

Flavour as a Window to New Physics at LHC - CERN Workshop

BSM Aspects of Flavour Physics
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The question of Flavour
has been with us for many { years
decades!

It started with Rabi's question

Who ordered the muon?

The Flavour problem

or

the Mass Problem :

One of the most open problems
in Particle Physics

- Why is there a family replication?
why 3?
- How to understand the pattern
of quark masses and mixing?

$$V_{us} \approx \lambda ; V_{cb} \approx \lambda^2 ; V_{ub} \approx \lambda^3$$

$$\lambda \approx .2$$

- Why is there large lepton mixing to be contrasted to small quark mixing?
- How to understand the smallness of neutrino masses?
Seesaw? How to test?
- What is the origin of CP violation?
Is CP explicitly or spontaneously broken?
- What are the "connections" among the above questions?
Is large mixing a "reflection" of the seesaw?

The 22 Parameters (assuming Majorana Neutrinos) [3]

Quark Sector :

Quark masses : 6

CP violating phase : 1

Mixing angles : $\frac{3}{10}$

Lepton Sector

Neutrino masses : 3

Charged lepton masses : 3

CP violating Phases : 3

Mixing Angles : $\frac{3}{12}$

10 + 12 = 22 parameters!

Great Progress in the determination of the parameters from Experiment and Theory

$$V_{CKM} = \begin{bmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{bmatrix}$$

$$|V_{ub}| \rightarrow s_{13} ; |V_{us}| \rightarrow s_{12} ; |V_{cb}| \rightarrow s_{23}$$

Central values (Assuming 3x3 unitarity)

$$\begin{bmatrix} |V_{ud}| = .9738 & |V_{us}| = .2272 & |V_{ub}| = 3.96 \times 10^{-3} \\ |V_{cd}| = .2271 & |V_{cs}| = .9729 & |V_{cb}| = 4.221 \times 10^{-2} \\ |V_{td}| = 8.14 \times 10^{-3} & |V_{ts}| = 4.161 \times 10^{-3} & |V_{tb}| = .999 \end{bmatrix}$$

Impressive success of the SM

Once s_{12} , s_{23} , s_{13} are fixed by

V_{us} , V_{cb} , V_{ub}

One has to fit with only one parameter

(6) a large number of physical quantities:

E_K

$$\beta \equiv \arg[-V_{cd} V_{cb}^* V_{td}^* V_{tb}]$$

$B_d - \bar{B}_d$ mixing

$$\gamma \equiv \arg[-V_{ud} V_{ub}^* V_{cd}^* V_{cb}]$$

$B_s - \bar{B}_s$ mixing

etc ...

- Note that the extraction of $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, γ , from experiment is not affected by the possible of New Physics beyond the SM
- Extraction of $|V_{td}|$, $|V_{ts}|$, β from experiment may be affected by the presence of New Physics

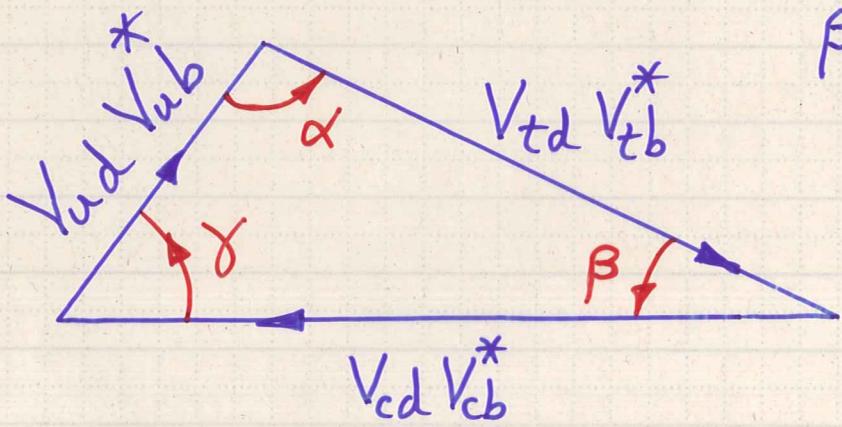
Traditional way of testing the SM : The "unitarity triangle"

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

$$\alpha \equiv \arg(-V_{td} V_{tb}^* V_{ud} V_{ub})$$

$$\beta \equiv \arg(-V_{cd} V_{cb}^* V_{td}^* V_{tb})$$

$$\gamma \equiv \arg(-V_{ud} V_{ub}^* V_{cd}^* V_{cb})$$

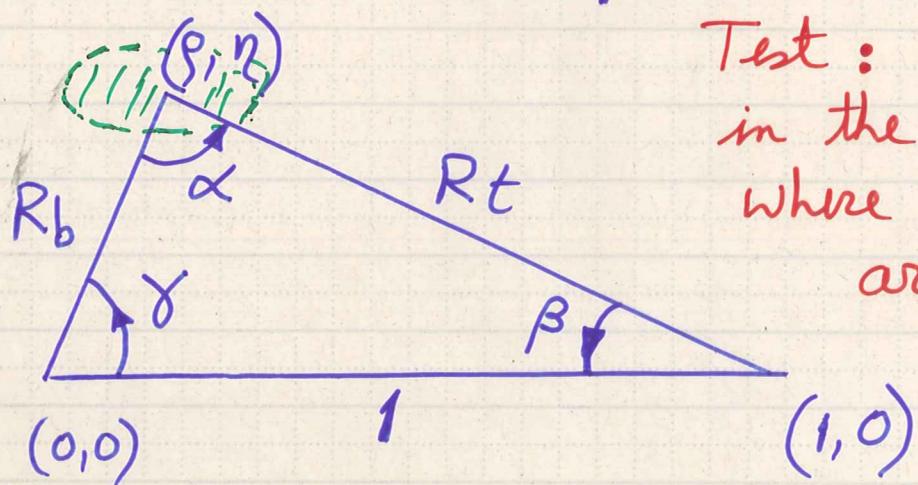


$$\alpha + \beta + \gamma = \arg(-1) = \pi \quad \text{by definition!!}$$

Rescaling :

