Simplified Glauber-Gribov model for Geant4 hadron cross sections

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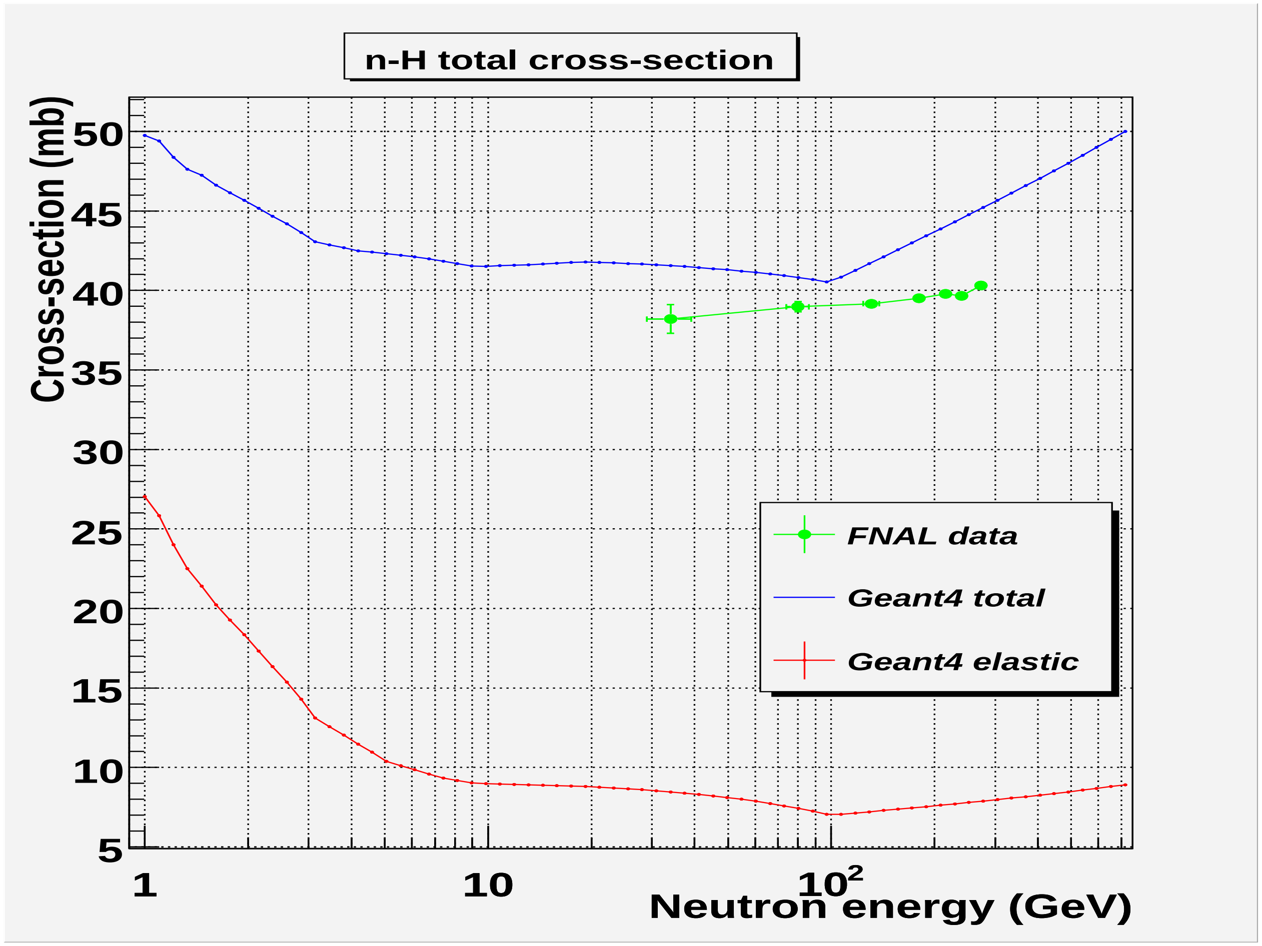
Abstract

