Family Symmetries and Neutrino Mixing

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Outline

1. Introduction
   - The data
   - Family symmetries
   - Neutrinos
2. Model
   - Overview
   - Mass terms and TBM
   - Vacuum alignment
3. Viability
   - Angle predictions
   - Conclusion

Standard model: Yukawa couplings

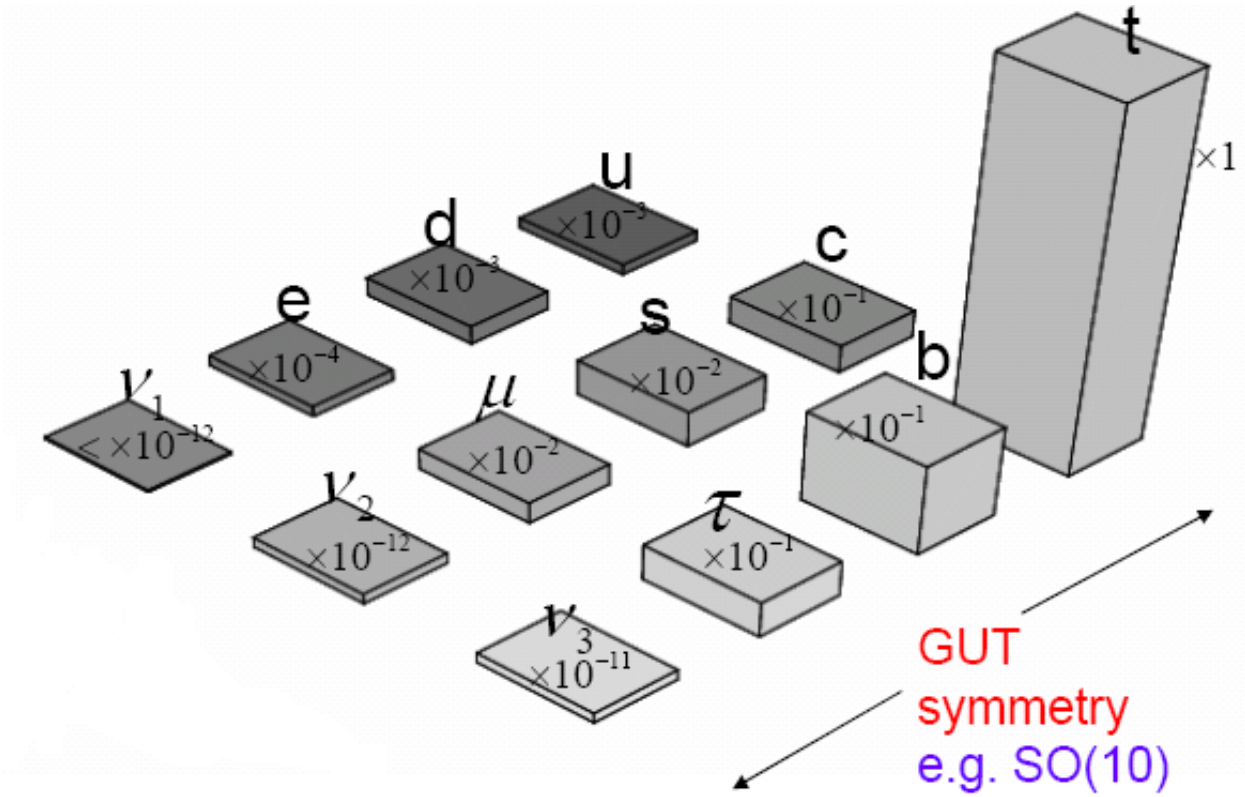
Down quark Yukawa

$$L_d = Y_d^{ij} Q_i d_j^c H_d$$

Down quark mass matrix

$$M_d^{ij} = Y_d^{ij} \langle H_d \rangle$$

Summary of data: masses



Summary of data: quark mixing

Wolfenstein

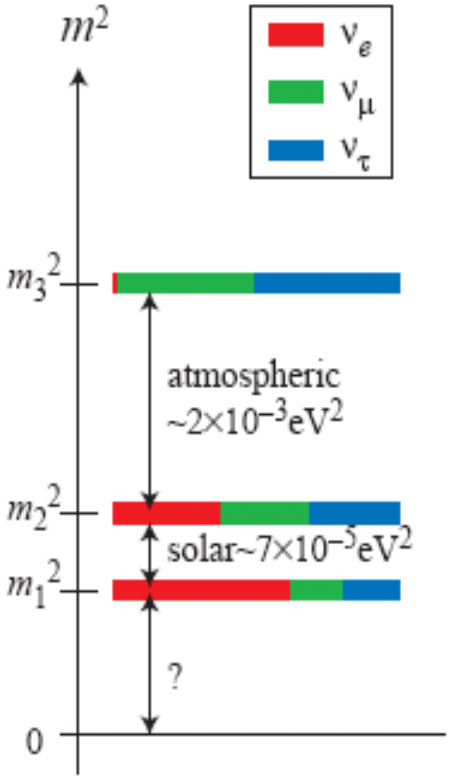
$$V_{CKM} \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ \lambda^3 & -\lambda^2 & 1 \end{pmatrix}$$

