}(x_A,y_\perp,\mu) = -\frac{x_A P^+}{2}\int \frac{dy^-}{2\pi}\, e^{-i\frac{1}{2}x_A y^- P^+} \sum_{X_n} \langle PS_A|\, B_{n\perp}^{\mu,a}(y^-,y_\perp)\,|X_n\rangle\, \langle X_n|\, B_{n\perp}^{\nu,a}(0)\,|PS_A\rangle$$

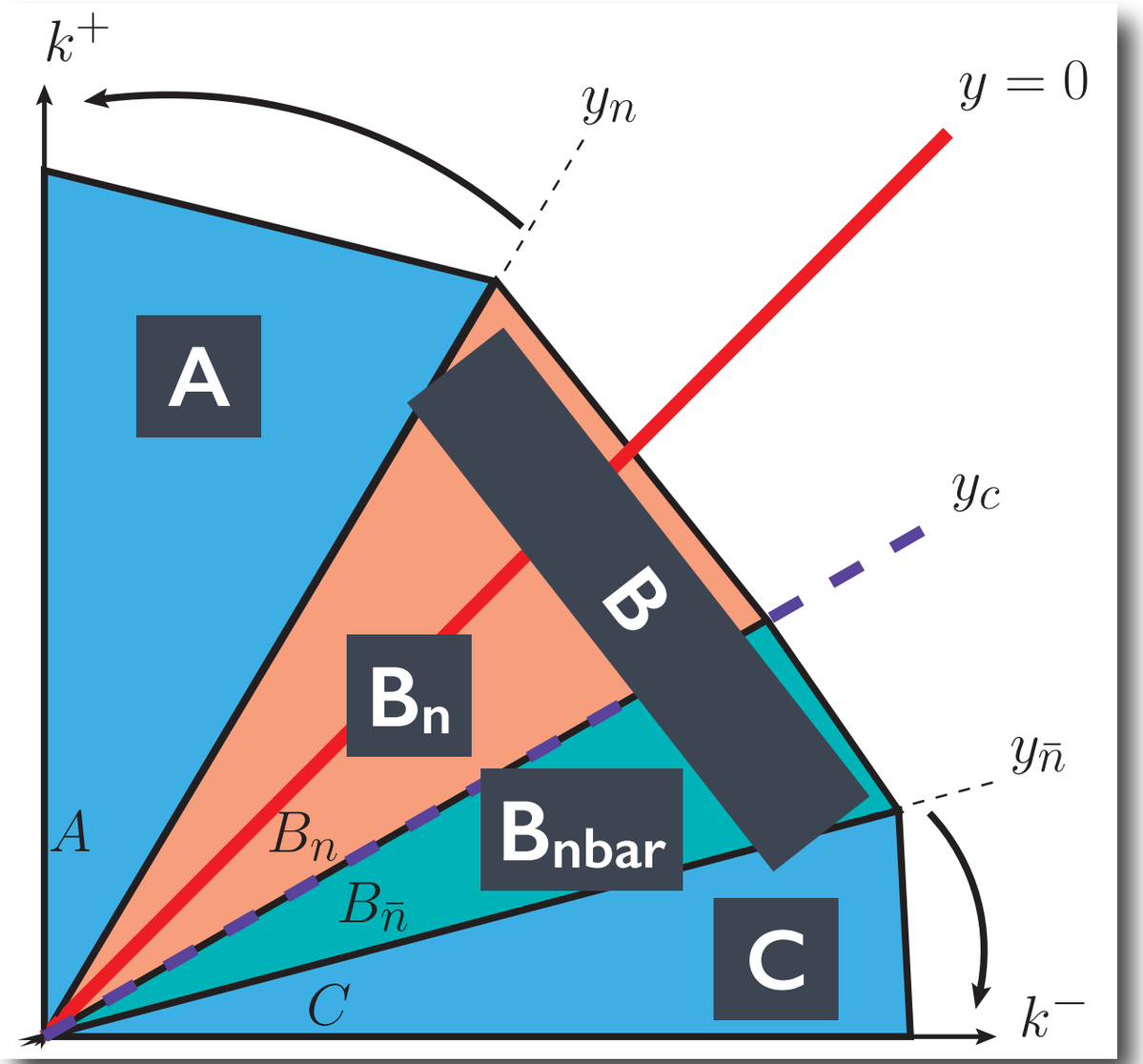
$$S(y_\perp,\mu) = \frac{1}{N_c^2-1}\sum_{X_s} \langle 0|\,\left(\mathcal{S}_n^\dagger \mathcal{S}_{\bar{n}}\right)^{ab}(y_\perp)\,|X_s\rangle\, \langle X_s|\,\left(\mathcal{S}_{\bar{n}}^\dagger \mathcal{S}_n\right)^{ba}(0)\,|0\rangle$$