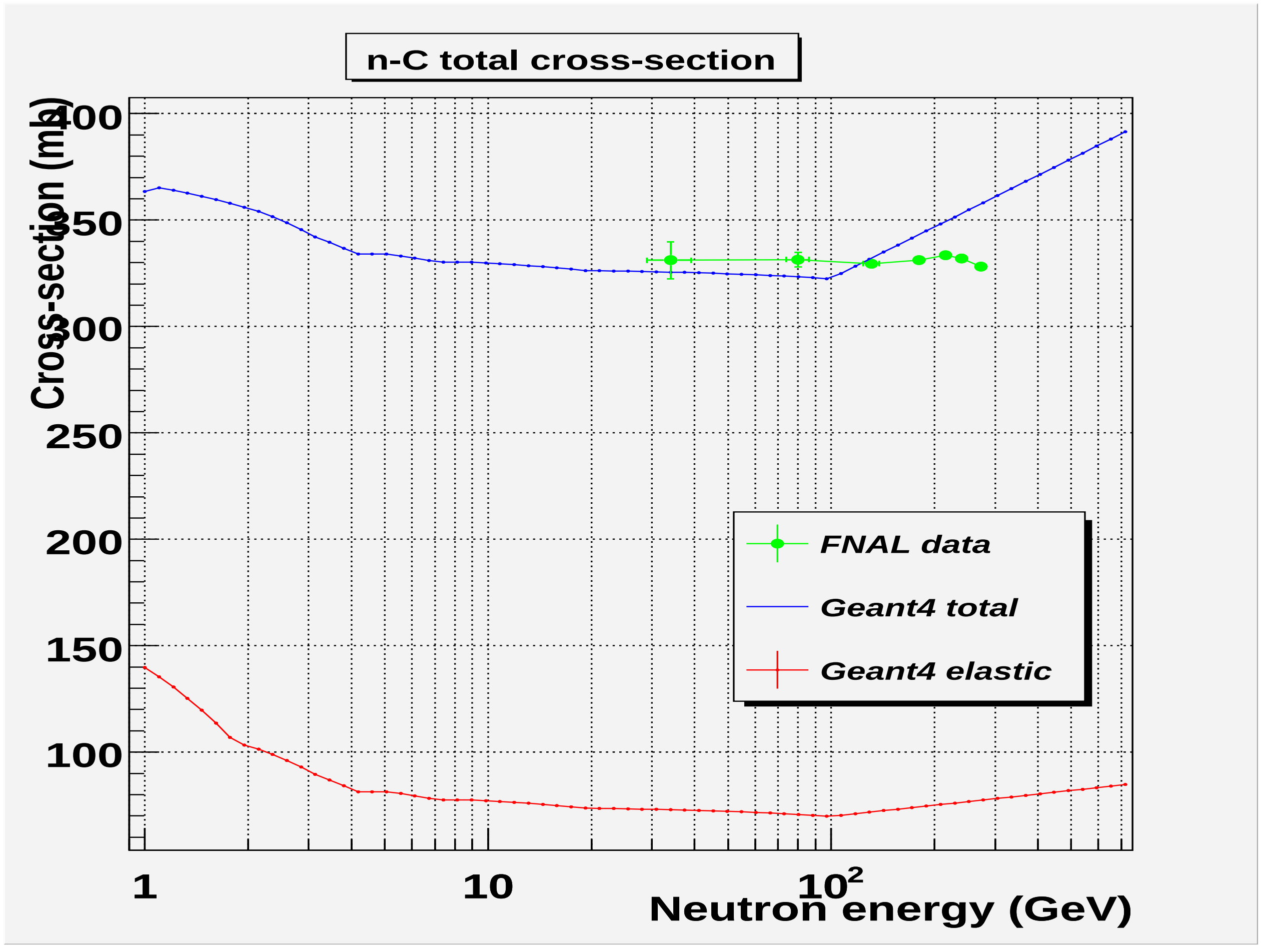
The current status of Glauber-Gribov model for Geant4 hadron cross sections with the hadron energy above few GeV is presented. The inelastic, elastic and total cross sections are simulated and comparison with experimental data is discussed.

1 Introduction

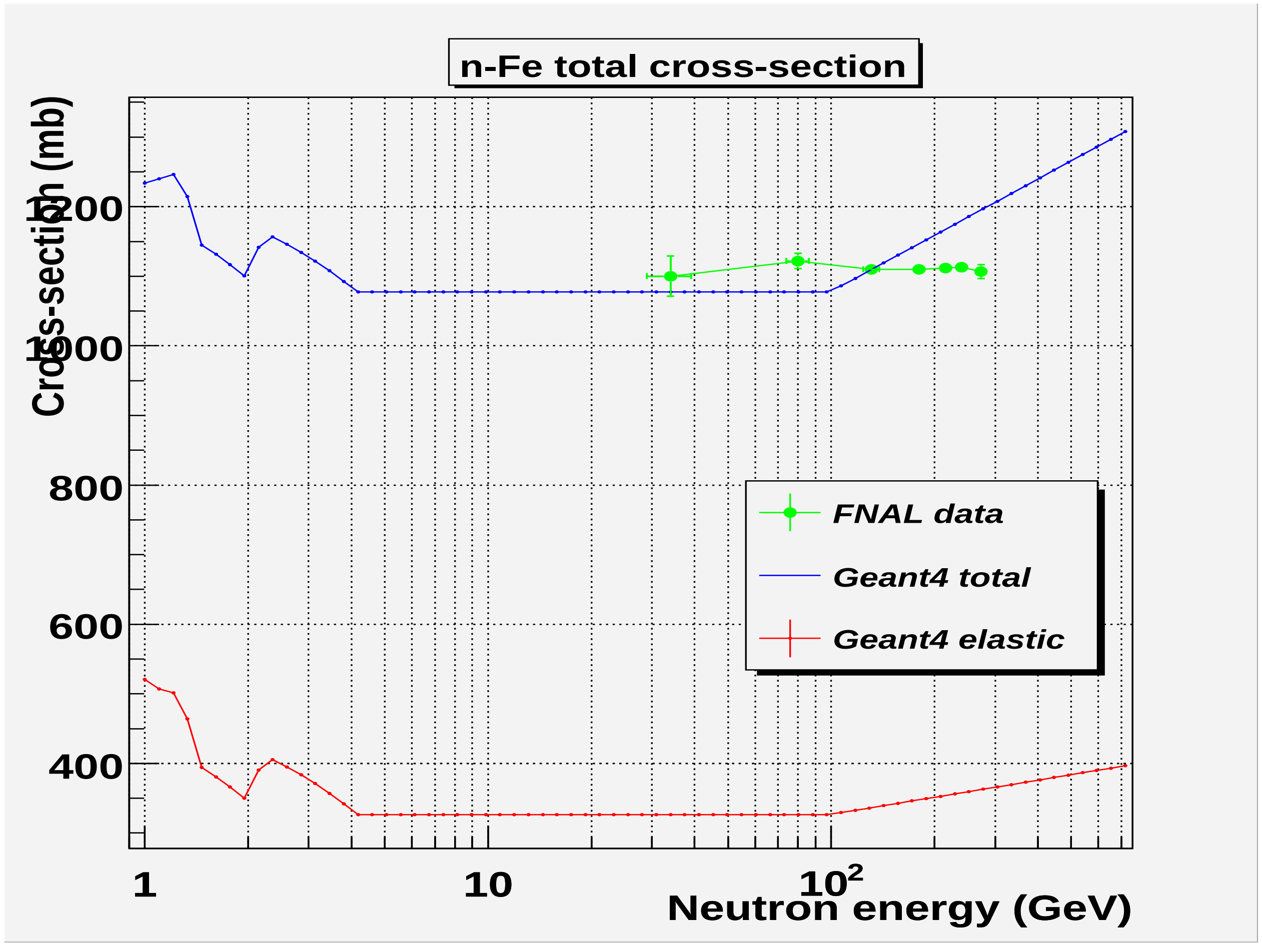
Here it is reported on verification of total and elastic cross-sections for hadrons on different targets. The class responsible for the majority of use cases for elastic and inelastic cross sections in Geant4 was historically inherited from Geisha. This is class G4HadronicCrossSections. Below are few examples of its predictions versus FNAL experimental data for total cross sections of neutron on different nucleus targets.



