CERN, 2 June '08

Neutrino Mixing in a Grand Unified Model

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Outline

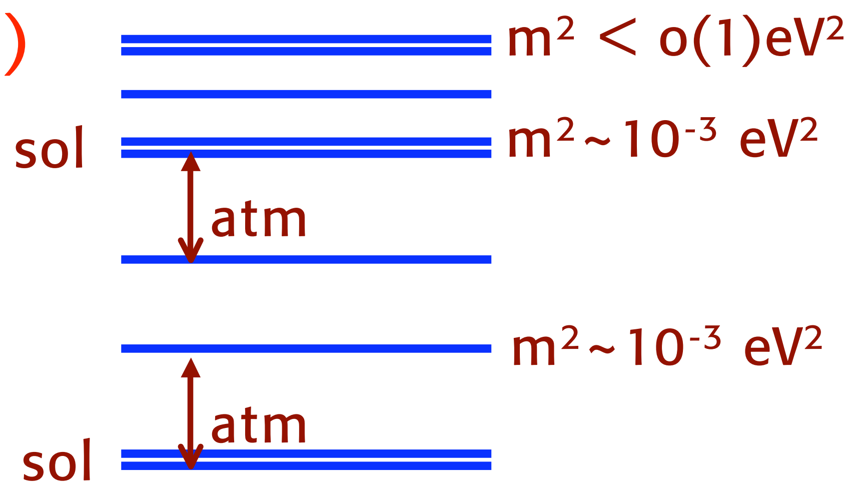
- Update on the data
- Tri-bimaximal mixing
- A4 as a flavour group for TB mixing
- Problems with quarks
- Problems with GUT's
- A solution: a GUT model with A4

The current experimental situation on ν masses and mixings has much improved but is still incomplete

- what is the absolute scale of ν masses?
- value of θ_{13}......
- no detection of $0\nu\beta\beta$ (proof that ν's are Majorana)
- pattern of spectrum

3 light ν's are OK (MiniBoone)

- Degenerate ($m^2 \gg \Delta m^2$)
- Inverse hierarchy
- Normal hierarchy

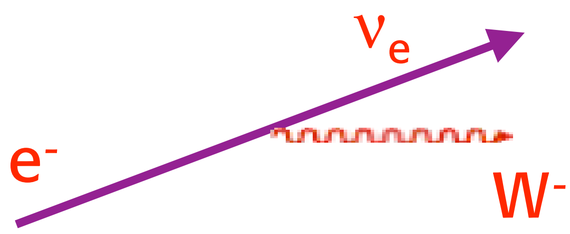


→ Different classes of models are still possible

3-ν Models

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U^+ \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

flavour — mass



$U = U_{P\text{-}MNS}$

Pontecorvo
Maki, Nakagawa, Sakata

In basis where e^-, μ^-, τ^- are diagonal:

δ: CP violation

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \sim$$

s = solar: large

CHOOZ: $|s_{13}| <\sim 0.2$

$$\sim \begin{pmatrix} c_{13}\,c_{12} & c_{13}\,s_{12} & s_{13}e^{-i\delta} \\ \dots & \dots & c_{13}\,s_{23} \\ \dots & \dots & c_{13}\,c_{23} \end{pmatrix}$$

atm.: ~ max

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

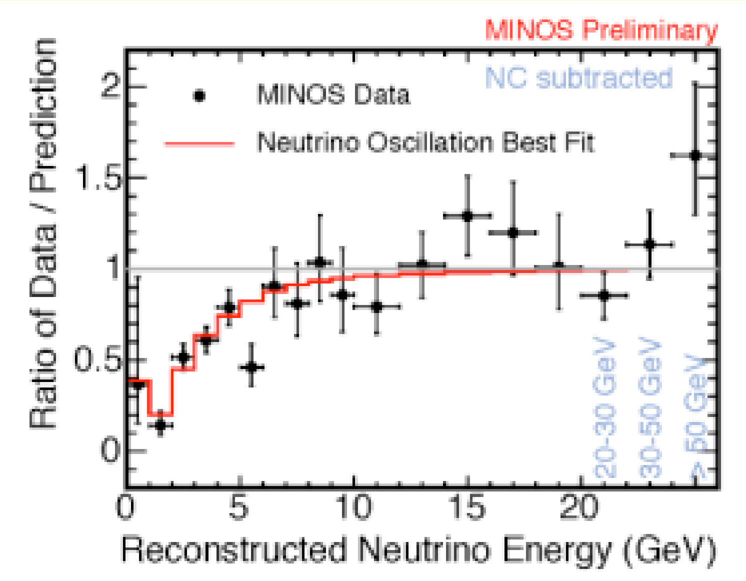
(some signs are conventional)

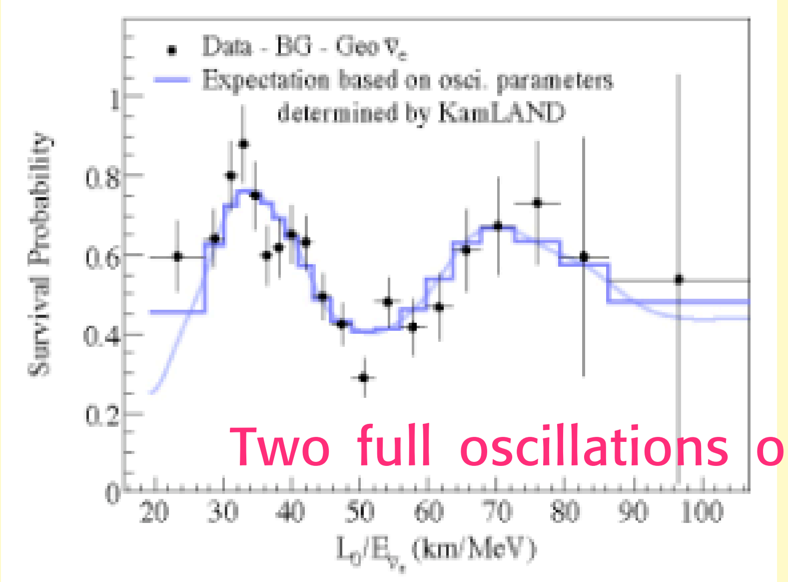
In general: $U = U^+_e U_\nu$

Latest from experiment

G.L. Fogli

MINOS 2007 (preliminary) and KamLAND 2008 data provide a better determination of the two independent neutrino oscillation frequencies:





Two full oscillations observed!

oscillations driven by $\Delta m^2 \sim 2.4 \times 10^{-3}$ eV^2

oscillations driven by $\delta m^2 \sim 7.6 \times 10^{-5}$ eV^2

(Recent solar neutrino results from Borexino 2007 and SK-phase II 2008 do not affect yet the global analysis of neutrino mass/mixing parameters)

G.L. Fogli et al

2008 parameter summary at 2σ level (95 % CL)

atm.	$\delta m^2/\mathrm{eV}^2$	$=$	$2.38 \pm 0.27\ 10^{-3}$
solar	$\lvert\Delta m^2\rvert/\mathrm{eV}^2$	$=$	$7.66 \pm 0.35\ 10^{-5}$
	$\sin^2\theta_{12}$	$=$	$0.326\,^{+0.05}_{-0.04}$
	$\sin^2\theta_{23}$	$=$	$0.45\,^{+0.16}_{-0.09}$
	$\sin^2\theta_{13}$	$<$	3.2×10^{-2}

(Addendum to hep-ph/0608060, in preparation)

Some recent work by our group

G.A., F. Feruglio, I. Masina, hep-ph/0402155,
G.A., F. Feruglio, hep-ph/0504165,hep-ph/0512103,
G.A, R. Franceschini, hep-ph/051220,
G.A., F. Feruglio, Y. Lin, hep-ph/0610165;
F. Feruglio, C. Hagedorn, Y. Lin, L. Merlo, hep-ph/0702194

In particular

G.A., F. Feruglio, C. Hagedorn, 0802.0090[hep-ph]

Reviews:

G.A., F. Feruglio, New J.Phys.6:106,2004 [hep-ph/0405048],
G.A., hep-ph/0410101, F. Feruglio, hep-ph/0410131,
G.A, hep-ph/061111, hep-ph/0705.0860.

General remarks

- After KamLAND, SNO.... not too much hierarchy is needed for ν masses:

$$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/30$$

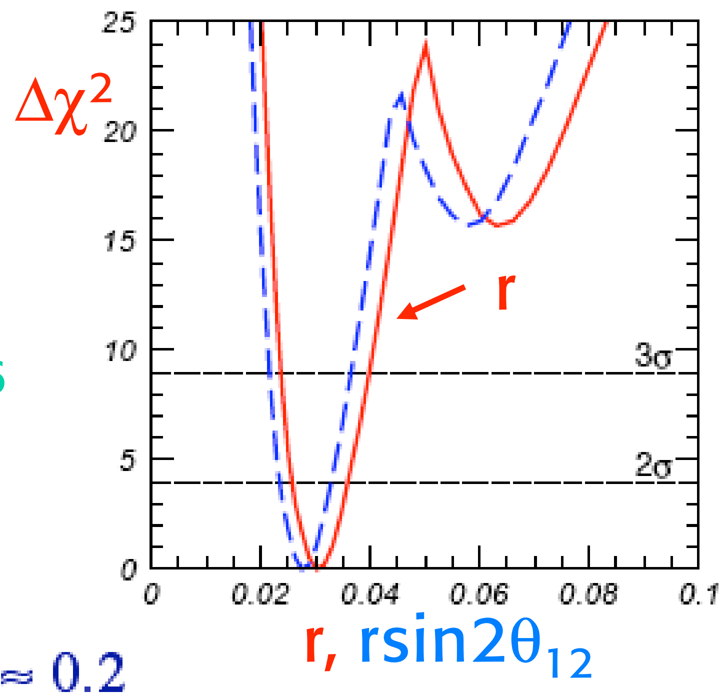
Only a few years ago could be as small as 10^{-8}!

Precisely at 3σ: $0.024 < r < 0.040$

Maltoni et al '06

or

$m_{heaviest} < 0.2$ - 0.7 eV
$m_{next} > \sim 8\ 10^{-3}$ eV



- For a hierarchical spectrum: $\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$

Comparable to $\lambda_C = \sin\theta_C$: $\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

Suggests the same "hierarchy" parameters for q, l, ν (small powers of λ_C) ⟶ e.g. θ_{13} not too small!

- Still large space for non maximal 23 mixing

  2-σ interval $0.36 < \sin^2\theta_{23} < 0.61$

  Maximal θ_{23} theoretically hard

- θ_{13} not necessarily too small
  probably accessible to exp.

  Very small θ_{13} theoretically hard

In the model we will discuss here $\theta_{23} - \pi/4$ and θ_{13} typically are expected of $o(\lambda_C^2)$.

For a long time people considered limiting models with $\theta_{13}= 0$ and θ_{23} maximal

The most general mass matrix for $\theta_{13}= 0$ and θ_{23} maximal is given by
(after ch. lepton diagonalization!!!):

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

Neglecting Majorana phases it depends on 4 real parameters (3 mass eigenvalues and 1 mixing angle: θ_{12})

Inspired models based on $\mu-\tau$ symmetry

Grimus, Lavoura..., Ma,.... Mohapatra, Nasri, Hai-Bo Yu ....

Actually, at present, since KamLAND, the most accurately known angle is θ_{12}

G.L.Fogli et al'08

At ~2σ:

$$\sin^2\theta_{12} = 0.326\,^{+0.05}_{-0.04}$$

By adding $\sin^2\theta_{12} \sim 1/3$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Harrison, Perkins, Scott '02

⊕ Some additional ingredient other than μ–τ symmetry needed!

