Rare kaon decays as probes of beyond SM physics

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CERN TH Flavour Instiute,
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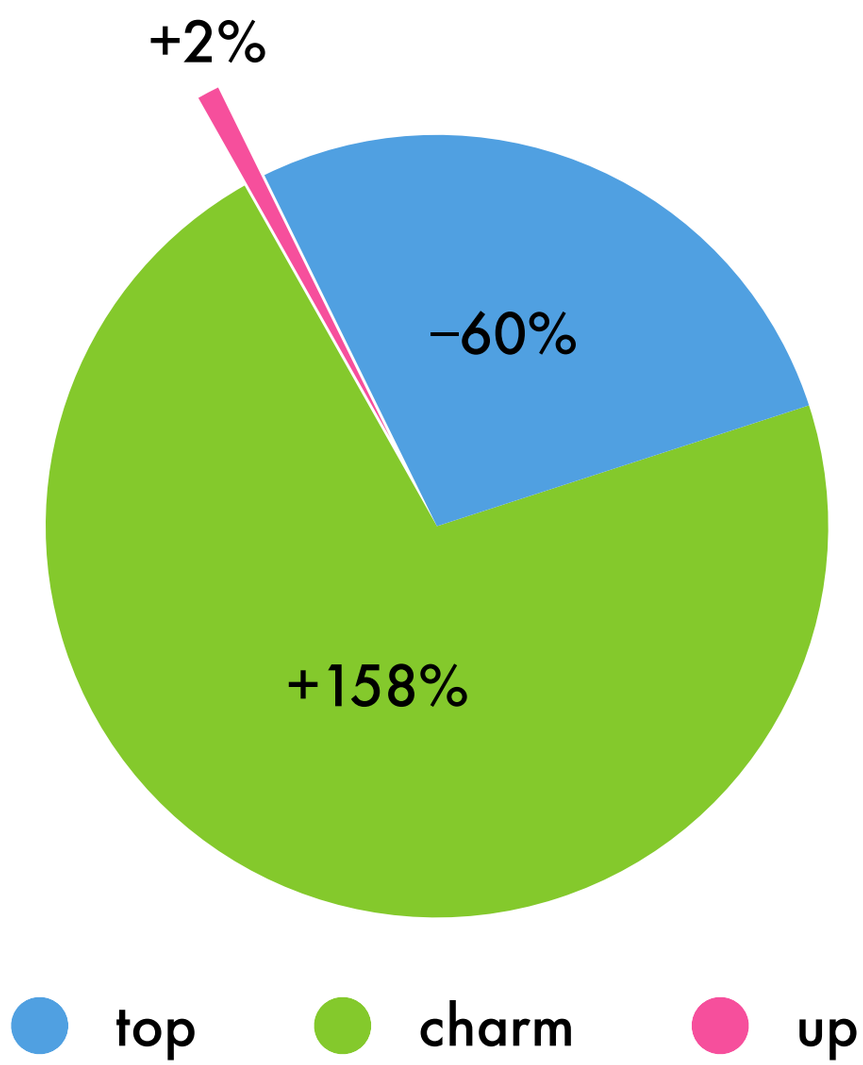
Warm-up: basic facts about $b \to s\gamma$

$$\mathcal{A}_{\rm SM}(b \to s\gamma) = \sum_{q=u,c,t} V_{qb}^* V_{qs}\, K_{\rm SM}^q \propto \ln\frac{m_b}{M_W}\lambda^2 + \ln\frac{m_b}{M_W}\lambda^2 + \ln\frac{m_b}{M_W}\lambda^4$$



$$Q_7 = \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma_{\mu\nu} b_R) F^{\mu\nu}$$

- tensor current not conserved: $b \to s\gamma$ depends logarithmically on scale where Q_7 is generated

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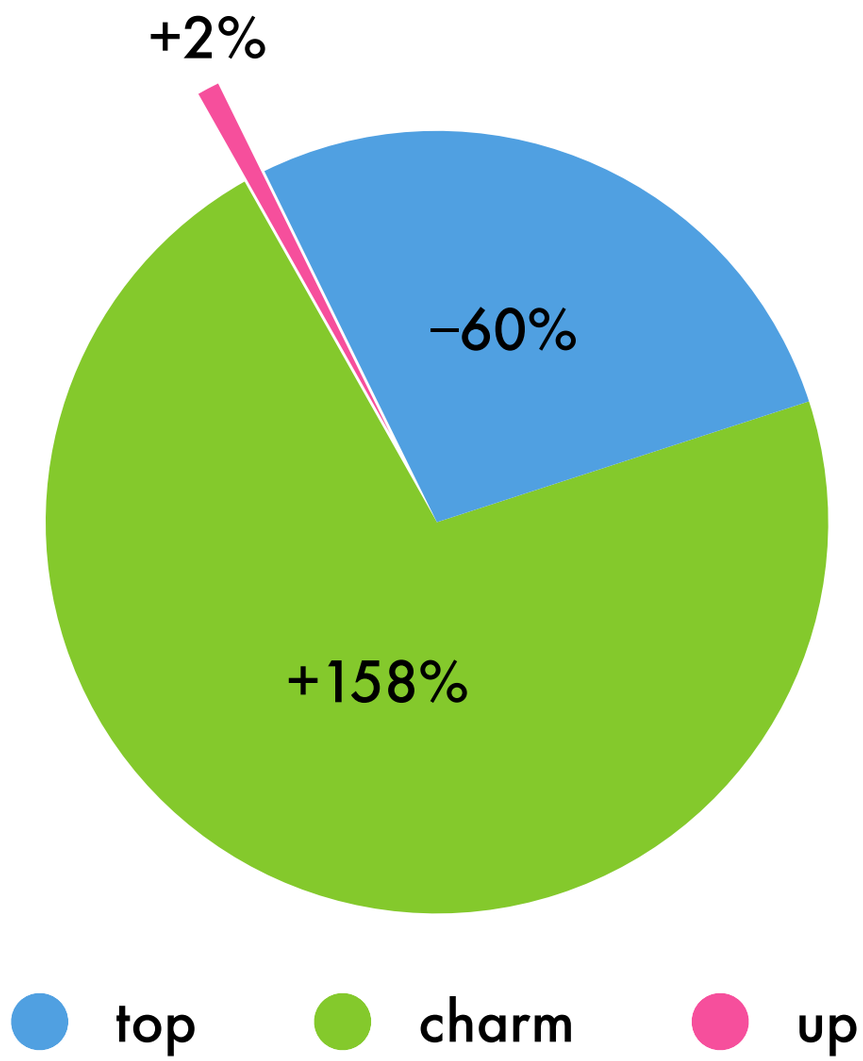
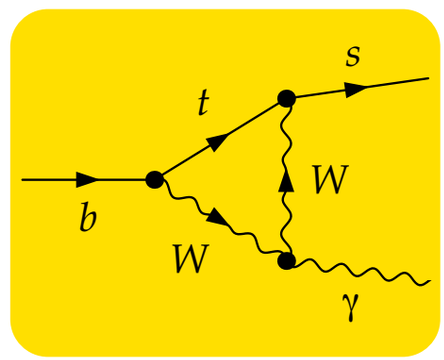
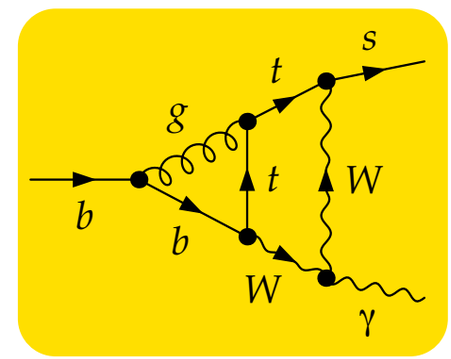




$\propto$ const.



$\propto \alpha_s \ln\frac{m_b}{M_W}$

- gluonic corrections lead to mild logarithmic GIM suppression of $b \to s\gamma$ amplitude

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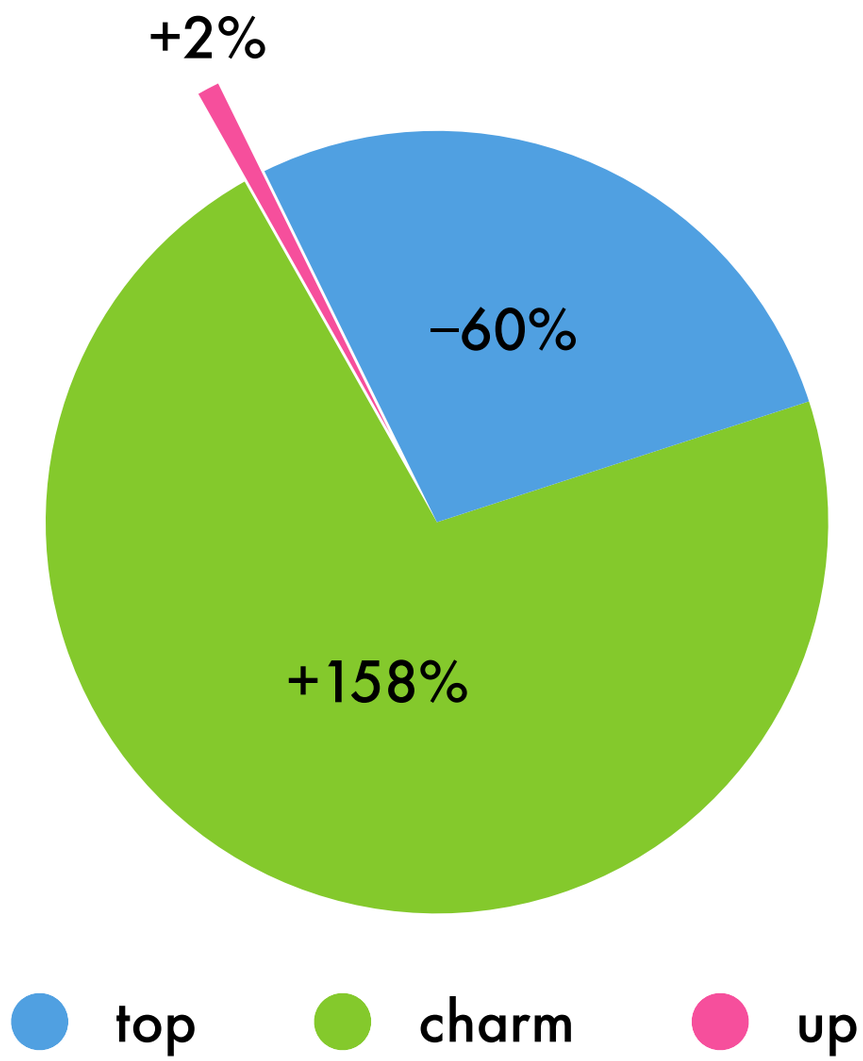
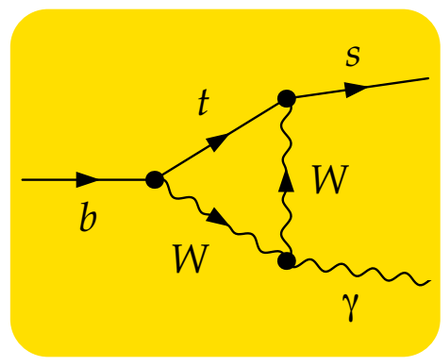
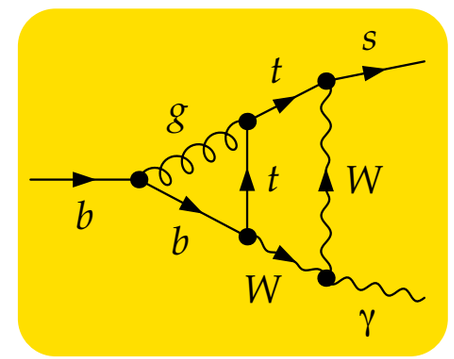




$\propto$ const.



$\propto \alpha_s \ln\frac{m_b}{M_W}$

- sensitivity on high scale physics "swamped" by renormalization group effects

Warm-up: basic facts about $b \to s\gamma$

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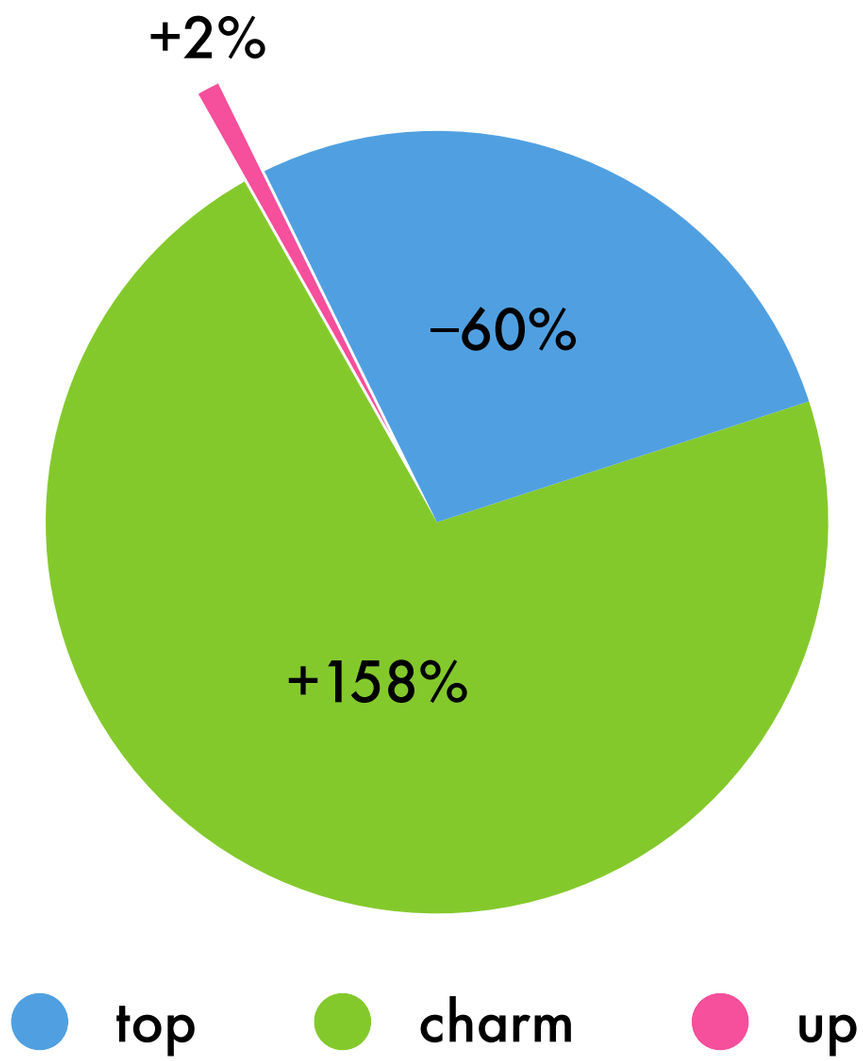
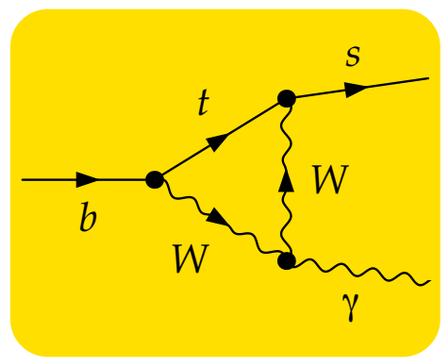
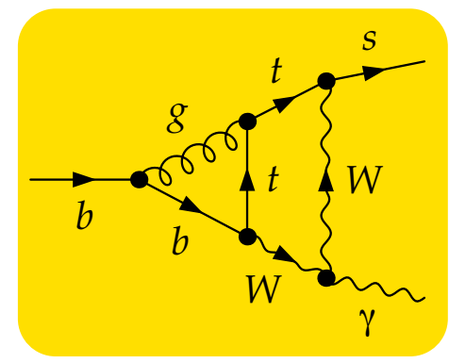




$\propto$ const.

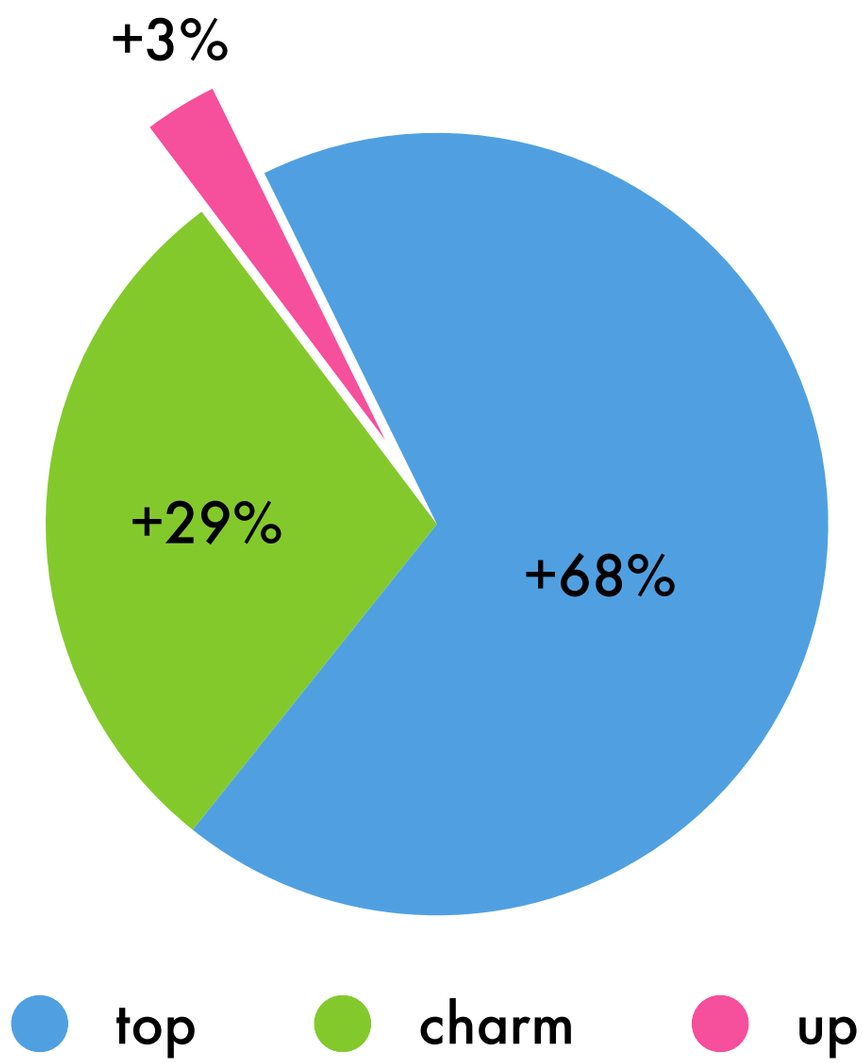


$\propto \alpha_s \ln\frac{m_b}{M_W}$

- thus: $b \to s\gamma$ wonderful QCD laboratory, providing stringent constraints on BSM physics

Warm-up: basic facts about $s \to d\nu\bar{\nu}$

$$\mathcal{A}_{\rm SM}(s \to d\nu\bar{\nu}) = \sum_{q=u,c,t} V_{qs}^* V_{qd}\, X_{\rm SM}^q \propto \frac{m_t^2}{M_W^2}(\lambda^5 + i\lambda^5) + \frac{m_c^2}{M_W^2}\ln\frac{m_c}{M_W}\lambda + \frac{\Lambda^2}{M_W^2}\lambda$$



$$Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \bar{\nu}_L)$$

- V–A current is conserved: large logarithms appear only in charm sector

Warm-up: basic facts about $s \to d\nu\bar{\nu}$

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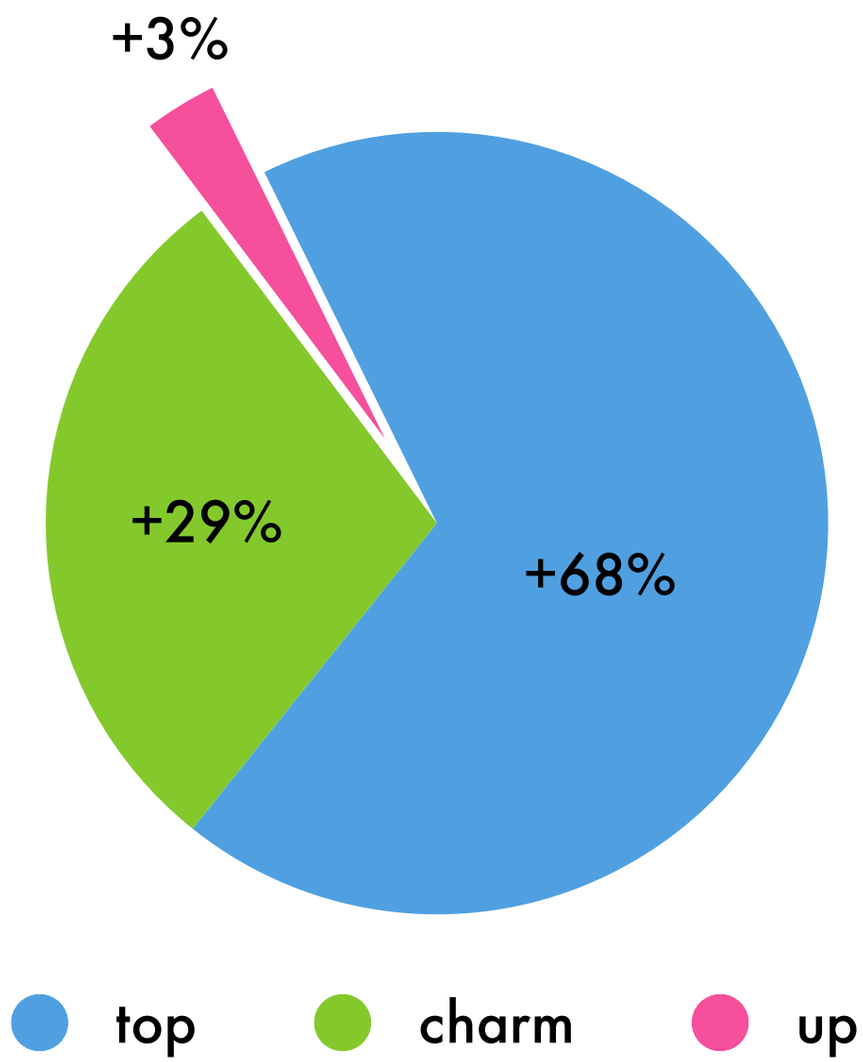
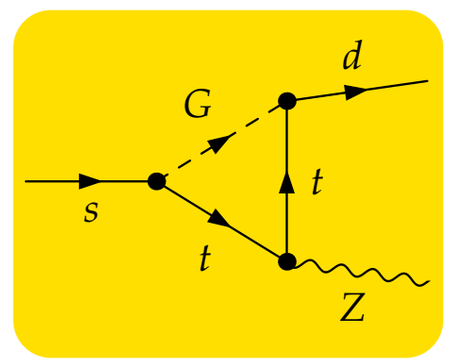
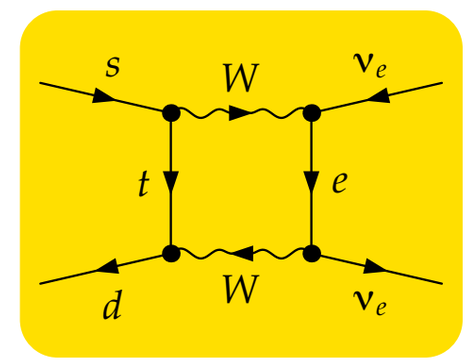




$$\propto \frac{m_t^2}{M_W^2}$$

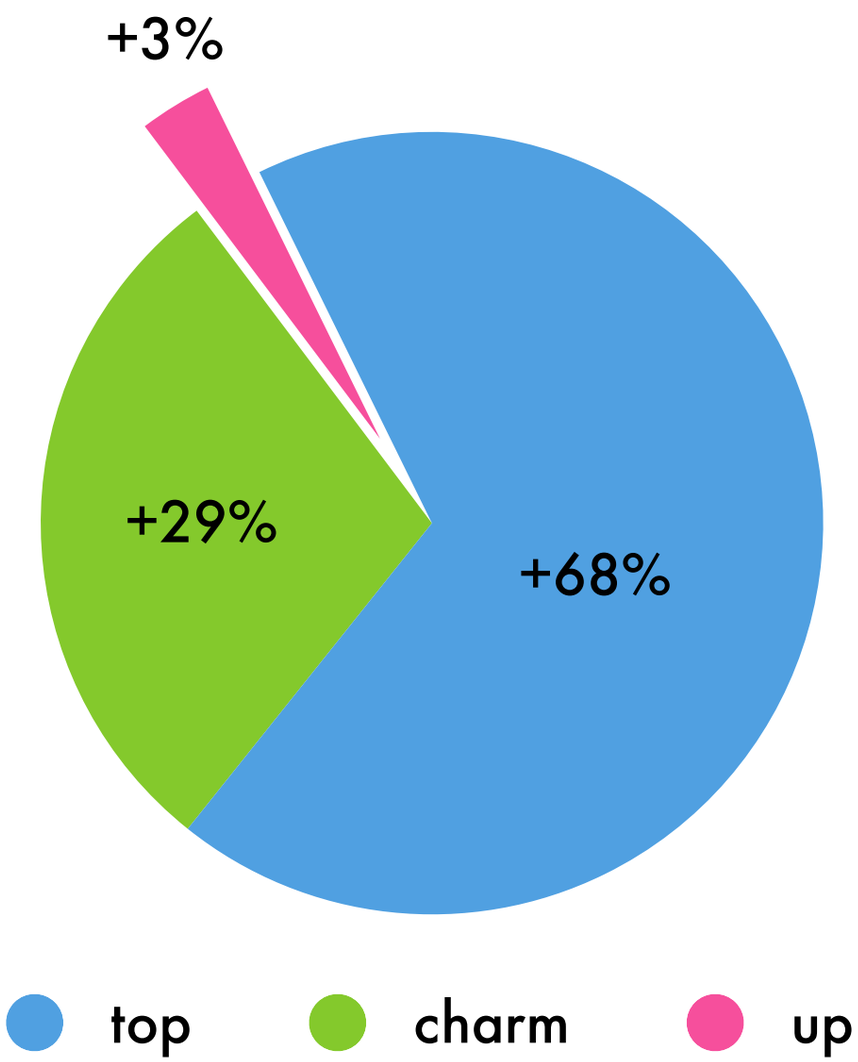


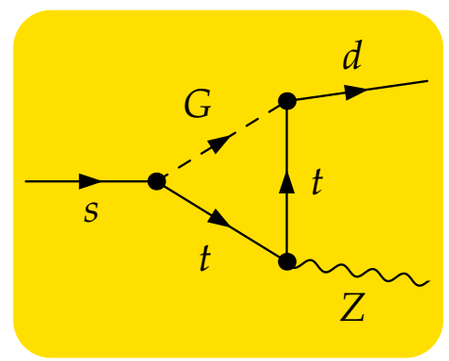
$$\propto \text{const.}$$

- $SU(2)_L$ breaking in Z-penguin amplitude leads to power-like GIM mechanism

Warm-up: basic facts about $s \to d\nu\bar{\nu}$

$$\mathcal{A}_{\rm SM}(s \to d\nu\bar{\nu}) = \sum_{q=u,c,t} V_{qs}^* V_{qd}\, X_{\rm SM}^q \propto \frac{m_t^2}{M_W^2}(\lambda^5 + i\lambda^5) + \frac{m_c^2}{M_W^2}\ln\frac{m_c}{M_W}\lambda + \frac{\Lambda^2}{M_W^2}\lambda$$





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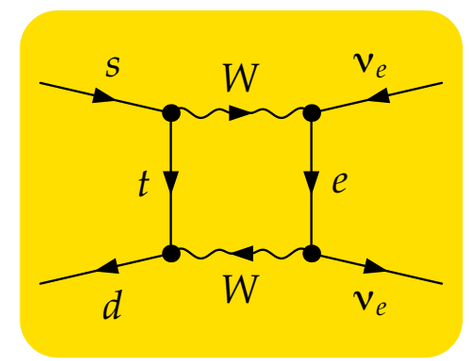
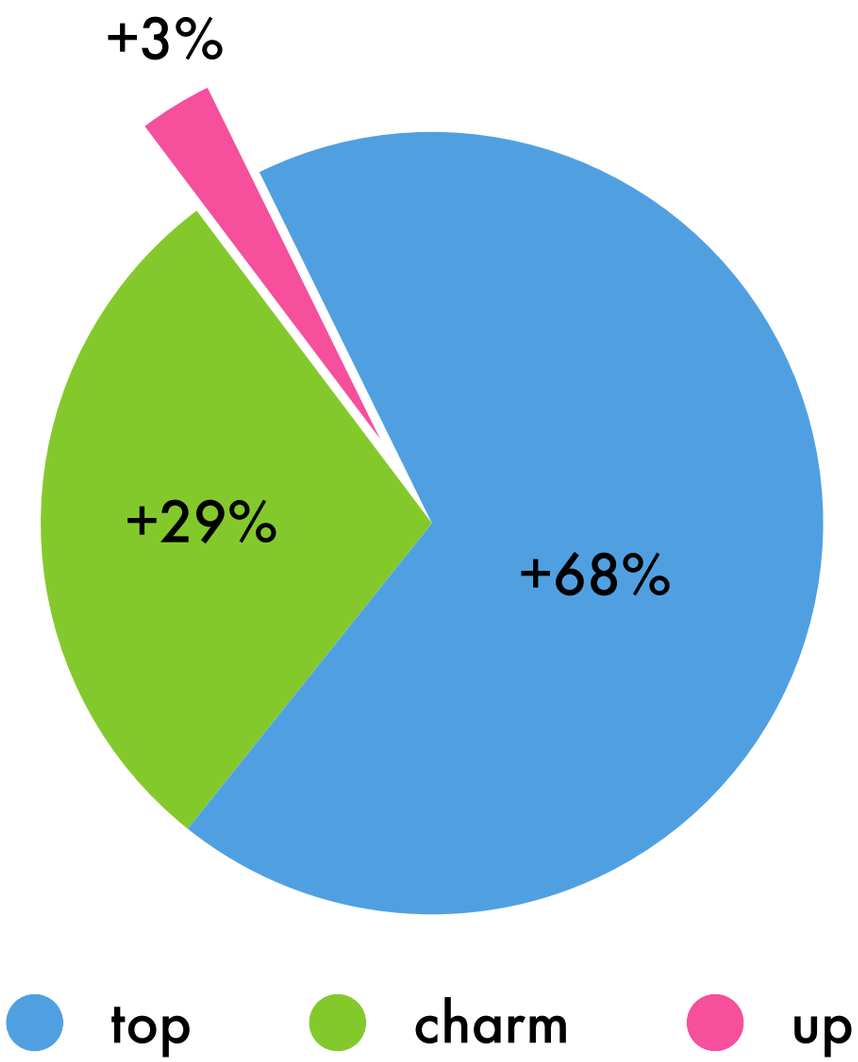