$$\text{Define : } R_t = |V_{td} V_{tb}| / |V_{cd} V_{cb}|$$

$$R_b = |V_{ud} V_{ub}| / |V_{cd} V_{cb}|$$



Test : Find a region in the ρ, η plane where all constraints are satisfied

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An alternative way of testing
the SM and its CKM mechanism
for mixing and CP violation.

One can show that in a CKM matrix
of arbitrary size, there are only
4 independent rephasing invariant
phases, which can be chosen to be:

$$\gamma \equiv \arg(-V_{ud} V_{cb} V_{ub}^* V_{cd}^*)$$

$$\beta \equiv \arg(-V_{cd} V_{tb} V_{cb}^* V_{td}^*)$$

$$\chi = \arg(-V_{cb} V_{ts} V_{cs}^* V_{tb}^*)$$

$$9-5=4$$

$$\chi' = \arg(-V_{us} V_{cd} V_{ud}^* V_{cs}^*)$$

A convenient phase convention:

$$\arg V^{\text{CKM}} = \begin{bmatrix} 0 & \chi' & -\gamma & \cdot & \cdot & \cdot \\ \pi & 0 & 0 & \cdot & \cdot & \cdot \\ -\beta & \pi + \gamma & 0 & \cdot & \cdot & \cdot \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots \end{bmatrix}$$

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One has 13 rephasing invariant quantities in the 3×3 sector of a CKM matrix of arbitrary size:

9 moduli + 4 rephasing inv. phases.

The SM with 3 generations predicts exact relations among these quantities:

Examples:

(db)

$$\frac{|V_{ub}|}{|V_{td}|} = \frac{\sin \beta}{\sin \delta} \frac{|V_{tb}|}{|V_{ud}|}$$

(ct)

$$\sin \chi = \frac{|V_{td}|}{|V_{ts}|} \frac{|V_{cd}|}{|V_{cs}|} \sin \beta$$

(uc)

$$\sin \chi' = \frac{|V_{ub}| |V_{cb}|}{|V_{us}| |V_{cs}|} \sin \delta$$

(db)

$$|V_{ub}| = \frac{|V_{cd}| |V_{cb}|}{|V_{ud}|} \frac{\sin \beta}{\sin(\delta + \beta)}$$

Important point :

Prior to the measurement of δ

there was no solid evidence for the fact that V^{CKM} is complex, independently of the presence of New Physics.

One can calculate :

$$|\text{Im } Q| = F(4 \text{ independent moduli})$$

where Q is any invariant quartet of V^{CKM}

Can take as independent "moduli":

$$|V_{us}|, |V_{cb}|, |V_{ub}|, |V_{td}|$$

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix}$$

General formula for $\text{Im } Q$

$$|\text{Im } Q|^2 = |V_{\alpha i}|^2 |V_{\beta j}|^2 |V_{\alpha j}|^2 |V_{\beta i}|^2 - \\ - (\text{Re } Q_{\alpha i \beta j})^2$$

$$\text{Re } Q_{\alpha i \beta j} = \frac{1}{2} \left\{ 1 - |V_{\alpha i}|^2 - |V_{\beta j}|^2 - |V_{\alpha j}|^2 - |V_{\beta i}|^2 + |V_{\alpha i}|^2 |V_{\beta j}|^2 + |V_{\alpha j}|^2 |V_{\beta i}|^2 \right\}$$

$$Q_{\alpha i \beta j} = V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*$$

If one uses the present "data" on
 $|V_{us}|, |V_{cb}|, |V_{ub}|, |V_{td}|$

one obtains indeed

$$\text{Im } Q \neq 0$$

However, this conclusion is **not valid**
because **New Physics** may contribute
to $B_d - \bar{B}_d$ mixing, thus affecting
the extraction of $|V_{td}|$ from the
experimental value of $B_d - \bar{B}_d$ mixing

The measurement of

$$\gamma = \arg(-V_{ud} V_{ub}^* V_{cd}^* V_{cb})$$

provides clear evidence for a complex CKM matrix, even if one allows for New Physics contributions to

$B_d - \bar{B}_d$, $B_s - \bar{B}_s$ mixings

EPS prize to Kobayashi and Maskawa!!

- There is room for New Physics, say at 20% level in $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing

- Contributions to χ could be large !!

The Leptonic Sector

$$[\bar{e} \bar{\mu} \bar{\tau}] \gamma_\mu V^{\text{PMNS}} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} W^\mu + \text{h.c.}$$

$$V^{\text{PMNS}} = \begin{bmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ \cdot & \cdot & s_{23} c_{13} \\ \cdot & \cdot & c_{23} c_{13} \end{bmatrix} \begin{bmatrix} 1 \\ e^{i\alpha} \\ e^{i\beta} \end{bmatrix}$$

$\delta \rightarrow$ Dirac phase

$\alpha, \beta \rightarrow$ Majorana phases

The appearance of Majorana phases reflects the fact that the phases of Majorana fields cannot be rephased.