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Comparison with experiment:

At 1σ:

G.L.Fogli et al'08

$\sin^2\theta_{12} = 1/3$: 0.31-0.35

$\sin^2\theta_{23} = 1/2$: 0.40-0.53

$\sin^2\theta_{13} = 0$: < 0.02

The HPS mixing is clearly a very good approx. to the data!

Also called:
Tri-Bimaximal mixing

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

By adding $\sin^2\theta_{12} \sim 1/3$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

Tribimaximal Mixing

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

$$\sin^2 2\theta_{12} = \frac{8y^2}{(x-w-z)^2 + 8y^2}$$

$m_1 = x - y$
$m_2 = x + 2y$
$m_3 = x - y + 2v$

The 3 remaining parameters are the mass eigenvalues

Tribimaximal Mixing

A simple mixing matrix compatible with all present data

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

In the basis of diagonal ch. leptons:

$$m_\nu = U \mathrm{diag}(m_1, m_2, m_3) U^T$$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{m_2}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{m_1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Eigenvectors: $m_3 \to \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\quad m_2 \to \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad m_1 \to \frac{1}{\sqrt{6}}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Note: mixing angles independent of mass eigenvalues

- For the HPS mixing matrix all mixing angles are fixed to particularly symmetric values

Sparked interest in constructing models that can naturally produce this highly ordered structure

Models based on the A4 discrete symmetry (even permutations of 1234) offer a minimal solution

Ma...;
GA, Feruglio hep-ph/0504165, hep-ph/0512103
GA, Feruglio, Lin hep-ph/0610165.......
Y. Lin, 0804.2867 [hep-ph]

Larger finite groups: T′, $\Delta(27)$....

Feruglio et al
Chen, Mahanthappa
Frampton, Kephart ....

Alternative models based on $SU(3)_F$ or $SO(3)_F$ or their finite subgroups

Verzielas, G. Ross
....

King ....

Lindner-Manchester '07

List of models with flavor symmetries

(incomplete, by symmetry):

S_3: Pakvasa et al. (1978) Derman (1979), Ma (2000), Kubo et al. (2003), Chen et al. (2004), Grimus et al. (2005), Dermisek et al. (2005), Mohapatra et al. (2006), ...

S4: Pakvasa et al. (1979), Derman et al. (1979), Lee et al. (1994), Mohapatra et al. (2004), Ma (2006), Hagedorn, ML and Mohapatra (2006), Caravaglios et al. (2006), ...

A_4: Wyler (1979), Ma et al. (2001), Babu et al. (2003), Altarelli et al. (2005,2006), He et al. (2006) ...

D_4: Seidl (2003), Grimus et al. (2003,2004), Kobayashi et al. (2005), ...

D_5: Ma (2004), Hagedorn et al. (2006).

D_n: Chen et al. (2005), Kajiyama et al. (2007), Frampton et al. (1995,1996,2000), Frigerio et al. (2005), Babu et al. (2005), Kubo (2005), ...

T': Frampton et al. (1994,2007), Aranda et al. (1999,2000), Feruglio et al. (2007), Chen and Mahanthappa (2007)

Δ_n: Kaplan et al. (1994), Chou et al. (1997), de Medeiros Varzielas et al. (2005), ...

T_7: Luhn et al.

A4

A4 is the discrete group of even perm's of 4 objects.
(the inv. group of a tetrahedron). It has $4!/2 = 12$ elements.

An element is abcd which means 1234 --> abcd

C_1: $1 = 1234$
C_2: $T = 2314$ $ST = 4132$ $TS = 3241$ $STS = 1423$
C_3: $T^2 = 3124$ $ST^2 = 4213$ $T^2S = 2431$ $TST = 1342$
C_4: $S = 4321$ $T^2ST = 3412$ $TST^2 = 2143$

Thus A4 transf.s can be written as:

$1, T, S, ST, TS, T^2, TST, STS, ST^2, T^2S, T^2ST, TST^2$

with: $S^2 = T^3 = (ST)^3 = 1$ [$(TS)^3 = 1$ also follows]

x, x' in same class if

C_1, C_2, C_3, C_4 are equivalence classes [$x' \sim gxg^{-1}$] g: group element

A4 has 4 inequivalent irreducible representations:
a triplet and 3 different singlets

$3, 1, 1', 1''$

(promising for 3 generations!)

Note:

as many representations as equivalence classes

$\sum d_i^2 = 12 \qquad 9+1+1+1=12$

Note: many models tried S3
S3 has no triplets but only 2 , 1, 1′
A4 is better in the lepton sector

Mohapatra, Nasri, Yu
Koide
Kubo et al
Kaneko et al
Caravaglios et al
Morisi
Picariello......

Three singlet inequivalent represent'ns:

$$\omega = \exp i\frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$
$$\omega^3 = 1$$
$$1 + \omega + \omega^2 = 0$$
$$\omega^2 = \omega^*$$

Recall:

$S^2 = T^3 = (ST)^3 = 1$

$$\begin{cases} 1: & S=1,\ T=1 \\ 1': & S=1,\ T=\omega \\ 1'': & S=1,\ T=\omega^2 \end{cases}$$

The only indep. 3-dim represent'n is obtained by:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(S-diag basis)

An equivalent form:

$$VV^\dagger = V^\dagger V = 1$$

$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

(T-diag basis)

Cabibbo '78

A4 has only 4 irreducible inequivalent represt'ns: 1,1',1",3

Table of Multiplication:
1'x1'=1"; 1"x1"=1';1'x1"=1
3x3=1+1'+1"+3+3

A4 is well fit for 3 families!