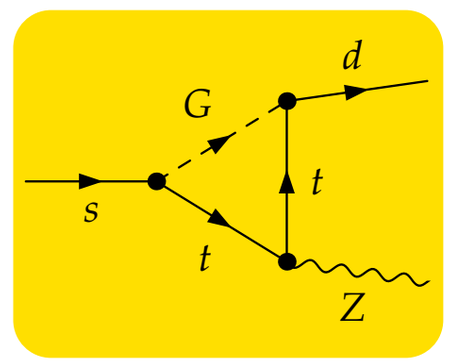
$$\propto \text{const.}$$

- large *CP* violating phase in dominant short-distance contribution due to top

Warm-up: basic facts about $s \to d\nu\bar{\nu}$

$$\mathcal{A}_{\rm SM}(s \to d\nu\bar{\nu}) = \sum_{q=u,c,t} V_{qs}^* V_{qd}\, X_{\rm SM}^q \propto \frac{m_t^2}{M_W^2}(\lambda^5 + i\lambda^5) + \frac{m_c^2}{M_W^2}\ln\frac{m_c}{M_W}\lambda + \frac{\Lambda^2}{M_W^2}\lambda$$





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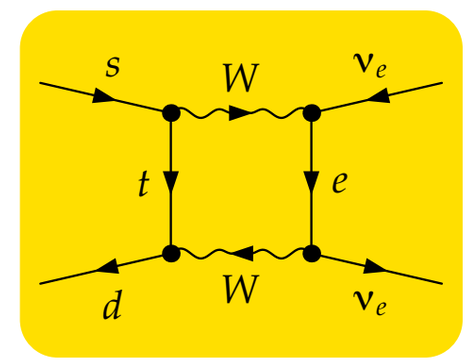


$$\propto \text{const.}$$

- thus: $s \to d\nu\bar{\nu}$ exceptional tool to discover non-MFV physics where hard GIM is not active

Next 45 minutes or so ...*

- Fundamentals: concise review of SM predictions of rare kaon decays
- Upset: rare kaon decays within constrained minimal flavor-violation (CMFV) & MFV
- Success stories: studies of beyond MFV effects in MSSM, littlest Higgs with *T*-parity (LHT), *Z'* (3-3-1) & Randall-Sundrum (RS) model
- Lively discussion

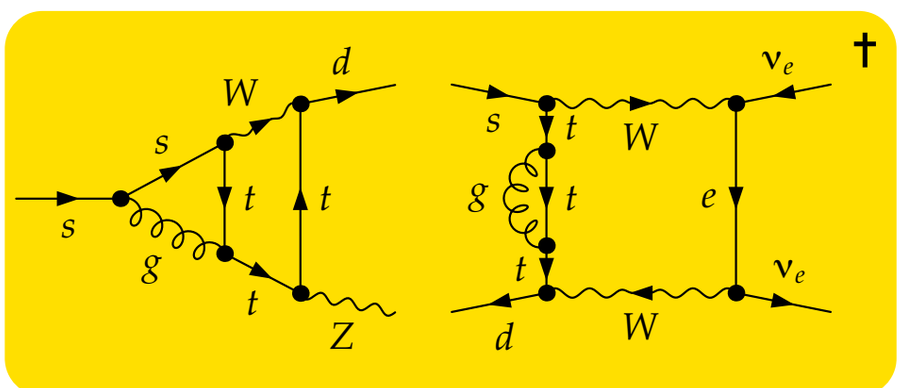
*my apologies: discussed topics reflect interests of speaker

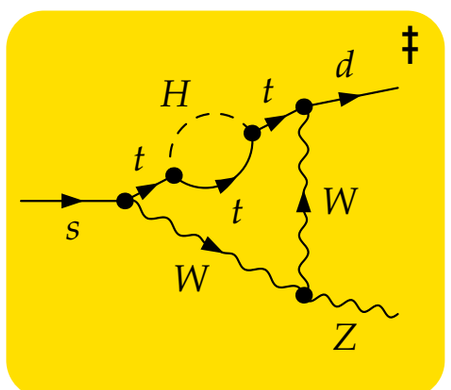
SM prediction of $K_L \to \pi^0 \nu\bar{\nu}$: general structure

$$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) = \kappa_L \left[\frac{\mathrm{Im}(V_{ts}^* V_{td})}{\lambda^5} X \right]^2 = (2.71 \pm 0.35) \times 10^{-11}$$

$$\kappa_L = (2.229 \pm 0.017) \times 10^{-10} \left(\frac{\lambda}{0.225} \right)^8 \quad *$$

$$X = 1.464 \pm 0.015_{m_t} \pm 0.013_{\mu_t} \overset{?}{\pm} 0.015^{\ddagger}_{\mathrm{EW}}$$





- $K \to \pi\nu\bar{\nu}$ matrix elements have been extracted precisely from K_{l3} data using χPT at $N^{3/2}LO$
- short-distance effects have been calculated to $O(\alpha_s)$ & $O(m_t^2 \alpha)$

*Mescia & Smith '07 †Misiak & Urban '99, Buchalla & Buras '99 ‡Buchalla & Buras '97

SM prediction of $K_L \to \pi^0 \nu\bar{\nu}$: error budget

$$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) = \kappa_L \left[\frac{\mathrm{Im}(V_{ts}^* V_{td})}{\lambda^5} X \right]^2 = (2.71 \pm 0.35) \times 10^{-11}$$

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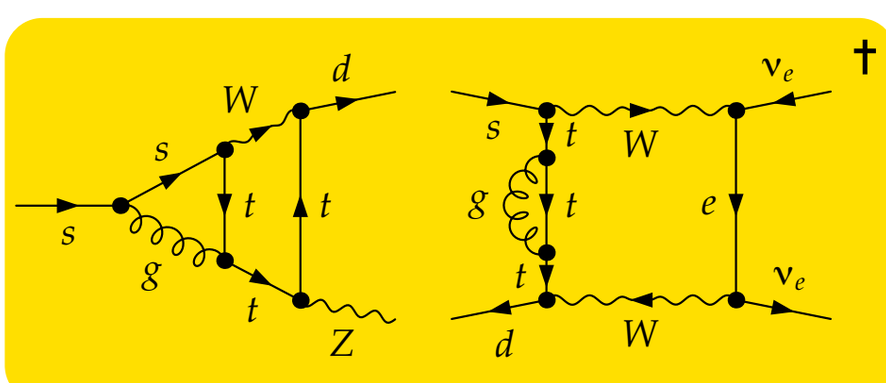
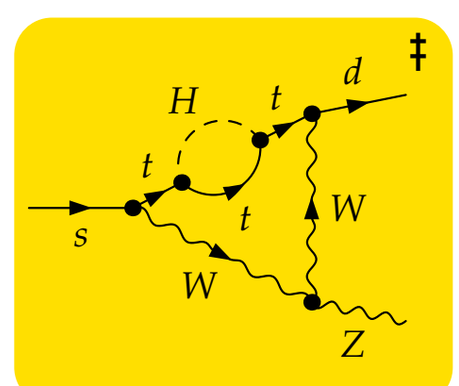
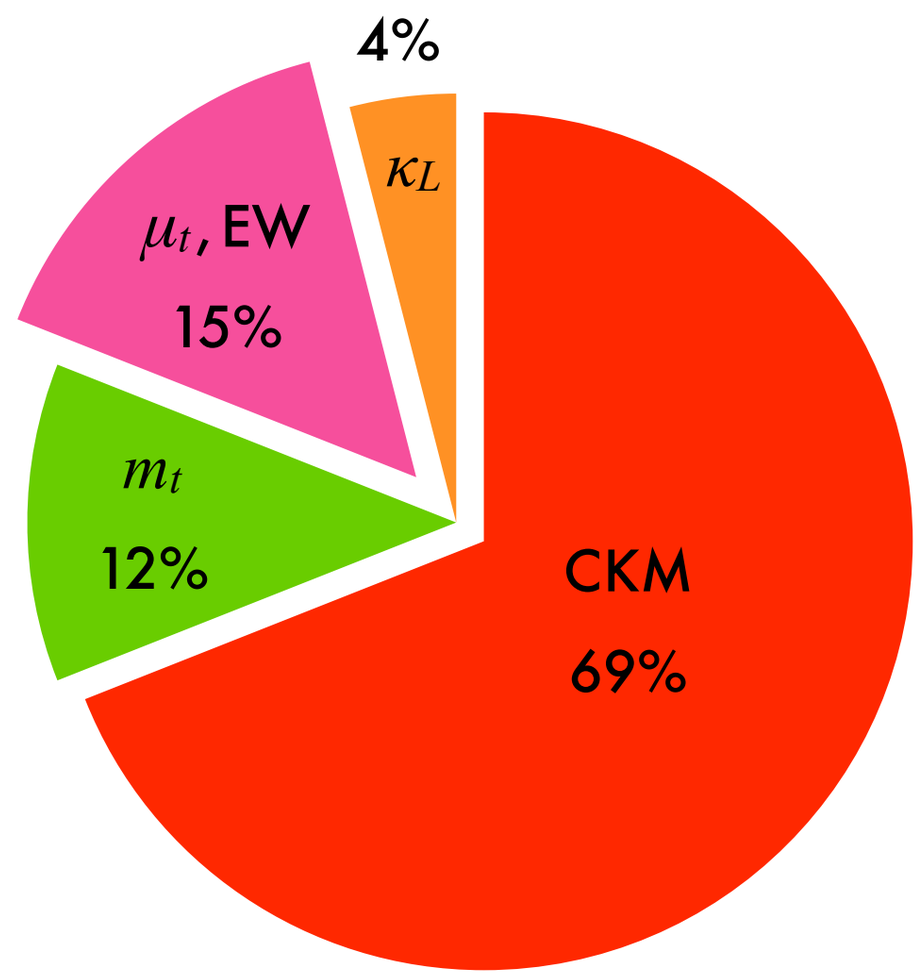






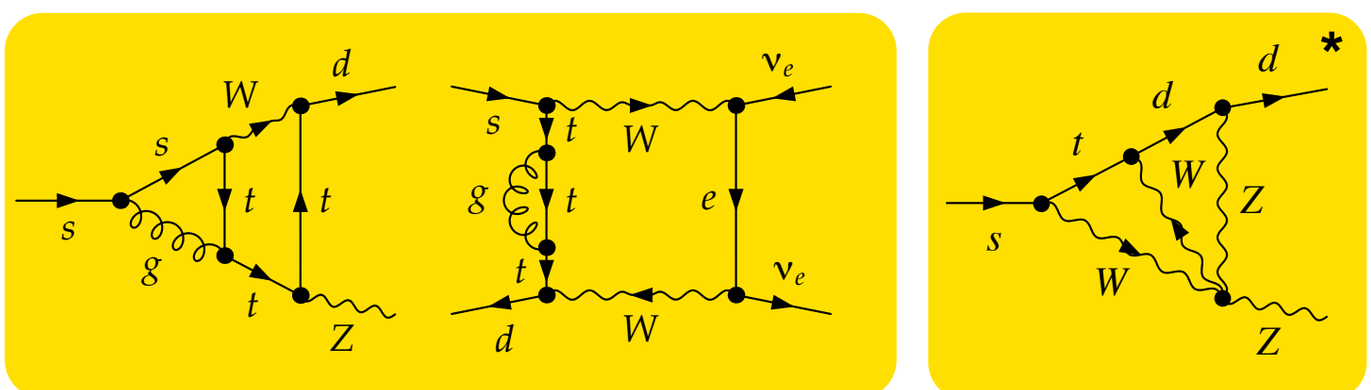
*Mescia & Smith '07 †Misiak & Urban '99, Buchalla & Buras '99 ‡Buchalla & Buras '97

SM prediction of $K_L \to \pi^0 \nu\bar{\nu}$: upshot

$$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) = \kappa_L \left[\frac{\mathrm{Im}(V_{ts}^* V_{td})}{\lambda^5} X \right]^2 = (2.71 \pm 0.35) \times 10^{-11}$$

$$\kappa_L = (2.229 \pm 0.017) \times 10^{-10} \left(\frac{\lambda}{0.225} \right)^8$$

$$X = 1.464 \pm 0.015_{m_t} \pm 0.013_{\mu_t} \overset{?}{\pm} 0.015_{\mathrm{EW}}$$



- unkown NNLO QCD & NLO EW* corrections dominate theory error
- within SM amount of *CP* violation could be determined with unmatched precision

*Brod & Gorbahn, work in progress

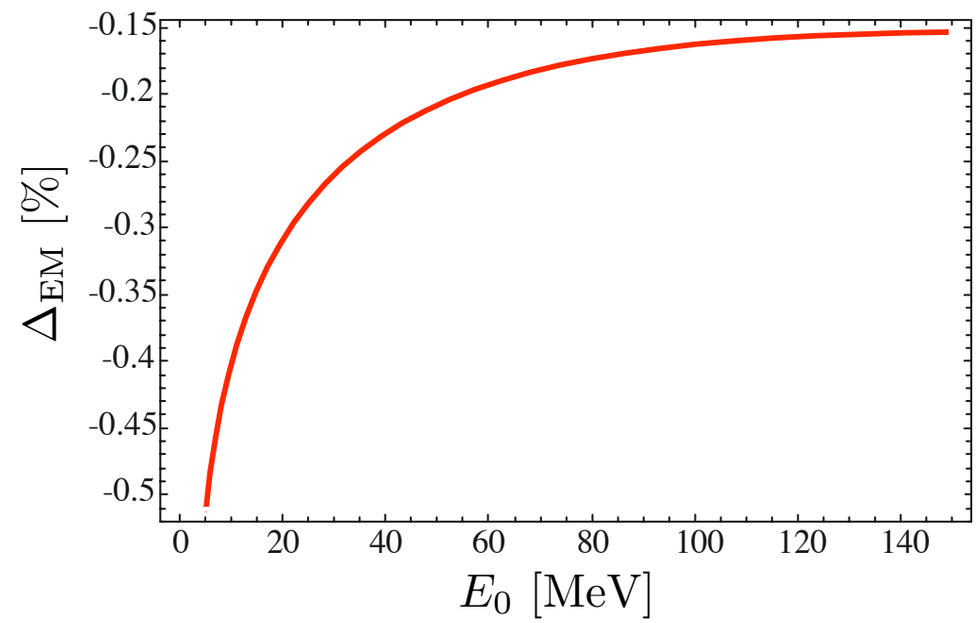
SM prediction of $K^+ \to \pi^+ \nu\bar{\nu}$: κ_+ & Δ_{EM}

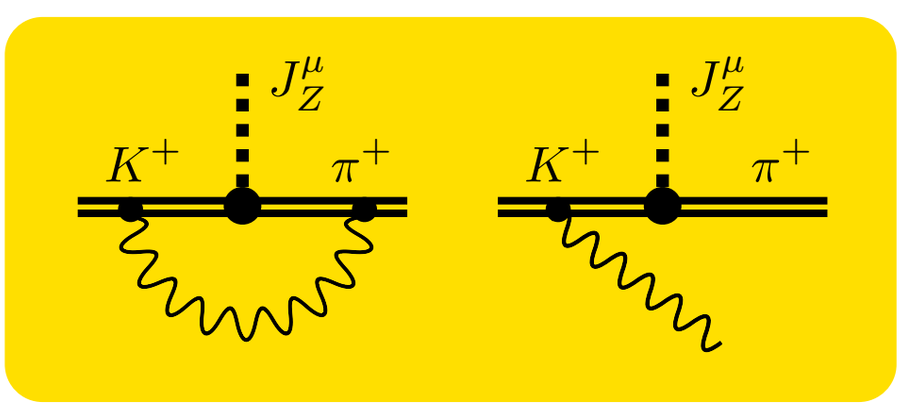
$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}(\gamma)) = \kappa_+ (1 + \Delta_{\rm EM}) \left| \frac{x}{\lambda^5} \right|^2$$

$$\Delta_{\rm EM}(E_\gamma < 20\,{\rm MeV}) = -0.003^*$$

$$x = V_{ts}^* V_{td} X + \lambda^4 \,{\rm Re}(V_{cs}^* V_{cd})(P_c + \delta P_{c,u})$$

$$\kappa_+ = (5.168 \pm 0.025) \times 10^{-11} \left(\frac{\lambda}{0.225} \right)^8 \quad *$$





- long-distance QED corrections at $O(p^2\alpha)$ in χPT are known now

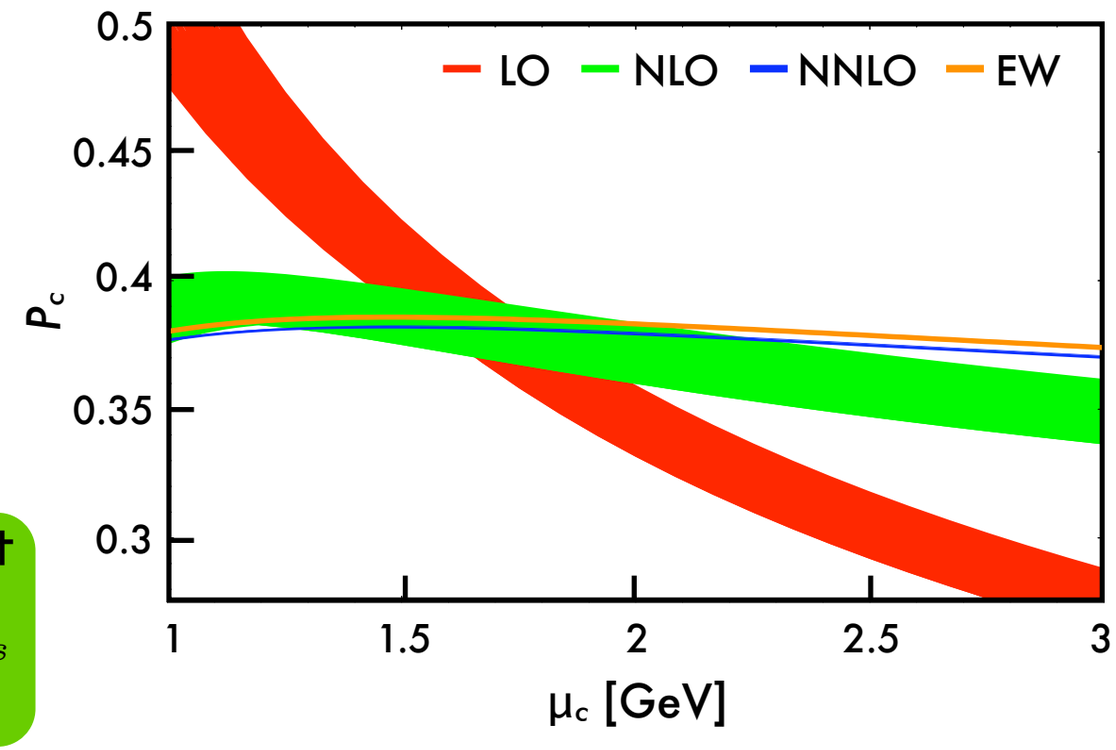
*Mescia & Smith '07

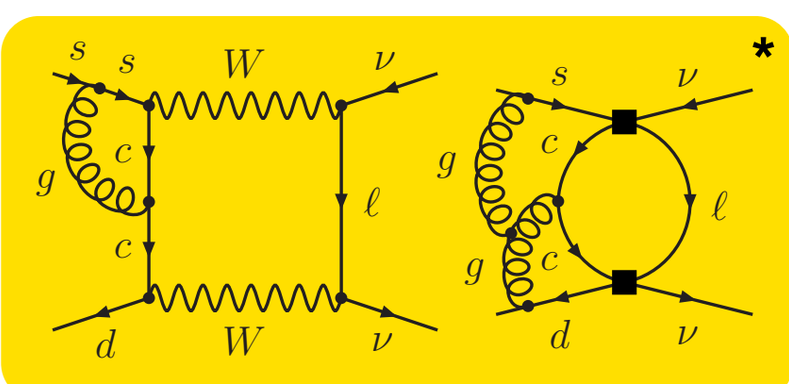
SM prediction of $K^+ \to \pi^+ \nu\bar{\nu}$: P_c

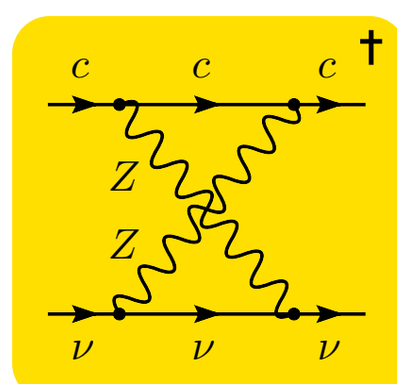
$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}(\gamma)) = \kappa_+ (1 + \Delta_{\rm EM}) \left| \frac{x}{\lambda^5} \right|^2$$

$$x = V_{ts}^* V_{td} X + \lambda^4 \, {\rm Re}(V_{cs}^* V_{cd})(P_c + \delta P_{c,u})$$

$$P_c = 0.372 \pm 0.008_{\rm pert} \overset{\ddagger}{\pm} 0.008_{m_c} \pm 0.009_{\alpha_s}$$ †







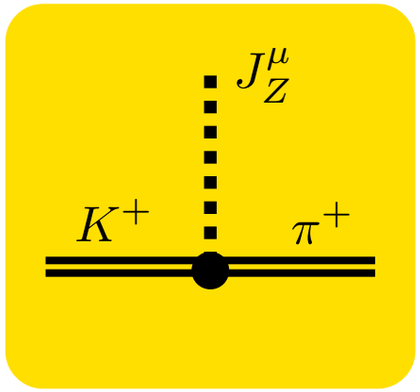
- inclusion of NNLO QCD & EW effects reduce theory error on P_c by factor of 4

*Buras et al. '05, '06 †Brod & Gorbahn '08 ‡Using $m_c(m_c)$ = (1.286 ± 0.013) GeV

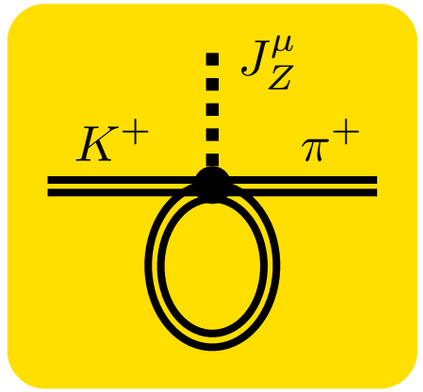
SM prediction of $K^+ \to \pi^+ \nu\bar{\nu}$: $\delta P_{c,u}$

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}(\gamma)) = \kappa_+ (1 + \Delta_{\rm EM}) \left| \frac{x}{\lambda^5} \right|^2$$

$$x = V_{ts}^* V_{td} X + \lambda^4 \, {\rm Re}(V_{cs}^* V_{cd})(P_c + \delta P_{c,u})$$



$O(G_F^2 p^2)$



$O(G_F^2 p^4)$

$$\delta P_{c,u} = 0.04 \pm 0.02$$ *

- effects scale as $O(\pi^2 F_\pi^2/m_c^2) \approx 5\%$ & enhance SM branching ratio by 6%

- theoretical uncertainty related to non-perturbative effects due to charm & up quarks may be reduced by dedicated lattice analysis†

*Isidori et al. '05 †Isidori et al. '05

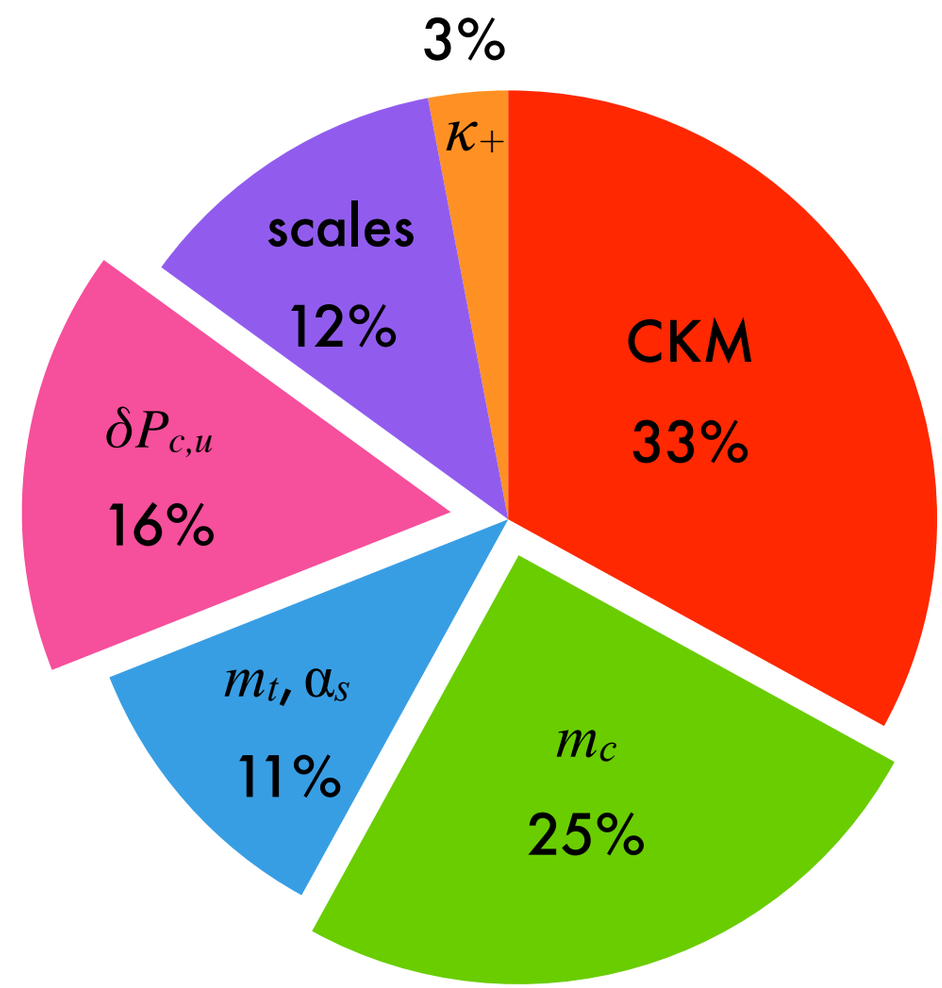
SM prediction(s) of $K^+ \to \pi^+ \nu\bar{\nu}$: error budget

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}(\gamma)) = \{8.57 \pm 0.93, 8.51 \pm 0.73, 8.04 \pm 0.98\} \times 10^{-11}$$

$$m_c(m_c) = (1.30 \pm 0.05)\,\mathrm{GeV}$$

$$m_c(m_c) = (1.286 \pm 0.013)\,\mathrm{GeV}^{*}$$

$$m_c(m_c) = (1.224 \pm 0.017 \pm 0.054)\,\mathrm{GeV}^{\dagger}$$



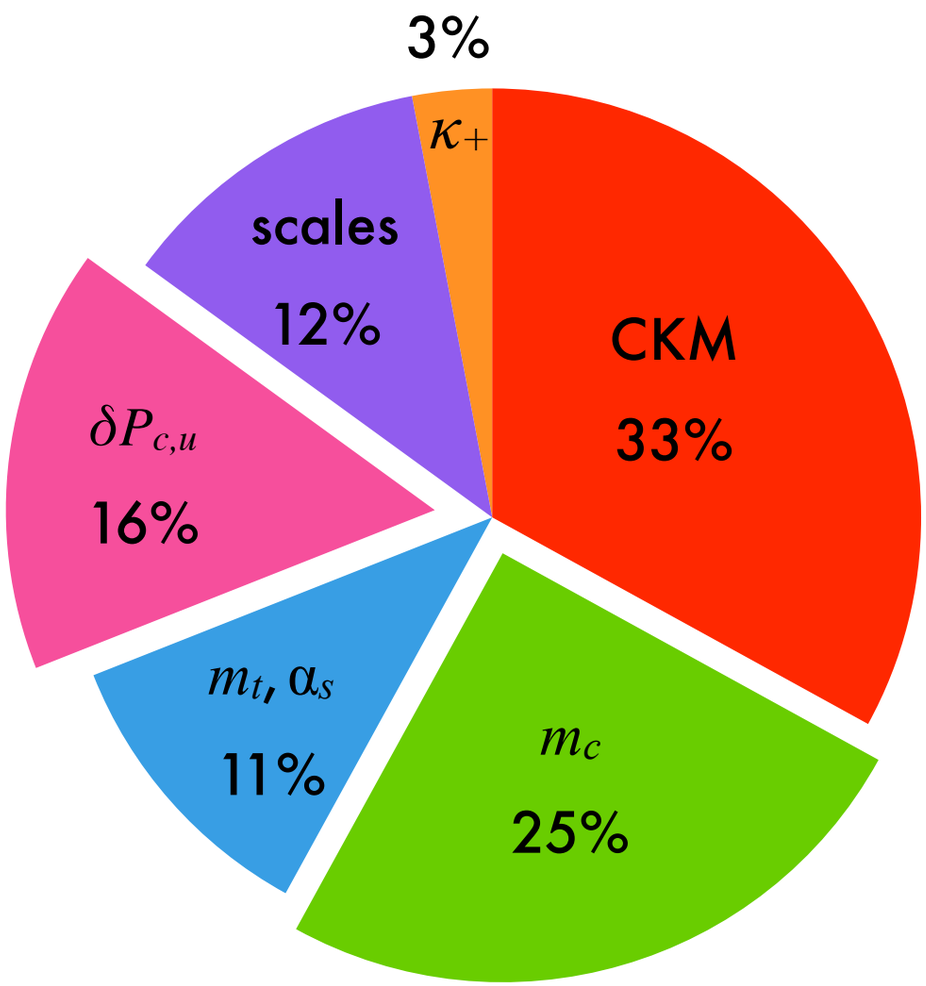
*Kühn et al. '07 †Hoang & Manohar '05

SM prediction of $K^+ \to \pi^+ \nu\bar{\nu}$: upshot

- theoretical progress in $K^+ \to \pi^+ \nu\bar{\nu}$ closely related to precision determination of charm mass

- better knowledge of long-distance effects highly desirable*

- $K^+ \to \pi^+ \nu\bar{\nu}$ new field of interesting physical applications for lattice community



*"If you have to shoot, shoot! Don't talk." – Tuco in *Il buono, Il brutto, Il cattivo*