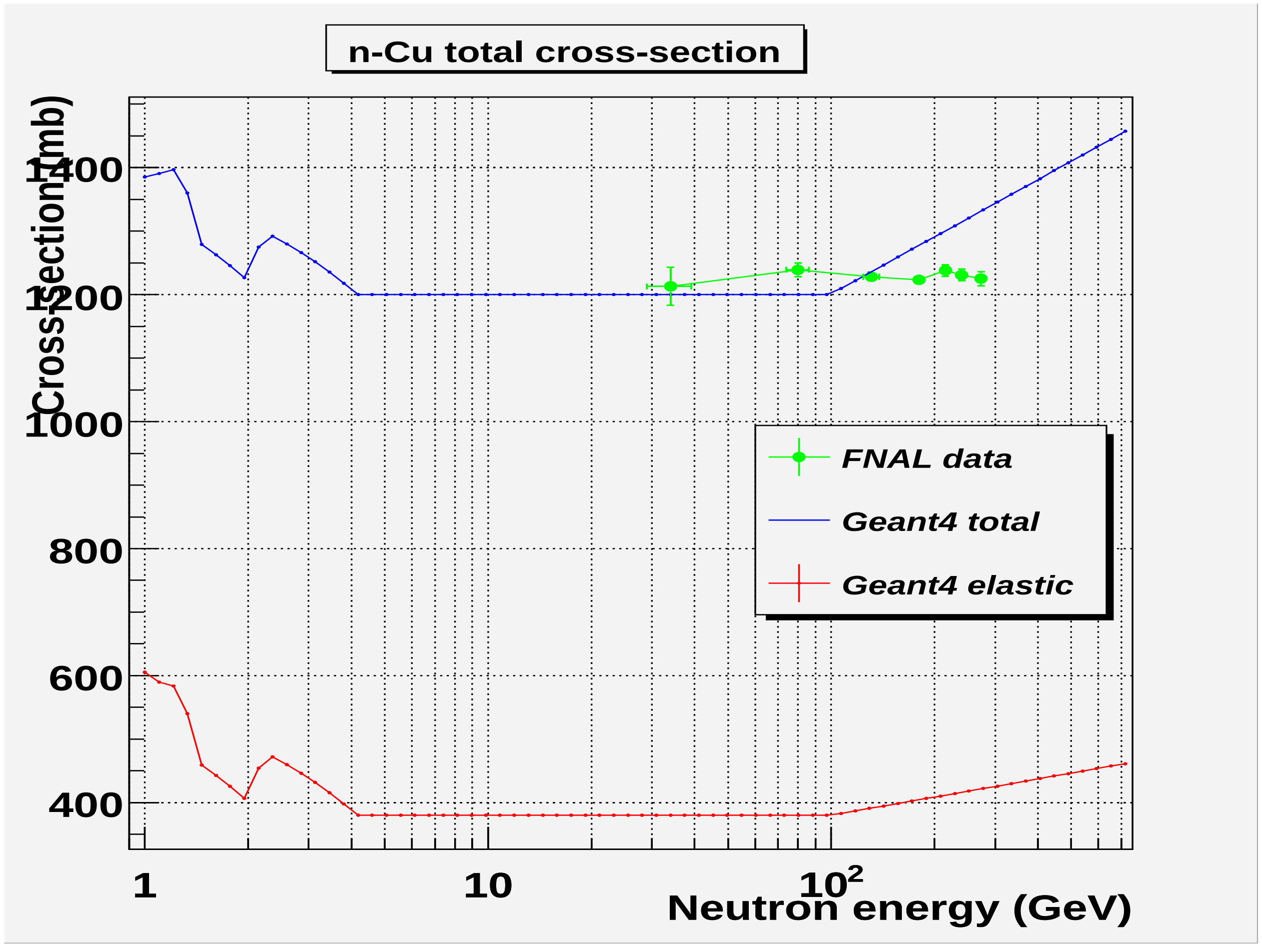
Elastic and total cross sections of neutrons on hydrogen target versus the neutron energy in the laboratory frame.



Elastic and total cross sections of neutrons on carbon target versus the neutron energy in the laboratory frame.



Elastic and total cross sections of neutrons on iron target versus the neutron energy in the laboratory frame.



Elastic and total cross sections of neutrons on copper target versus the neutron energy in the laboratory frame.