Ch. leptons l ~ 3

e^c, μ^c, τ^c ~ 1, 1", 1'

In the S-diag basis consider 3: (a_1,a_2,a_3) —S→ $(a_1,-a_2,-a_3)$; —T→ (a_2,a_3,a_1)

For $3_1=(a_1,a_2,a_3)$, $3_2=(b_1,b_2,b_3)$ we have in $3_1 x 3_2$:

$$1 = a_1b_1 + a_2b_2 + a_3b_3$$

$$1' = a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3$$

$$1" = a_1b_1 + \omega a_2b_2 + \omega^2 a_3b_3$$

$$3 \sim (a_2b_3, a_3b_1, a_1b_2)$$

$$3 \sim (a_3b_2, a_1b_3, a_2b_1)$$

e.g. $1" = a_1b_1+\omega a_2b_2+\omega^2 a_3b_3 \xrightarrow{T} a_2b_2+\omega a_3b_3+\omega^2 a_1b_1 =$
$= \omega^2 [a_1b_1+\omega a_2b_2+\omega^2 a_3b_3]$

while, under S, 1" is inv.

In the T-diagonal basis we have:

$$VV^\dagger = V^\dagger V = 1$$

$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

Cabibbo '78

For $3_1=(a_1,a_2,a_3)$, $3_2=(b_1,b_2,b_3)$ we have in $3_1 x 3_2$:

$$1 = a_1b_1 + a_2b_3 + a_3b_2$$

$$1' = a_3b_3 + a_1b_2 + a_2b_1$$

$$1'' = a_2b_2 + a_1b_3 + a_3b_3$$

We will see that in this basis the charged leptons are diagonal

$$3_{symm} \sim \frac{1}{3}(2a_1b_1 - a_2b_3 - a_3b_2, 2a_3b_3 - a_1b_2 - a_2b_1, 2a_2b_2 - a_1b_3 - a_3b_1)$$

$$3_{antisymm} \sim \frac{1}{2}(a_2b_3 - a_3b_2, a_1b_2 - a_2b_1, a_1b_3 - a_3b_1)$$

Under A4 the most common classification is:

lepton doublets $l \sim 3$

$e^c, \mu^c, \tau^c \sim 1, 1'', 1'$ respectively

A4 breaking gauge singlet flavons $\phi_S, \phi_T, \xi, (\xi') \sim 3, 3, 1, (1)$
For SUSY version: driving fields $\phi'_S, \phi'_T, \xi_0 \sim 3, 3, 1$

with the alignment:

!!!

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u \quad , \quad \langle \tilde{\xi} \rangle = 0$$

In all versions there are additional symmetries:

- e.g. a broken $U(1)_F$ symmetry to ensure hierarchy of charged lepton masses
- one or more discrete parities to restrict allowed couplings

Structure of the model (a 4-dim SUSY version)

GA, Feruglio, hep-ph/0512103

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + (x_a \xi + \tilde{x}_a \tilde{\xi})(ll) + x_b(\varphi_S ll) + h.c. + ...$$

shorthand: Higgs and cut-off scale Λ omitted, e.g.:

$$y_e e^c(\varphi l) \sim y_e e^c(\varphi l) h_d/\Lambda, \qquad x_a \xi(ll) \sim x_a \xi(l h_u l h_u)/\Lambda^2$$

In T-diag basis:

with this alignment:

!!!

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u \quad , \quad \langle \tilde{\xi} \rangle = 0$$

Ch. leptons are diagonal

$$m_l = v_T \frac{v_d}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

ν's are tri-bimaximal

$$m_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a + 2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a - b/3 \\ -b/3 & a - b/3 & 2b/3 \end{pmatrix}$$

$$a \equiv x_a \frac{u}{\Lambda} \qquad b \equiv x_b \frac{v_T}{\Lambda}$$

recall:

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

Extension to quarks

If we take all fermion doublets as 3 and all singlets as 1, 1′, 1″ (as for leptons): $Q_i \sim 3$, $u^c, d^c \sim 1$, $c^c, s^c \sim 1'$, $t^c, b^c \sim 1''$

Then u and d quark mass matrices, like for charged leptons, are BOTH diagonal in the T-diagonal basis

As a result V_{CKM} is unity: $V_{CKM} = U_u^+ U_d \sim 1$

So, in first approx. (broken by loops and higher dim operators), ν mixings are HPS and quark mixings ~identity

Corrections are far too small to reproduce quark mixings eg λ_C (for leptons, corrections cannot exceed $o(\lambda_C^2)$). But even those are essentially the same for u and d quarks)

A4 is simple and economic for leptons

One problem is how to extend the model to quarks

Also one would like a GUT model with all fermion masses and mixings reproduced, which includes TB mixing for ν's from A4

NOT straightforward to embed these models in a GUT: for A4 to commute with SU(5) one needs

If $l \sim 3$ then all $F_i \sim 5_i^* \sim 3$, so that $d^c_i \sim 3$
if $e^c, \mu^c, \tau^c \sim 1, 1'', 1'$ then all $T_i \sim 10_i \sim 1, 1'', 1'$

Widespread feeling that A4 cannot be unified in a satisfactory way.
Here we show a counterexample

Recent directions of research:

- Different (larger) finite groups

  Ma;
  Kobayashi et al;
  Luhn, Nasri, Ramond [$\Delta(3n^2)$];
  .....

- Trying to improve the quark mixings

  Carr, Frampton
  Feruglio et al
  Frampton, Kephart.....

- Construct GUT models with approximate tribimaximal mixing

  Ma, Sawanaka, Tanimoto; Ma;
  Morisi, Picarello, Torrente Lujan; Bazzocchi et al;
  de Madeiros Verzielas, King, Ross [$\Delta(27)$];
  King, Malinsky [$SU(4)_C x SU(2)_L x SU(2)_R$]; Antusch et al;
  Chen, Mahanthappa .....

Here is our A4 GUT model (0802.0090[hep-ph])

A SUSY SU(5) Grand Unified Model of Tri-Bimaximal Mixing from A_4

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Abstract

We discuss a grand unified model based on SUSY SU(5) in extra dimensions and on the flavour group $A_4 \times U(1)$ which, besides reproducing tri-bimaximal mixing for neutrinos with the accuracy required by the data, also leads to a natural description of the observed pattern of quark masses and mixings.