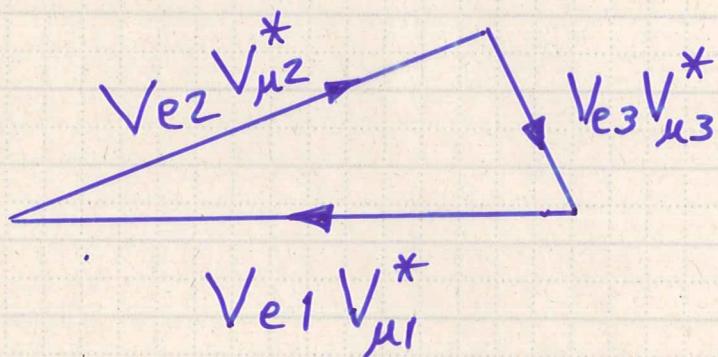
Dirac and Majorana Unitarity

Triangles

$$(\bar{e} \bar{\mu} \bar{\tau}) \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \gamma^\mu \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = w_\mu$$

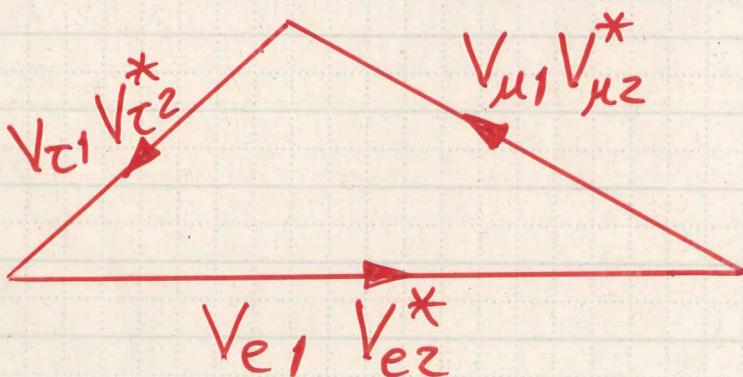
VPMNS

Dirac unitarity triangles



Orientation of triangle has no physical meaning

Majorana Unitarity triangle



Orientation of the triangle does have physical meaning

Current experimental knowledge on neutrino masses and mixing

$$\Delta m_{21}^2 = 8.0^{+4}_{-3} \times 10^{-5} \text{ eV}^2$$

$$\sin^2(2\theta_{12}) = .86^{+.03}_{-.04}$$

$$|\Delta m_{32}^2| = (1.9 \text{ to } 3.0) \times 10^{-3} \text{ eV}^2$$

$$\sin^2(2\theta_{23}) > .92$$

$$\sin^2 \theta_{13} < .05$$

Main features :

- 2 of the mixing angles are large :
 θ_{23} can be maximal (i.e. $\pi/4$)
 θ_{12} is large but not maximal
- No experimental knowledge on leptonic CP violation

"Impossibility" of determining the neutrino mass matrix through feasible experiments 15

In the Weak-Basis (WB) where ^{the} charged lepton mass matrix is diagonal, real, the neutrino mass matrix is characterized by 9 parameters :

neutrino masses	-----	3
mixing angles	-----	3
Dirac phase	-----	1
Majorana phases	-----	2
		<hr/>
		9

From "feasible experiments" :

mass differences	---	2
Mixing angles	---	3
Dirac CP viol. phase	---	1
Double beta decay	-----	1

Sugestion from Glashow, Frampton, Mafartia:
Use some "input from theory"

Eg. "Texture zeros" → GFM
postulate $\det m = 0$ → F.Gonzalez, R.Joaquim,
T.Yanagida, GCB

Contrast between the experimental situation in the quark and lepton sectors

quark sector → { Overdetermination of V_{CKM} but ...
Hadronic uncertainty

Lepton Sector

{ No Hadronic uncertainties
but
Not sufficient experimental data !!

17 A minimal extension of the SM
with naturally small neutrino masses

$$SM + \nu_{1R}, \nu_{2R}, \nu_{3R}$$

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= - \left[\bar{\nu}_L^\circ m_D \nu_R^\circ + \frac{1}{2} \nu_R^{\circ T} C M_R \nu_R^\circ + \right. \\ &\quad \left. + \bar{\ell}_L^\circ m_\ell \ell_R^\circ \right] + h.c. \\ &= - \left[\frac{1}{2} n_L^T C M^* n_L + \bar{\ell}_L^\circ m_\ell \ell_R^\circ \right] + h.c. \end{aligned}$$

$$n_L = (\nu_L^\circ, (\nu_R^\circ)^c) \quad ; \quad V^T M^* V = D$$

$$M = \begin{bmatrix} 0 & m_D \\ m_D^T & M_R \end{bmatrix} \rightarrow 6 \times 6 \text{ matrix}$$

$$V = \begin{bmatrix} K & R \\ S & T \end{bmatrix} \quad ; \quad D = \begin{bmatrix} d & 0 \\ 0 & D \end{bmatrix}$$

$$D = \text{diag}(m_1, m_2, m_3, M_1, M_2, M_3)$$

$$S, R \neq 0 \Rightarrow K = V^{\text{PMNS}}$$

does not satisfy 3×3 unitarity !!