The reason for such a behavior of G4HadronCrossSections is:

```
void G4HadronCrossSections::CalcScatteringCrossSections(
const G4DynamicParticle* aParticle, const G4Element* anElement)
{
......
// Correction factor for high (p > 100 GeV/c) energies:
   G4double corh = 1.;
   if (p > 100.) corh = 0.1085736156*std::log(p) + 0.5;
   sigel = corh*sigel;
   sigin = corh*sigin;
   // Convert cross section from mb to default units
   sigelastic = sigel*millibarn;
   siginelastic = sigin*millibarn;
   return;
}
```

Since GEANT4 has a set of additional models for inelastic cross section, the main motivation was to develop a model for either total or elastic cross sections.

2 Simplified Glauber-Gribov model for total and elastic cross sections

As was mentioned by B.Z. Kopeliovitch in 'Transparent Nuclei and Deuteron-Gold Collisions at HRIC' (nucl-th/0306044, Sep 5 , 2003), the Glauber model results in the following relations for the total σ_{tot}^{hA} and inelastic σ_{in}^{hA} cross sections of a hadron h on a nuclear target A, respectively:

$$\sigma_{tot}^{hA} = 2\int d^2b \left\{1 - \left\langle \exp\left[-\frac{1}{2}\sigma_{tot}^{hN} T_A^h(b)\right]\right\rangle\right\},$$

$$\sigma_{in}^{hA} = \int d^2b \left\{1 - \left\langle \exp\left[-\sigma_{tot}^{hN} T_A^h(b)\right]\right\rangle\right\},$$

$$\sigma_{el}^{hA} = \int d^2b \left|1 - \left\langle \exp\left[-\frac{1}{2}\sigma_{tot}^{hN} T_A^h(b)\right]\right\rangle\right|^2,$$

where b is the impact parameter, σ_{tot}^{hN} is the total cross section of the hadron on a nucleon and $T_A^h(b)$ is the hadron profile function. Here $\langle \ldots \rangle$ means a proper averaging of hadron eigenstates including Gribov inelastic shadowing correction and, $\sigma_{tot}^{hA} = \sigma_{in}^{hA} + \sigma_{el}^{hA}$.

Applying the light-cone dipole approximation for the correction calculation, one can perform averaging:

$$\left\langle \exp\left[-\sigma_{tot}^{hN} T_A^h(b)\right]\right\rangle = \frac{1}{1+\sigma_{tot}^{hN} T_A^h(b)}.$$

This explicitly demonstates how Gribov correction makes nuclei more transparent (power for exponent). Then the cross sections read:

$$\sigma_{tot}^{hA} = \int d^2 b \frac{\sigma_{tot}^{hN} T_A^h(b)}{1+\frac{1}{2}\sigma_{tot}^{hN} T_A^h(b)}, \quad \sigma_{in}^{hA} = \int d^2 b \frac{\sigma_{tot}^{hN} T_A^h(b)}{1+\sigma_{tot}^{hN} T_A^h(b)},$$

For practical calculations in a wide range of hadrons and nuclei, we make two simplifications for the hadron profile function $T_A^h(b)$ calculation according to the Glauber model:

$$T_A^h(b) = \frac{2}{\sigma_{tot}^{hN}} \int d^2 s \mathrm{Re}\Gamma^{hN}(s) T_A(\mathbf{b}-\mathbf{s}), \quad s(x,y), \quad T_A(b) = A\int_{-\infty}^{\infty} d\,z \rho(b,z),$$

where $T_A(b)$ is the nuclear thickness function, A is the number of nucleons in the target nucleus and $\rho(b,z)$ is the nuclear density normalised on unit (z is the distance along the hadron trajectory at the impact parameter b).

The elastic scattering amplitude $i\Gamma^{hN}(s)$ on a nucleon can be used in exponential form:

$$\mathrm{Re}\Gamma^{hN}(s) = \frac{\sigma_{tot}^{hN}}{4\pi B_{hN}} \exp\left(-\frac{s^2}{2B_{hN}}\right),$$

where B_{hN} is the slope of the differential hN elastic cross section. The latter relation can be simplified assuming small slope B_{hN} or small size of a nucleon compared to the nucleous:

$$\mathrm{Re}\Gamma^{hN}(s) \ (B_{hN} \to 0) \to \frac{\sigma_{tot}^{hN}}{2}\delta(x)\delta(y),$$

$$T_A^h(b) \ (B_{hN} \to 0) \sim T_A(b) = A\int_{-\infty}^{\infty} d\,z\rho(b,z),$$

The second step is simplified Gaussian representation of $\rho(b,z)$:

$$\rho(b,z) = \frac{1}{(R\sqrt{\pi})^3} \exp\left\{-\frac{b^2+z^2}{R^2}\right\},$$

where R is the nuclear radius, allows us to calculate analytically the cross section integrals in respect of b and z.