SUSY-SU(5) GUT with A4

Key ingredients:

- SUSY

  In general SUSY is crucial for coupling unification and p decay
  Specifically it makes simpler to implement the required alignment

- GUT's in 5 dimensions

  In general GUT's in ED are most natural and effective
  Here also contribute to fermion hierarchies

- Extended flavour symmetry: $A4xU(1)xZ_3xU(1)_R$

  $U(1)_R$ is a standard ingredient of SUSY GUT's in ED

  Hall-Nomura'01

GUT's in extra dimensions

- Minimal SUSY-SU(5), -SO(10) models are in trouble
- More realistic models are possible but they tend to be baroque (e.g. large Higgs representations)

Recently a new idea has been developed and looks promising:
unification in extra dimensions

Kawamura
GA, Feruglio
Hall, Nomura;
Hebecker, March-Russell;
Hall, March-Russell, Okui, Smith
Asaka, Buchmuller, Covi
....

Factorised metric

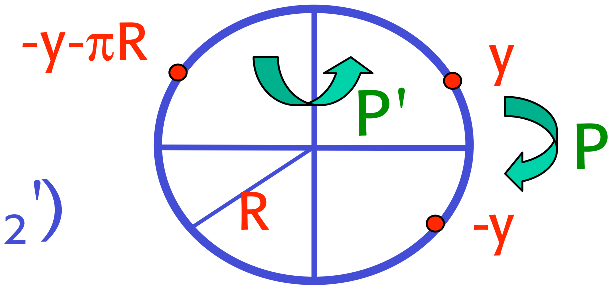
$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + h_{ij}(y) dy^i dy^j$$

The compactification radius $R \sim 1/M_{GUT}$ (not so large!)

Virtues:

- No baroque large Higgs representations
- SUSY and SU(5) breaking by orbifolding
- Doublet-triplet splitting problem solved
- New handles for p decay, flavour hierarchies

Symmetry breaking by orbifolding



5-dim theory with compatified $x_5=y$ $S/(Z_2 x Z_2')$

P and P' break the symmetries of 5-dim theory

On the branes located at the fixed points y=0 and y= -πR/2 the symmetry is reduced

Z_2-> P: y ⟺ -y

Z_2'-> P': y' ⟺ -y'

y'=y + πR/2

or y ⟺ -y- πR

$$\phi_{++}(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_n \phi_{++}^{(2n)}(x_\mu) \cos\frac{2ny}{R}$$

$$\phi_{+-}(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_n \phi_{+-}^{(2n+1)}(x_\mu) \cos\frac{2n+1}{R}y$$

$$\phi_{-+}(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_n \phi_{-+}^{(2n+1)}(x_\mu) \sin\frac{2n+1}{R}y$$

$$\phi_{--}(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_n \phi_{--}^{(2n+2)}(x_\mu) \sin\frac{2n+2}{R}y$$

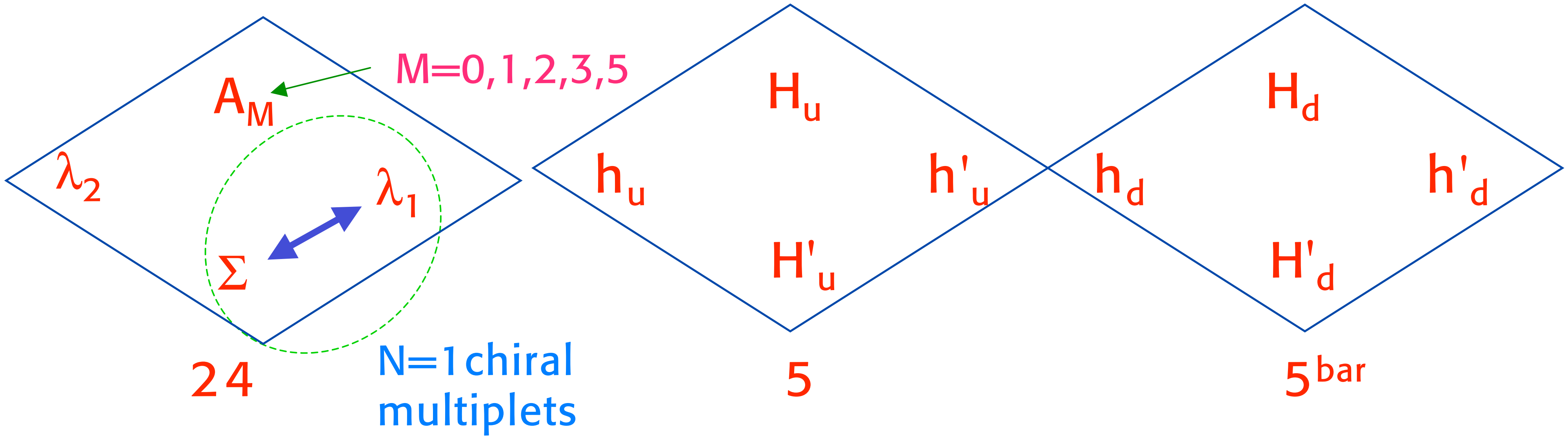
At y=0 only ϕ_{++} and ϕ_{+-} survive.