Individually they are ill-defined!! They contain rapidity divergences (RDs)…

Definition of TMDPDFs: Cancellation of RDs

MGE, Idilbi, Scimemi JHEP'12, PLB'13

- Pictorially, the relevant (anti-)collinear and soft modes are represented as:



$$k_n \sim (1, \lambda^2, \lambda)$$
$$k_{\bar{n}} \sim (\lambda^2, 1, \lambda)$$
$$k_s \sim (\lambda, \lambda, \lambda)$$

$$y = \frac{1}{2}\ln\left|\frac{k^+}{k^-}\right|$$

- **Naive collinear = A+B**
- **Soft = B**
- **Naive anticollinear = C+B**
- (Pure collinear = A)
- (Pure anticollinear = C)
- Each piece is boost invariant and depends on the difference of rapidities at the borders.
- **x-section = (A+B) + (C+B) - B = A+B+C**

- **Divergences at yn and ynbar as spurius…**
- **(Anti-)Collinear and Soft are ill-defined!!!**

So in order to <u>cancel rapidity divergences,</u> we define the TMDPDFs as:

$$G^{\mu\nu}_{g/A}(x_A, \boldsymbol{k}_{n\perp}, S_A; \zeta_A, \mu^2) = A + B_n$$
$$G^{\mu\nu}_{g/B}(x_B, \boldsymbol{k}_{\bar{n}\perp}, S_B; \zeta_B, \mu^2) = C + B_{\bar{n}}$$

Definition of TMDPDFs: Cancellation of RDs

The goal is to <u>cancel Rapidity Divergences</u>. The particular regulator is irrelevant!!

MGE, Idilbi, Scimemi JHEP'12, PLB'13

- Rapidity regulator I: Δ-regulator (MGE, Idilbi, Scimemi JHEP'12)

$$\tilde{G}^{\mu\nu}_{g/A}(x_A, \boldsymbol{b}_T, S_A; \zeta_A, \mu^2) = \tilde{J}^{\mu\nu}_n(x_A, \boldsymbol{b}_T, S_A; Q^2, \mu^2; \Delta^+)\, \tilde{S}^{-1}_+(b_T; \zeta_B, \mu^2; \Delta^+)$$

$$\zeta_A = Q^2/\alpha \qquad \zeta_B = Q^2\alpha$$

- Rapidity regulator II: rapidity-regulator (eta) (Chiu, Jain, Neill, Rothstein PRL'12)

$$\tilde{G}^{\mu\nu}_{g/A}(x_A, \boldsymbol{b}_T, S_A; \zeta_A, \mu^2) = \tilde{J}^{\mu\nu\,(0)}_n(x_A, \boldsymbol{b}_T, S_A; Q^2, \mu^2; \nu_-; \eta)\, \tilde{S}_-(b_T; \mu^2; \alpha\nu_-; \eta)$$

$$\zeta_A = Q^2/\alpha \qquad \zeta_B = Q^2\alpha$$

- Rapidity regulator III: "combining integrands" (Collins'11)

$$\tilde{G}^{\mu\nu}_{g/A}(x_A, \boldsymbol{b}_T, S_A; \zeta_A, \mu^2) = \lim_{\substack{y_n \to +\infty \\ y_{\bar{n}} \to -\infty}} \tilde{J}^{\mu\nu}_n(x_A, \boldsymbol{b}_T, S_A; \mu^2; y_{\bar{n}}) \sqrt{\frac{\tilde{S}(y_n, y_c)}{\tilde{S}(y_c, y_{\bar{n}})\, \tilde{S}(y_n, y_{\bar{n}})}}$$

$$\zeta_A = (p^+)^2 e^{-2y_c} \qquad \zeta_B = (\bar{p}^-)^2 e^{+2y_c}$$

- **One could also use off-shellnesses, masses, "real Δ's", analytic regulator, etc… Yet they all mean (pictorially):**

$$\tilde{G}^{\mu\nu}_{g/A}(x_A, \boldsymbol{b}_T, S_A; \zeta_A, \mu^2) = A + B_n$$

Previous slide!