Then the cross sections read:

$$\sigma_{tot}^{hA} = 2\pi R^2 \ln\left[1 + \frac{A\sigma_{tot}^{hN}}{2\pi R^2}\right],$$

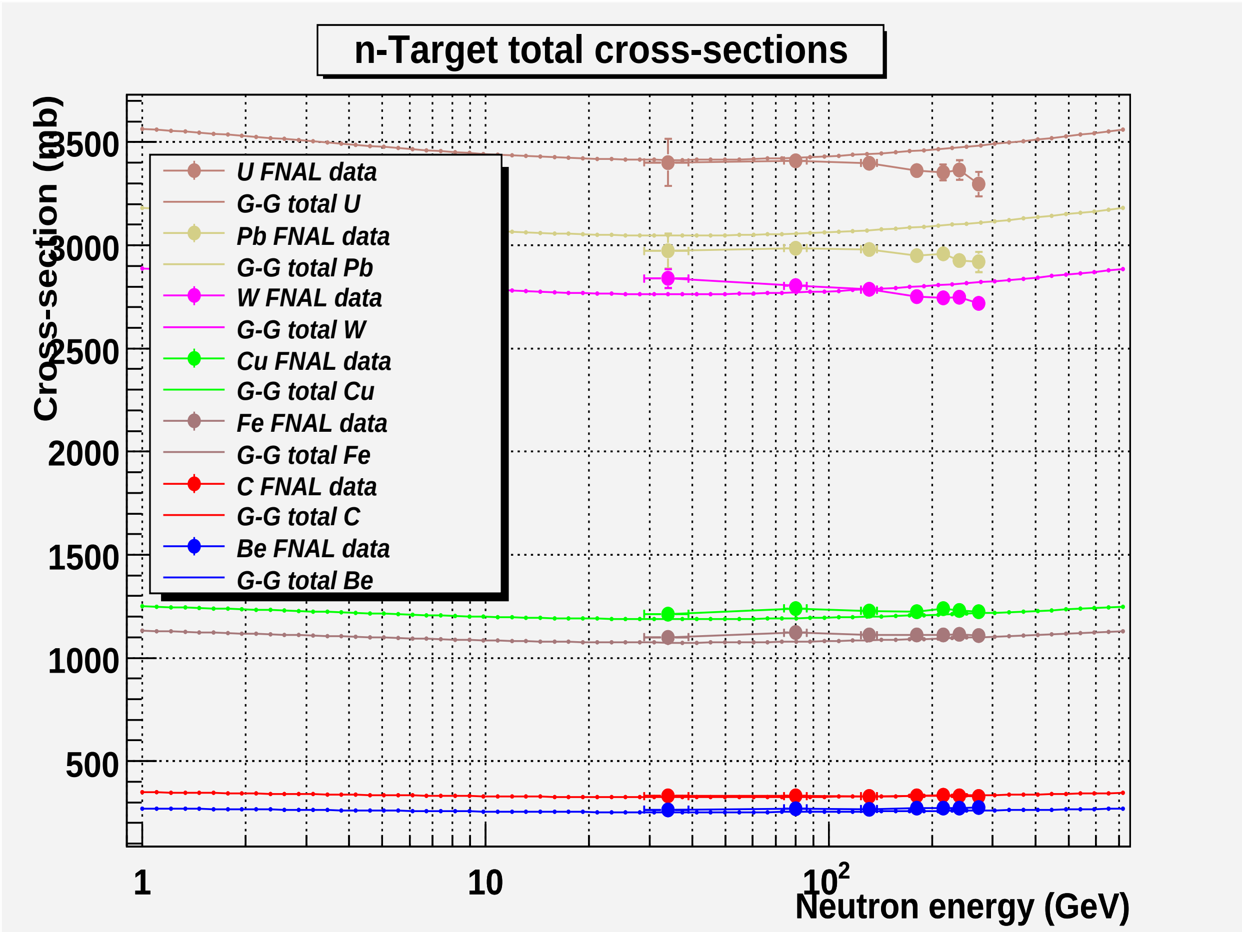
$$\sigma_{in}^{hA} = \pi R^2 \ln\left[1 + \frac{A\sigma_{tot}^{hN}}{\pi R^2}\right].$$

Where σ_{in}^{hA} is the corresponding inelastic cross section.

The model is reduced to the selection of σ_{tot}^{hN} and $R(A)$ values. We use the latest edition of Particle Data Group parametrization for σ_{tot}^{hN}, including the total cross sections of p, $\bar{p}$, n, $\pi^{\pm}$, $K^{\pm}$ and Σ^{-} on protons and neutrons (http://pdg.lbl.gov/2006/reviews/hadronicrpp.pdf). For known cross sections on proton and neutron, $A\sigma_{tot}^{hN} = N_p\sigma_{tot}^{hp} + N_n\sigma_{tot}^{hn}$, where N_p and N_n are the number of protons and neutrons in the nucleus. The nuclear radius is parametrized by:

$$R(A) = r_o A^{\frac{1}{3}} f(A), \quad r_o \sim 1.1\ fm,$$

where for $A > 21$, $f(A) < 1$, while in the opposite case $3 < A < 21$, $f(A) > 1$ in the limits of 20%. Below are the model predictions for the total neutron cross sections on different targets.