SM predictions of $K_L \to \pi^0 l^+ l^-$: general structure*

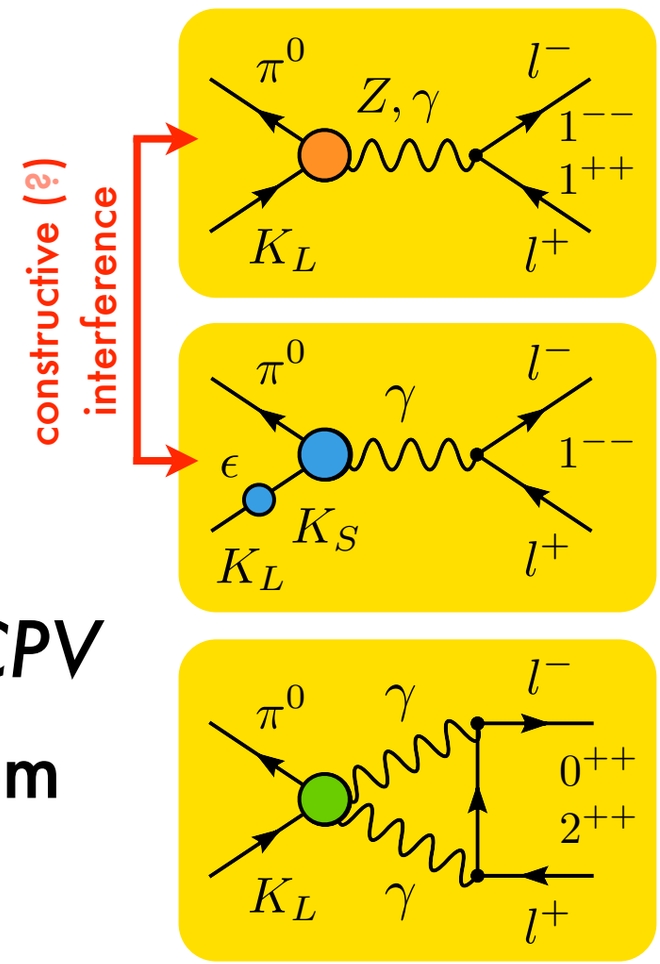
$$\mathcal{B}(K_L \to \pi^0 l^+ l^-) = \left[C^l_{\rm dir}\kappa^2 \pm C^l_{\rm int}|a_S|\kappa + C^l_{\rm ind}|a_S|^2 + C^l_{\gamma\gamma} \right] \times 10^{-12}$$

$$\kappa = \frac{{\rm Im}\,(V^*_{ts}V_{td})}{10^{-4}}\,, \quad |a_S| = 1.2 \pm 0.2\,, \quad y_{7V} \approx 0.73\,, \quad y_{7A} \approx -0.68$$

$$C^e_{\rm dir} \approx 2.3\,(y^2_{7V} + y^2_{7A})\,, \qquad C^\mu_{\rm dir} \approx 0.55\,(y^2_{7V} + 2.33\,y^2_{7A})\,,$$

$$C^e_{\rm int} \approx 8.3\,y_{7V}\,, \qquad C^\mu_{\rm int} \approx 1.9\,y_{7V}\,,$$

$$C^e_{\rm ind} \approx 14.5, \quad C^e_{\gamma\gamma} \approx 0 \qquad C^\mu_{\rm ind} \approx 3.4, \quad C^\mu_{\gamma\gamma} \approx 5.2$$



- three sizable contributions: short-distance direct *CPV* (sensitive to BSM), long-distance indirect *CPV* (from $K_S \to \pi^0 l^+ l^-$ data) & *CPC* one (from $K_L \to \pi^0 \gamma\gamma$ data)

*Buchalla et al. '03, Isidori et al. '04

SM predictions of $K_L \to \pi^0 l^+ l^-$: general structure*

$$\mathcal{B}(K_L \to \pi^0 l^+ l^-) = \left[C^l_{\rm dir}\kappa^2 \pm C^l_{\rm int}|a_S|\kappa + C^l_{\rm ind}|a_S|^2 + C^l_{\gamma\gamma} \right] \times 10^{-12}$$

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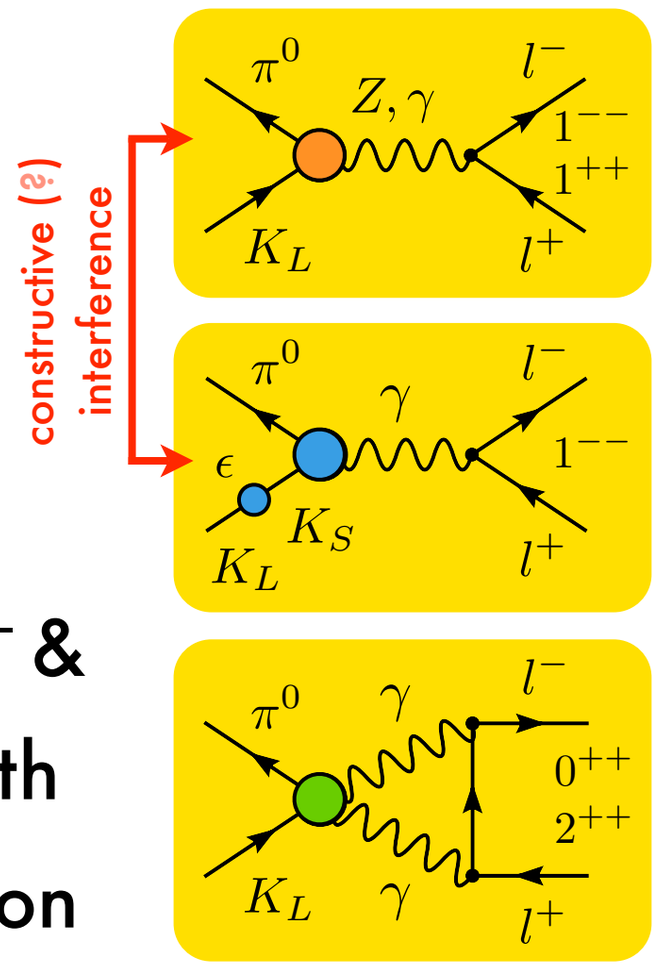
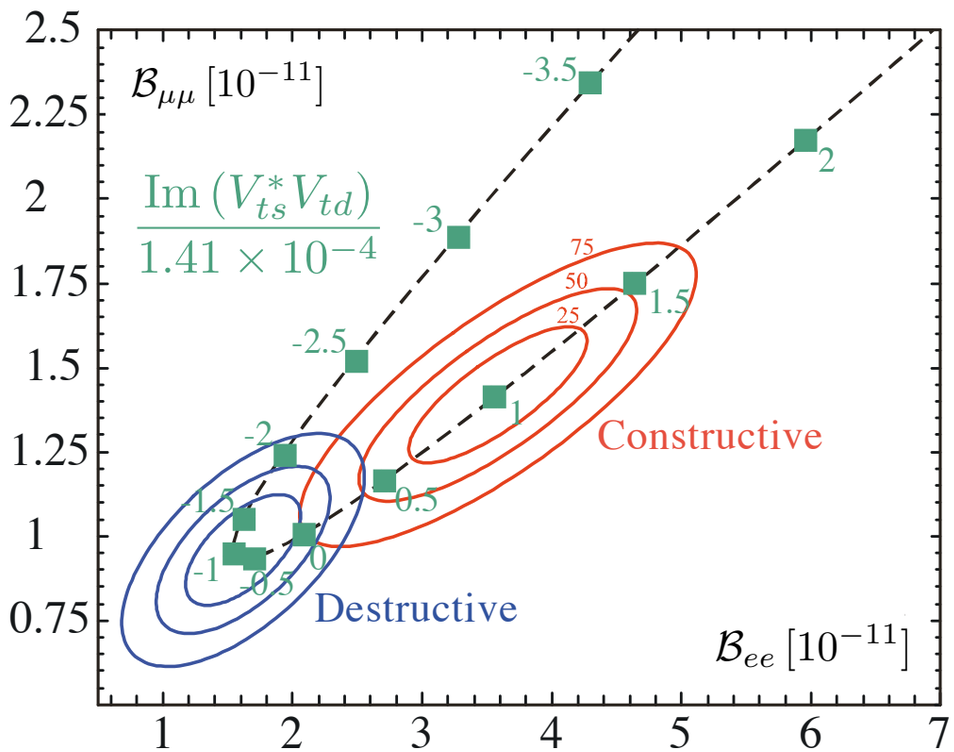
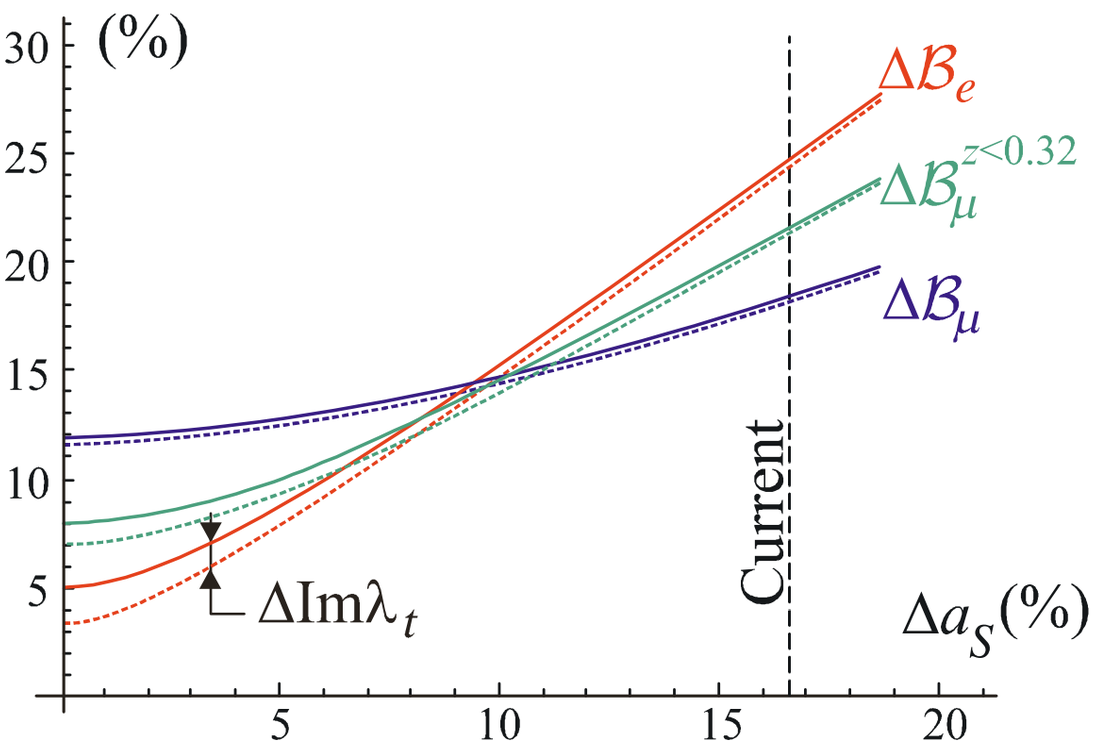


- phase-space suppression of factor 4 between e^+e^- & $\mu^+\mu^-$; contribution $K_L \to \pi^0(\gamma\gamma)_{J=0,2} \to \pi^0(l^+l^-)_{J=0,2}$ with $J = 0^{++}$ negligible for e^+e^- due to helicity suppression

*Buchalla et al. '03, Isidori et al. '04

SM predictions of $K_L \to \pi^0 l^+ l^-$: upshot





- two types of short-distance contributions, namely $y_{7V,7A}$, can be disentangled by measuring both e^+e^- & $\mu^+\mu^-$ rate*
- error on χPT counterterm a_S is currently most limiting factor & better measurements of $K_S \to \pi^0 l^+ l^-$ would be welcome†

*Mescia et al. '06 †Smith '07

Minimal flavor-violation (MFV)

In limit of vanishing Yukawa couplings $Y_{U,D}$ SM aquires a global symmetry $G_F = U(3)^5 \supset SU(3)_Q \times SU(3)_U \times SU(3)_D$

$$\mathcal{L}_{\text{Yukawa}} = \bar{Q}_L Y_D D_R \phi + \bar{Q}_L Y_U U_R \phi_c + \text{h.c.}$$

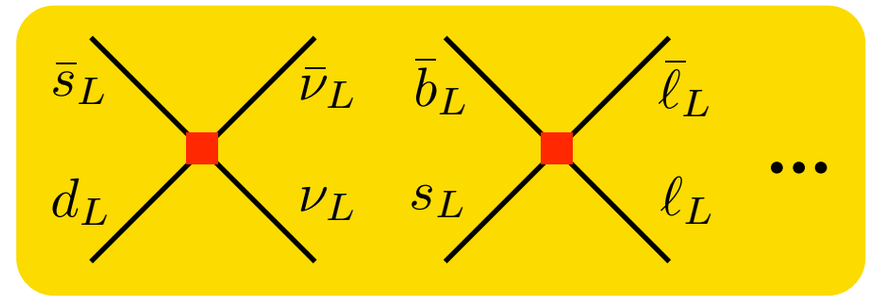
If Yukawa couplings $Y_{U,D}$ transform as $Y_U \sim (3,\bar{3},1)$ & $Y_D \sim (3,1,\bar{3})$ global symmetry G_F is restored

MFV = "effective theory constructed from SM fields & Yukawa couplings $Y_{U,D}$ that is invariant under G_F"*

*D'Ambrosio et al. '02

Typical FCNC dimension-six operator

$$(\bar{Q}_L^i\,(Y_U Y_U^\dagger)_{ij}\,Q_L^j)(\bar{L}_L L_L)$$



$$(Y_U Y_U^\dagger)_{ij} = V^\dagger \mathrm{diag}(y_u^2, y_c^2, y_t^2) V \approx y_t^2 V_{ti}^* V_{tj}$$

$y_t^2 V_{ti}^* V_{tj}$ is effective coupling ruling all FCNCs with external down-type quarks: $K \to \pi\nu\bar{\nu}$, $\bar{B} \to X_s\gamma$, $\bar{B} \to X_s l^+ l^-$, ...

Only flavor-independent magnitude of FCNC amplitudes can be modified by BSM contributions to higher-dimensional operators*

*phase measurements $a(B \to \psi K_s)$, $\Delta M_{B_s}/\Delta M_{B_d}$, ... unaffected in MFV

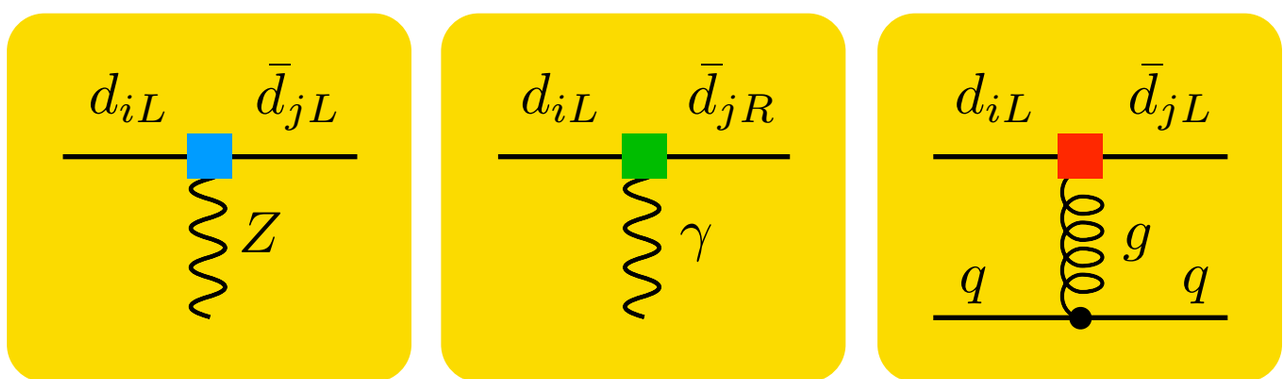
Constrained MFV (CMFV)*

CMFV ≡ "MFV & no other operators beyond SM ones"*

CMFV = "MFV under assumption of single ϕ doublet";

large $\tan\beta$ effects in 2HDM/MSSM not covered by CMFV

- D=4 effective FCNC Z-vertex: $C = C_{SM} + \Delta C$
- D=5 (chromo)magnetic operators: $C_7^{eff} = C_{7,SM}^{eff} + \Delta C_7^{eff}$, ...
- D=6 subleading penguins and EW boxes: $E = E_{SM} + \Delta E$, ...



*Buras et al. '00, ..., Blanke et al. '06

CMFV parameters

$$X(v) = C(v) + B^{\nu\nu}(v)$$
$$Y(v) = C(v) + B^{\ell\ell}(v)$$
$$Z(v) = C(v) + \frac{1}{4}D(v)$$

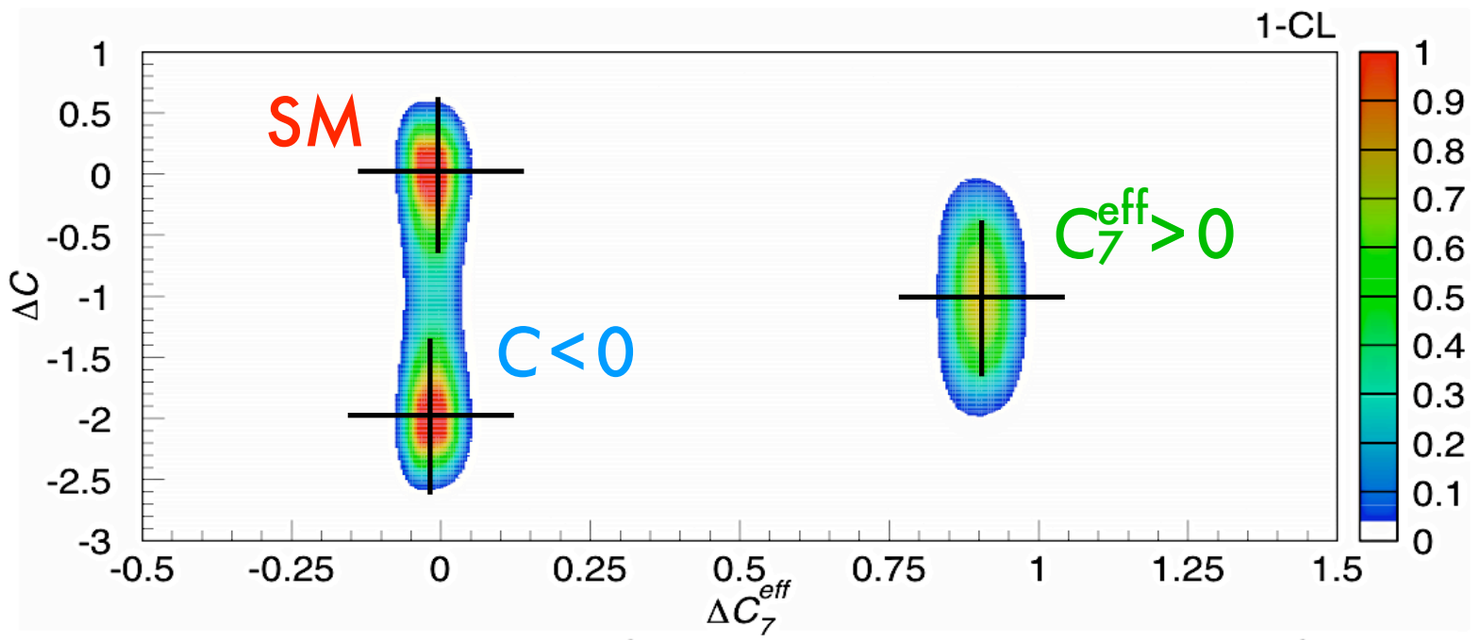
K-$\bar{K}$ mixing ($\lvert\epsilon_K\rvert$)	$S(v)$
$B_{d,s}$-$\bar{B}_{d,s}$ mixing ($\Delta M_{B_{d,s}}$)	$S(v)$
$K \to \pi\nu\bar{\nu}, \bar{B} \to X_{d,s}\nu\bar{\nu}$	$X(v)$
$K_L \to \mu^+\mu^-, B_{d,s} \to \mu^+\mu^-$	$Y(v)$
$K_L \to \pi^0\ell^+\ell^-$	$Y(v), Z(v), E(v)$
$\epsilon'/\epsilon, \lvert\Delta S\rvert = 1$	$X(v), Y(v), Z(v), E(v)$
non-leptonic $\lvert\Delta B\rvert = 1$	$X(v), Y(v), Z(v), E(v), E'(v)$
$\bar{B} \to X_s\gamma$	$D'(v), E'(v)$
$\bar{B} \to X_s g$	$E'(v)$
$\bar{B} \to X_s\ell^+\ell^-$	$Y(v), Z(v), E(v), D'(v), E'(v)$

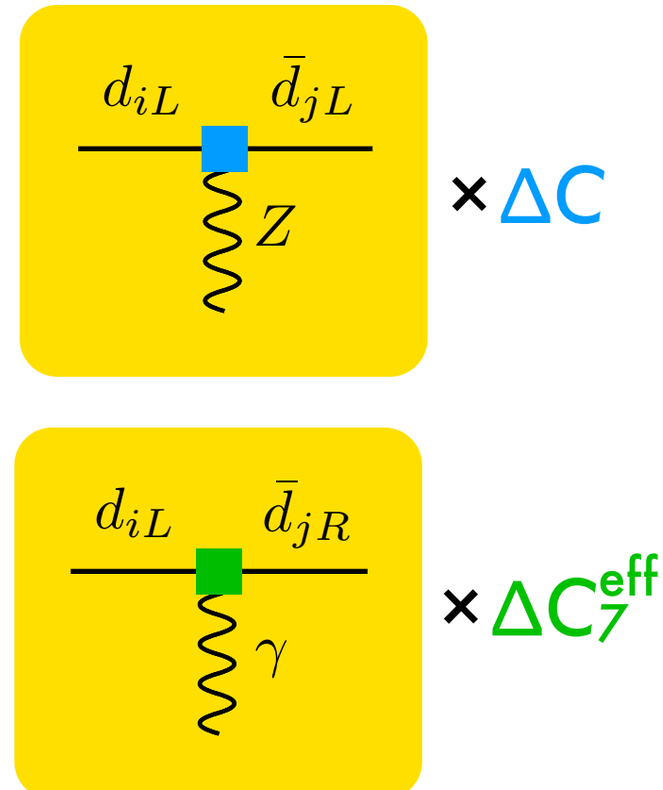
drop as $O(10^{-2})$

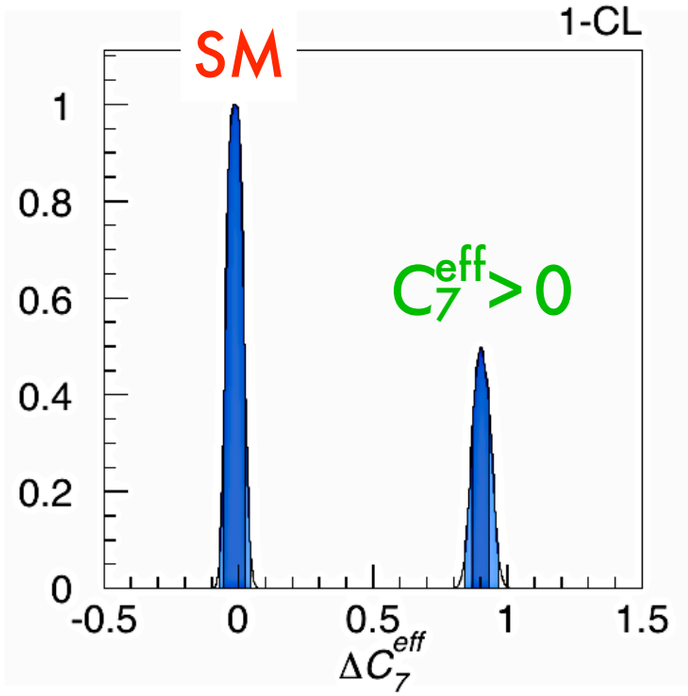
dominated by Z-penguin

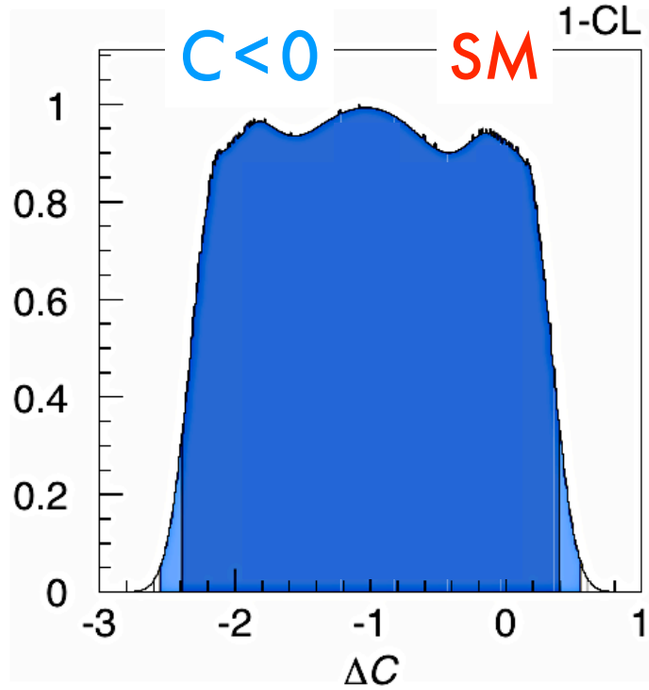
trade for $C_7^{\rm eff}(\mu_b)$

CMFV: combining $\bar{B}\to X_s\gamma$ & $\bar{B}\to X_s l^+ l^-$





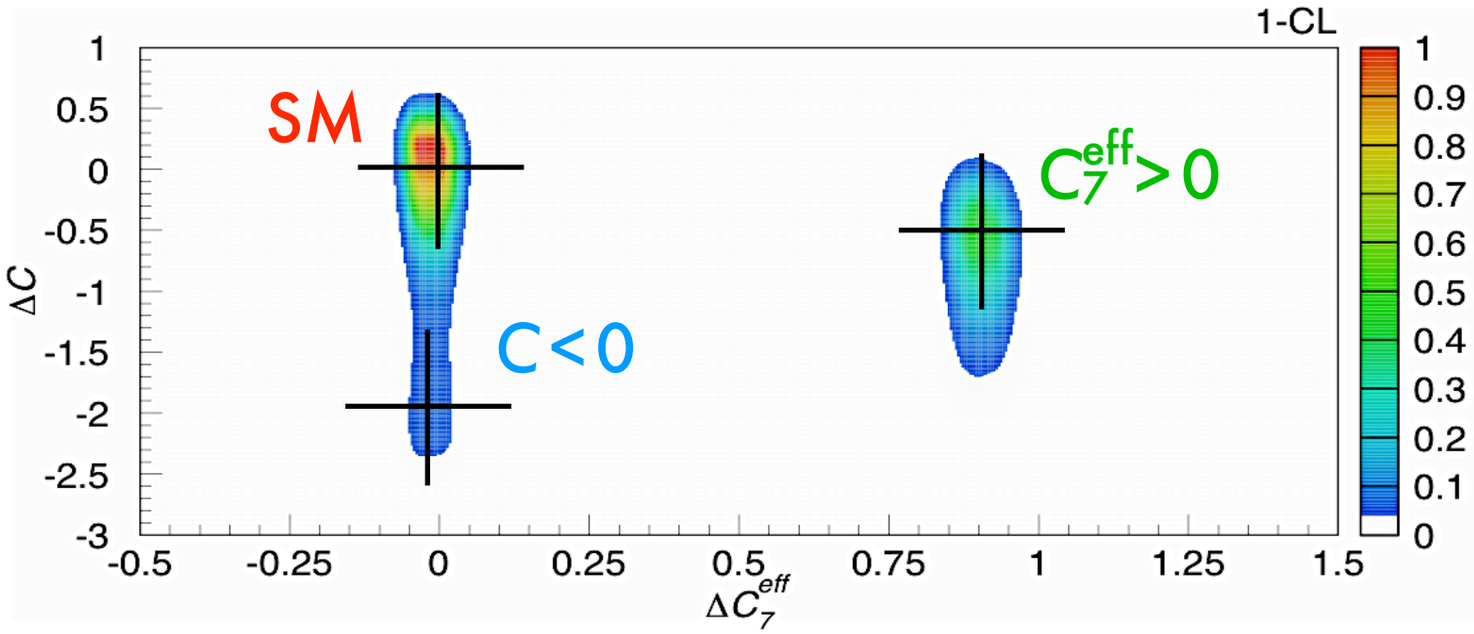


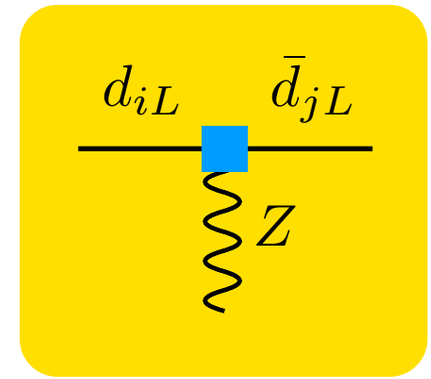


- opposite sign of C_7^{eff} disfavored by $\bar{B}\to X_s l^+ l^-$ measurements*