ϕ_{++} massless

SUSY-SU(5) in extra dimensions

- In 5 dim. the theory is symmetric under N=2 SUSY and SU(5)

Gauge 24 + Higgs $5+5^{bar}$: N=2 supermultiplets in the bulk



- Compactification by $S/(Z_2 x Z_2')$ $1/R \sim M_{GUT}$

N=2 SUSY-SU(5) -> N=1 SUSY-SU(3)xSU(2)xU(1)

- Matter 10, 5^{bar}, 1 on the brane (e.g. $x_5=y=0$) or in the bulk (many possible variations)

P breaks N=2 SUSY down to N=1 SUSY
but conserves SU(5): on 5 of SU(5) P=(+,+,+,+,+)

P' breaks SU(5) P'=(-,-,-,+,+) $P'T^aP'=T^a$, $P'T^\alpha P'= -T^\alpha$
(T^a: span 3x2x1, T^α: all other SU(5) gen.'s)

P P'	bulk field	mass
++	$A^a_\mu, \lambda^a_2, H^D_u, H^D_d$ ← Doublet	2n/R
+ -	$A^\alpha_\mu, \lambda^\alpha_2, H^T_u, H^T_d$ ← Triplet	(2n+1)/R
- +	$A^\alpha_5, \Sigma^\alpha, \lambda^\alpha_1, H'^T_u, H'^T_d$	(2n+1)/R
- -	$A^a_5, \Sigma^a, \lambda^a_1, H'^D_u, H'^D_d$	(2n+2)/R

Note:

$$\partial_5 = (-,-)$$

Gauge parameters are also y dep.

$$U = \exp[i\xi^a(x_\mu, y)T^a + i\xi^\alpha(x_\mu, y)T^\alpha]$$

$$\xi^a(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum \xi^a(x_\mu)\cos\frac{2ny}{R}$$

$$\xi^\alpha(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_n \xi^\alpha(x_\mu)\cos\frac{2n+1}{R}y$$

} both not zero at y=0

$$U = \exp[i\xi^{a}(x_{\mu}, y)T^{a} + i\xi^{\alpha}(x_{\mu}, y)T^{\alpha}]$$

$$\xi^{a}(x_{\mu}, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_{n} \xi^{a}(x_{\mu}) \cos\frac{2ny}{R}$$

$$\xi^{\alpha}(x_{\mu}, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_{n} \xi^{\alpha}(x_{\mu}) \cos\frac{2n+1}{R}y$$

At y=0 both ξ^a and ξ^α not 0: so full SU(5) gauge transf.s, while at $y=\pi R/2$ only SU(3)xSU(2)xU(1).

Virtues:

- No baroque 24 Higgs to break SU(5)
- $A^{a(0)}_{\mu}, \lambda^{a(0)}_{2}$ massless N=1 multiplet
- $A^{a(2n)}_{\mu}$ eat $\partial_5 A^{a(2n)}_5$ and become massive (n>0)
- Doublet-Triplet splitting automatic and natural:

$H^{D(0)}_{u,d}$ massless, $H^{T(0)}_{u,d}$ $m \sim 1/R \sim m_{GUT}$

$U(1)_R$ symmetry is a remnant of the $SU(2)_R$ of N=2 SUSY bulk action before compactification: going from N=2 to N=1 SUSY in 4 dim reduces $SU(2)_R$ down to $U(1)_R$

Hall-Nomura'01

When N=1 SUSY is broken by terms of order m_{soft}, $U(1)_R$ is also broken and only R-parity is left

At y=0 only terms in the superpotential w with $U(1)_R$ charge +2 are allowed (to compensate the -2 of $d^2\theta$):

$$\int d^4x \int_0^{\pi R} dy \int d^2\theta \; w(x)\delta(y) + h.c. = \int d^4x \int d^2\theta \; w(x) + h.c.$$

$U(1)_R$ forbids the relevant coloured Higgsino vertices and prevents fast p decay

SUSY-SU(5) GUT with A4

Key ingredients:

- GUT's in 5 dimensions
- Extended flavour symmetry: $A4 \times U(1) \times Z_3 \times U(1)_R$

Froggatt-Nielsen

Reduces to R-parity when SUSY is broken at m_{soft}

Keeps ϕ_S and ϕ_T separate

Field	N	F	T_1	T_2	T_3	H_5	$H_{\bar{5}}$	φ_T	φ_S	$\xi,\ \tilde{\xi}$	θ	θ''	φ_0^T	φ_0^S	ξ_0
SU(5)	1	$\bar{5}$	10	10	10	5	$\bar{5}$	1	1	1	1	1	1	1	1
A_4	3	3	$1''$	$1'$	1	1	$1'$	3	3	1	1	$1''$	3	3	1
U(1)	0	0	3	1	0	0	0	0	0	0	-1	-1	0	0	0
Z_3	ω	ω	ω	ω	ω	ω	ω	1	ω	ω	1	1	1	ω	ω
$U(1)_R$	1	1	1	1	1	0	0	0	0	0	0	0	2	2	2

U(1) breaking flavons

driving fields for alignment

● : in bulk

ED effects contribute to the fermion mass hierarchies

A bulk field is related to its zero mode by: $B = \frac{1}{\sqrt{\pi R}} B^0 + \ldots$

This produces a suppression parameter for couplings with bulk fields $s \equiv \frac{1}{\sqrt{\pi R \Lambda}} < 1$

Λ: UV cutoff

- In bulk: N=2 SUSY Yang-Mills fields + H_5, H_5^{bar} + T_1, T_2, T_1', T_2'

(doubling of bulk fermions to obtain chiral massless states at y=0)

also crucial to avoid too strict mass relations for 1,2 families:

(b-τ unification only for 3rd family)

- All other fields on brane at y=0 (in particular N, F, T_3)

Superpotential terms on the brane
($T_{1,2}$ represent either $T_{1,2}$ or $T'_{1,2}$)

Up masses

$$w_{up} = \frac{1}{\Lambda^{1/2}} H_5 T_3 T_3 + \frac{\theta''}{\Lambda^2} H_5 T_2 T_3 + \frac{\theta''^2}{\Lambda^{7/2}} H_5 T_2 T_2 + \frac{\theta\theta''^2}{\Lambda^4} H_5 T_1 T_3$$
$$+ \frac{\theta^4}{\Lambda^{11/2}} H_5 T_1 T_2 + \frac{\theta\theta''^3}{\Lambda^{11/2}} H_5 T_1 T_2 + \frac{\theta^5\theta''}{\Lambda^{15/2}} H_5 T_1 T_1 + \frac{\theta^2\theta''^4}{\Lambda^{15/2}} H_5 T_1 T_1$$