No Problem : Violations of

3×3 unitarity are naturally suppressed

One can derive :

$$S^+ = -K^+ m_D M_R^{-1} ; R = m_D T^* \bar{D}'$$

$$-K^+ m \frac{1}{M} m^T K^* = d$$

↳ famous seesaw formula

From unitarity of 6×6 unitary matrix V , one obtains

$$KK^+ = (1\!/\!0)_3 - \underbrace{RR^+}_{}$$

of order m^2/M^2

$$K^+ K = (1\!/\!0)_3 - \underbrace{S^+ S}_{}$$

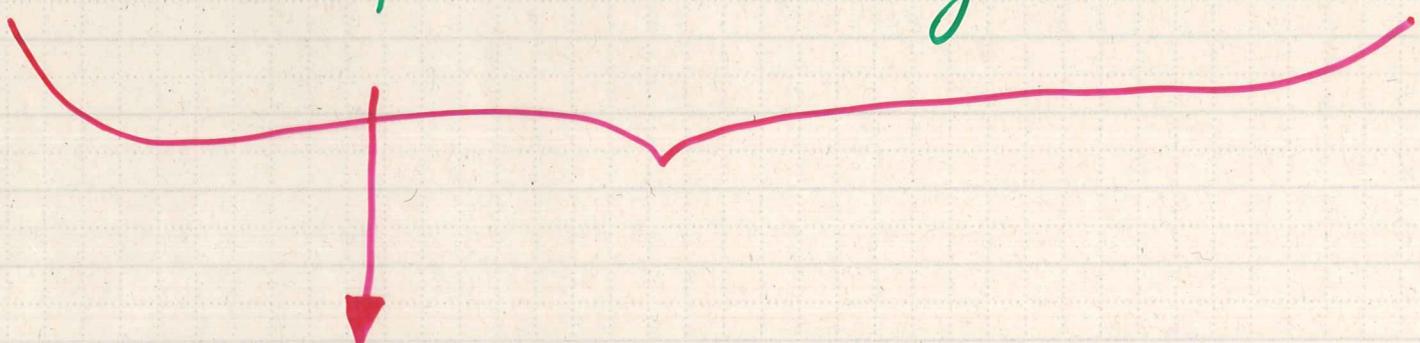
of order m^2/M^2

19)

This minimal extension of the SM with naturally small neutrino masses

provides an example of a :

self-consistent model with naturally small violations of 3×3 unitarity in the fermion mixing matrix



Obviously, entirely analogous models can be constructed in the quark sector, where there are naturally small violations of 3×3 unitarity of V_{CKM}

Mixing and CP violation in the leptonic sector of this minimal extension of the SM.

Without loss of generality one can go to the weak basis where:

$m_l \rightarrow$ diagonal, real

$M_R \rightarrow$ diagonal, real

In this basis, all mixing and CP violation is controlled by m_D

$m_D \rightarrow 3 \times 3$ complex matrix

3 of the phases can be eliminated by simultaneous rephasing of

γ_L° and λ_L° . So altogether,

one has 6 CP violating phases

m_D can be written as:

$$m_D = U H$$

where U is a unitary matrix
and H is a Hermitian matrix

$$m_D = P_f \hat{U}_\rho P_\alpha \hat{H}_\sigma P_\beta$$

$$P_f = \text{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})$$

$\hat{U}_\rho \rightarrow$ like CKM

$$P_\alpha = \text{diag}(1, e^{i\alpha_1}, e^{i\alpha_2})$$

\hat{H}_σ contains only one phase

$$\sigma = \arg \left[\begin{bmatrix} \hat{H}_{12} & \hat{H}_{23} & \hat{H}_{31} \end{bmatrix} \right]$$

The phases P_f can be eliminated

One is left with

$$\beta, \alpha_1, \alpha_2, \sigma, \beta_1, \beta_2$$



six phases

22

Is there any direct connection
between CP violation observable
through neutrino oscillations

(Dirac phase in $\sqrt{\nu_{\text{PMNS}}}$)

and CP violation needed for
leptogenesis?

Answer: In general no, but
a connection may arise with
some "theoretical input", like
e.g. texture zeros in m_D .

Reason: phases entering in
leptogenesis (flavour blind) are
those in $m_D^+ m_D^- \rightarrow \sigma, \beta_1, \beta_2$

All phases appear in $m_{\text{eff}}^+ m_{\text{eff}}^-$.

We need a theory of
Lepton Flavours !!

23 Can one use the data on fermion masses and mixing to [16] infer from it a Symmetry Principle?

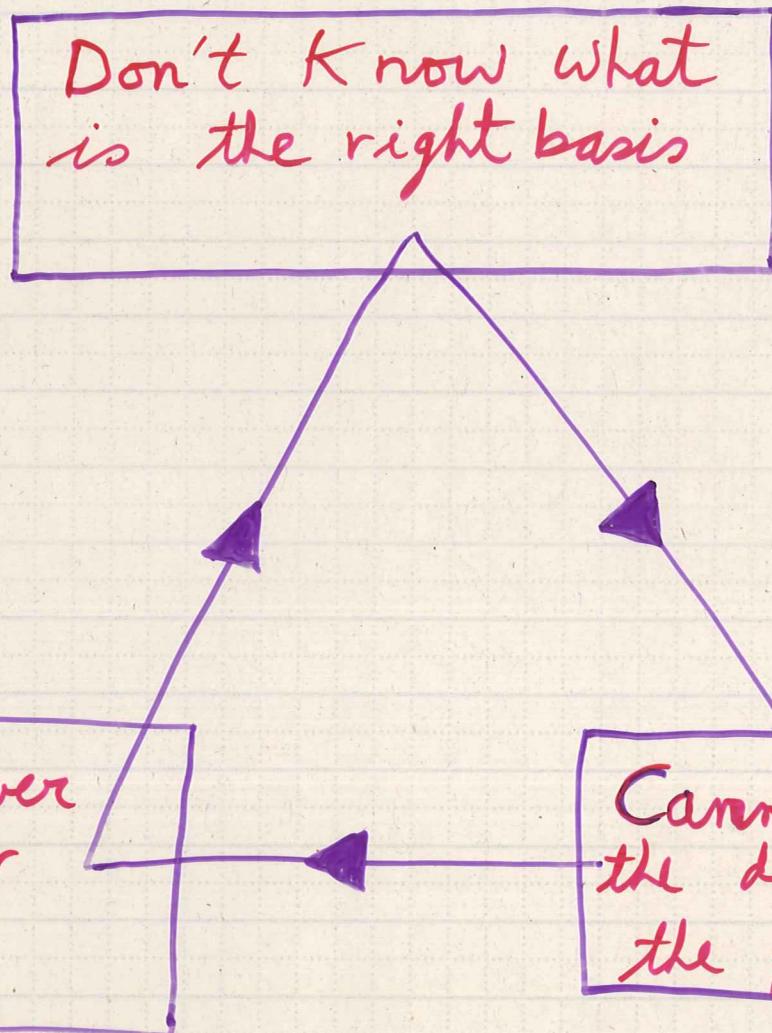
How to decipher the "Flavour Code"?