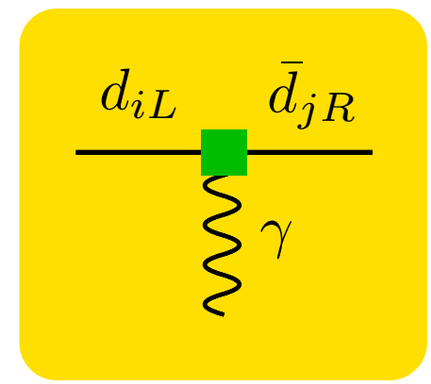
*Gambino et al. '04

CMFV: combining $\bar{B}\to X_s\gamma, l^+l^-$ & $K^+\to\pi^+\nu\bar{\nu}$

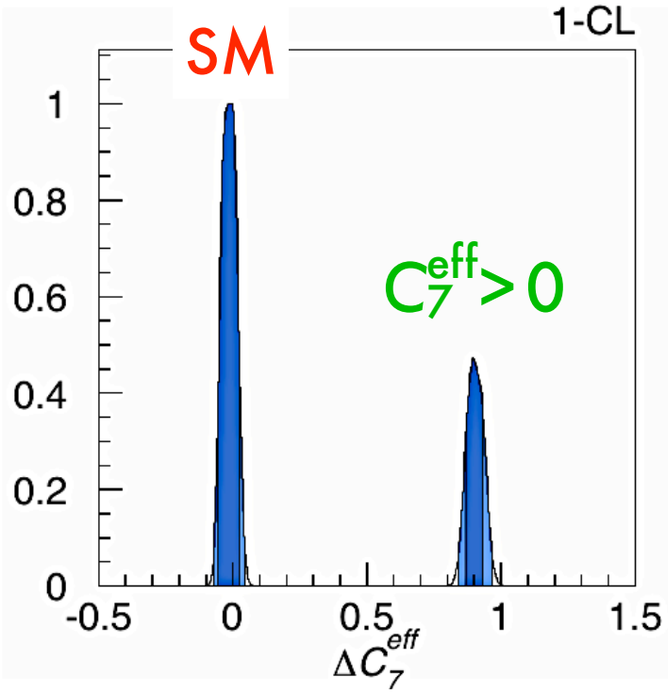


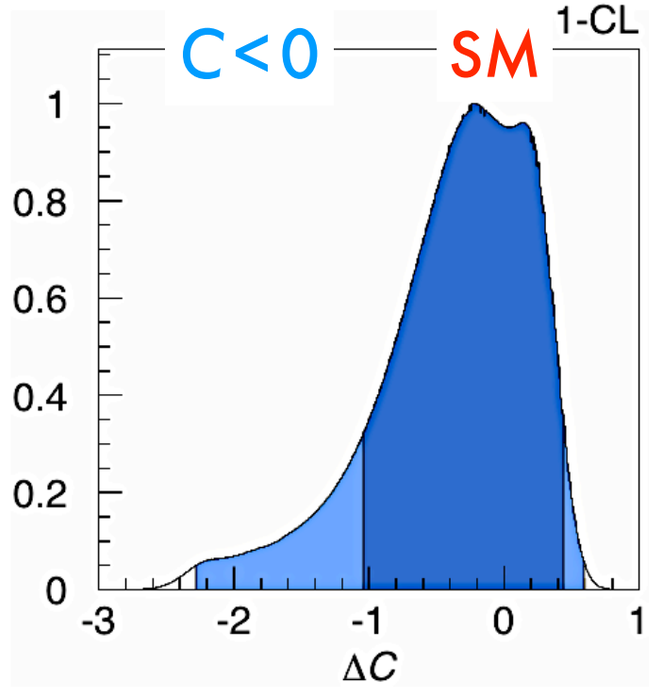


$\times\Delta C$



$\times\Delta C_7^{eff}$

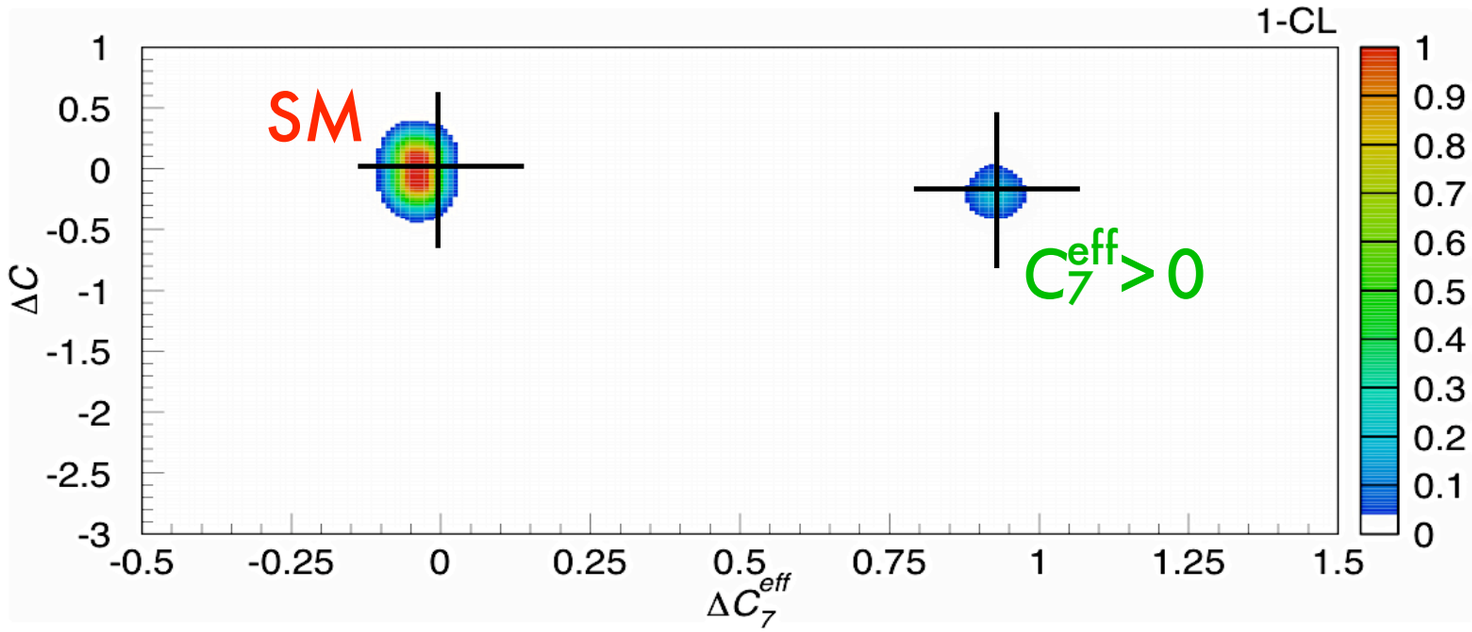


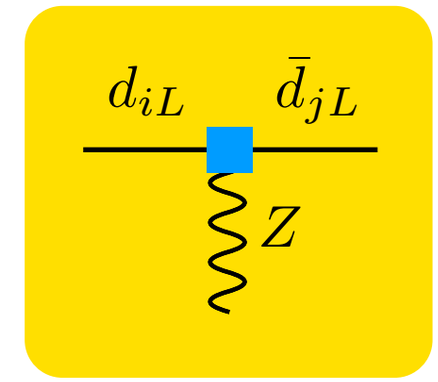


- large destructive Z-penguin allowed by flavor constraints*

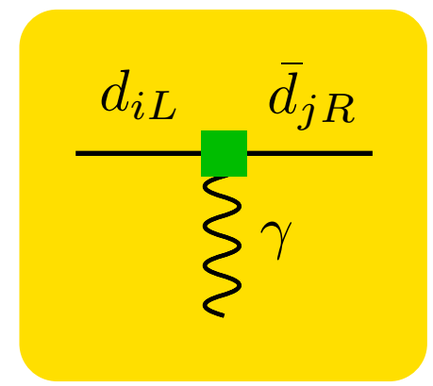
*Bobeth et al. '05

CMFV: combining $\bar{B}\to X_s\gamma, l^+l^-$ & $K^+\to\pi^+\nu\bar{\nu}$

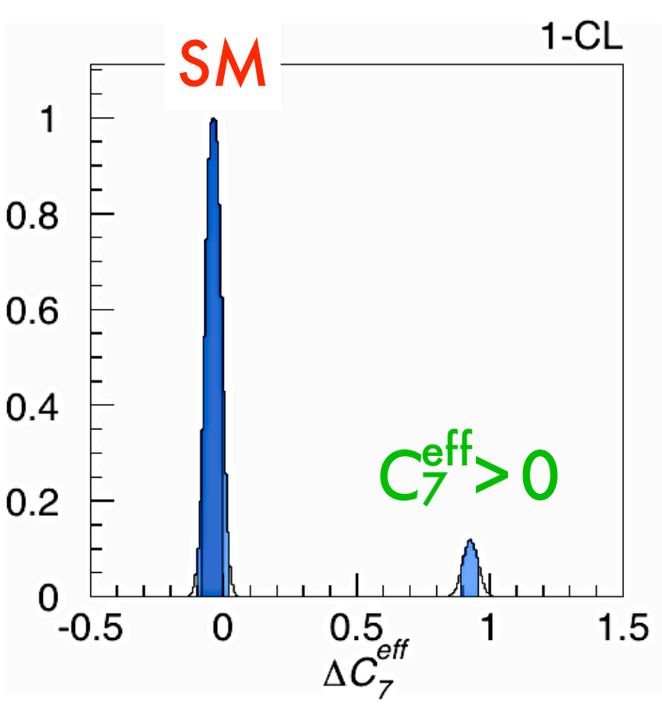


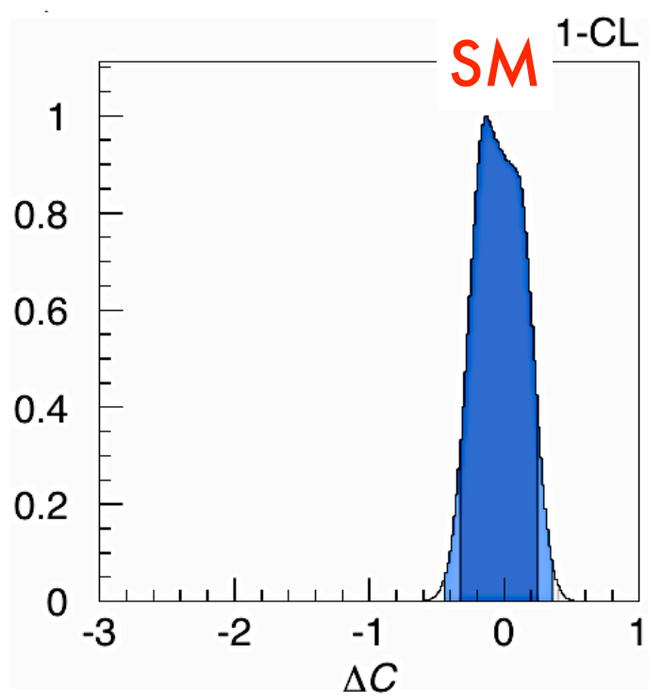


$\times\Delta C$



$\times\Delta C_7^{eff}$

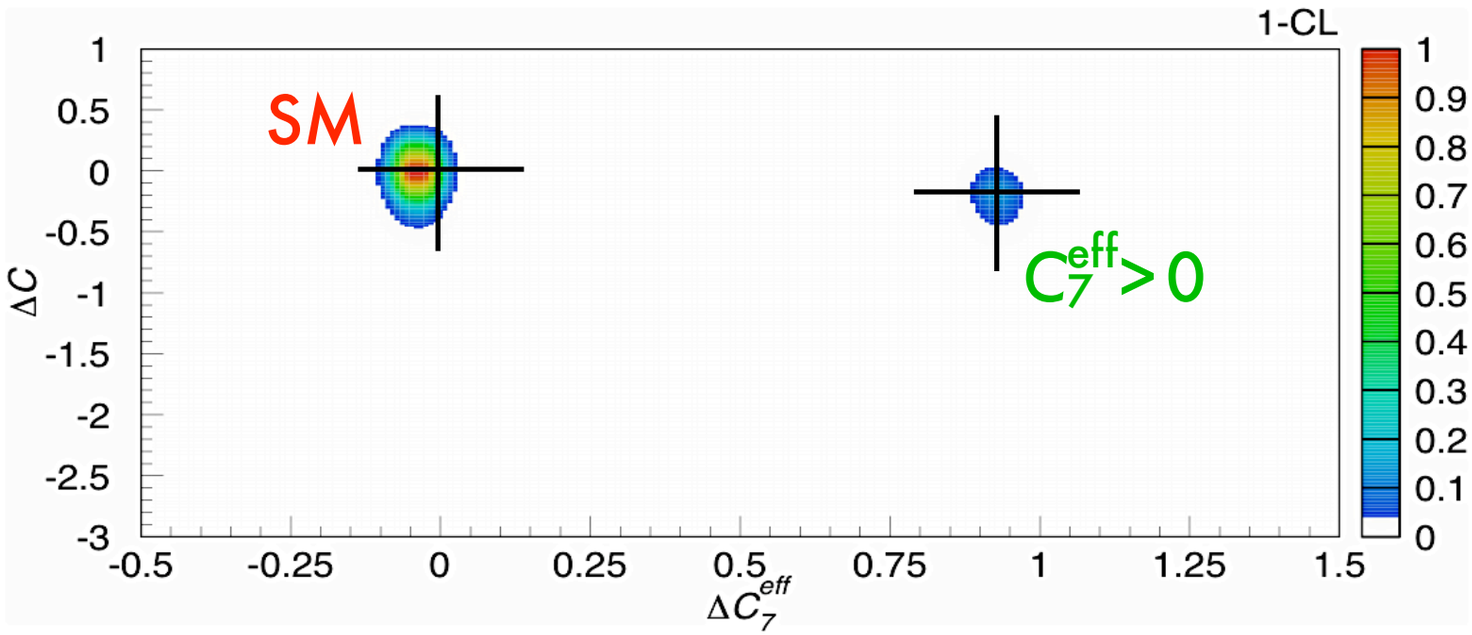


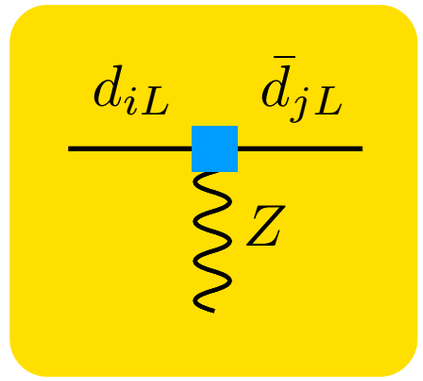


- 2015 (?): measurement of $K^+\to\pi^+\nu\bar{\nu}$ close to SM excludes $\Delta C\approx -2$*

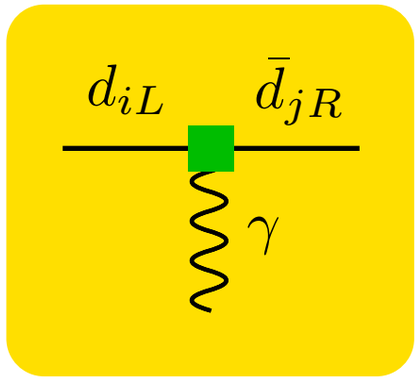
*Bobeth et al. '05

CMFV: combining $\bar{B}\to X_s\gamma, l^+l^-$ & $Z\to b\bar{b}$

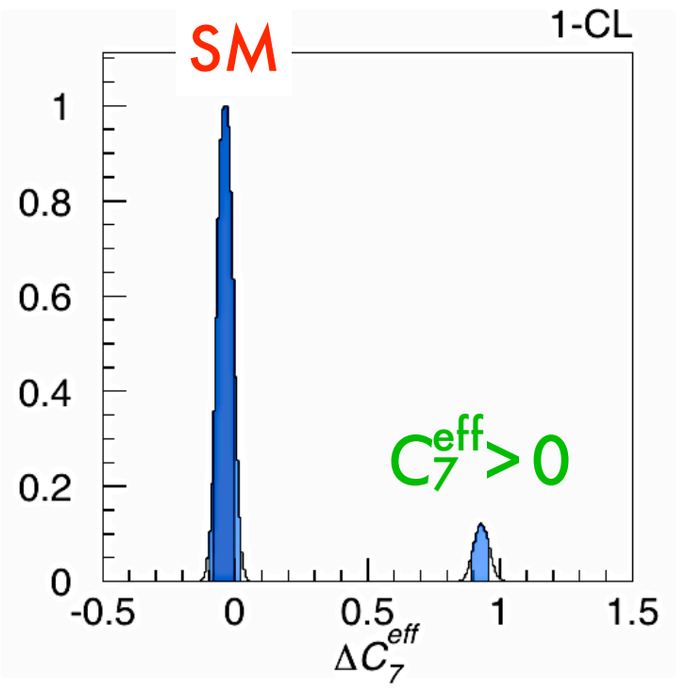


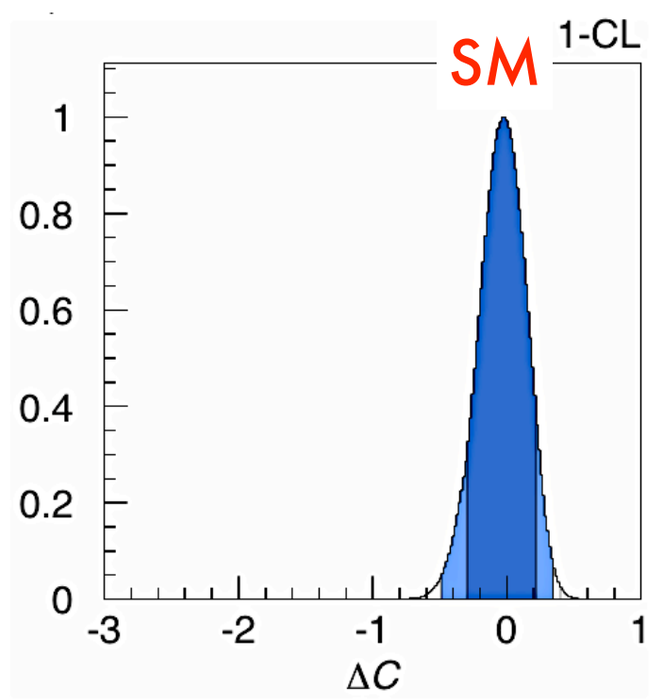


$\times\Delta C$



$\times\Delta C_7^{eff}$

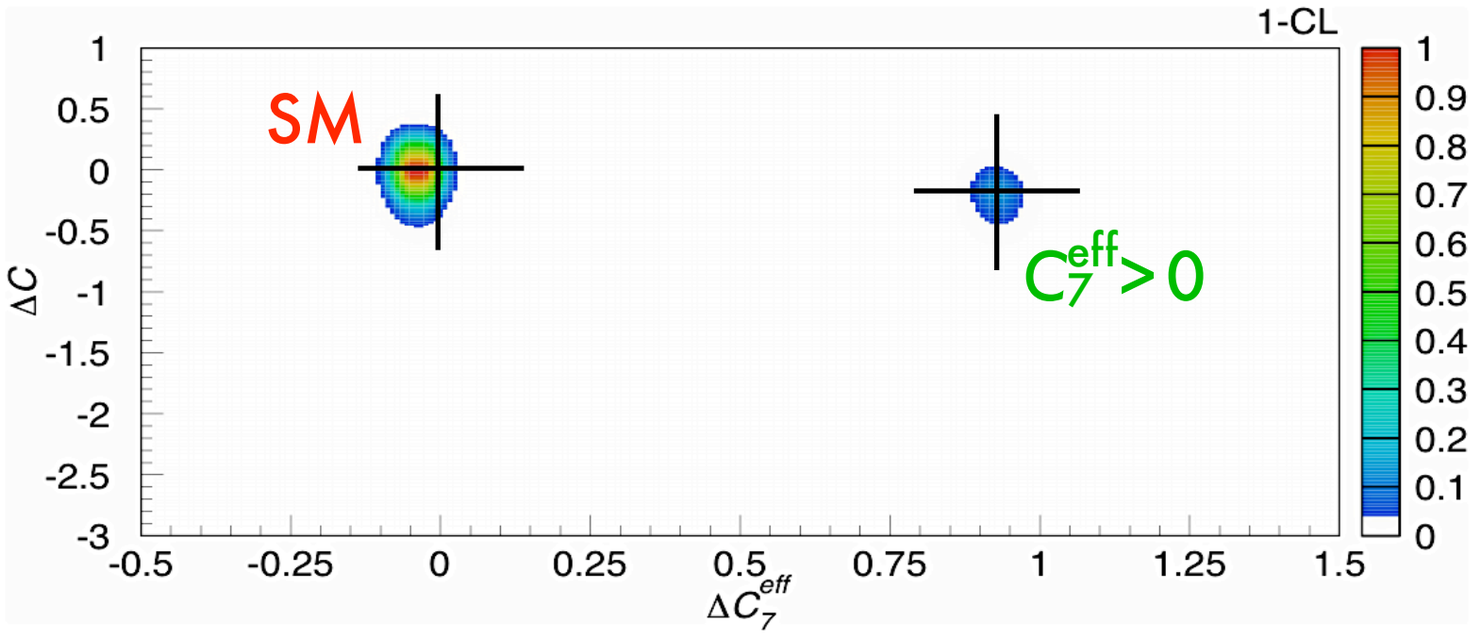


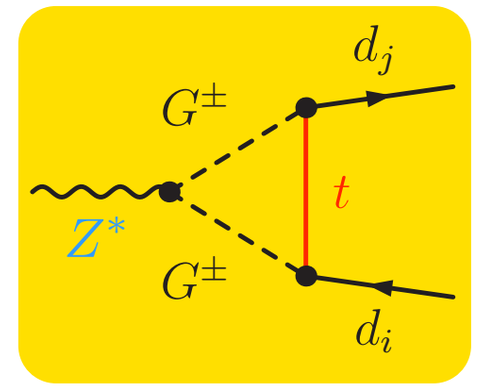


- existing $Z\to b\bar{b}$ data rule out large CMFV Z-penguin*

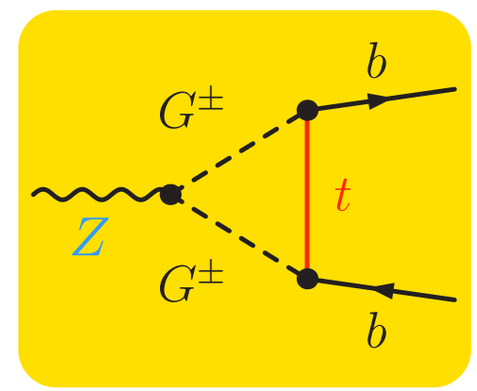
*UH & Weiler '07

CMFV: combining $\bar{B}\to X_s\gamma, l^+l^-$ & $Z\to b\bar{b}$

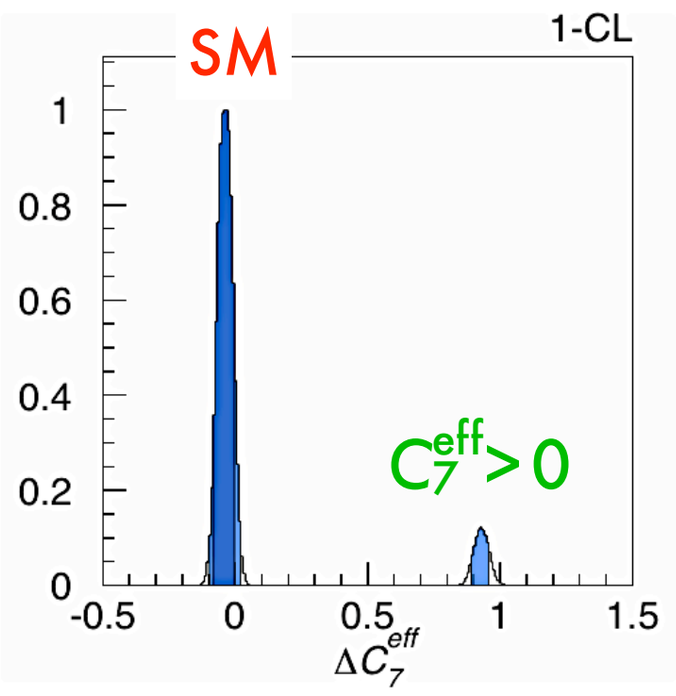


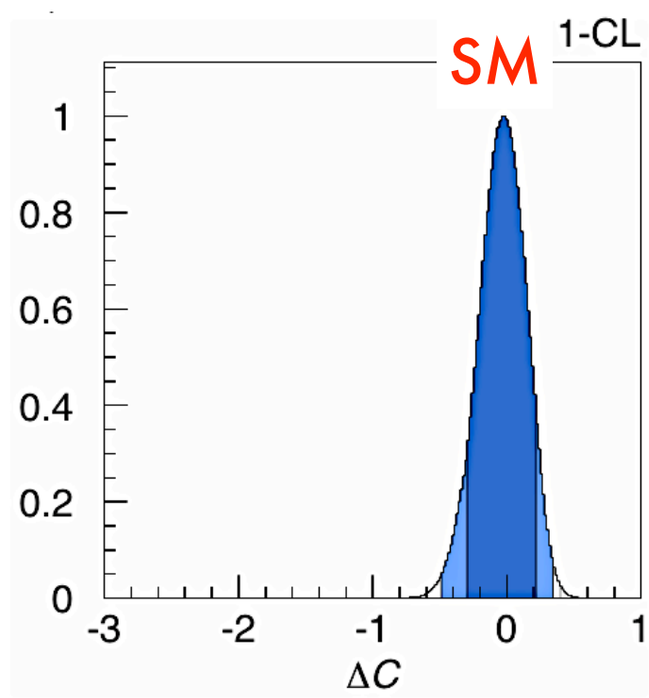


$\propto V_{tj}^* V_{ti} \frac{m_t^2}{M_W^2}$



$\propto |V_{tb}|^2 \frac{m_t^2}{M_W^2}$

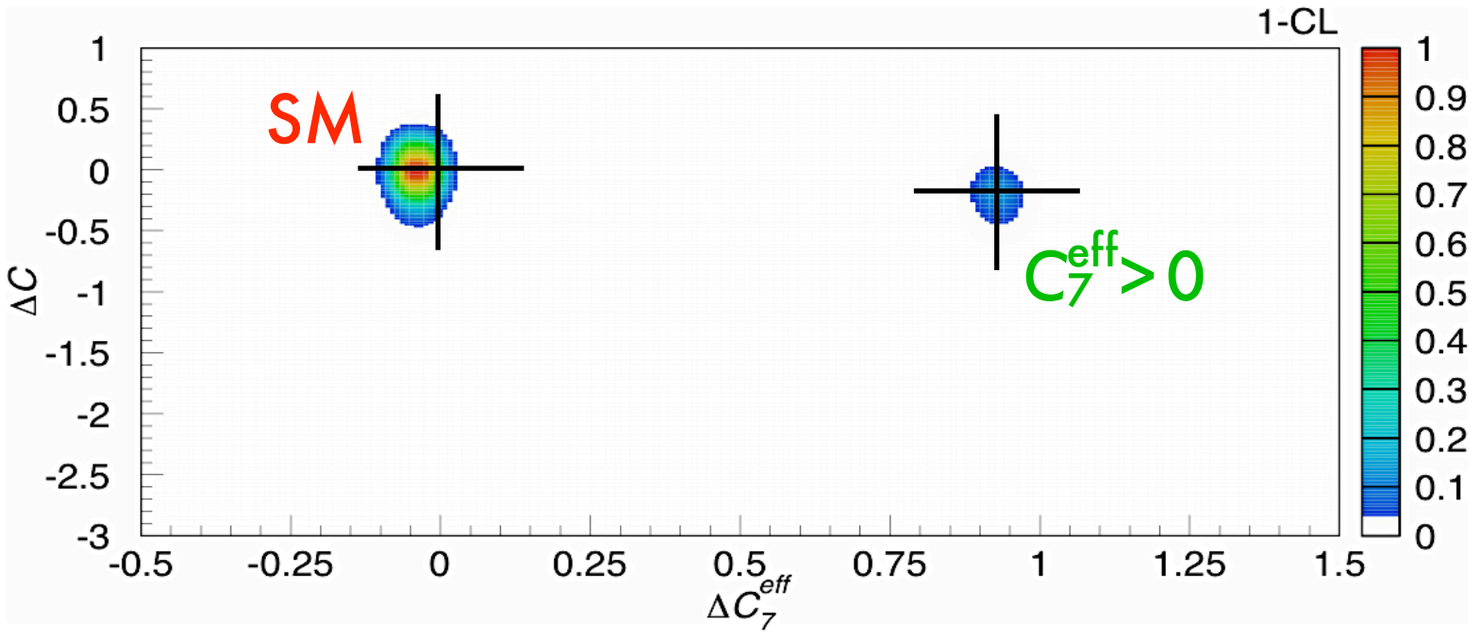


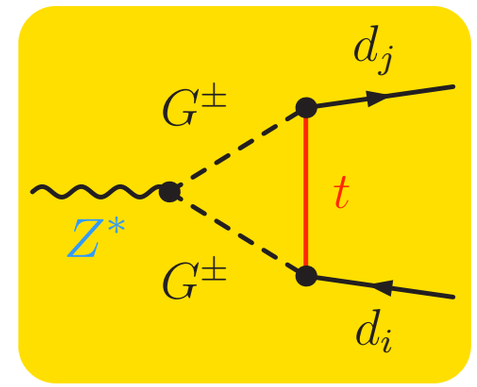


- based on observation that in CMFV $Z\to b\bar{b}$, $d_i\bar{d}_j$ are "identical"*