$\lambda \simeq 0.23$

Summary of data: lepton mixing

HPS / Tri-bi-maximal (TBM)

$$V_{PMNS} = \begin{pmatrix} -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}$$



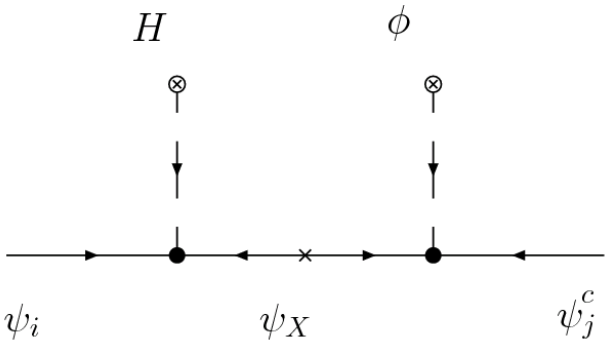
Broken?

Breaking expectations

- Flavon(s) ϕ, charged under the F.S.
- Hierarchical structure suggests $\langle\phi\rangle \neq 0$
- Suppresions arise from exchange of messengers (ψ_X) with large masses

Froggatt-Nielsen

$$m^{ij} \propto \frac{\langle\phi\rangle\langle H\rangle}{M_X}$$



Abelian?

Simple $U(1)$

Field	$U(1)$
H	**0**
ϕ	$-$**1**
d_3	**0**
d_3^c	**0**
d_2	**1**
d_2^c	**1**
d_1	**2**
d_1^c	**2**

Respective mass matrix

$$M_d = m_b \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}$$

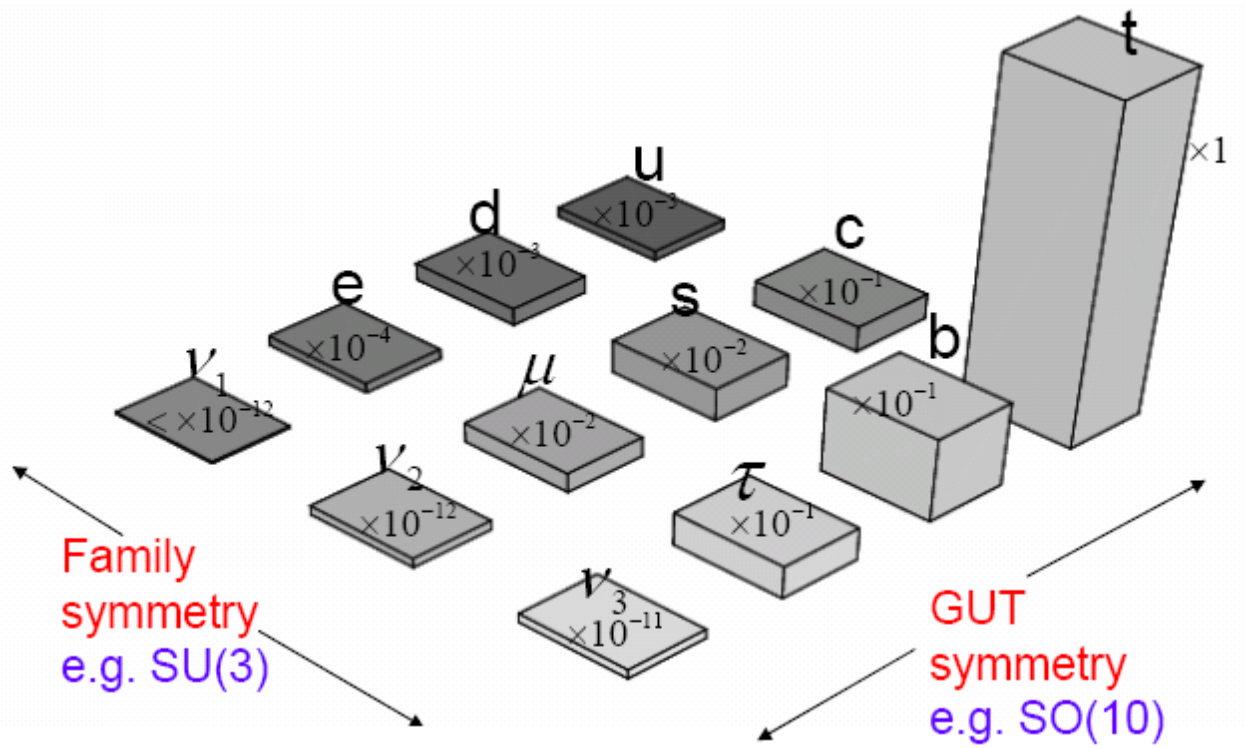
$$\frac{\langle \phi \rangle}{M_X} = \epsilon$$

Non-Abelian?