One of the Major difficulties is the "Ambiguity in the choice of Weak Basis"

Even if there is a "symmetry principle" behind flavour, in what weak-Basis will it be "transparent"?

Of course "once discovered" the flavour symmetry will dictate the "natural weak-basis" but we do not know the symmetry...

The Vicious Triangle



Weak-Basis transformations in the SM

Quark Sector

Consider a WB, i.e. a basis where all the gauge currents are flavour diagonal.

One can make a WB transformation:

$$u_L \rightarrow W_L u_L$$

$$d_L \rightarrow W_L d_L$$

$$u_R \rightarrow W_R^u u_R$$

$$d_R \rightarrow W_R^d d_R$$

under this basis transformations the Yukawa couplings transform as:

$$Y_d \rightarrow Y'_d = W_L^+ Y_d W_R^d$$

$$Y_u \rightarrow Y'_u = W_L^+ Y_u W_R^u$$

(Y_d, Y_u) , (Y'_d, Y'_u) contain the same physics!

Analogous WB transformations can be done in the leptonic sector.

Early Attempts at understanding [18] fermion masses and mixings

$$\theta_c = \sqrt{m_d/m_s} \quad \text{Gatto et al}$$

Weinberg (in "The problem of Mass")

$$M_u = \begin{bmatrix} 0 & a_u \\ a_u & c_u \end{bmatrix} ; \quad M_d = \begin{bmatrix} 0 & a_d \\ a_d & c_d \end{bmatrix}$$



$$|V_{us}| = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\phi} \sqrt{\frac{m_u}{m_c}} \right| \quad (1)$$

Fritzsch - generalization for 3 generations

$$M_u = \begin{bmatrix} 0 & a_u & 0 \\ a_u & 0 & b_u \\ 0 & b_u & c_u \end{bmatrix} ; \quad M_d = \begin{bmatrix} 0 & a_d & 0 \\ a_d & 0 & b_d \\ 0 & b_d & c_d \end{bmatrix}$$

One obtains (1), but also :

$$|V_{cb}| = \sqrt{\frac{m_s}{m_b}} - e^{i\sigma} \sqrt{\frac{m_c}{m_t}}$$

Excluded by the observed size of $V_{cb} \approx \frac{m_s}{m_b}$
and the large value of the top quark
mass. \rightarrow it was determined by experiment

27 Important point : 119

In order for the Fritzsch ansatz to have predictive power one has to have simultaneously hermitian M_u, M_d and the texture zeros.

Starting from arbitrary Y_u, Y_d (or M_u, M_d) in the SM, can one create "zeros" by making WB transformations? Yes !!

It has been shown that starting from arbitrary M_u, M_d , one can make WB transformations so that the Fritzsch zeros are obtained.

L.Lavoura, F.Mota, G.C.B.

$$M_u = \begin{bmatrix} 0 & a_u & 0 \\ a'u & 0 & b_u \\ 0 & b'_u & c_u \end{bmatrix}; M_d = \begin{bmatrix} 0 & ad & 0 \\ a'd & 0 & bd \\ 0 & b'd & cd \end{bmatrix}$$

Note that in the context of the SM, it is very difficult to have a symmetry which constrains M_u, M_d to be hermitian

Can one create a $(1,1)$ zero in both M_u, M_d in a WB where M_u, M_d are hermitian, by making WB transformations? Yes!!

- Start from arbitrary M_u, M_d
- Make WB transformations which lead to M'_u, M'_d hermitian
- Create a $(1,1)$ zero in both M_u, M_d by making WB transformations

$$M''_u = W_L^+ M'_u W_L$$

$$M''_d = W_L^+ M'_d W_L$$

One can choose W_L , so that

$$(M''_u)_{11} = (M''_d)_{11} = 0$$

Why the $(1, 1)$ zeros are "special" [21]

Consider diagonalization of M_d :

$$U^+ M_d U = d = \text{diag}(m_d, m_s, m_b)$$

$$M_d = U d U^+$$

$$(M_d)_{11} = 0 \Rightarrow m_d |U_{11}|^2 + m_s |U_{12}|^2 + m_b |U_{13}|^2 = 0$$

If U_{13} is "small", for example

$$U_{13} \approx O(m_d/m_b)$$

Then, one obtains:

$$\frac{|U_{12}|^2}{|U_{11}|^2} = \left| \frac{m_d}{m_s} \right| \quad \begin{pmatrix} 0 & a \\ a & b \end{pmatrix}$$

Similarly for the up sector



$$|V_{us}| = \sqrt{\frac{m_d}{m_s}} - e^{i\varphi} \sqrt{\frac{m_u}{m_c}}$$

P. Ramond, R.G. Roberts, G.G. Ross have developed a systematic analysis of quark mass matrices which, starting with the measured values of quark masses and mixing angles, allows for a model-independent search for all possible (symmetric or hermitian) mass matrices having texture zeros at unification scale.