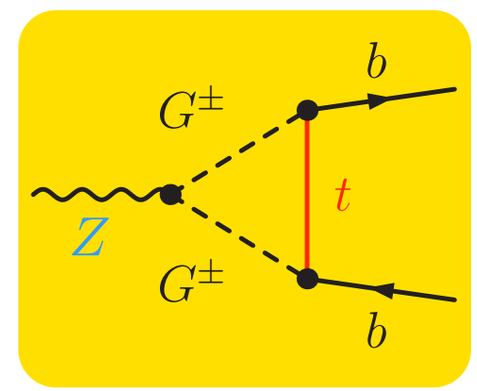
*UH & Weiler '07

CMFV: combining $\bar{B}\to X_s\gamma, l^+l^-$ & $Z\to b\bar{b}$

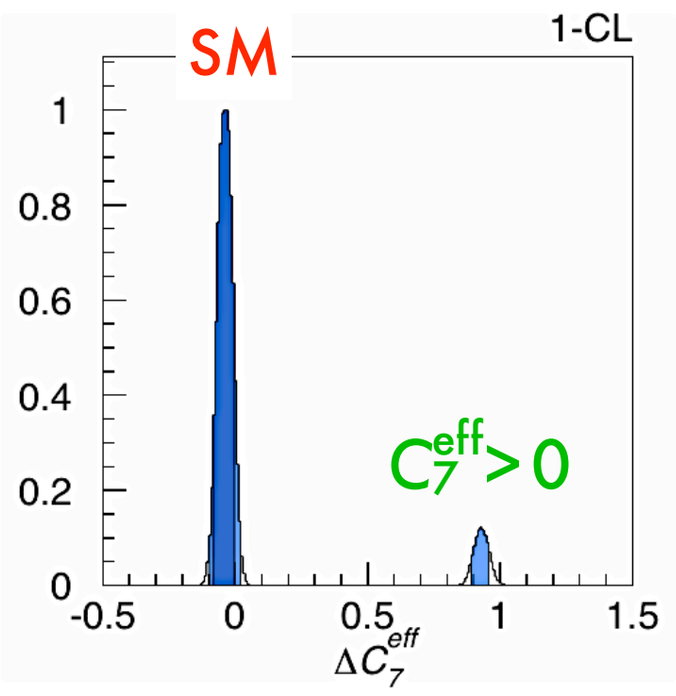


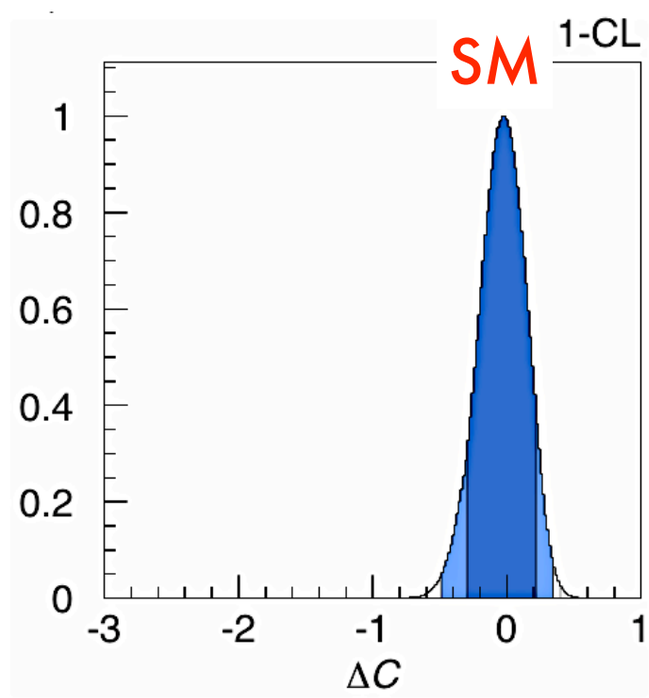


$\propto V_{tj}^* V_{ti} \frac{m_t^2}{M_W^2}$



$\propto |V_{tb}|^2 \frac{m_t^2}{M_W^2}$

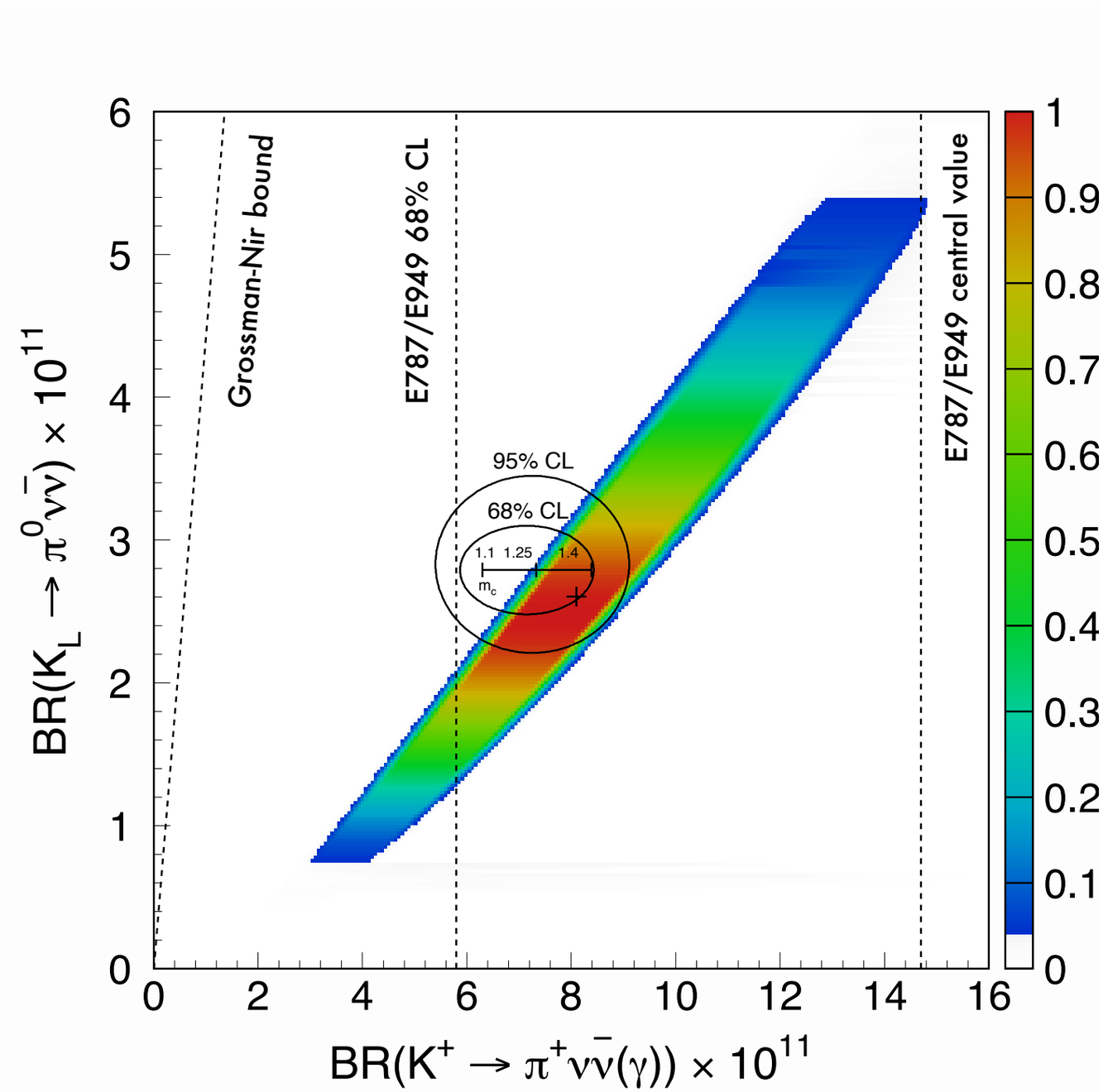




- correlation shown to hold in CMFV 2HDM, MSSM, UED5 & LHT*

*UH & Weiler '07

CMFV: $K^+ \to \pi^+ \nu\bar{\nu}$ vs. $K_L \to \pi^0 \nu\bar{\nu}$



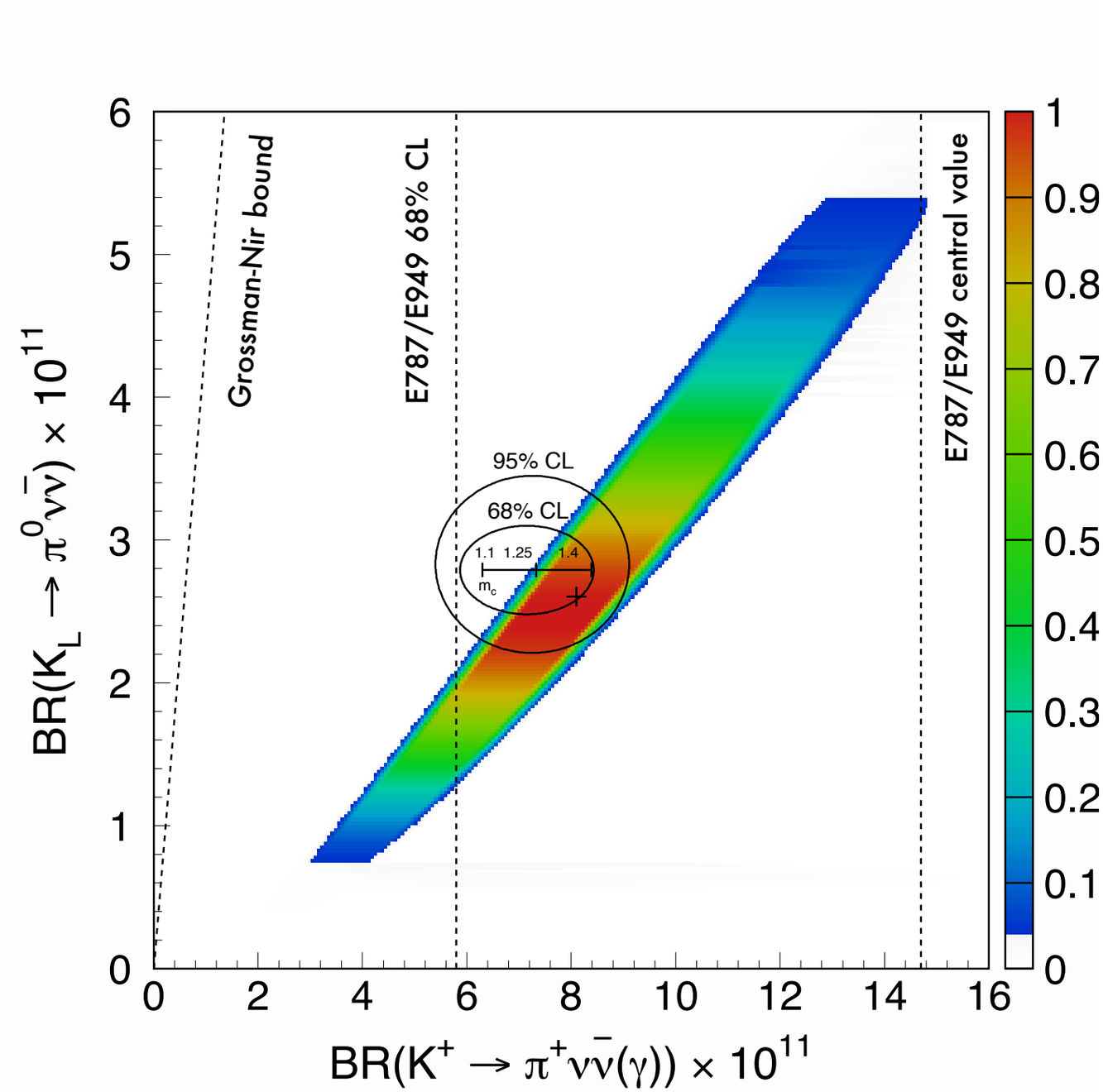
$$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})_{\rm CMFV} \overset{\dagger}{=} [1.55, 4.38] \times 10^{-11} \quad (95\%\,{\rm CL})$$

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})_{\rm CMFV} \overset{\dagger}{=} [4.29, 10.72] \times 10^{-11} \quad (95\%\,{\rm CL})$$

- violation of bounds may signal new flavor & *CP* violation || new operators || sizable box effects

*UH & Weiler '07; UH '07 †Assuming vanishing box contributions

MFV: $K^+ \to \pi^+ \nu\bar{\nu}$ vs. $K_L \to \pi^0 \nu\bar{\nu}$



$$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})_{\rm MFV} = [1.55, 4.38] \times 10^{-11} \quad (95\%\,{\rm CL})$$

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})_{\rm MFV} = [4.29, 10.72] \times 10^{-11} \quad (95\%\,{\rm CL})$$

- bounds satisfied in MFV MSSM with $\tan\beta < 30$ ($\Delta B \in [-30^*, 25^\dagger]\%$), UED5 ($\Delta B < +5\%$)‡ & LH ($\Delta B < +15\%$)†

*Buras et al. '01 †Isidori et al. '06 ‡Buras et al. '02 †Bardeen et al. '04

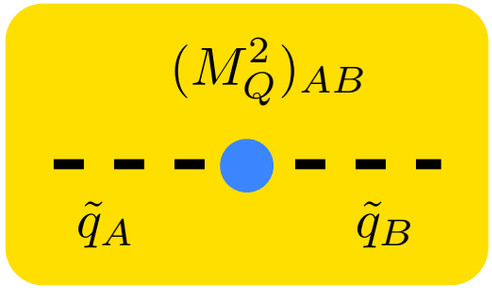
Anatomy of MSSM contributions

- physical 6×6 squark mass matrices after EWSB have non-trivial flavor structure

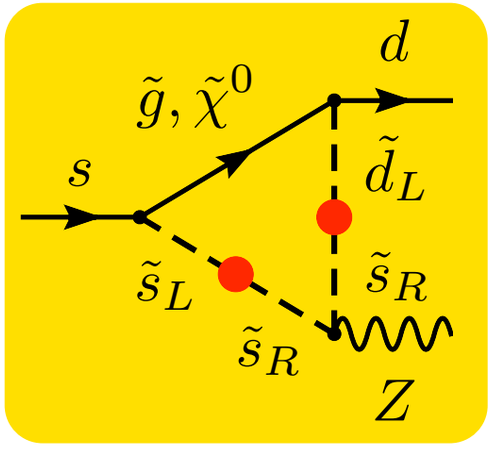
$$\mathcal{M}^2_{D/U} = \begin{pmatrix} (M^2_Q)_{LL} & (M^2_{D/U})_{LR} \\ (M^2_{D/U})^\dagger_{LR} & (M^2_U)_{RR} \end{pmatrix}$$

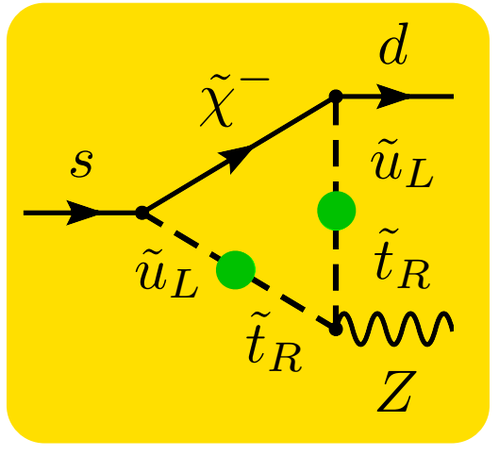
$$(M^2_{D/U})_{LR} = M_{d/u}\,(A_{D/U} - \mu \tan\beta / \cot\beta)$$

- after "diagonalizing" interaction vertices, "flavor non-diagonality" resides in squark propagators & can be described by mass insertions (MIs)



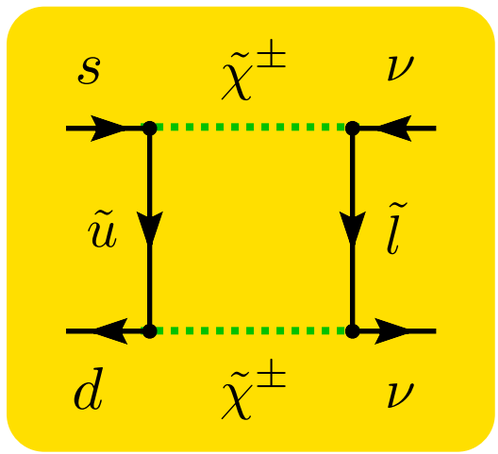
- $SU(2)_L$ breaking induced only via Z-penguins envolving $\tilde{g}/\tilde{\chi}^0$-$\tilde{d}$ || $\tilde{\chi}^\pm$-$\tilde{u}$ exchange & 2 MIs*

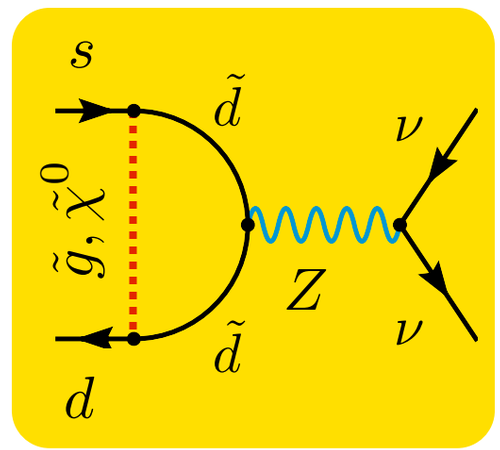


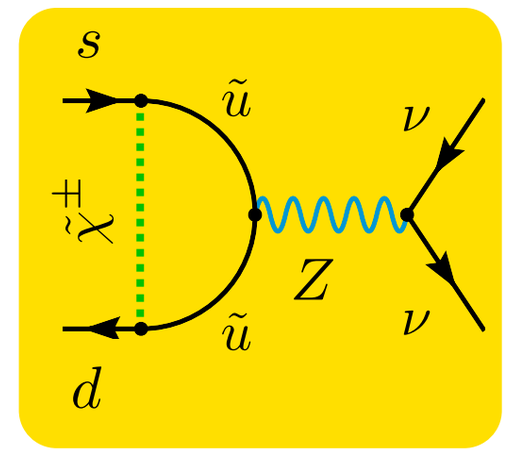


*Colangelo & Isidori '98

Anatomy of MSSM contributions cont'd*







- boxes do not require $SU(2)_L$ breaking, but are suppressed by additional heavy propagators & therefore numerically subleading
- Z-penguins with down $(LR)^2$ insertions are tightly constrained by $\Delta S = 2$ transitions, while those involving up $(LR)^2$ insertions are not
- dominant contributions to $K \to \pi\nu\bar{\nu}$ arises typically from $\tilde{\chi}^{\pm}$-$\tilde{u}$ diagrams

$$\left(X^{\tilde{\chi}^{\pm}}_{\rm MSSM}\right)_L \propto \frac{(M_U^2)^{31}_{LR}(M_U^2)^{32*}_{LR}}{\tilde{m}^4}$$

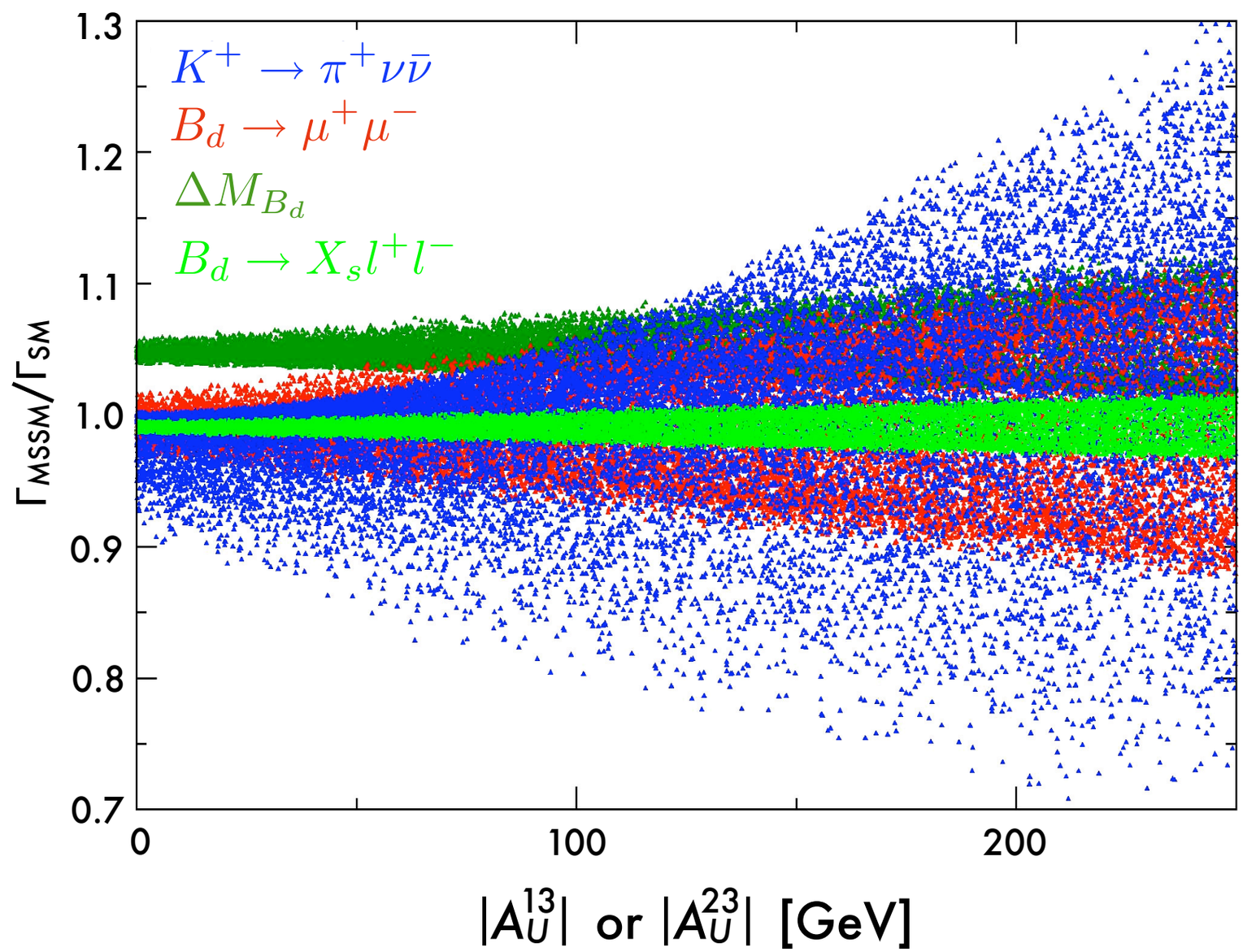
*Nir & Worah '97, Buras et al. '97, Colangelo & Isidori '98

Sensitivity to trilinear A_U terms*

- due to quadratic dependence, $s \to d\nu\bar{\nu}$ is best probe of A_U terms

fixed MSSM spectrum:

$\tan\beta \in [2, 4]$
$\mu = (500 \pm 10)$ GeV
$M_2 = (300 \pm 10)$ GeV
$\tilde{m}_{u_R} = (600 \pm 20)$ GeV
$\tilde{m}_{q_L} = (800 \pm 20)$ GeV
$\tilde{m} = 2$ TeV



- determination of A_U terms hard for LHCb/SuperB due to limited sensitivity of *B*-physics

*Isidori et al. '06

Sensitivity to trilinear A_U terms*

- same conclusion holds in case of *CP*-violating *K*- & *B*-observables

fixed MSSM spectrum:

$\tan\beta \in [2, 4]$
$\mu = (500 \pm 10)$ GeV
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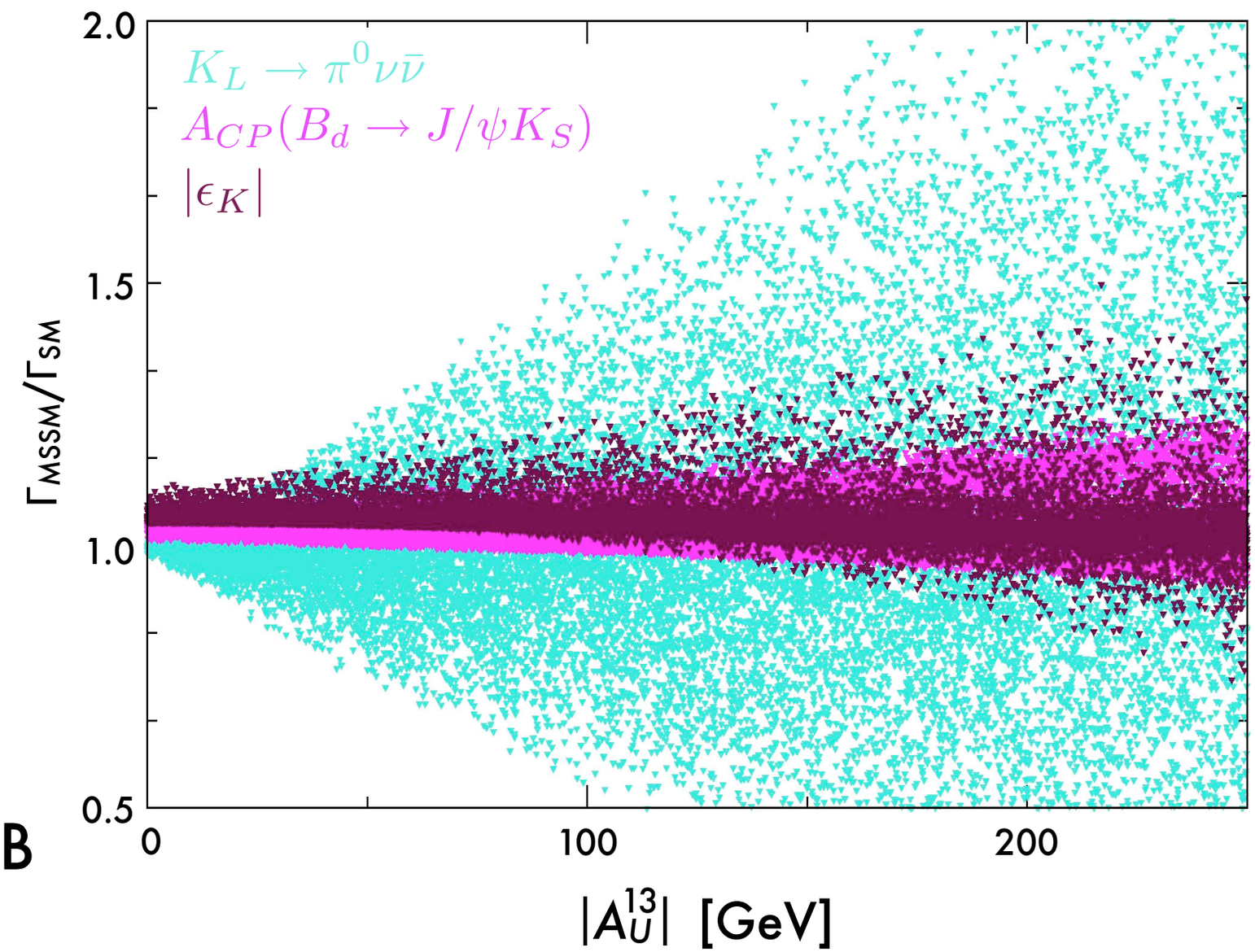
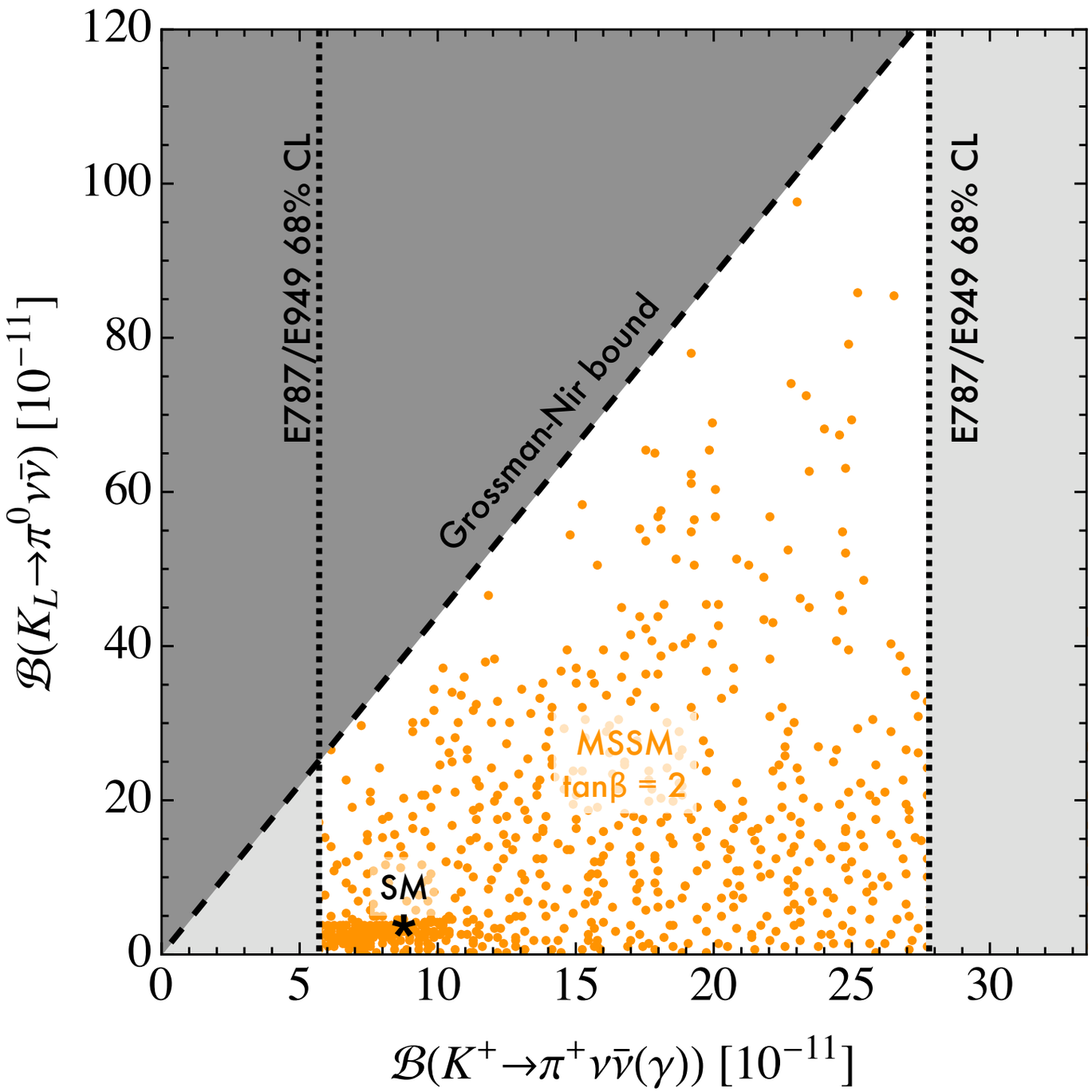
$\tilde{m}_{q_L} = (800 \pm 20)$ GeV
$\tilde{m} = 2$ TeV

- small impact of A_U on $\sin\beta$ & $|\epsilon_K|$ implies complementarity of $K \to \pi\nu\bar{\nu}$ & LHCb/SuperB



*Isidori et al. '06

MSSM & Grossman-Nir bound



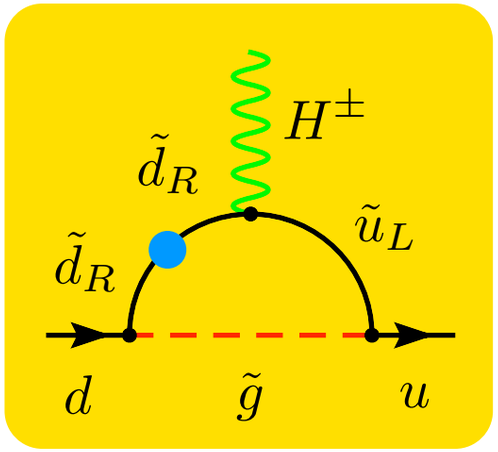
adaptive VEGAS scan* involving all MSSM parameters to search for maximal effects compatible with *K*-, *B*-, EW precision measurements, UFB & CCB stability bounds

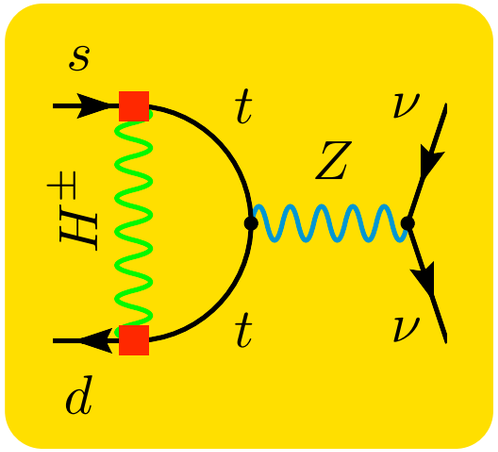
- enhancement by factor 30 possible in $K_L \to \pi^0 \nu\bar{\nu}$ & Grossman-Nir bound, *i.e.*, $B(K_L) \lesssim 4.4\ B(K^+)$,† can be saturated‡
- such spectacular effects in reach of E391a

*Brein '04 †Grossman & Nir '97 ‡Buras et al. '04

MSSM at large tanβ

- effects from neutral Higgs & charginos diagrams negligible both within & beyond MFV*
- beyond MFV strong sensitivity to higher order effects in charged Higgs penguin†
- in constrast to case of $B_{d,s}\to\mu^+\mu^-$, charged Higgs corrections in $K\to\pi\nu\bar{\nu}$ decouple slowly due to logarithmic dependence on $M_{H^\pm}$