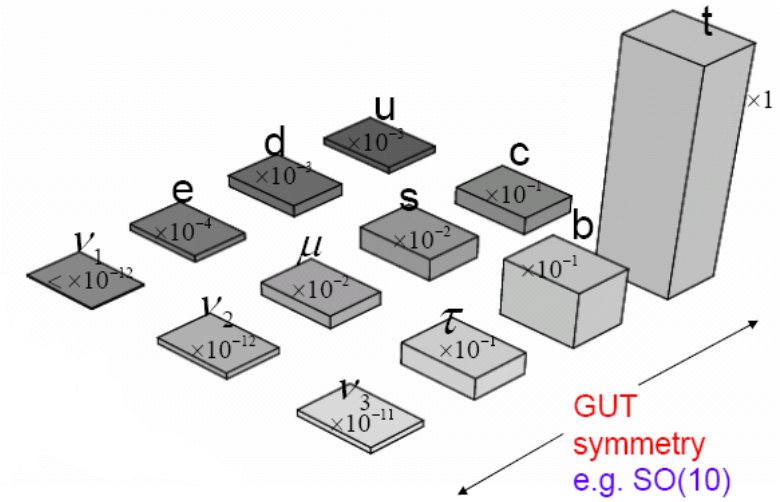
Reasons

- 3 generations
- Lepton mixing strongly suggests non-Abelian
- SM+ν_R: F.S. $\subset SU(3)^6$; $SO(10)$: F.S. $\subset SU(3)$

$SO(10) \times SU(3)$?

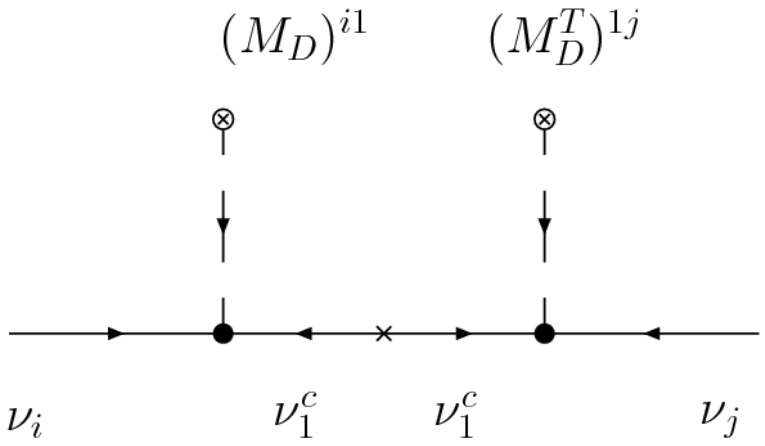


Seesaw mechanism



Type I formula

$$m_\nu = (M_D)\,(M_{RR})^{-1}\,(M_D)^T$$



Seesaw mechanism: masses

Seesaw formula

$$m_\nu = (M_D)\,(M_{RR})^{-1}\,(M_D)^T$$

Type I (1 generation)

$$M_\nu = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

if $\det(M_\nu) = -m^2$; $\mathrm{tr}(M_\nu) = M \gg m$:

$$M_\nu \simeq \begin{pmatrix} -m^2/M & 0 \\ 0 & M \end{pmatrix}$$

Seesaw mechanism: mixing

Type I returns (2 generations)

$$m_\nu = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}^{-1} \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix}$$

$$= 1/M_1 \begin{pmatrix} a_1^2 & a_1 a_2 \\ a_2 a_1 & a_2^2 \end{pmatrix} + 1/M_2 \begin{pmatrix} b_1^2 & b_1 b_2 \\ b_2 b_1 & b_2^2 \end{pmatrix}$$

With $M_1 < M_2$ and $(a_1, a_2).(b_1, b_2) = 0$:

- heaviest state $\propto (a_1, a_2)$, mass $\propto 1/M_1$
- lightest state $\propto (b_1, b_2)$, mass $\propto 1/M_2$

Which representations?

The tools

- Abelian symmetry
- Non-Abelian, no "triplet"
- Non-Abelian, with "triplet"

Which fermions?

The ingredients

- Just quarks
- Just leptons
- All of the above

Which embedding?