Found 5 possible solutions

All of them have a zero in the (1, 1) position !!

3)

The 5 solutions found
by Ramond, Roberts and Ross

Solution

 Y_u Y_d

1

$$\begin{bmatrix} O & C & O \\ C & B & O \\ O & O & A \end{bmatrix}$$

$$\begin{bmatrix} O & F & O \\ F^* & E & E' \\ O & E' & D \end{bmatrix}$$

2

$$\begin{bmatrix} O & C & O \\ C & O & B \\ O & B & A \end{bmatrix}$$

$$\begin{bmatrix} O & F & O \\ F^* & E & E' \\ O & E'^* & D \end{bmatrix}$$

3

$$\begin{bmatrix} O & O & C \\ O & B & O \\ C & O & A \end{bmatrix}$$

$$\begin{bmatrix} O & F & O \\ F^* & E & E' \\ O & E' & D \end{bmatrix}$$

4

$$\begin{bmatrix} O & C & O \\ C & B & B' \\ O & B' & A \end{bmatrix}$$

$$\begin{bmatrix} O & F & O \\ F^* & E & O \\ O & O & D \end{bmatrix}$$

5

$$\begin{bmatrix} O & O & C \\ O & B & B' \\ C & B' & A \end{bmatrix}$$

$$\begin{bmatrix} O & F & O \\ F^* & E & O \\ O & O & D \end{bmatrix}$$

For an extensive list of references:

- Guido Altarelli, Ferruccio Feruglio,
New J. Phys 6 : 106, 2004, hep-ph/0405048
Leptonic Sector
- P. Ramond, R.G. Roberts, G.G. Ross
Nuclear Phys. B 406 (1993)

A

33/ So far, we do not have a

Standard Theory of Flavour (STF)

$$SU(3)_C \times SU(2)_L \times U(1) \times G_{\text{Family}}$$

Gauge Invariance does not constrain the family structure of Yukawa couplings

Y_u, Y_d, Y_e are arbitrary complex matrices

Suppose we had a STF. How could one check that it is the right one?

How can one test it, experimentally?

B₃₄) Obvious requirements for an STF:

- It should fit the experimental value of fermion masses and mixings with

$$N_{\text{parameters}} \ll 22 !$$

Is this sufficient to test an STF?

A possible requirement:

The STF should make predictions for New Physics.

These predictions should be such that the STF can be ruled out!

Avoid theories that

Are Not Even Wrong! W. Pauli

Two Old Dogmas in Flavour Physics

- No Z -mediated FCNC at tree level

Natural Flavour conservation (NFC)

Generalization of GIM

- No Higgs mediated FCNC

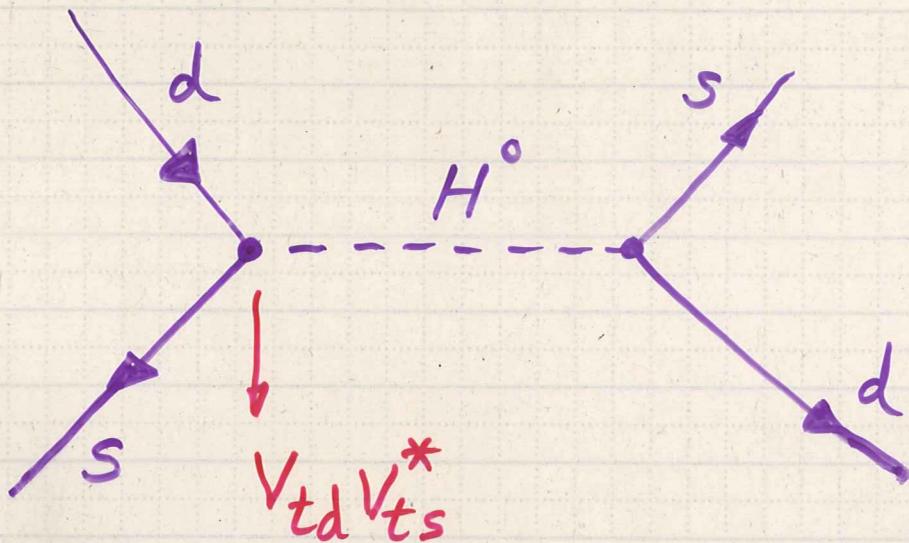
NFC in the Higgs sector

S.L.Glashow, S.Weinberg

Can one have a "reasonable model" where these two dogmas are violated?

Yes !!

- Z -mediated FCNC
 - They are present in the minimal extension of the SM, where neutrinos acquire mass through the seesaw mechanism.
 - Models with iso singlet heavy quarks
 - It is possible to have multi-Higgs theories where there are naturally suppressed but non-vanishing FCNC



L. Lavoura, W. Grimus, G.C.B

L. Hall, S. Weinberg
A.S. Joshipura, S. Rindani

- It is likely that a STF will violate the dogma of NFC in the Higgs sector

Gatto et al no-go theorem
on "Naturality groups"

^a
38) Taking into account the impressive agreement of the SM with experimental data, is there any room for New Physics?