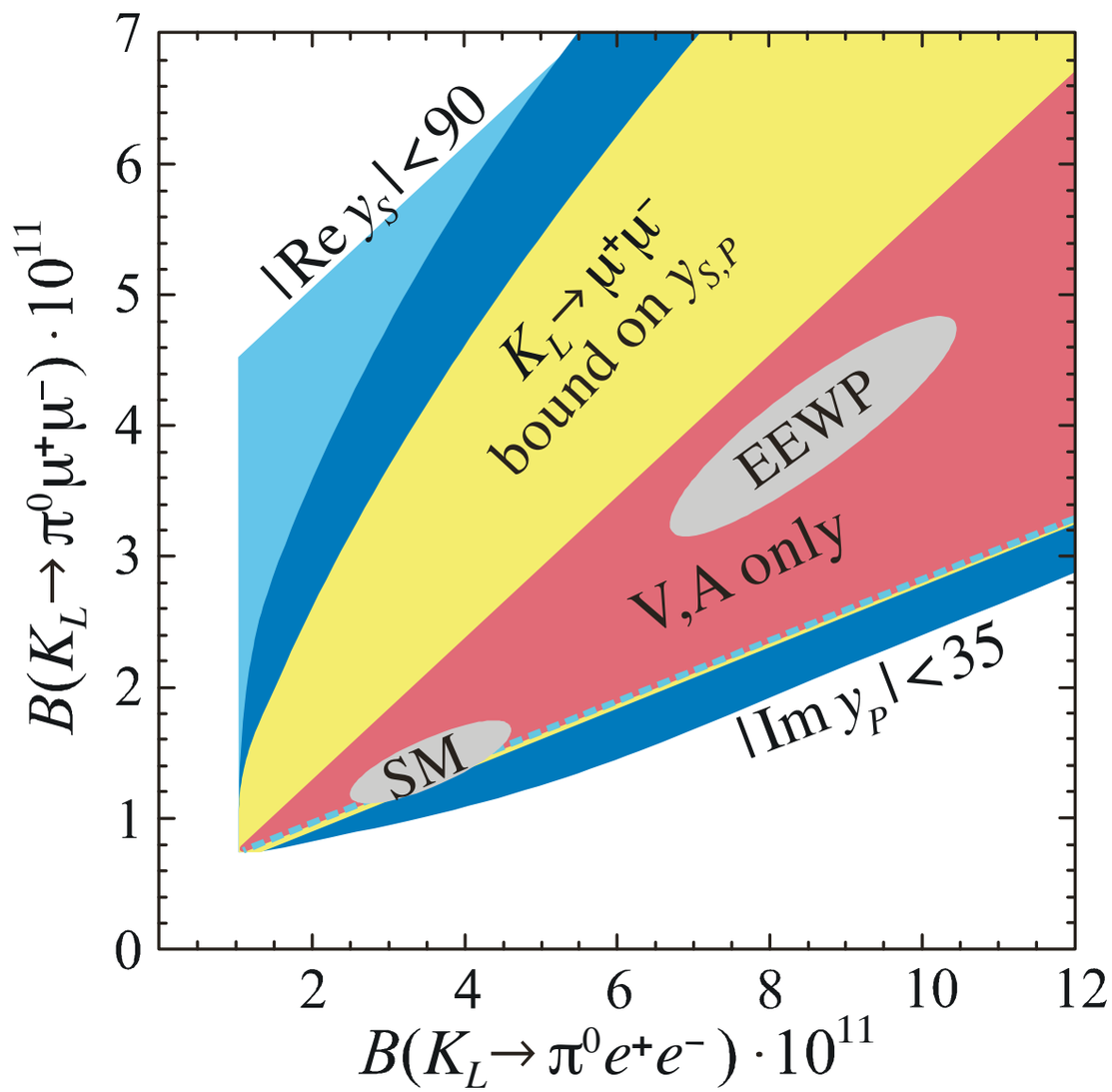


$$\left(X^{H^\pm}_{\rm MSSM}\right)_R \propto \frac{(M_D^2)^{31}_{RR}(M_D^2)^{32}_{RR}}{\tilde{m}^4} \times \frac{m_t^2}{M_{H^\pm}^2}\ln\frac{m_t^2}{M_{H^\pm}^2}\,\frac{m_b^2\tan^4\beta}{M_W^2}$$

*Isidori et al. '06 †Isidori & Paradisi '06

Anatomy of $K_L \to \pi^0 l^+ l^-$ in MSSM beyond MFV*

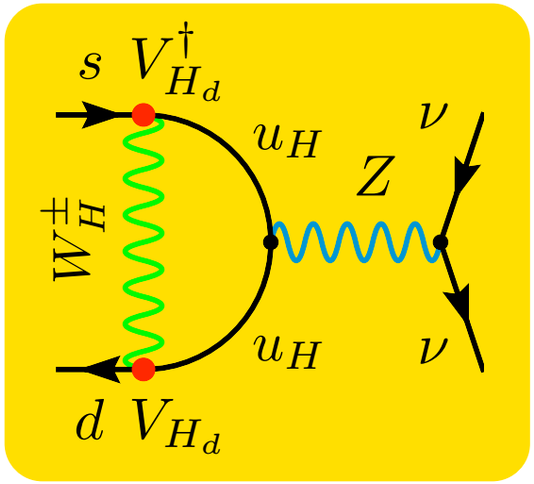
- effects in QCD penguins entirely fixed by data, while effects in EW penguins follow pattern of $K \to \pi\nu\bar{\nu}$ but tend to be smaller
- at large tanβ, scalar & pseudo-scalar operators $Q_S = (\bar{s}d)(\bar{l}l)$ & $Q_P = (\bar{s}d)(\bar{l}\gamma_5 l)$ are enhanced
- being helicity suppressed, y_S & y_P affect only $\mu^+\mu^-$ mode & can lead to enhancement that is correlated with effects in $K_L \to \mu^+\mu^-$



*Mescia et al. '06

Flavor of littlest Higgs with *T*-parity (LHT)*

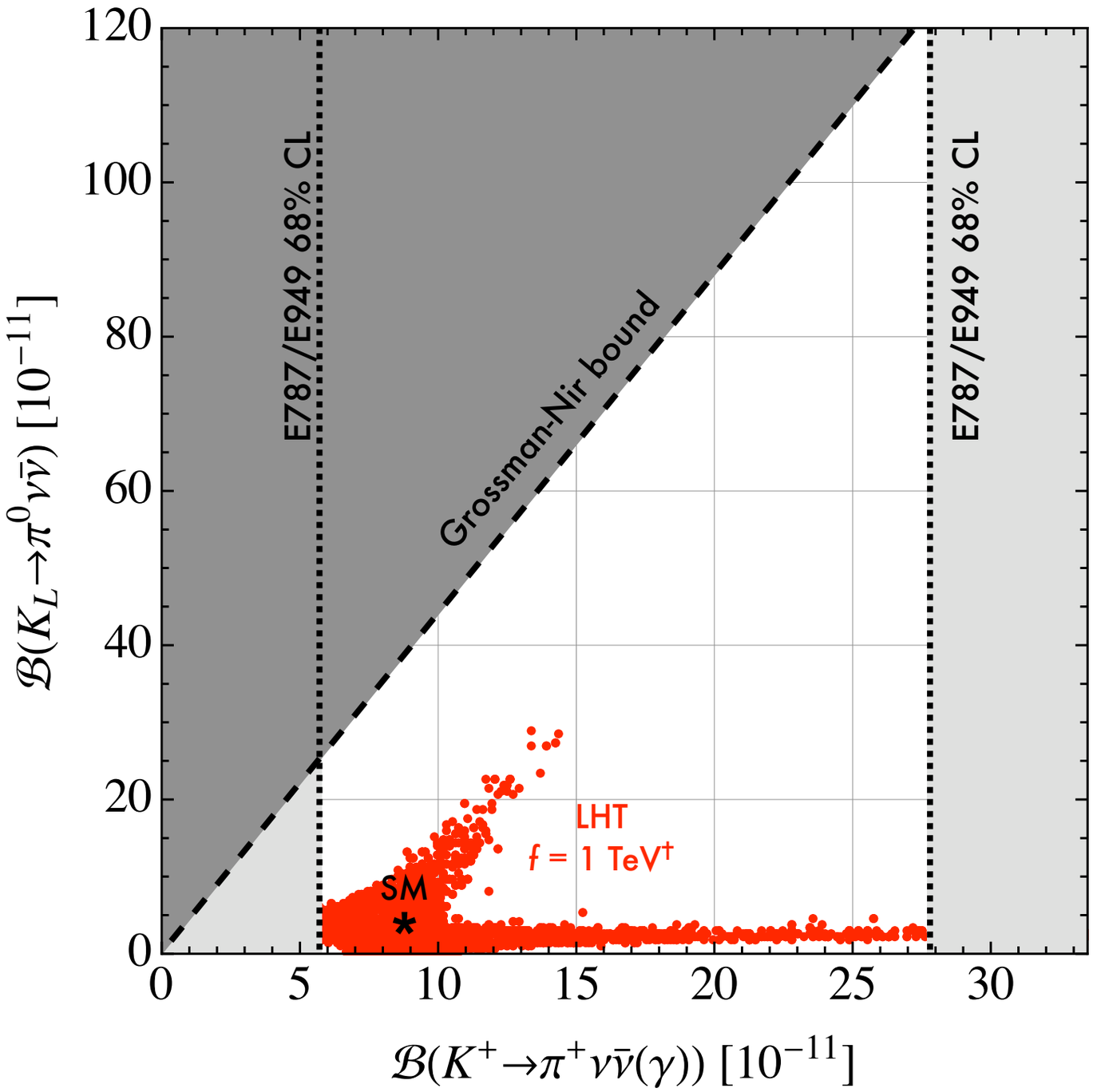
- mirror quarks d_H & u_H introduce additional flavor interactions governed by new mixing matrix V_{H_d} containing 3 angles & 3 phases



- since λ^5 suppression is in general not present in V_{H_d}, for non-degenerate d_H & u_H, sizable $SU(2)_L$ breaking effects in Z-penguin are expected having potential significant impact on rare K-decays
- left-over $1/\epsilon$ pole in Z-penguin reflects sensitivity to unknown UV completion of LHT; studies only take into account flavor-universal logarithmic piece but neglect possible constant non-universal parts

*Blanke et al. 3×'06, '07, '08

LHT: $K^+ \to \pi^+ \nu\bar{\nu}$ vs. $K_L \to \pi^0 \nu\bar{\nu}$ *

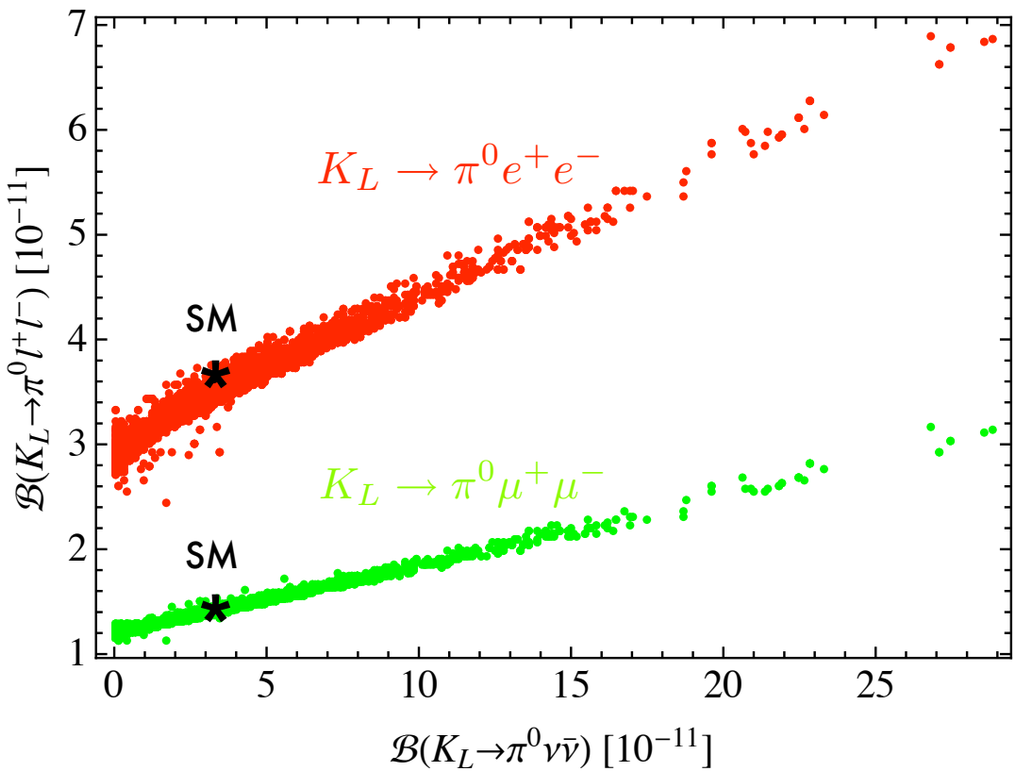


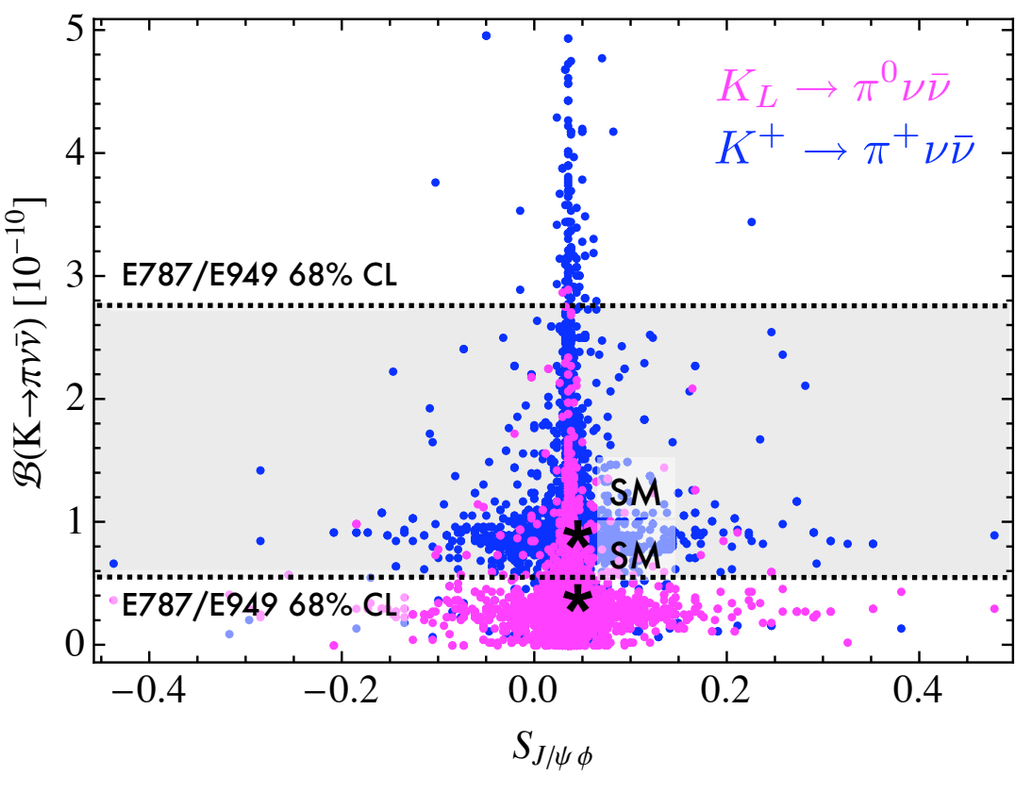
all input parameters taken to be flatly distributed within their 1σ ranges & $|\epsilon_K|$, $\Delta M_{B_{d,s}}$, $S_{J/\psi K_S}$ & ΔM_K required to lie within 1σ & 30% of central values

- $|\epsilon_K|$ allows for 2 branches: either $B(K^+)$ or $B(K_L)$ can be enhanced by factor 5 or 10 with $B(K_L) \approx B_{SM}(K_L)$ or $B(K^+) \approx (2-3)\, B_{SM}(K^+)$
- precise theoretical value for ϵ'/ϵ would strongly constrain $K \to \pi\nu\bar{\nu}$

*Blanke et al. '06, '07, '08 †For f = 3 TeV effects in $K_L \to \pi^0 \nu\bar{\nu}$ are smaller by factor 5

LHT: interesting correlations*



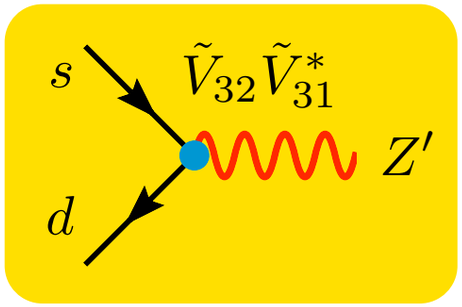


- $K_L \to \pi^0 l^+ l^-$ modes are strongly correlated with $K_L \to \pi^0 \nu\bar{\nu}$ & enhancements of factors 2 are possible
- *CP*-asymmetry $S_{J/\psi\phi}$ can reach values up to 0.4, although values > 0.2 seem "unlikely"
- for $S_{J/\psi\phi} \approx 0.2$, both $K \to \pi\nu\bar{\nu}$ & $K_L \to \pi^0 l^+ l^-$ modes could still be enhanced by factors 3 & 1.5

*Blanke et al. '06, '07, '08

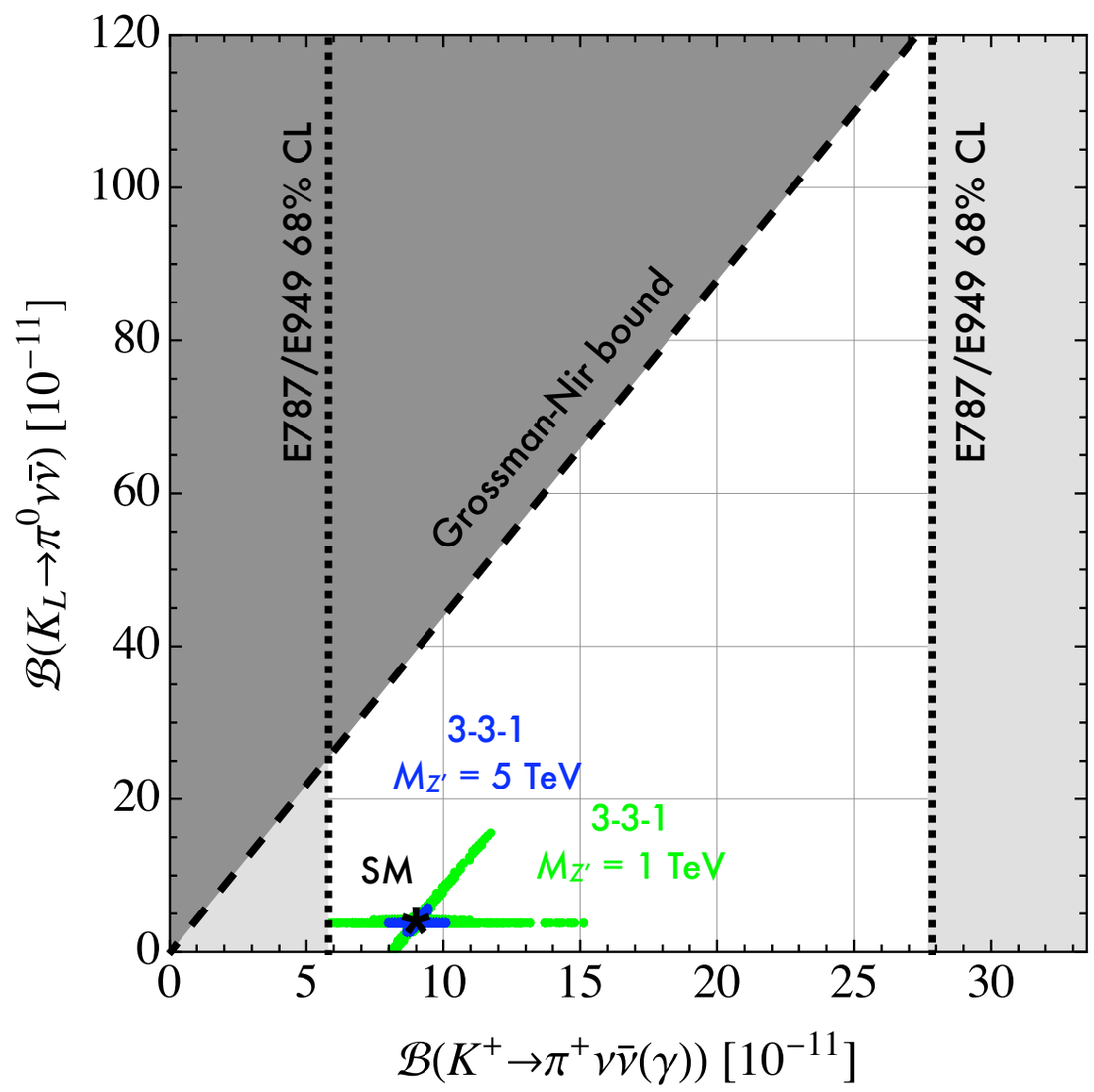
Minimal 3-3-1 model: $K \to \pi\nu\bar{\nu}$ & $K_L \to \pi^0 l^+ l^-$ *

- FCNCs induced by exchange of heavy Z′ gauge boson which involves new mixing matrix $\tilde{V}$



- pattern of effects similar to LHT, but smaller due to leptophobic nature of 3-3-1 model:† $B(K^+) \lesssim 2\, B_{SM}(K^+)$ & $B(K_L) \lesssim 4\, B_{SM}(K_L)$

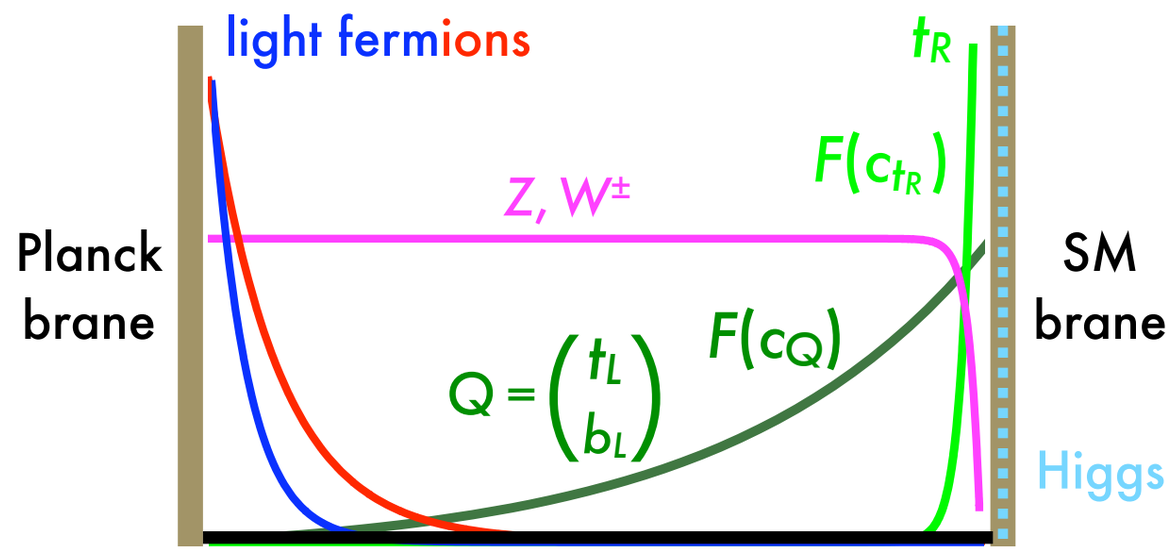
- $K_L \to \pi^0 l^+ l^-$ are strongly correlated with each other & with $K_L \to \pi^0 \nu\bar{\nu}$ & can be enhanced by factor 3†



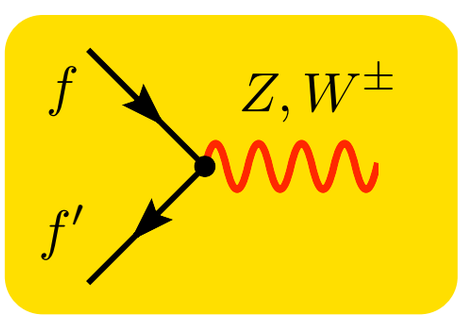
*Promberger et al. '07 †For $M_{Z'}$ = 1 TeV

Flavor of flavor in Randall-Sundrum (RS) model*

- SM fermions masses & mixings are generated without flavor symmetries by localizing fields at different points in warped extra dimension



- different sources of flavor-violation:

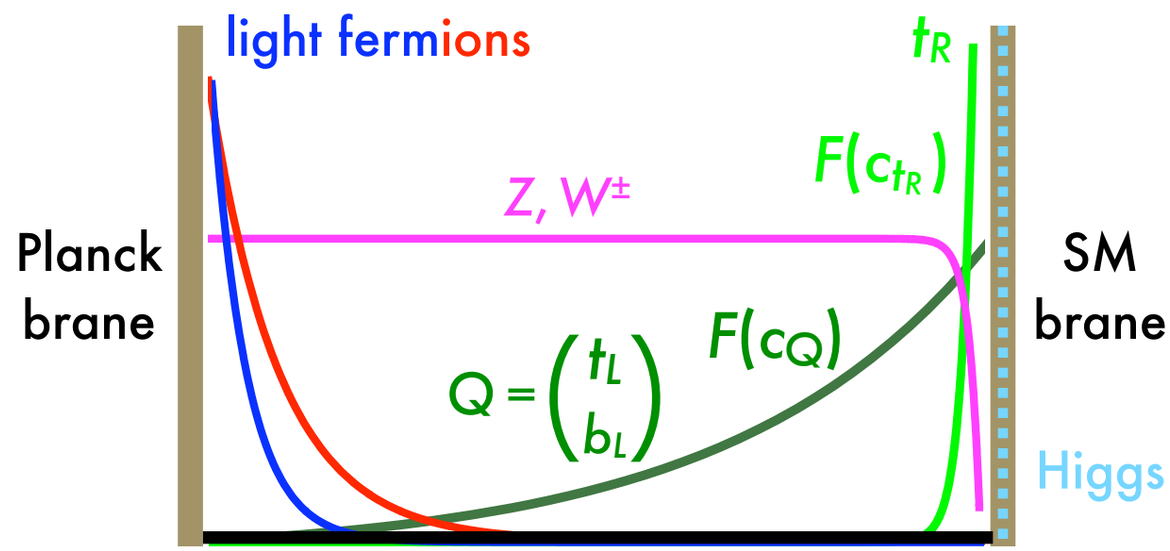


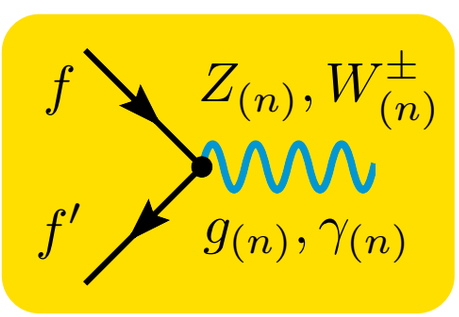
  - non-orthonormality of fermion profiles leading to mixing of $SU(2)_L$ doublets & singlets
  - modification of SM gauge boson profiles due to EWSB on SM brane

*Huber & Shafi '99, ..., Csaki et al. '08

Flavor of flavor in Randall-Sundrum (RS) model*

- SM fermions masses & mixings are generated without flavor symmetries by localizing fields at different points in warped extra dimension





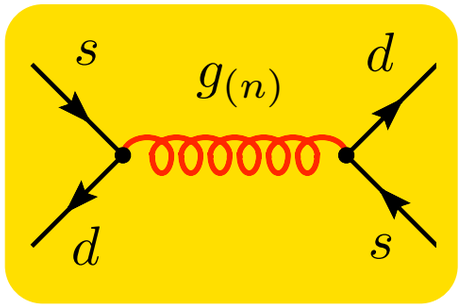
- effects from non-trivial KK gauge boson profiles
- dominant corrections in general from KK gluons

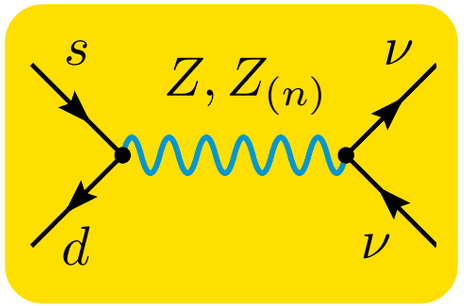
- fact that all flavor-violating effects "live" close to SM brane implies suppression of FCNCs by $F(c_i)F(c_j)$, dubbed RS-GIM mechanism†

*Huber & Shafi '99, ..., Csaki et al. '08 †Agashe et al. '04, '05

"Anticorrelation" between $|\epsilon_K|$ & $K\to\pi\nu\bar{\nu}$*

- all $s\to d$ transitions governed by only 4 couplings, namely $(\Delta_{D,d})_{12}$ & $(\delta_{D,d})_{12}$





- invariance of quark masses & mixings under $F(c_i)\to\xi F(c_i)$ & $Y_q\to\xi^{-2}Y_q$ might allow to achieve observable effects in $K\to\pi\nu\bar{\nu}$ while avoiding disastrous corrections to $|\epsilon_K|$, since $\Delta_{D,d}\to\xi^2\Delta_{D,d}$ while $\delta_{D,d}\to\xi^{-2}\delta_{D,d}$

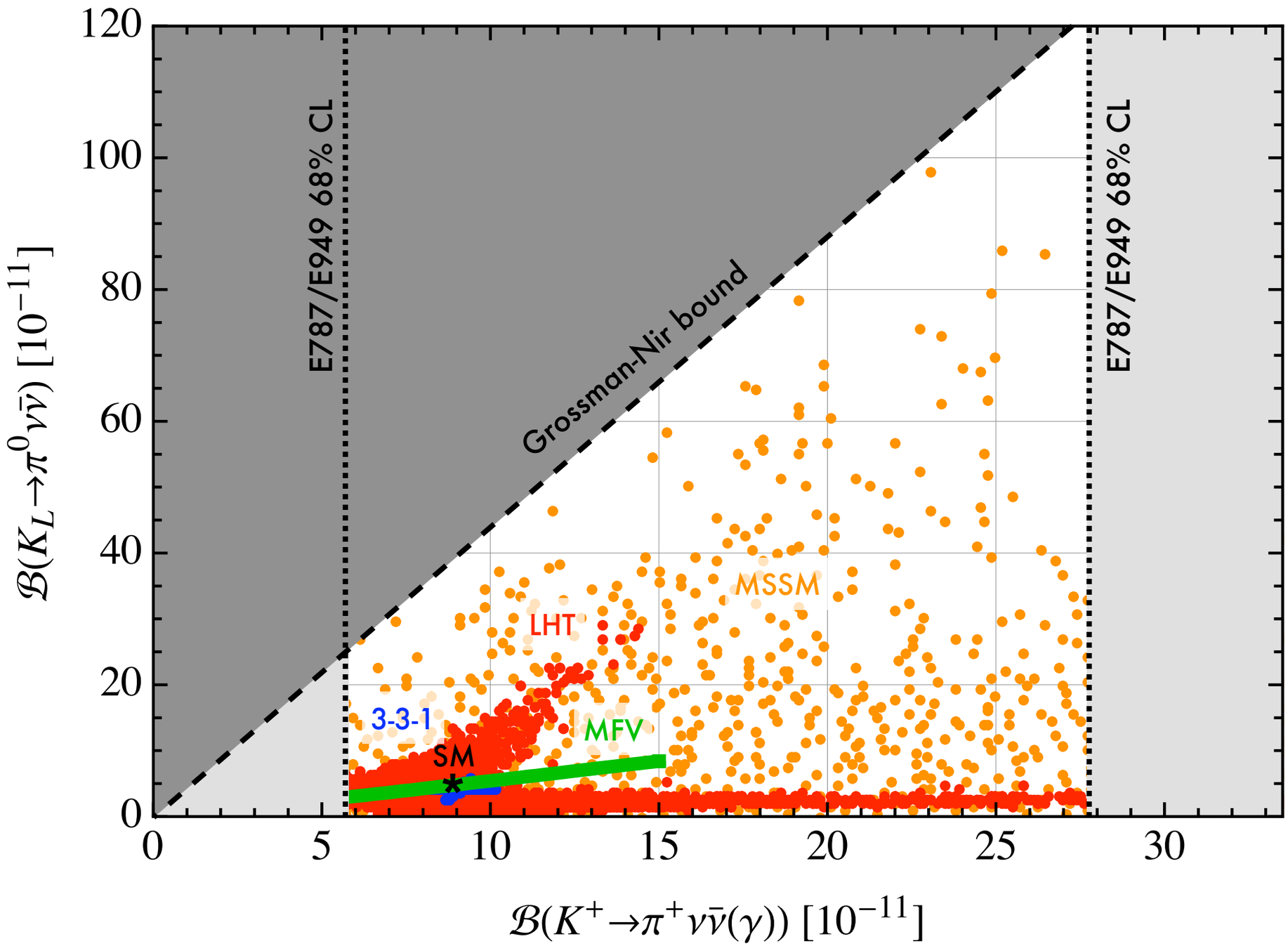
$$(X_{\rm RS})_L \approx \frac{4.4\times10^{-3}\,|V_{ts}^*V_{td}|}{M_{\rm KK}^2\,{\rm TeV}^{-2}}\left[15.7\,\frac{(\Delta_D)_{12}}{10^{-6}}+\frac{M_{\rm KK}^2}{M_Z^2}\,\frac{(\delta_d)_{12}}{10^{-6}}\right]^\dagger$$

$$|\epsilon_K|_{\rm RS} \approx -\frac{4\times10^{-3}\,{\rm Im}\left[(\Delta_D)_{12}(\Delta_d)_{12}-10^{-3}\left((\Delta_D)_{12}^2+(\Delta_d)_{12}^2\right)\right]}{10^{-12}\,M_{\rm KK}^2\,{\rm TeV}^{-2}}$$