Down and charged lepton masses

$$w_{down} = \frac{1}{\Lambda^{3/2}} H_{\bar{5}} (F\varphi_T)'' T_3 + \frac{\theta}{\Lambda^3} H_{\bar{5}} (F\varphi_T)' T_2 + \frac{\theta^3}{\Lambda^5} H_{\bar{5}} (F\varphi_T) T_1 + \frac{\theta''^3}{\Lambda^5} H_{\bar{5}} (F\varphi_T) T_1$$
$$+ \frac{\theta''}{\Lambda^3} H_{\bar{5}} (F\varphi_T)'' T_2 + \frac{\theta^2\theta''}{\Lambda^5} H_{\bar{5}} (F\varphi_T)' T_1 + \frac{\theta\theta''^2}{\Lambda^5} H_{\bar{5}} (F\varphi_T)'' T_1 + \ldots \quad ,$$

Neutrino masses from see-saw
(correct relation bewteen m_ν and M_{GUT})

$$w_\nu = \frac{y^D}{\Lambda^{1/2}} H_5 (NF) + (x_a \xi + \tilde{x}_a \tilde{\xi})(NN) + x_b (\varphi_S NN)$$

$$m_u = \begin{pmatrix} s^2t^5t'' + s^2t^2t''^4 & s^2t^4 + s^2tt''^3 & stt''^2 \\ s^2t^4 + s^2tt''^3 & s^2t''^2 & st'' \\ stt''^2 & st'' & 1 \end{pmatrix} sv_u^0 \sim \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \lambda v_u^0$$

dots=0 in 1st approx

fixed by higher dim operators & corrections to alignment (see later)

$$m_d = \begin{pmatrix} st^3 + st''^3 & \dots & \dots \\ st^2t'' & st & \dots \\ stt''^2 & st'' & 1 \end{pmatrix} v_T sv_d^0 \sim \begin{pmatrix} \lambda^4 & \dots & \dots \\ \lambda^4 & \lambda^2 & \dots \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} v_T \lambda v_d^0$$

$$m_e = \begin{pmatrix} st^3 + st''^3 & st^2t'' & stt''^2 \\ \dots & st & st'' \\ \dots & \dots & 1 \end{pmatrix} v_T sv_d^0 \sim \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \dots & \lambda^2 & \lambda^2 \\ \dots & \dots & 1 \end{pmatrix} v_T \lambda v_d^0$$

with

$$\frac{\langle \varphi_T \rangle}{\Lambda} = (v_T, 0, 0) \quad , \quad \frac{\langle \varphi_S \rangle}{\Lambda} = (v_S, v_S, v_S) \quad , \quad \frac{\langle \xi \rangle}{\Lambda} = u \quad \frac{\langle \theta \rangle}{\Lambda} = t \quad , \quad \frac{\langle \theta'' \rangle}{\Lambda} = t''$$

$s \sim t \sim t'' \sim \lambda \sim 0.22$ $\quad$ $v_T \sim \lambda^2 \sim m_b/m_t$ $\quad$ $v_S,\ u \sim \lambda^2$

For ν's after see-saw

$$m_\nu = \frac{1}{3a(a+b)} \begin{pmatrix} 3a+b & b & b \\ b & \frac{2ab+b^2}{b-a} & \frac{b^2-ab-3a^2}{b-a} \\ b & \frac{b^2-ab-3a^2}{b-a} & \frac{2ab+b^2}{b-a} \end{pmatrix} \frac{s^2(v_u^0)^2}{\Lambda}$$

with

$$a \equiv \frac{2x_a u}{(y^D)^2} \quad , \qquad b \equiv \frac{2x_b v_S}{(y^D)^2}$$

m_ν is of the form

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix} \longrightarrow U = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

charged lepton diagonalization for dots=0 contributes λ^4, λ^8, λ^4 terms to 12, 13, 23

with

$$m_1 = \frac{1}{(a+b)} \quad , \qquad m_2 = \frac{1}{a} \quad , \qquad m_3 = \frac{1}{(b-a)}$$

or

$$\frac{2}{m_2} = \frac{1}{m_1} - \frac{1}{m_3}$$

$$r \equiv \Delta m^2_{sol}/\Delta m^2_{atm}$$

$$\Delta m^2_{sol} \equiv |m_2|^2 - |m_1|^2$$

$$\Delta m^2_{atm} \equiv \left||m_3|^2 - |m_1|^2\right|$$

$$r = \frac{|1-z|^2|z+\bar{z}+|z|^2|}{2|z+\bar{z}|}$$

$$z \equiv \frac{b}{a}$$

For z~+1 a viable normal hierarchy spectrum
while z~-2 would give an inverse hierarchy solution

z~+1, normal hierarchy
is the most natural:

$$\sqrt{\Delta m^2_{atm}} \approx \frac{s^2 (v_u^0)^2}{|a|\Lambda\sqrt{r}}$$

$$\sum_i |m_i| \approx (0.06 - 0.07)\ \mathrm{eV}$$

$$|m_{ee}| \approx 0.007\ \mathrm{eV}$$

$$\begin{aligned} |m_1|^2 &= \frac{1}{3}\Delta m^2_{atm}\ r + \ldots \\ |m_2|^2 &= \frac{4}{3}\Delta m^2_{atm}\ r + \ldots \\ |m_3|^2 &= \left(1 + \frac{r}{3}\right)\Delta m^2_{atm} + \ldots \\ |m_{ee}|^2 &= \frac{16}{27}\Delta m^2_{atm}\ r + \ldots \quad , \end{aligned}$$

The model crucially depends on the precise vev alignment

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u \quad , \quad \langle \tilde{\xi} \rangle = 0$$

One more singlet is needed for vacuum alignment: then one is chosen as the combination with vev=0

This version: a SUSY model with driving fields and a superpotential where all terms allowed by symmetry are present

In a natural model

- all terms allowed by symmetry are present
- all correct'ns are under control and can be made negligible