(For a nice review and complete list of references, see Robert Fleischer, arXiv: 0802.2882 - Feb. 2008)

- The Standard Model with its built-in CKM mechanism for mixing and CP violation gives the dominant contribution to the Physics of the unitarity triangle.
- One may have small NP contributions to some of the observables entering in the unitarity triangle.

b) _{39/} Assume 3×3 unitarity, but allow for NP contributions to $B_d - \bar{B}_d$, $B_s - \bar{B}_s$ mixings:

$$M_{12}^{(q)} = (M_{12}^{(q)})^{\text{SM}} r_q^2 e^{-2i\phi_q}$$

$q = d, s$

$$\tan \phi_d = \frac{R_u \sin(\delta + \bar{\beta}) - \sin \bar{\beta}}{\cos(\bar{\beta}) - R_u \cos(\delta + \bar{\beta})}$$

$$R_u = \frac{|V_{ud}| |V_{ub}|}{|V_{cd}| |V_{cb}|}$$

Similar expression for ϕ_s :

$$\tan \phi_s = \frac{\sin \bar{\chi} - C \sin(\delta - \bar{\chi})}{C \cos(\delta - \bar{\chi}) + \cos \bar{\chi}}$$

$$C = \frac{|V_{us}| |V_{ub}|}{|V_{cs}| |V_{cb}|}; \quad \begin{array}{l} \text{To detect, small} \\ \phi_d \text{ needs great} \\ \text{accuracy in the} \\ \text{measurement of } \frac{|V_{ub}|}{|V_{cb}|}, \bar{\beta}, \delta \end{array}$$

40/ Fortunately there is Physics beyond the unitarity triangle.

For example the SM predicts :

$$\sin \chi = \frac{|V_{ub}| |V_{us}|}{|V_{cb}| |V_{cs}|} \sin(\gamma - \chi + \chi') \quad \text{Exact!!}$$

$$\sin \chi = \frac{|V_{cd}|}{|V_{cs}|} \frac{|V_{td}|}{|V_{ts}|} \sin \beta$$

\Rightarrow In the SM one has :

$$\chi = O(\lambda^2)$$

Can one have "reasonable models" where

$$\chi = O(\lambda) ?$$

Yes!

In most of tests of the Flavour sector of the SM

3×3 unitarity of V_{CKM} plays a crucial rôle.

Can one have a self-consistent extension of the SM, where there are naturally small violations of 3×3 unitarity? Yes!

Consider an extension of the SM, with the addition of a $Q = 2/3$ isosinglet quark.

T_L

T_R

} \rightarrow both isosinglets under $SU(2)_L$

2/42 In the mass eigenstate basis, the charged and neutral currents can be written:

$$\mathcal{L}_W = -g/\sqrt{2} \bar{u}_L \gamma^\mu V d_L W_\mu^+ + h.c.$$

$$\mathcal{L}_Z = -\frac{g}{2 \cos \theta_W} \left[\bar{u}_L \gamma^\mu (V V^\dagger) u_L - \bar{d}_L \gamma^\mu d_L - 2 \sin^2 \theta_W J_{em}^\mu \right] Z_\mu$$

Let U be the 4×4 unitary matrix U which enters in the diagonalization of the up-type mass matrix (in the weak-basis where down quark mass matrix is diagonal, real)

$$U = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & U_{14} \\ V_{cd} & V_{cs} & V_{cb} & U_{24} \\ V_{td} & V_{ts} & V_{tb} & U_{34} \\ V_{Td} & V_{Ts} & V_{Tb} & U_{44} \end{pmatrix} \quad V V^\dagger \neq 1$$

$V \rightarrow 4 \times 3$ matrix entering in charged currents

43

There are naturally small violations of 3×3 unitarity leading to small FCNC couplings in the up quark sector.

$$(\bar{u} \bar{c} \bar{s} \bar{T})_L \begin{bmatrix} 1 - |U_{14}|^2 & -U_{14} U_{24}^* & -U_{14} U_{34}^* & -U_{14} U_{44}^* \\ -U_{14} U_{24} & 1 - |U_{24}|^2 & -U_{24} U_{34}^* & -U_{24} U_{44}^* \\ -U_{14} U_{34} & -U_{24} U_{34} & 1 - |U_{34}|^2 & -U_{34} U_{44}^* \\ -U_{14} U_{44} & -U_{24} U_{44} & -U_{34} U_{44} & 1 - |U_{44}|^2 \end{bmatrix} \cdot \gamma_\mu \begin{bmatrix} u \\ c \\ t \\ T \end{bmatrix}_L Z^\mu$$

In this extension of the SM, one may have

$$\sin \chi = O(1)$$

From orthogonality of the second and third columns of V , one obtains

$$\sin \chi = \frac{|V_{ub}| |V_{us}|}{|V_{cb}| |V_{cs}|} \sin(\delta - \chi + \chi') + \boxed{\frac{|V_{tb}| |V_{ts}|}{|V_{cb}| |V_{cs}|} \sin(\sigma - \chi)}$$

NP

In order to obtain χ of order λ one needs :

$$V_{tb} = O(1)$$

$$V_{ts} = O(\lambda^2)$$

$$\sigma \equiv \arg(V_{ts} V_{cb} V_{tb}^* V_{cs}^*) = O(1)$$

From orthogonality of second and third rows :

$$\sin \chi = \frac{|V_{cd}| |V_{td}|}{|V_{cs}| |V_{ts}|} \sin \beta + \frac{|U_{24}| |U_{34}|}{|V_{cs}| |V_{ts}|} \sin \delta$$

$$\delta = \arg(V_{tb}^* U_{24}^* V_{cb} U_{34})$$

χ of order λ requires

$$|U_{24}| |U_{34}| = O(\lambda^3)$$

$$\sin \delta = O(1)$$

Clear cut prediction :

FCNC couplings of the type

$$\bar{c}_L \gamma^\mu t_L Z_u$$

are proportional to :

$$|U_{24} U_{34}|$$

On the other hand, in order to have

$$\chi = O(\lambda)$$

one requires in the context of the model

$$|U_{24} U_{34}| \simeq \lambda^3$$

\Rightarrow this leads to rare top decays

$t \rightarrow c Z$ at rates such that
they can be observed at LHC !!