Cross-Section in terms of TMDPDFs

- The cross-section in terms of well-defined TMDPDFs is given by:

$$\frac{d\sigma}{dyd^2q_\perp} = \sigma_0(\mu)\, C_t^2(m_t^2,\mu) H(m_H^2,\mu) \frac{m_H^2}{\tau s} \frac{2}{(2\pi)^2} \int d^2y_\perp\, e^{i\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp} \times \tilde{G}^{\mu\nu}_{g/A}(x_A,\boldsymbol{b}_\perp,S_A;\zeta_A,\mu)\, \tilde{G}_{g/B\,\mu\nu}(x_B,\boldsymbol{b}_\perp,S_B;\zeta_B,\mu)$$

- We know the evolution of TMDPDFs at NNLL (universal evolution kernel):

$$\tilde{G}^{\mu\nu\,[pol]}_{g/A}(x_n,\boldsymbol{b}_\perp,S_A;\zeta_{A,f},\mu_f^2) = \tilde{G}^{\mu\nu\,[pol]}_{g/A}(x_n,\boldsymbol{b}_\perp,S_A;\zeta_{A,i},\mu_i^2)\, \tilde{R}^g\left(b_T;\zeta_{A,i},\mu_i^2,\zeta_{A,f},\mu_f^2\right)$$

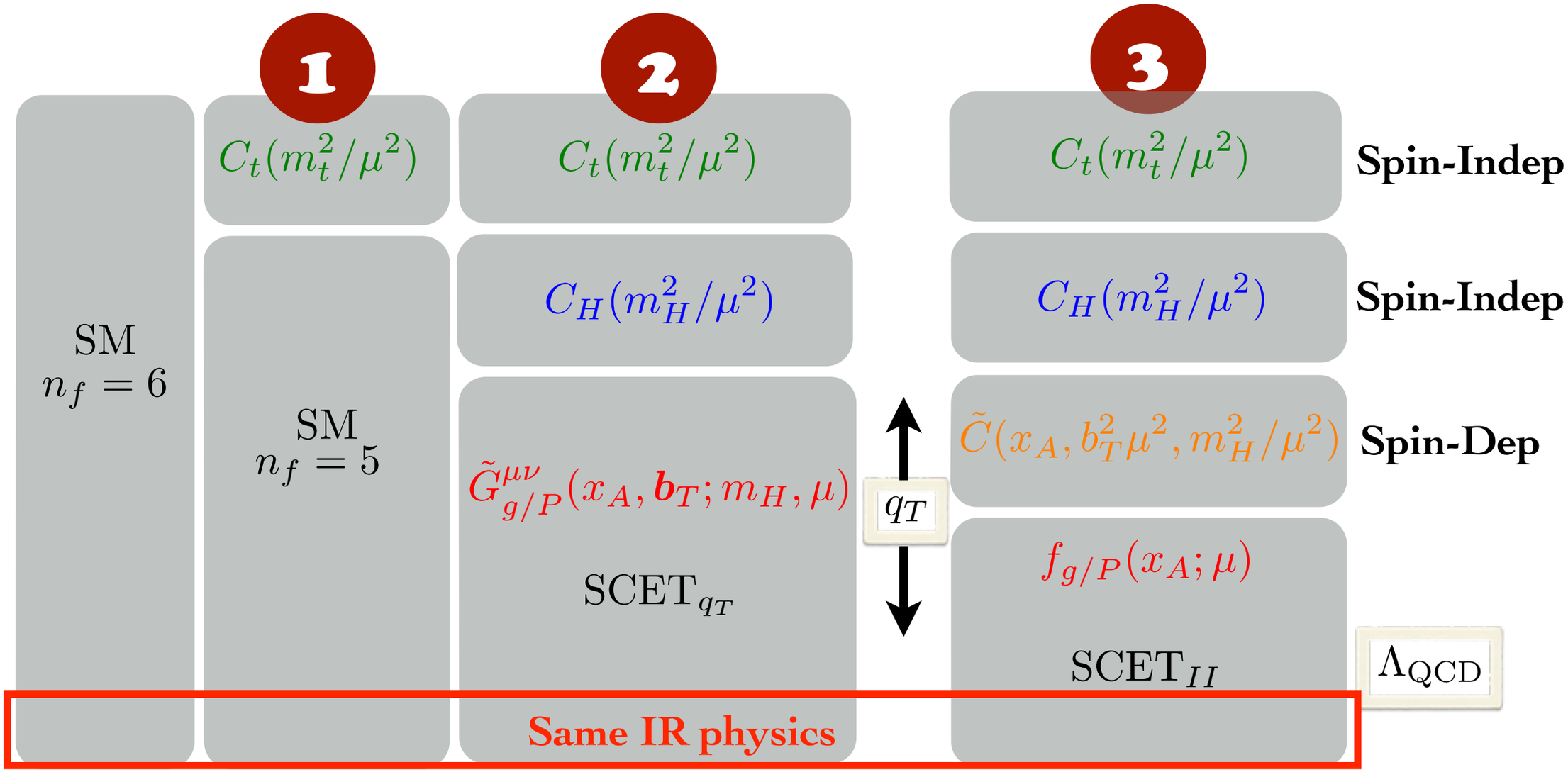
$$\tilde{R}^g\left(b_T;\zeta_{A,i},\mu_i^2,\zeta_{A,f},\mu_f^2\right) = \exp\left\{\int_{\mu_i}^{\mu_f}\frac{d\bar{\mu}}{\bar{\mu}}\,\gamma_G\left(\alpha_s(\bar{\mu}),\ln\frac{\zeta_{A,f}}{\bar{\mu}^2}\right)\right\}\left(\frac{\zeta_{A,f}}{\zeta_{A,i}}\right)^{-D_g(b_T;\mu_i)}$$