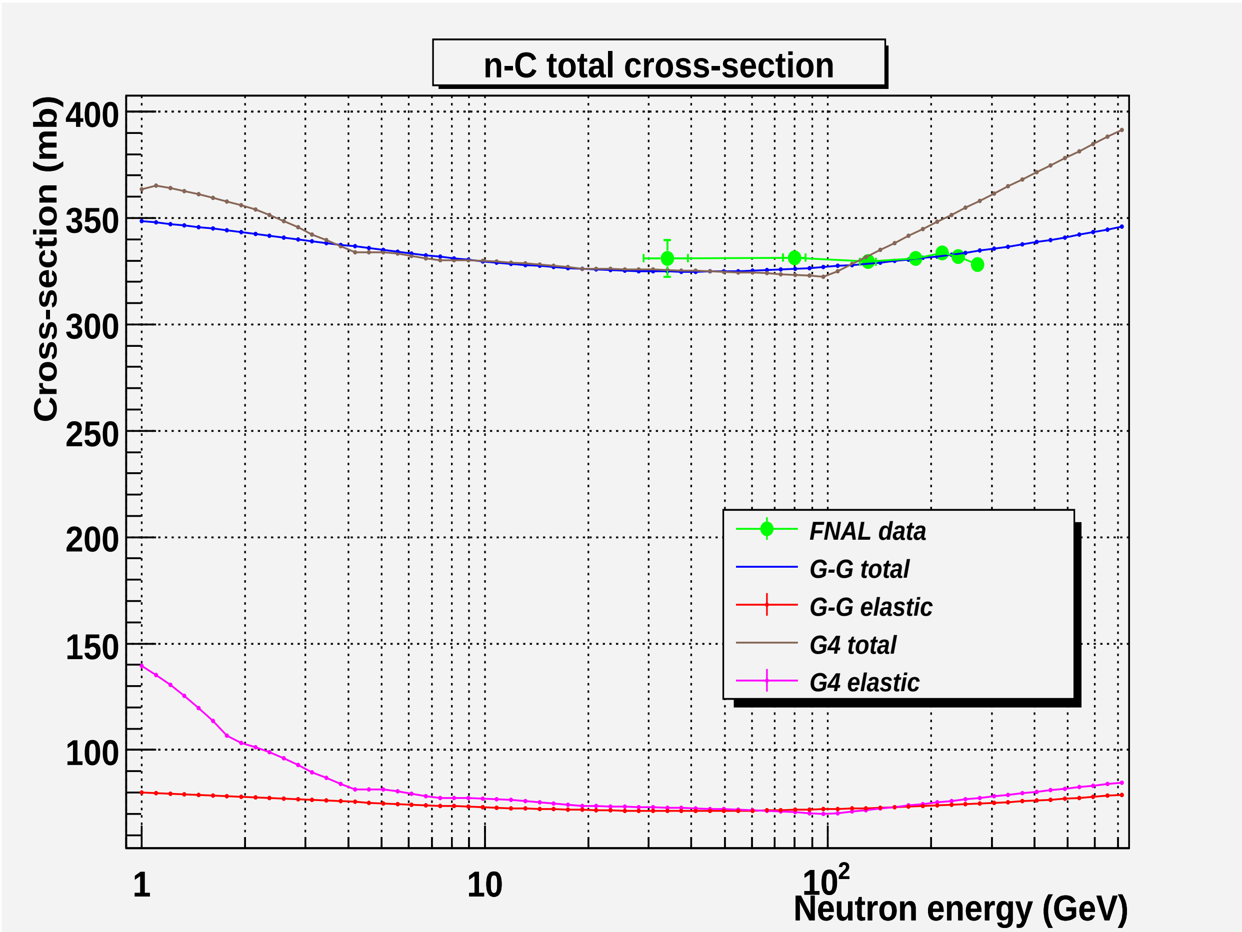


Total cross sections of neutrons on berillium, carbon, iron, copper, tangsten, lead and uranium target versus the neutron energy in the laboratory frame. (Data from: P.V.R. Murthy et al, Neutron Total Cross Sections on Nuclei at FermiLab Energies, Nucl.Phys B92 (1975), 269-308).

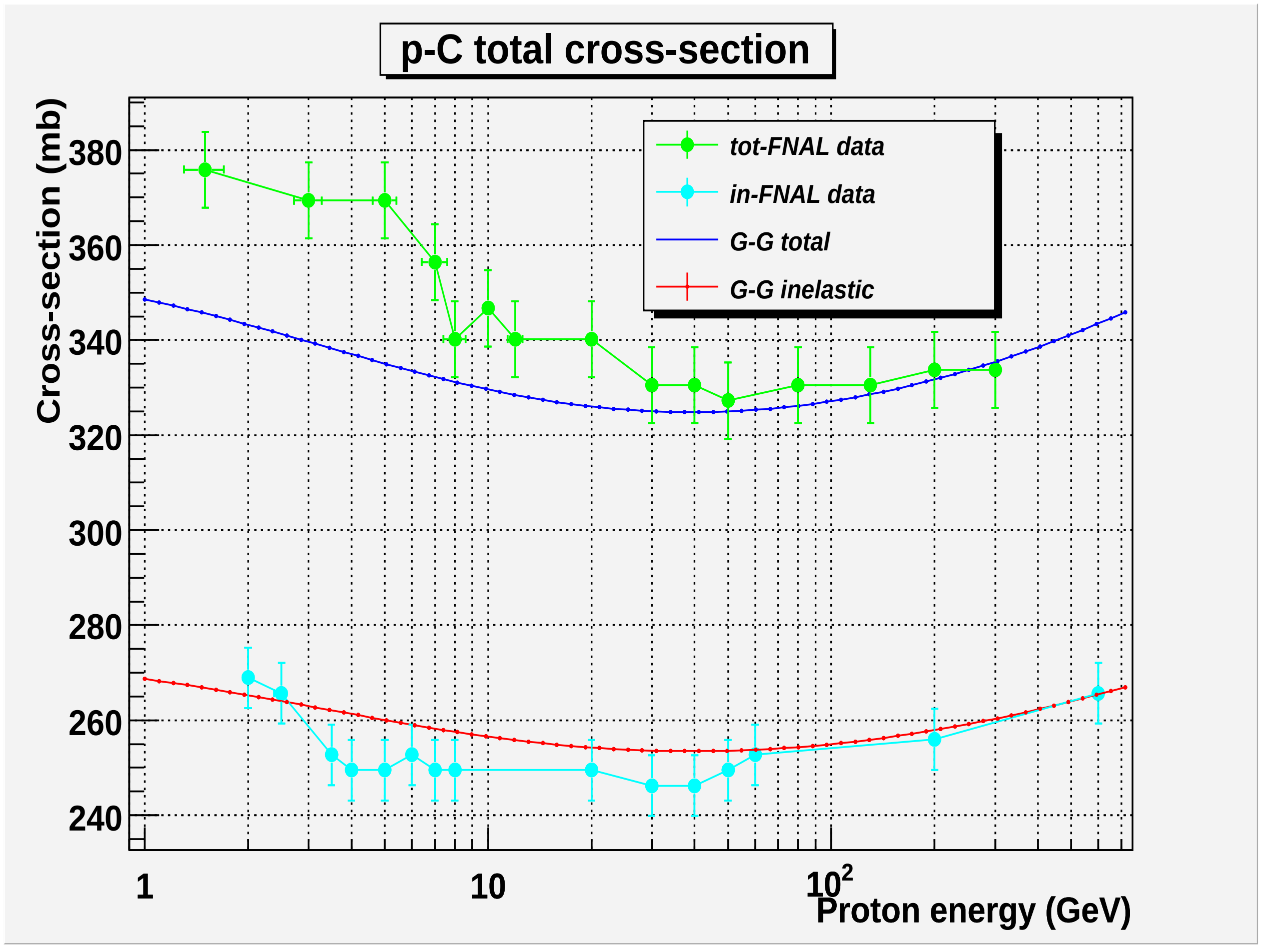
The table of experimental and simulated total cross sections
for different hadrons and targets