*Casagrande et al., work in progress

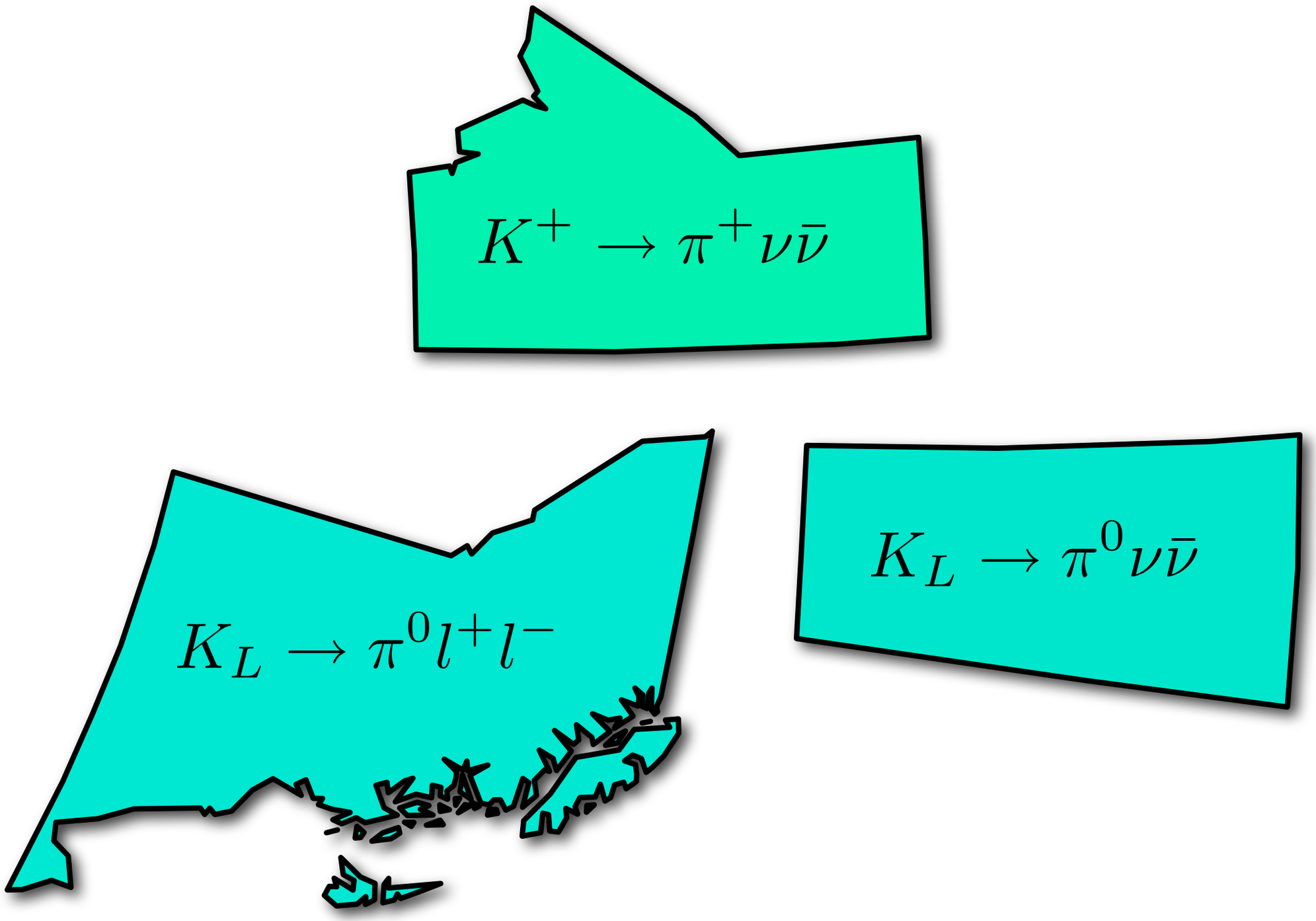
†Similar expression with $D\leftrightarrow d$ holds for $(X_{RS})_R$

Mescia's plot reloaded

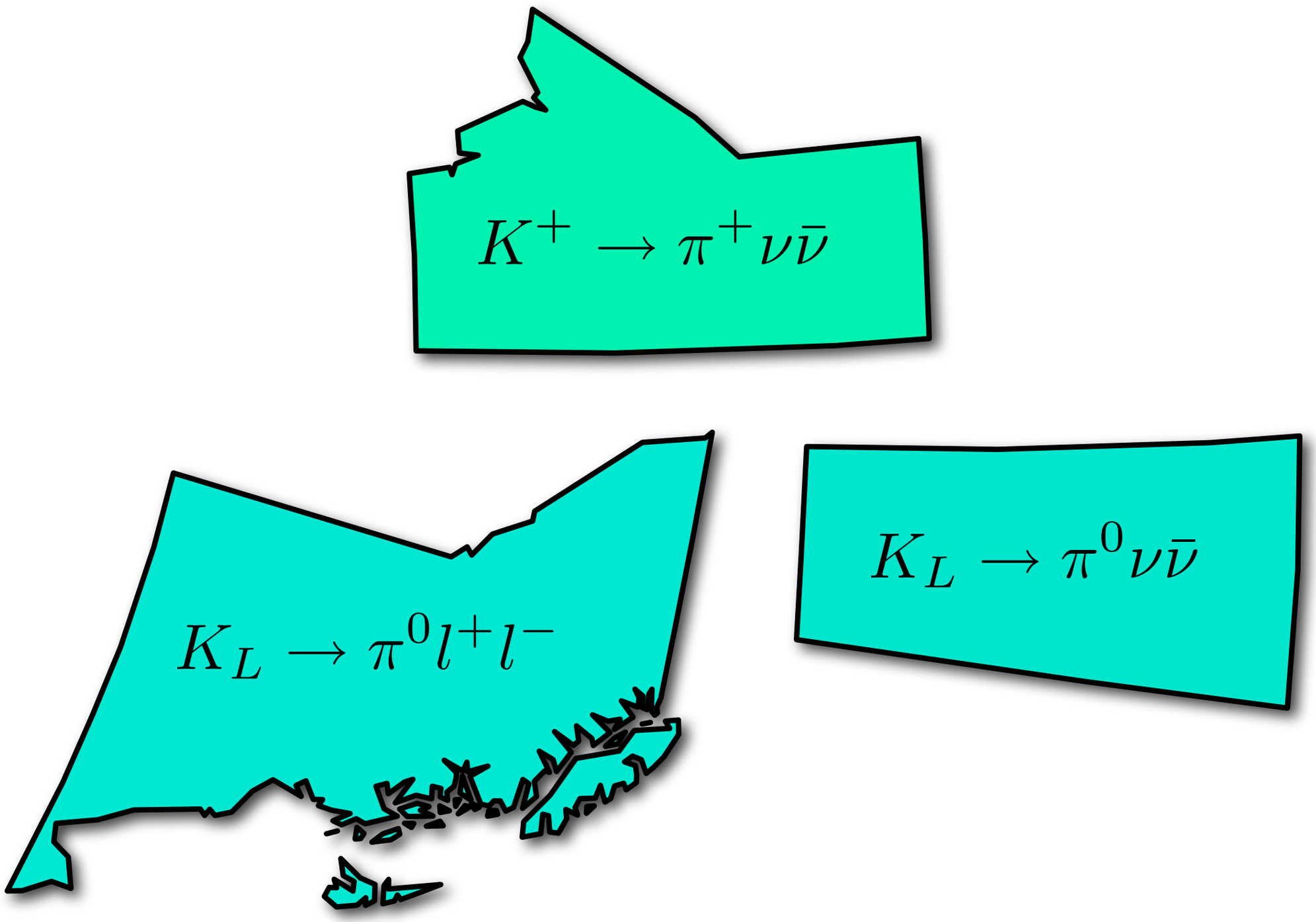


large & correlated signals could be seen in rare *K*-decays

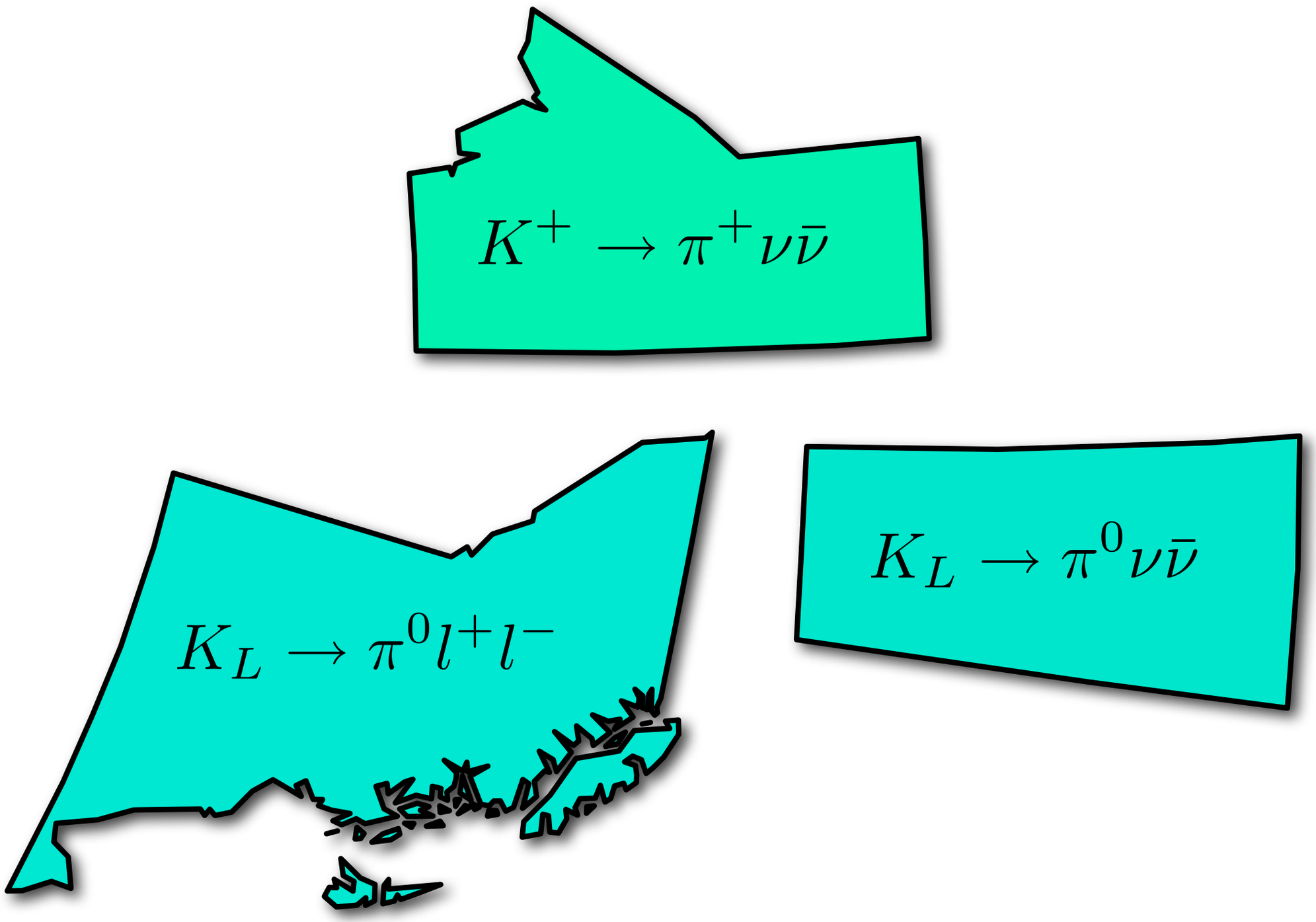
However, ...



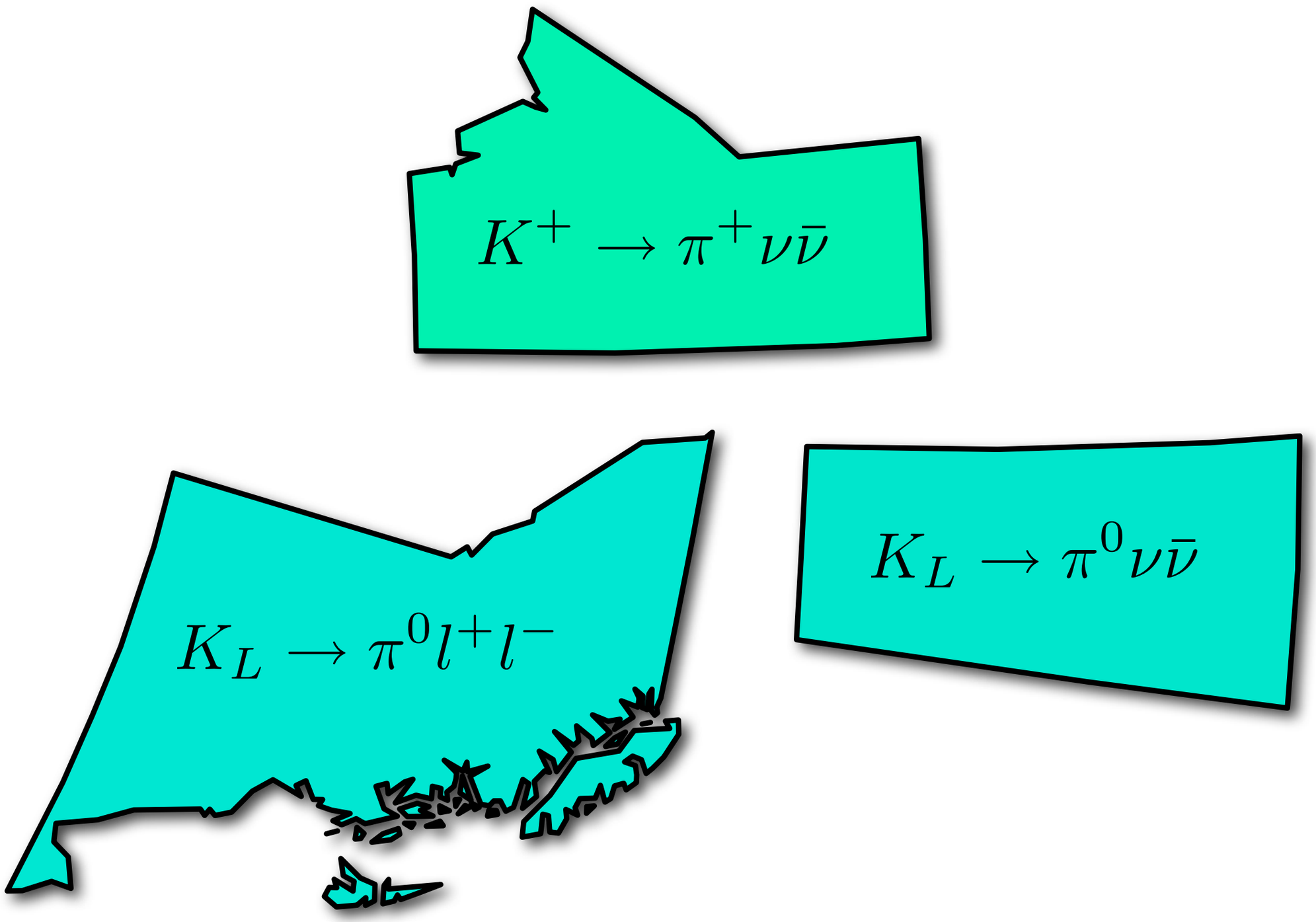
whether or not spectacular effects are present ...



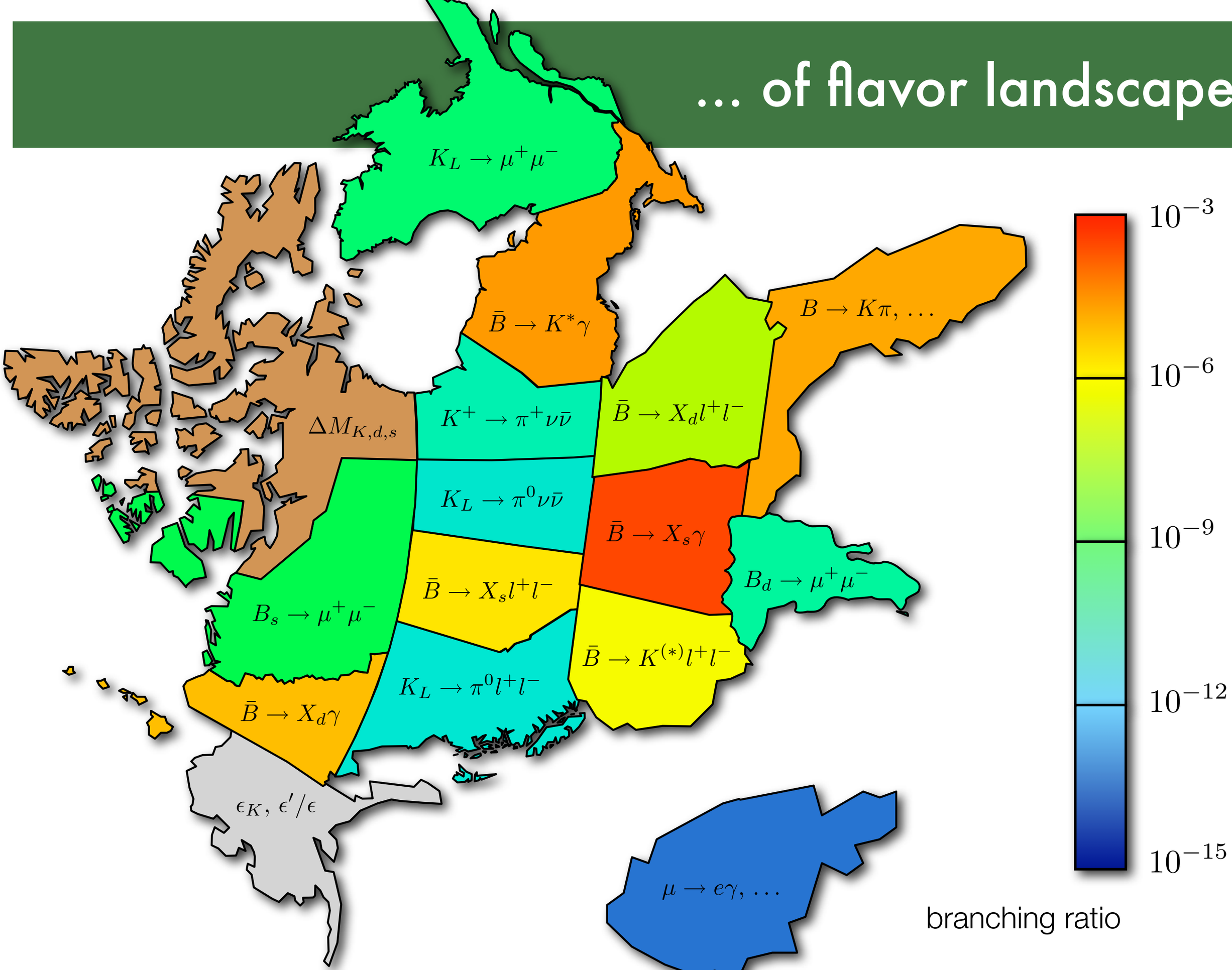
can only be determined by measurements ...



thereby exploring unmapped territories ...



... of flavor landscape



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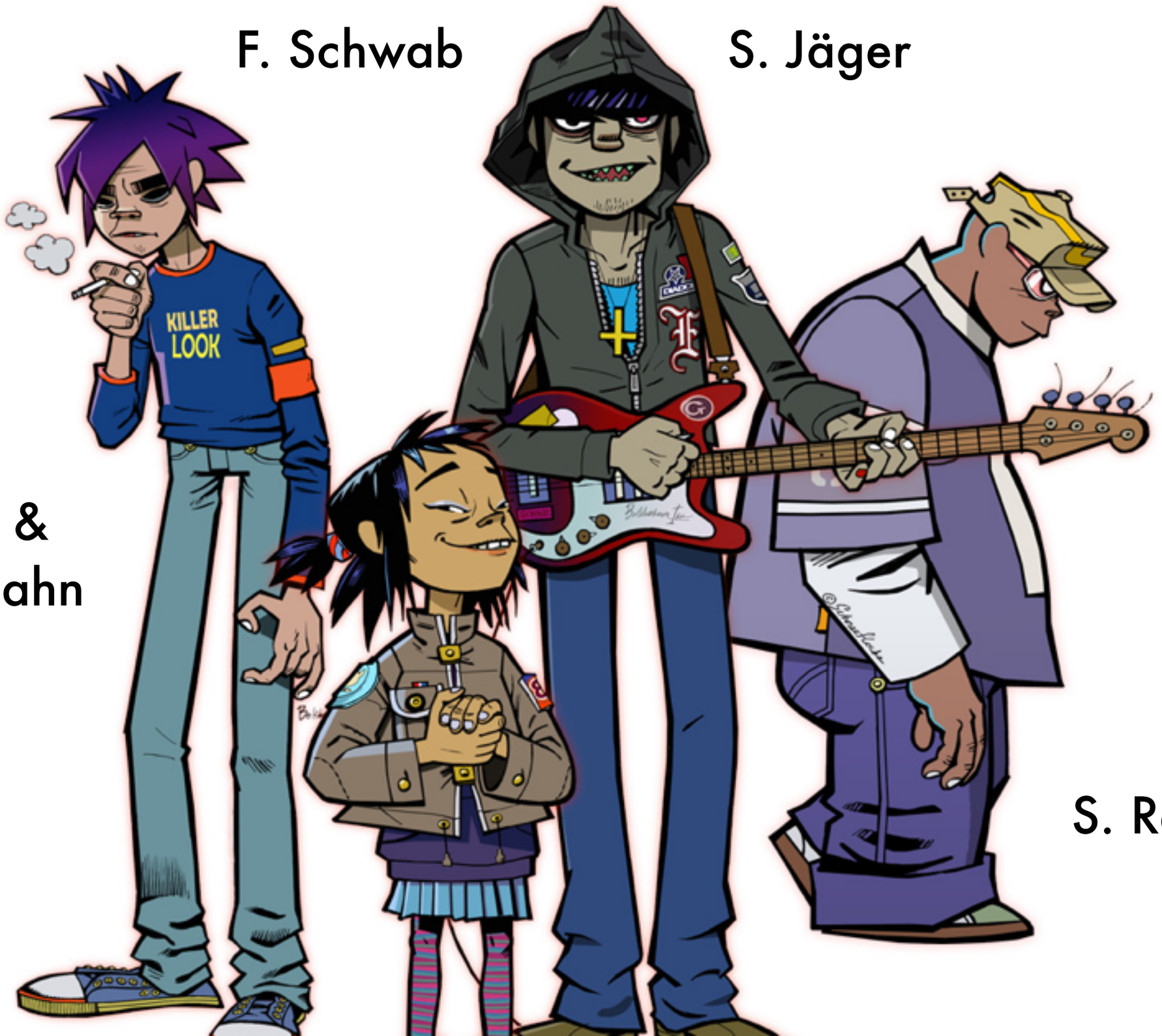


F. Schwab

S. Jäger

J. Brod &
M. Gorbahn

S. Recksiegel

$K \to \pi\nu\bar{\nu}$ matrix elements from K_{l3} in χPT

$$\kappa_L = \frac{G_F^2 m_K^5 \alpha^2(M_Z)}{256\,\pi^5 s_W^4}\,\lambda^8\,(\tau_L|\lambda \times f_+^{K^0\pi^+}(0)|^2)_{\rm exp}\left(\frac{r\,r_K}{r_{0+}}\right)^2 \mathcal{I}_\nu^0\,,$$

$$\kappa_+ = \frac{G_F^2 m_K^5 \alpha^2(M_Z)}{256\,\pi^5 s_W^4}\,\lambda^8\,\tau_+\,(r_K|\lambda \times f_+^{K^0\pi^+}(0)|^2)_{\rm exp}\,\mathcal{I}_\nu^+$$

$$r_K = \frac{f_+^{K^+\pi^+}(0)}{f_+^{K^0\pi^+}(0)} = 1.0015 \pm 0.0007^{\dagger}$$

$$r_{0+} = \frac{f_+^{K^+\pi^{0(+)}}(q^2)}{f_+^{K^0\pi^{+(0)}}(q^2)} = 1.0238 \pm 0.0035^{\dagger}$$

$$r = \frac{f_+^{K^+\pi^0}(q^2)\,f_+^{K^0\pi^0}(q^2)}{f_+^{K^0\pi^+}(q^2)\,f_+^{K^+\pi^+}(q^2)} = 1.0000 \pm 0.0002^{\dagger}$$

$$\epsilon^{(2)} \propto \frac{m_u - m_d}{m_s}$$

- classic χPT analysis of $O(p^2\epsilon^{(2)})$* isospin-breaking effects very recently extended to $O(p^4\epsilon^{(2)})$ & partially $O(p^6\epsilon^{(2)})^{\dagger}$

*Marciano & Parsa '96 †Mescia & Smith '07

K_L & K_+: main messages*

- overall uncertainties on $K_L \to \pi^0 \nu\bar{\nu}$ & $K^+ \to \pi^+ \nu\bar{\nu}$ matrix elements are reduced by factor 4 & 7
- further reduction of errors possible with better data on K_{l3} slopes & K_{l3} branching ratios

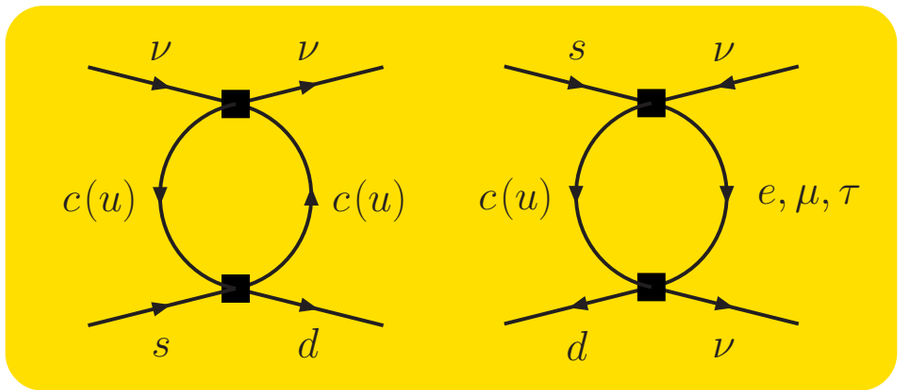
	$(r_{0+})_{theo}$	$(r_{0+})^{KLOE}_{exp}$	$(r_{0+})_{exp}$	τ_+	$f(0)_{K_{l3}}$	$\mathcal{I}$	r_K	r	future (?)
K_L	2.229 ± 0.017	2.229 ± 0.036	2.190 ± 0.018	–	77%	12%	9%	2%	± 0.013
K_+	5.168 ± 0.025	5.168 ± 0.025	5.168 ± 0.025	19%	43%	21%	17%	–	± 0.023

*Mescia & Smith '07

SM prediction of $K^+ \to \pi^+ \nu\bar{\nu}$: $\delta P_{c,u}$

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}(\gamma)) = \kappa_+ (1 + \Delta_{\rm EM}) \left| \frac{x}{\lambda^5} \right|^2$$

$$x = V_{ts}^* V_{td} X + \lambda^4 \, {\rm Re}(V_{cs}^* V_{cd})(P_c + \delta P_{c,u})$$



$$Q_1^{(8)} = (\bar{s}_L \gamma_\mu d_L) \partial^2 (\bar{\nu}_L \gamma^\mu \nu_L) \,,$$

$$Q_2^{(8)} = (\bar{s}_L \overleftarrow{D}_\nu \gamma_\mu \overrightarrow{D}^\nu d_L)(\bar{\nu}_L \gamma^\mu \nu_L) \,,$$

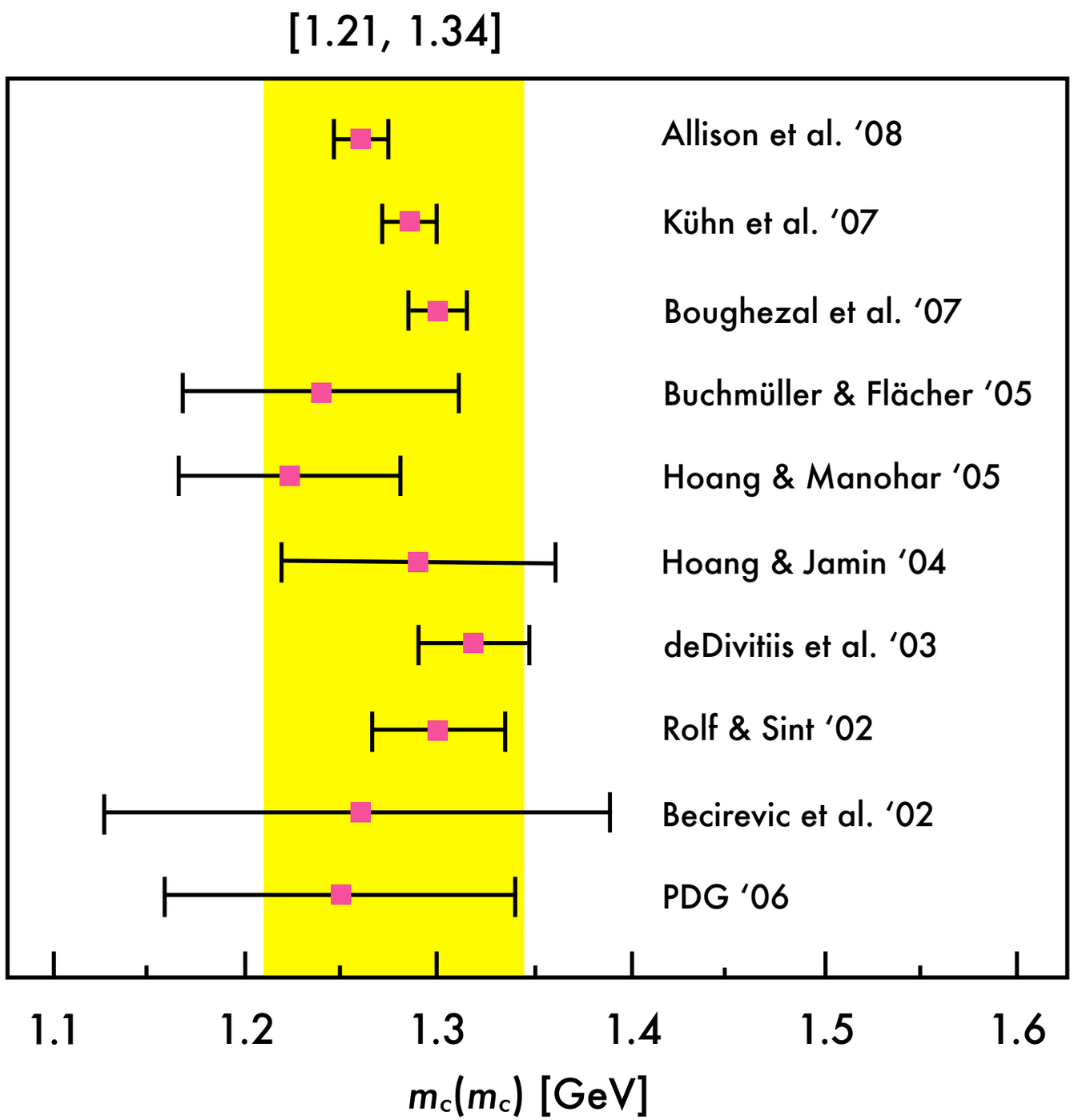
$$Q_3^{(8)} = (\bar{s}_L \overleftarrow{D}_\nu \gamma_\mu d_L)(\bar{\nu}_L (\overleftarrow{\partial^\nu} - \overrightarrow{\partial^\nu}) \gamma^\mu \nu_L)$$

*

- local charm effects due to dimension-eight operators naively of $O(m_K^2/m_c^2) \approx 15\%$
- genuine long-distance up effects of $O(\Lambda^2/m_c^2) \approx 10\%$

*Falk et al. '00

Recent determinations of charm mass



$m_c(m_c)$ [GeV]	method
1.266 ± 0.014	lattice, unquenched, staggered
1.286 ± 0.013	low-momentum sum rules, N^3LO
1.295 ± 0.015	low-momentum sum rules, N^3LO
1.24 ± 0.07	fit to *B*-decay distribution, $\alpha_s^2\beta_0$
1.224 ± 0.017 ± 0.054	fit to *B*-decay data, $\alpha_s^2\beta_0$
1.29 ± 0.07	NNLO moments
1.319 ± 0.028	lattice, quenched
1.301 ± 0.034	lattice, quenched
1.26 ± 0.04 ± 0.12	lattice, quenched
1.25 ± 0.09	PDG 2006

Future (?) CKM fit from $K \to \pi\nu\bar{\nu}$*

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) = (8.0 \pm 0.8) \times 10^{-11}$$

$$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) = (3.0 \pm 0.3) \times 10^{-11}$$

Future (?)