- The evolution itself contains some non-perturbative input (in the D term)
- The TMDPDFs have perturbative content at large k_T, so we can further re-factorize them…

Step 3:

Factorization of the q_T

$d\sigma =$



Re-factorization (unpolarized proton)

- Inside an unpolarized proton we can have unpolarized or linearly polarized gluons:

$$G^{\mu\nu[O]}_{g/A}(x,\boldsymbol{k}_{nT}) = -g_\perp^{\mu\nu} f_1^g(x,k_{nT}^2) + \frac{1}{2}\left(g_\perp^{\mu\nu} - \frac{2k_{n\perp}^\mu k_{n\perp}^\nu}{k_{n\perp}^2}\right) h_1^{\perp g}(x,k_{nT}^2)$$

$$\tilde{G}^{\mu\nu[O]}_{g/A}(x,\boldsymbol{b}_T) = -g_\perp^{\mu\nu} \tilde{f}_1^g(x,b_T^2) + \frac{1}{2}\left(g_\perp^{\mu\nu} - \frac{2b_\perp^\mu b_\perp^\nu}{b_\perp^2}\right) \tilde{h}_1^{\perp g}(x,b_T^2)$$

$$\tilde{f}_1^g(x,b_T^2) = \int \frac{d^2\boldsymbol{k}_{n\perp}}{(2\pi)^2}\, e^{-i\boldsymbol{k}_{n\perp}\cdot\boldsymbol{b}_\perp}\, f_1^g(x,k_{nT}^2)$$

$$\tilde{h}_1^{\perp g}(x,b_T^2) = -2\pi \int dk_{nT}\, k_{nT}\, J_2(k_{nT}b_T)\, h_1^{\perp g}(x,k_{nT}^2)$$

- The OPEs of (renormalized) unpolarized and linearly polarized gluon TMDPDFs are both given in terms of the (renormalized) collinear quark/gluon PDFs:

$$\tilde{f}^g_{1,g/A}(x,b_T;\zeta_A,\mu) = \left(\frac{\zeta_A b_T^2}{4e^{-2\gamma_E}}\right)^{-D_g(b_T;\mu)} \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{I}^f_{g\leftarrow j}(\bar{x},b_T;\mu)\, f_{1,j/A}(x/\bar{x};\mu) + \mathcal{O}(b_T\Lambda_{\rm QCD})$$

$$\tilde{h}^{\perp g}_{1,g/A}(x,b_T;\zeta_A,\mu) = \left(\frac{\zeta_A b_T^2}{4e^{-2\gamma_E}}\right)^{-D_g(b_T;\mu)} \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{I}^h_{g\leftarrow j}(\bar{x},b_T;\mu)\, f_{1,j/A}(x/\bar{x};\mu) + \mathcal{O}(b_T\Lambda_{\rm QCD})$$

Re-factorization (unpolarized proton)

- The cross-section is finally given by:

$$\frac{d\sigma}{dyd^2q_\perp} = \sigma_0(\mu)\, C_t^2(m_t^2,\mu) H(m_H^2,\mu) \frac{m_H^2}{\tau s} \frac{2}{(2\pi)^2} \int d^2y_\perp\, e^{i\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp} \left(\frac{m_H^2 b_T^2}{4e^{-2\gamma_E}}\right)^{-2D_g(b_T;\mu)} \sum_{i,j=q,\bar{q},g} \int_{x_A}^1 \frac{dx_1}{x_1} \int_{x_B}^1 \frac{dx_2}{x_2}$$
$$\times \sum_{k=f,h} \tilde{I}^i_{g\leftarrow j}(x_1,b_T;\mu)\, \tilde{I}^j_{g\leftarrow j}(x_2,b_T;\mu)\, f_{1,i/A}(x_A/x_1;\mu)\, f_{1,j/B}(x_B/x_2;\mu) + \mathcal{O}(b_T\Lambda_{\rm QCD})$$

- The non-perturbative inputs are the collinear PDFs and the large b contributions to the coefficients and the D term.

- There are different ways of parameterizing the non-perturbative input, separating perturbative and non-perturbative and resumming large logarithms (scale choice).

Time for discussion…