Cross section	P_{lab}, GeV/c	FNAL data, mb	GEANT4 G-G model, mb
$\sigma_{tot}(p\,Be)$	536	268.6 ± 2.5	266.9
$\sigma_{tot}(\pi^-\,Be)$	638	188.7 ± 1.5	188.4
$\sigma_{tot}(p\,C)$	457	333.6 ± 5.5	340.5
$\sigma_{tot}(p\,C)$	490	335.4 ± 5.5	341.7
$\sigma_{tot}(\pi^-\,C)$	591	234.1 ± 4.5	242.9
$\sigma_{tot}(\pi^-\,Cu)$	608	1032 ± 180	953.7

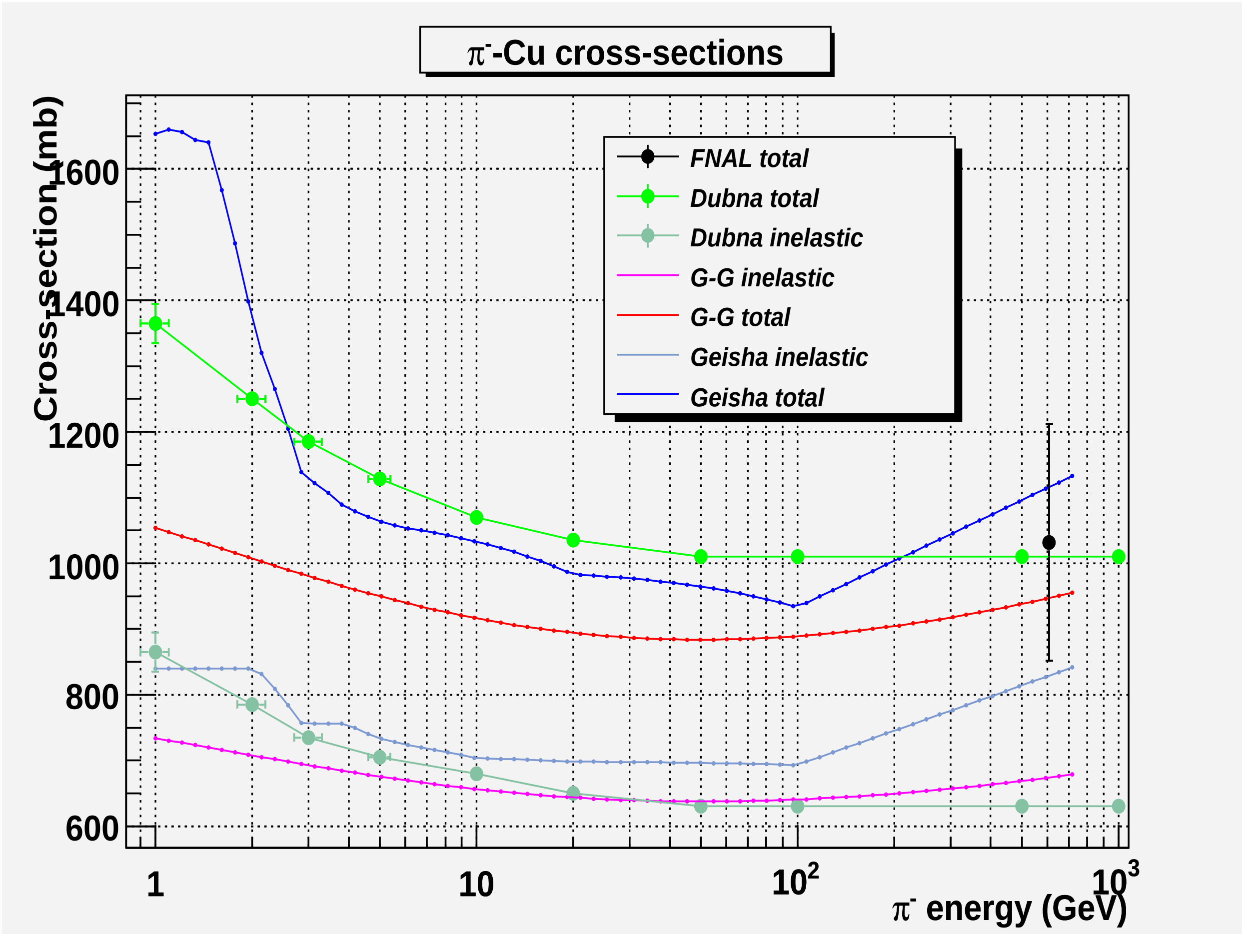
Data from: U. Dersch et al, Total Cross Sections Measurements with π^-, Σ^- and Protons on Nuclei and Nucleons around 600 GeV/c, hep-ex/9910052, Oct 25, 1999