In SUSY the alignment is simpler (driving fields)

The superpotential (at leading order) is very constrained:

$$w_d = M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T)$$
$$+ \; g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \tilde{\xi}(\varphi_0^S \varphi_S) + g_3 \xi_0(\varphi_S \varphi_S) + g_4 \xi_0 \xi^2 + g_5 \xi_0 \xi \tilde{\xi} + g_6 \xi_0 \tilde{\xi}^2$$

and the potential $V=V_F+V_D$

$$V_F = \sum_i \left| \frac{\partial w}{\partial \varphi_i} \right|^2 \qquad V_D = \frac{1}{2}(M_{FI}^2 - g_{FN}|\theta|^2 - g_{FN}|\theta''|^2 + \ldots)^2$$

The D-term arises from the Froggatt-Nielsen U(1) and $V_D=0$ implies

$$g_{FN}|\theta|^2 + g_{FN}|\theta''|^2 = M_{FI}^2$$

Data require $t=\theta/\Lambda$ and $t''=\theta''/\Lambda \sim o(\lambda)$

The driving field have zero vev. So the minimization of V_F is:

$$\frac{\partial w}{\partial \varphi_{01}^T} = M\varphi_{T1} + \frac{2g}{3}(\varphi_{T1}^2 - \varphi_{T2}\varphi_{T3}) = 0 \qquad \frac{\partial w}{\partial \varphi_{01}^S} = g_2\tilde{\xi}\varphi_{S1} + \frac{2g_1}{3}(\varphi_{S1}^2 - \varphi_{S2}\varphi_{S3}) = 0$$

$$\frac{\partial w}{\partial \varphi_{02}^T} = M\varphi_{T3} + \frac{2g}{3}(\varphi_{T2}^2 - \varphi_{T1}\varphi_{T3}) = 0 \qquad \frac{\partial w}{\partial \varphi_{02}^S} = g_2\tilde{\xi}\varphi_{S3} + \frac{2g_1}{3}(\varphi_{S2}^2 - \varphi_{S1}\varphi_{S3}) = 0$$

$$\frac{\partial w}{\partial \varphi_{03}^T} = M\varphi_{T2} + \frac{2g}{3}(\varphi_{T3}^2 - \varphi_{T1}\varphi_{T2}) = 0 \qquad \frac{\partial w}{\partial \varphi_{03}^S} = g_2\tilde{\xi}\varphi_{S2} + \frac{2g_1}{3}(\varphi_{S3}^2 - \varphi_{S1}\varphi_{S2}) = 0$$

$$\frac{\partial w}{\partial \xi_0} = g_4\xi^2 + g_5\xi\tilde{\xi} + g_6\tilde{\xi}^2 + g_3(\varphi_{S1}^2 + 2\varphi_{S2}\varphi_{S3}) = 0$$

Solution:

$$\langle\varphi_T\rangle = (v_T, 0, 0)\Lambda \quad , \quad v_T\Lambda = -\frac{3M}{2g}$$

$$\langle\varphi_S\rangle = (v_S, v_S, v_S)\Lambda \quad , \quad v_S = \frac{\tilde{g}_4}{3\tilde{g}_3}u$$

$$\langle\xi\rangle = u\Lambda \quad ,$$

$$\langle\tilde{\xi}\rangle = 0 \qquad g_3 \equiv 3\tilde{g}_3^2 \ , \ g_4 \equiv -\tilde{g}_4^2$$

Data require
v_S, v_T, u ~$o(\lambda^2)$

NLO corrections studied in detail

vevs

$$\langle\varphi_T\rangle/\Lambda = (v_T+\delta v_{T1}, \delta v_{T2}, \delta v_{T3}) \quad , \text{ with } \delta v_{T2} = \delta v_{T3}$$

$$\langle\varphi_S\rangle/\Lambda = (v_S+\delta v_1, v_S+\delta v_2, v_S+\delta v_3) \quad ,$$

$$\langle\xi\rangle/\Lambda = u \quad , \quad \langle\tilde{\xi}\rangle/\Lambda = \delta u'$$

and all δ's ~ o(λ^4)

m_u δm_u negligible (o(λ^4))

$m_{d.e}$

$$m_d = \begin{pmatrix} \lambda^4 & \dots & \dots \\ \lambda^4 & \lambda^2 & \dots \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} v_T \lambda v_d^0 \quad \text{-->} \quad \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} v_T \lambda v_d^0$$

Diagonalisation of ch leptons contributes o(λ^2) corr's to TB mixing values for all mixing angles

m_ν All 6 entries of the symmetric mass matrix after see-saw receive indep. corr's of order o(λ^2) and so do the 3 angles

Summarising

By taking $s \sim t \sim t'' \sim \lambda \sim 0.22$ $v_T \sim \lambda^2 \sim m_b/m_t$ $v_S, u \sim \lambda^2$

a good description of all quark and lepton masses is obtained.
As for all U(1) models only $o(\lambda^p)$ predictions can be given (modulo o(1) coeff.s)

TB mixing for neutrinos is reproduced in first approximation

Quark hierarchies force corrections to TB mixing to be $o(\lambda^2)$
(in particular we predict $\theta_{13} \sim o(\lambda^2)$, accessible at T2K).

A moderate fine tuning is needed to fix λ_C and r (nominally of $o(\lambda^2)$ and 1 respectively)

Normal hierarchy is favoured, degenerate ν's are excluded

Conclusion

The A4 approach to TB neutrino mixing is shown to be compatible with quark masses and mixings in a GUT model

The unification with quarks fixes the size of the expected deviations from TB mixing: all mixing angles should deviate by o(λ^2) from the TB values

A normal hierarchy spectrum is indicated with

$$\frac{2}{m_2} = \frac{1}{m_1} - \frac{1}{m_3}$$

$$\sum_i |m_i| \approx (0.06 - 0.07)\ \mathrm{eV}$$

$$|m_{ee}| \approx 0.007\ \mathrm{eV}$$