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 4.0\%$$

$$\sigma(\sin 2\beta) = \pm 0.024$$

$$\sigma(\gamma) = \pm 4.7^\circ$$

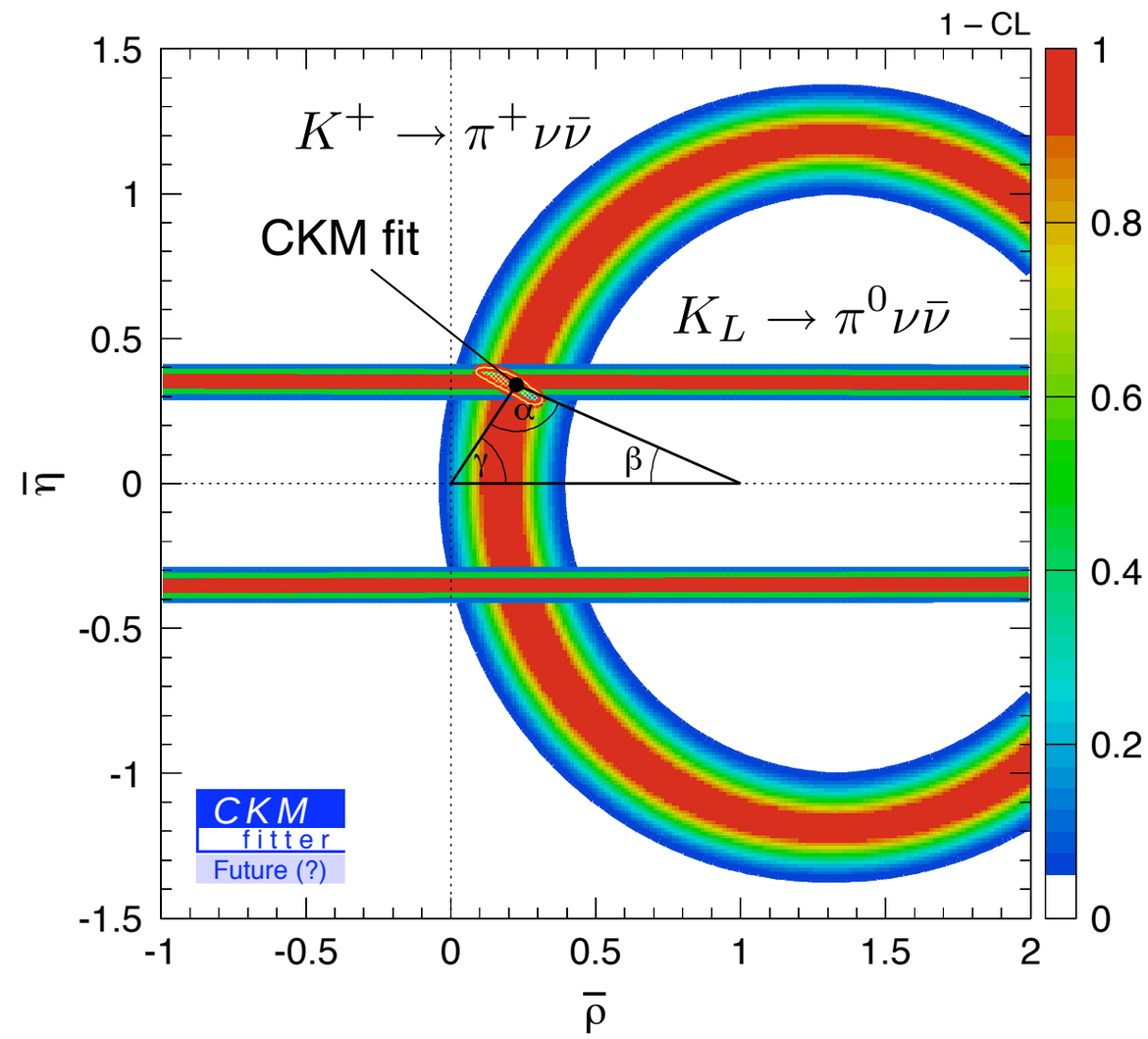
NLO
(theory error only)

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 1.0\%$$

$$\sigma(\sin 2\beta) = \pm 0.006$$

$$\sigma(\gamma) = \pm 1.2^\circ$$

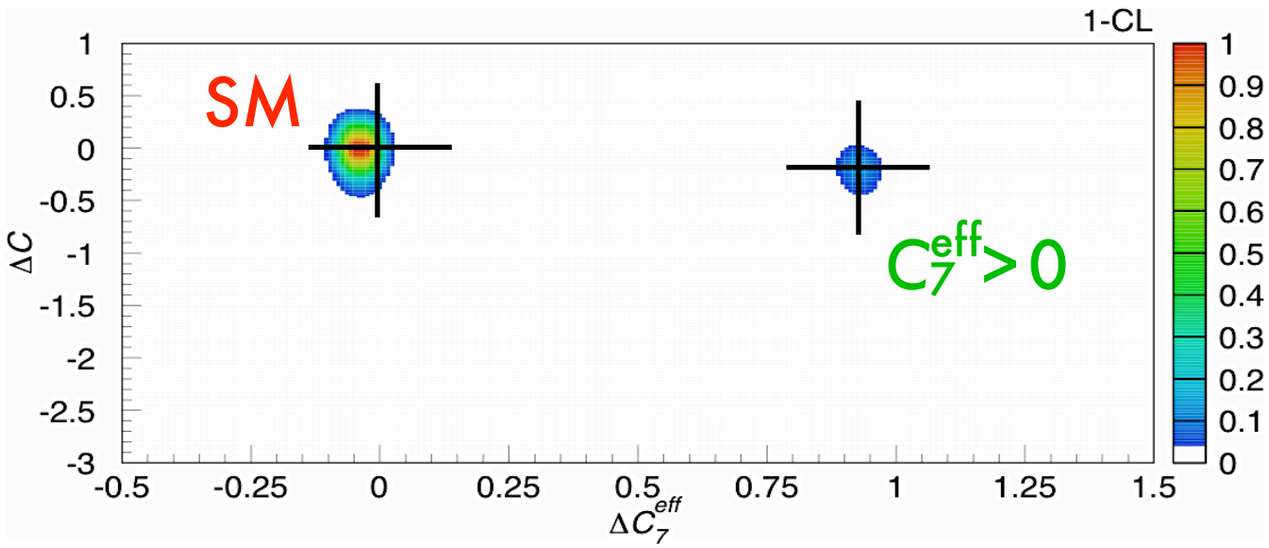
NNLO
(theory error only)

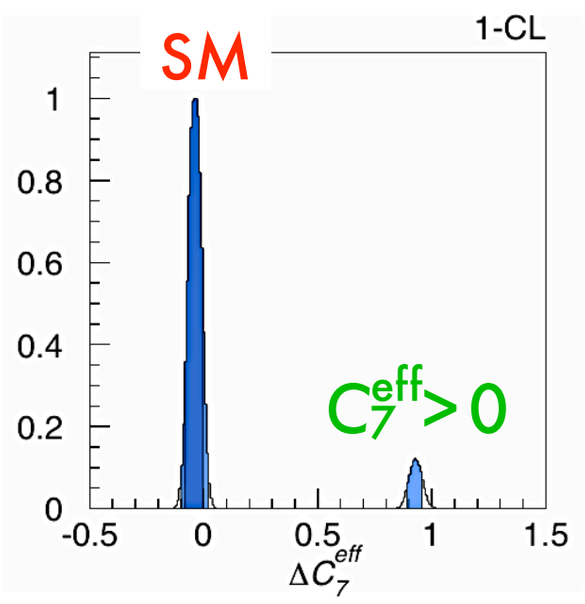


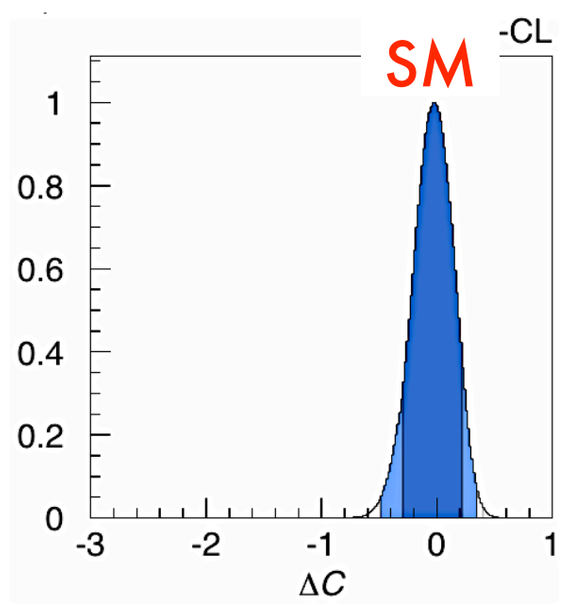
- nice CKM fit from $K \to \pi\nu\bar{\nu}$, almost comparable to present global analysis, but not ultimate goal

*Buras et al. '05, '06

Bounds on ΔC & $\Delta C_7^{\rm eff}$ from $\bar{B}\to X_s\gamma, l^+l^-$ & $Z\to b\bar{b}$







$$\Delta C = [-0.486, 0.366] \quad (95\%\ \mathrm{CL})$$

$$\Delta C_7^{\rm eff} = [-0.104, 0.026] \ \cup \ [0.891, 0.968] \quad (95\%\ \mathrm{CL})$$

- large corrections to off-shell photon penguin still allow for "wrong" sign of $C_7^{\rm eff}$

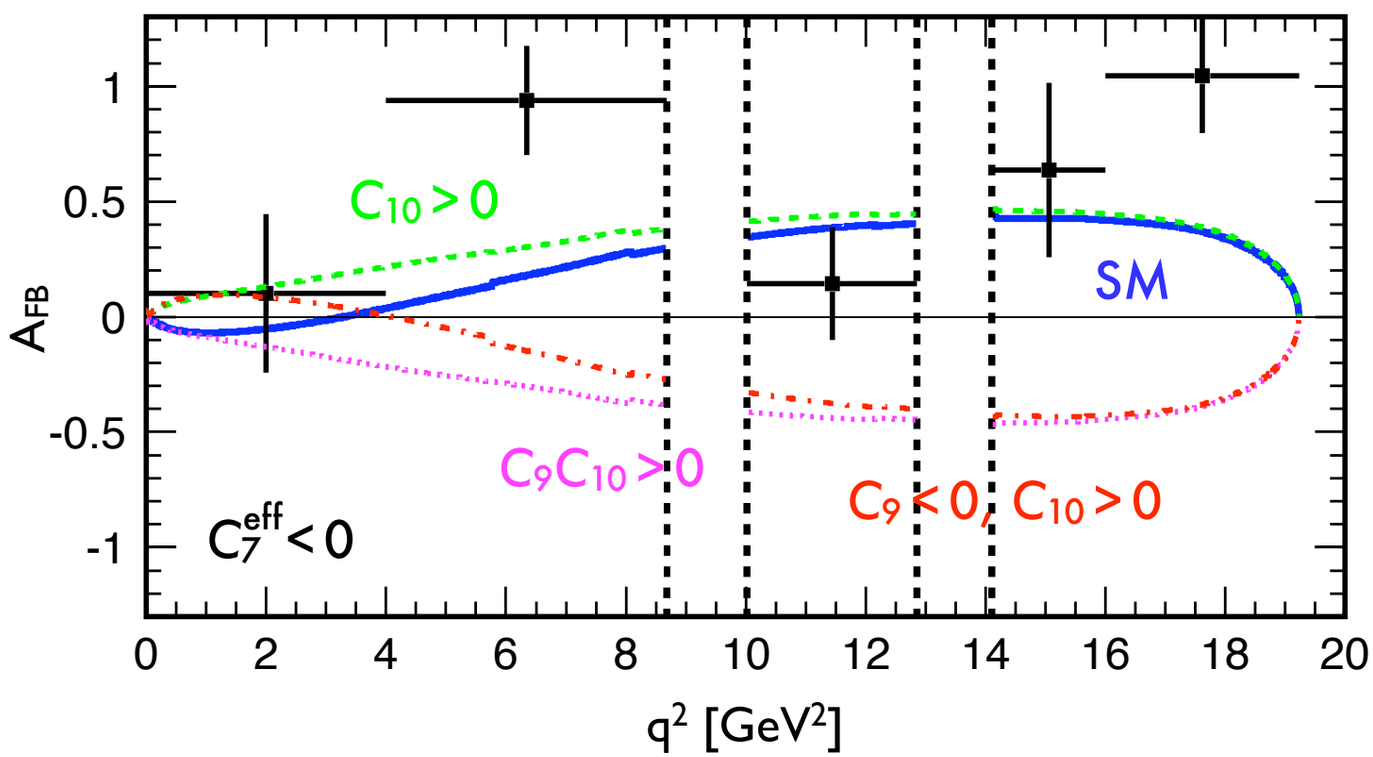
*UH & Weiler '07

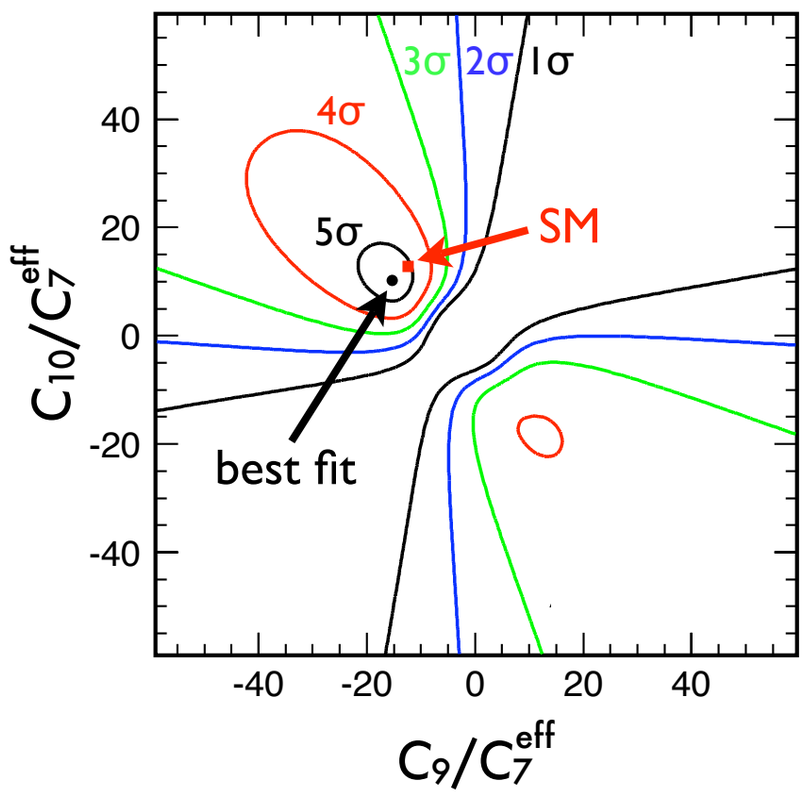
Lower and upper CMFV bounds†

Observable	CMFV (95% CL)	SM (95% CL)	Experiment
$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) \times 10^{11}$	$[4.24, 11.09]$	$[5.46, 9.41]$	$(14.7^{+13.0}_{-8.9})$
$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) \times 10^{11}$	$[1.56, 4.56]$	$[2.24, 3.59]$	$< 6.7 \times 10^4$ (90% CL)
$\mathcal{B}(K_L \to \mu^+\mu^-)_{\rm SD} \times 10^9$	$[0.30, 1.22]$	$[0.54, 0.88]$	–
$\mathcal{B}(\bar{B} \to X_d \nu\bar{\nu}) \times 10^6$	$[0.77, 2.00]$	$[1.24, 1.45]$	–
$\mathcal{B}(\bar{B} \to X_s \nu\bar{\nu}) \times 10^5$	$[1.88, 4.86]$	$[3.06, 3.48]$	< 64 (90% CL)
$\mathcal{B}(B_d \to \mu^+\mu^-) \times 10^{10}$	$[0.36, 2.03]$	$[0.87, 1.27]$	$< 1.8 \times 10^2$ (95% CL)
$\mathcal{B}(B_s \to \mu^+\mu^-) \times 10^9$	$[1.17, 6.67]$	$[2.92, 4.13]$	$< 5.8 \times 10^1$ (95% CL)

*UH & Weiler '07 †Assuming vanishing box contributions

Hunting Z-penguin with $\bar{B}\to K^* l^+ l^-$





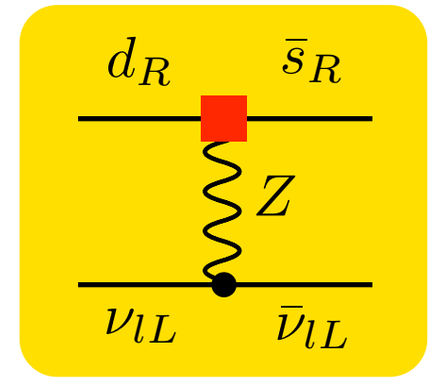
- forward-backward asymmetry in $\bar{B}\to K^* l^+ l^-$ excludes $C_9C_{10} > 0$ at 95% CL*
- hints towards exclusion of large destructive Z-penguin: $|\Delta C| \lesssim 1.5$

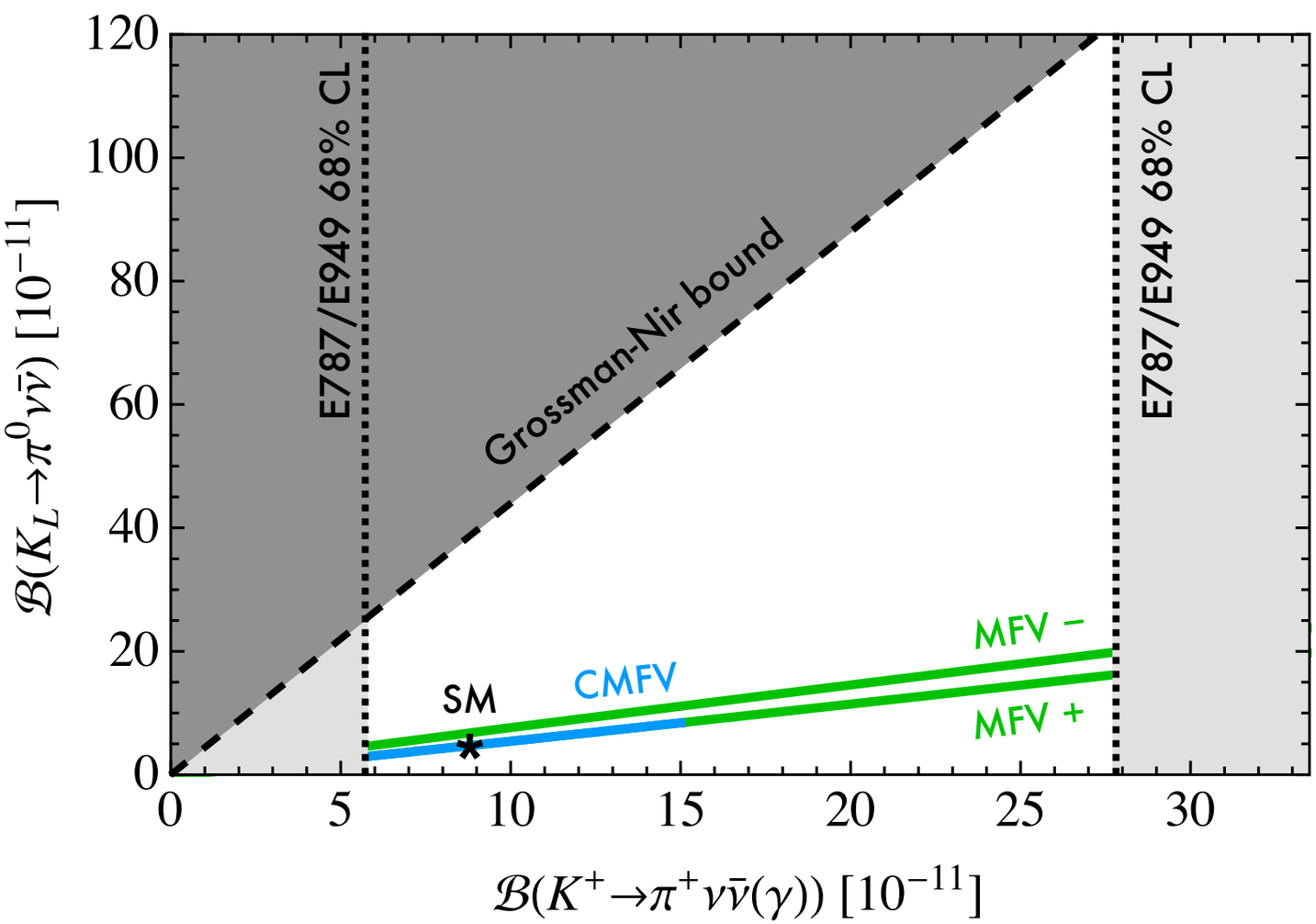
*Belle '05; BaBar '06, '08

MFV: $K^+ \to \pi^+ \nu\bar{\nu}$ vs. $K_L \to \pi^0 \nu\bar{\nu}$

$$\mathcal{H}_{\rm eff} \propto \sum_{l=e,\mu,\tau} \left[X_L(\bar{s}_L\gamma_\mu d_L) + X_R(\bar{s}_R\gamma_\mu d_R)\right](\bar{\nu}_{lL}\gamma^\mu\nu_{lL})$$

$$\sim \sum_{l=e,\mu,\tau} \left[X_L + X_R\right](\bar{s}_L\gamma_\mu d_L)(\bar{\nu}_{lL}\gamma^\mu\nu_{lL})$$





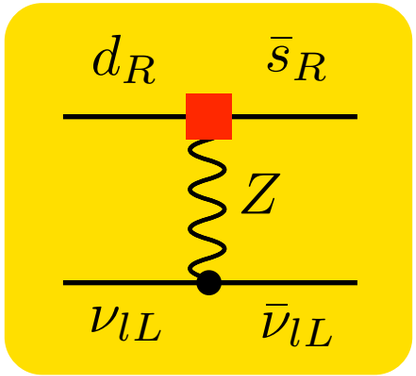
- sizable constructive model-independent MFV effects in $K^+ \to \pi^+ \nu\bar{\nu}$ difficult to obtain in any* explicit model due to chiral suppression

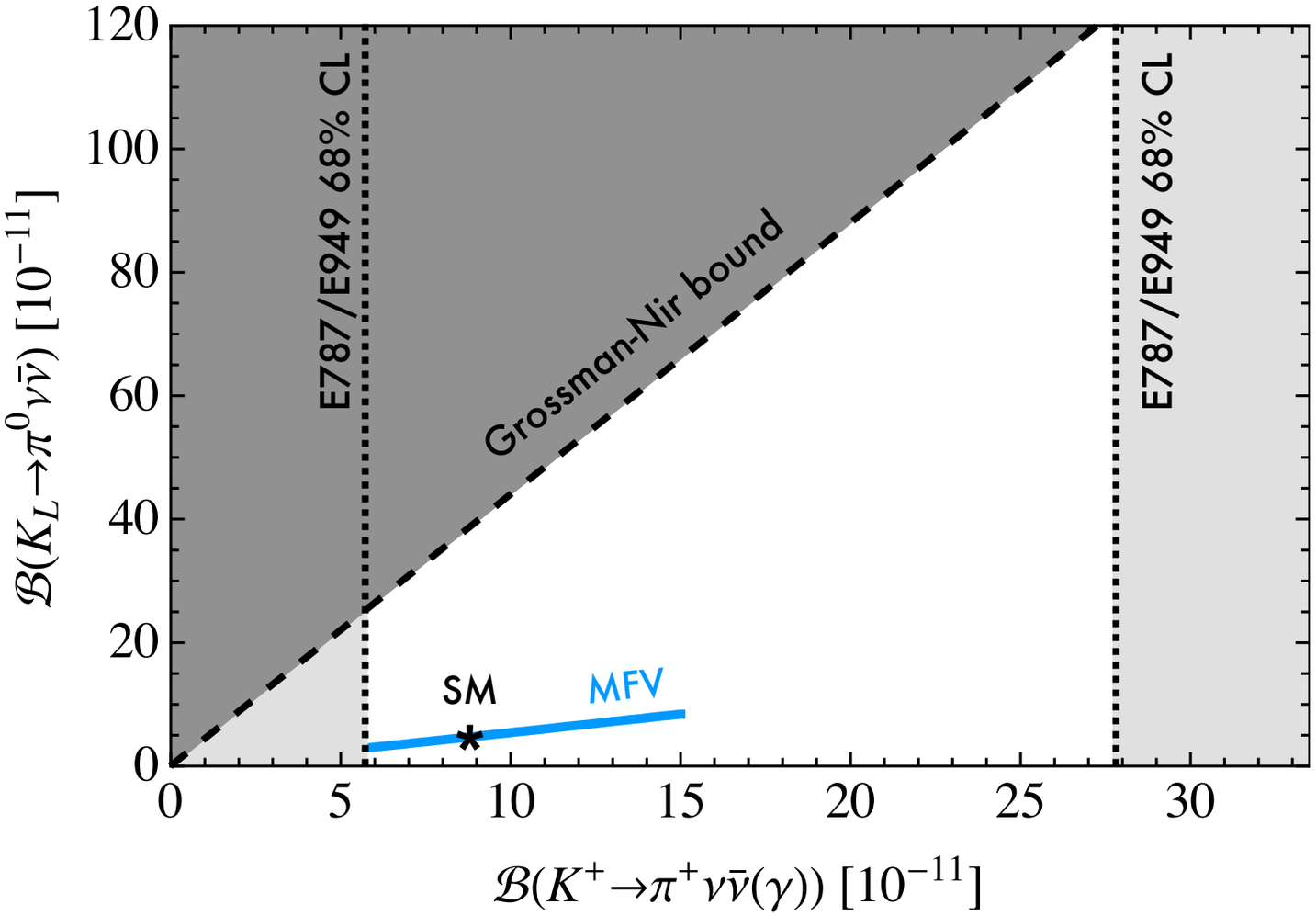
*to limited knowledge of speaker

MFV: $K^+ \to \pi^+ \nu\bar{\nu}$ vs. $K_L \to \pi^0 \nu\bar{\nu}$

$$\mathcal{H}_{\rm eff} \propto \sum_{l=e,\mu,\tau} \left[X_L(\bar{s}_L\gamma_\mu d_L) + X_R(\bar{s}_R\gamma_\mu d_R)\right](\bar{\nu}_{lL}\gamma^\mu\nu_{lL})$$

$$\sim \sum_{l=e,\mu,\tau} \left[X_L + X_R\right](\bar{s}_L\gamma_\mu d_L)(\bar{\nu}_{lL}\gamma^\mu\nu_{lL})$$





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