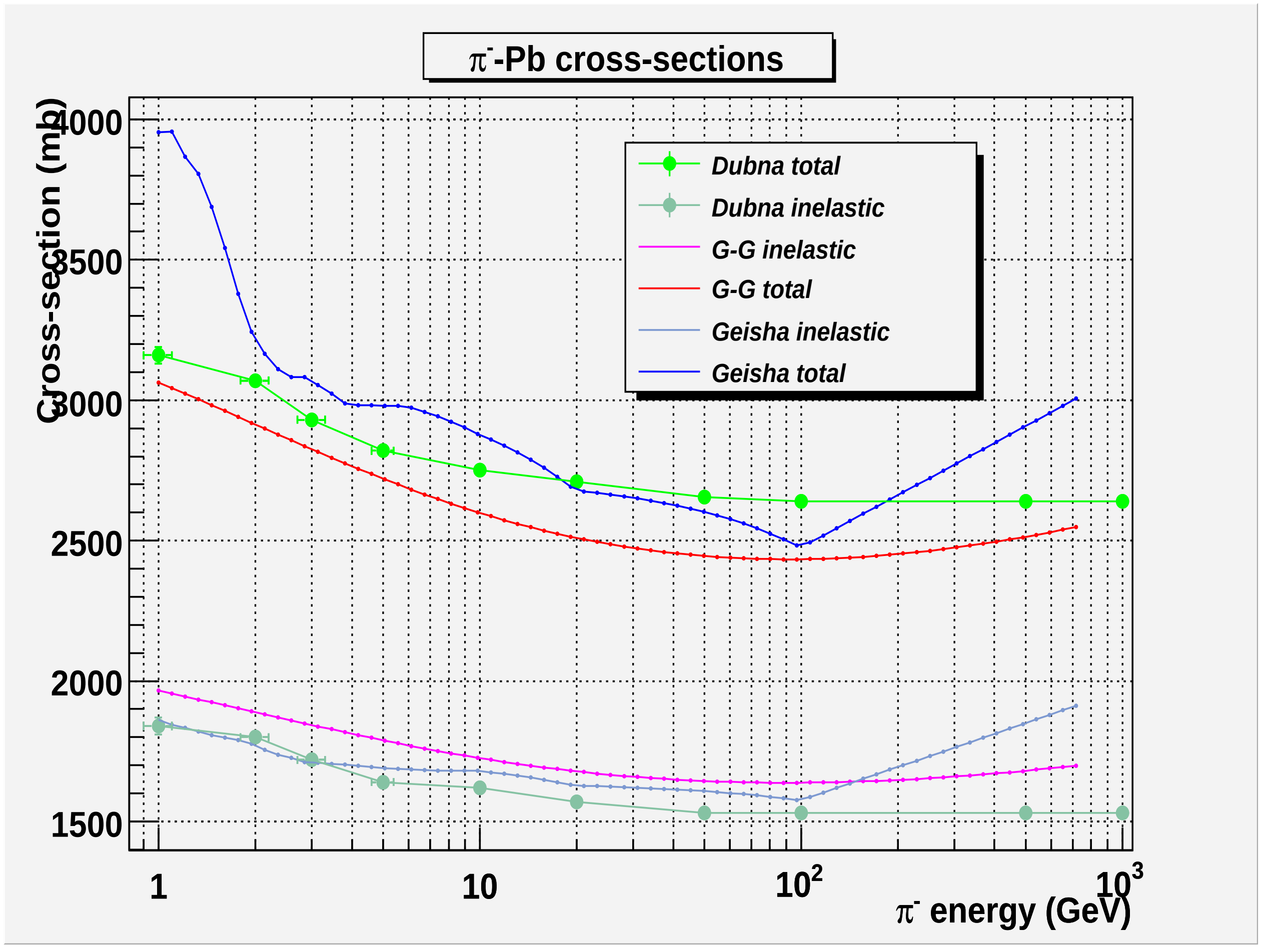
Elastic and total cross sections of neutrons on carbon target versus the neutron energy in the laboratory frame. Geisha (G4) and updated Glauber-Gribov (GG) models are shown.



Inelastic and total cross sections of protons on carbon target versus the proton energy in the laboratory frame. Updated Glauber-Gribov (GG) models is shown.



Inelastic and total cross sections of π^- on copper target versus the π^- energy in the laboratory frame. Different GEANT4 models are shown.



Inelastic and total cross sections of π^- on lead target versus the π^- energy in the laboratory frame. Different GEANT4 models are shown.

3 Summary and ToDo

1. Simple model for total and elastic cross sections based on the Glauber model with the Gribov correction estimated within the light cone dipole approximation was developed and implemented as class G4GlauberGribovCrossSection.

2. The model shows satisfactory agreement with experimental data for total cross sections in wide range of hadron energies (more than 30 GeV) for different targets.

3. The model tuning (it is very simple) and comparison with other data are in progress.