J. A. Aguilar-Saavedra, F. J. Botella, M. Nebot,
Nuc. Phys. B 706, 204 (2005) GCB

F. J. Botella, M. Nebot, GCB to appear
next week

Some interesting "flavoured" questions [24]

- Recent data has shown that V_{CKM} is complex. Does that mean that CP violation necessarily originates from complex Yukawa couplings, thus excluding the possibility of having spontaneous CP violation?

No!

- In the SM, V_{CKM} is a 3×3 unitary matrix. Can one have a "reasonable, self-consistent model where there are naturally small violations of 3×3 unitarity?"

Yes!

- Can physics at a high energy scale affect the effective fermion mass matrices at low energies?

Yes!

- There are various aspects of CP violation.
 - CP violation in the quark sector
 - CP violation in the lepton sector
 - CP violation needed for Leptogenesis
 - Strong CP problem

Axions have not been discovered!!

Can one construct a model where all these "manifestations" of CP violation have the same origin, while at the same time providing a possible solution to the strong CP problem?

Yes!

Consider an extension of the SM
with the addition of the following
fields :

$D \rightarrow Q = -\frac{1}{3}$ isosinglet quark

$S \rightarrow$ complex singlet scalar

$\nu_{R_i} \rightarrow$ 3 right-handed neutrinos

Introduce a Z_4 symmetry under
which :

$$D_{L,R} \rightarrow -D_{L,R}; \quad S \rightarrow -S$$

$$\left(\begin{matrix} \nu \\ l \end{matrix}\right)_L \rightarrow i \left(\begin{matrix} \nu \\ l \end{matrix}\right)_L; \quad l_R \rightarrow i l_R$$

$$\nu_R \rightarrow i \nu_R$$

49)

Impose CP invariance at Lagrangian level \rightarrow all couplings real

$SU(3) \times SU(2) \times U(1) \times Z_4$ Yukawa couplings :

$$(\bar{u} d)_L^i [Y_{ij}^d \phi^d d_R^j + Y_{ij}^u \tilde{\phi}^u u_R^j] -$$

$$\mu \bar{D}_L D_R - [f_i S + f'_i S^*] \bar{D}_L^i d_R^i + h.c.$$

Down quark mass matrix :

$$M_d = \begin{bmatrix} m_d & 0 \\ M_D & \mu \end{bmatrix} \xrightarrow{\text{forbidden by } Z_4 \text{ symmetry}}$$

$$(m_d)_{ij}^c = Y_{ij}^d v ; \quad M_D^i = [f_i V e^{i\alpha} + f'_i V \bar{e}^{-i\alpha}]$$

where $\langle S \rangle = V e^{i\alpha}$

50)

The 4×4 down quark mass matrix is diagonalized by :

$$U_L^+ M_d U_R = \begin{bmatrix} \bar{m} & 0 \\ 0 & \bar{M} \end{bmatrix}$$

$\bar{m} = \text{diag}(m_d, m_s, m_b)$; $\bar{M} \rightarrow$ physical heavy quark mass

$$U_L = \begin{bmatrix} K & R \\ S & T \end{bmatrix}; \quad K \rightarrow 3 \times 3 \text{ mixing matrix connecting standard quarks}$$

U_L is the matrix which diagonalizes $M_d M_d^\dagger$. One obtains

$$K \bar{m}^2 K^{-1} \approx m_d m_d^\dagger - \frac{m_d M_D M_D^\dagger m_d^\dagger}{M^2}$$

$$\text{where } M^2 = M_D M_D^\dagger + \mu^2 \approx \bar{M}^2$$

$$\begin{aligned} S &= -M_D m_d^\dagger K [M^2 - \bar{m}^2]^{-1} \\ &\approx -\frac{M_D m_d^\dagger K}{M^2} \left[1 + \frac{\bar{m}^2}{M^2} \right] \end{aligned}$$

$$U_L^+ U_L = \begin{bmatrix} K^+ & S^+ \\ R^+ & T^+ \end{bmatrix} \begin{bmatrix} K & R \\ S & T \end{bmatrix} = (\mathbb{I})_4$$

$$K^+ K = (\mathbb{I})_3 - S^+ S$$



Naturally suppressed deviations of
3x3 unitarity.

$$\left[m_{eff} \quad m_{eff}^+ \right] = m_d \quad m_d^+ - \frac{m_d \quad M_D \quad M_D \quad m_d}{M^2}$$

real due to
reality of Yukawa
couplings

complex, generic,
due to the
phase of $\langle S \rangle$

Important point :

The two contributions to $m_{eff} \quad m_{eff}^+$
are of the same order !!

Conclusions

- The Flavour Problem or the Mass Problem remains one of the most fascinating open problems in Particle Physics
- "Anticipating a New Golden Age"
Frank Wilczek
Abstract

The Standard Model of Fundamental Interactions is remarkably successful but it leaves an unfinished agenda. Several major questions seem ripe for exploration in the near future. I anticipate that the coming decade will be a Golden Age of discovery in Fundamental Physics.

- I hope Frank Wilczek is right!!