The work space

- Standard Model, no seesaw
- Effective field theory, no (explicit) seesaw
- SUSY GUT, with seesaw

Objectives

Ambitious model features

- Straightforward embedding into GUT (or string unification)
- Explanation of all fermion data (quarks, leptons)
- Explanation of (near) TBM lepton mixing

Family assignments

$\bar{\phi}$ and ψ

- The fermions ψ_i, ψ_j^c are triplets of the family symmetry
- The flavons $\bar{\phi}^i_A$ are anti-triplets
- Invariant mass terms: $\bar{\phi}^i_A \psi_i \bar{\phi}^j_B \psi_j^c H$

Desired vevs

$$\langle \bar{\phi}_3 \rangle \propto (0, 0, 1)$$

$$\langle \bar{\phi}_{23} \rangle \propto (0, 1, -1)$$

$$\langle \bar{\phi}_{123} \rangle \propto (1, 1, 1)$$

Seesaw revisited: Sequential Dominance

$M_1 < M_2 \ll M_3$

$$m_\nu = \begin{pmatrix} b_1 & c_1 & . \\ b_2 & c_2 & . \\ b_3 & c_3 & . \end{pmatrix} \begin{pmatrix} M_1^{-1} & 0 & 0 \\ 0 & M_2^{-1} & 0 \\ 0 & 0 & M_3^{-1} \end{pmatrix} \begin{pmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ . & . & . \end{pmatrix}$$

Wanted ν Dirac matrix?

$$M_D = \begin{pmatrix} b_1 & c_1 & . \\ b_2 & c_2 & . \\ b_3 & c_3 & . \end{pmatrix} \propto \begin{pmatrix} 0 & c & . \\ b & c & . \\ -b & c & . \end{pmatrix}$$

Mass matrices 1

Term by term

$$P_Y = (\bar{\phi}_3^i \psi_i)(\bar{\phi}_3^j \psi_j^c) H$$

Respective Dirac mass matrix

$$M_f = m_{f_3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Mass matrices 2

Term by term

$$+(\bar{\phi}^{i}_{123}\psi_i)(\bar{\phi}^{j}_{23}\psi^c_j)H$$

Respective Dirac mass matrix

$$+m_{f_3}\begin{pmatrix} 0 & \epsilon^3 & -\epsilon^3 \\ 0 & \epsilon^3 & -\epsilon^3 \\ 0 & \epsilon^3 & -\epsilon^3 \end{pmatrix}$$

Mass matrices 3

Term by term

$$+(\bar{\phi}^{i}_{23}\psi_i)(\bar{\phi}^{j}_{123}\psi^c_j)H$$

Respective Dirac mass matrix

$$+m_{f_3}\begin{pmatrix} 0 & 0 & 0 \\ \epsilon^3 & \epsilon^3 & \epsilon^3 \\ -\epsilon^3 & -\epsilon^3 & -\epsilon^3 \end{pmatrix}$$

Effective neutrino Lagrangian

Effective terms

$$P_\nu = \lambda_@(\bar{\phi}^i_{23}\nu_i)(\bar{\phi}^j_{23}\nu_j)$$

$$+\lambda_\odot(\bar{\phi}^i_{123}\nu_i)(\bar{\phi}^j_{123}\nu_j)$$

Enforced by effective Z_2 symmetry:

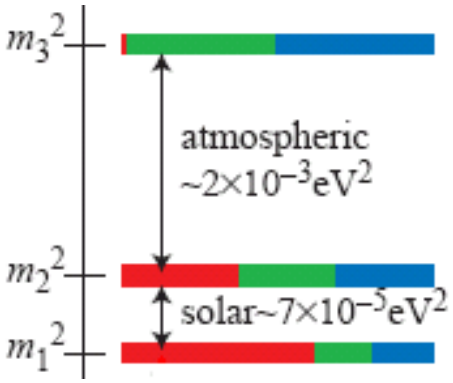
- $\bar{\phi}_{23} \to -\bar{\phi}_{23}$
- $\bar{\phi}_{123} \to \bar{\phi}_{123}$

Try to remember

Once again:

$$\langle\bar{\phi}_{23}\rangle \propto (0, 1, -1)$$

$$\langle\bar{\phi}_{123}\rangle \propto (1, 1, 1)$$



Getting the effective terms

Dirac + Majorana

$$P_Y = (\bar{\phi}^i_{23}\nu_i)(\bar{\phi}^j_{123}\nu^c_j)H \rightarrow @$$

$$+(\bar{\phi}^i_{123}\nu_i)(\bar{\phi}^j_{23}\nu^c_j)H \rightarrow \odot$$

$$+(\bar{\phi}^i_{3}\nu_i)(\bar{\phi}^j_{3}\nu^c_j)H \rightarrow decouples$$

$$P_M = (\bar{\phi}^i_{123}\nu^c_i)(\bar{\phi}^j_{123}\nu^c_j)(...) \rightarrow @$$

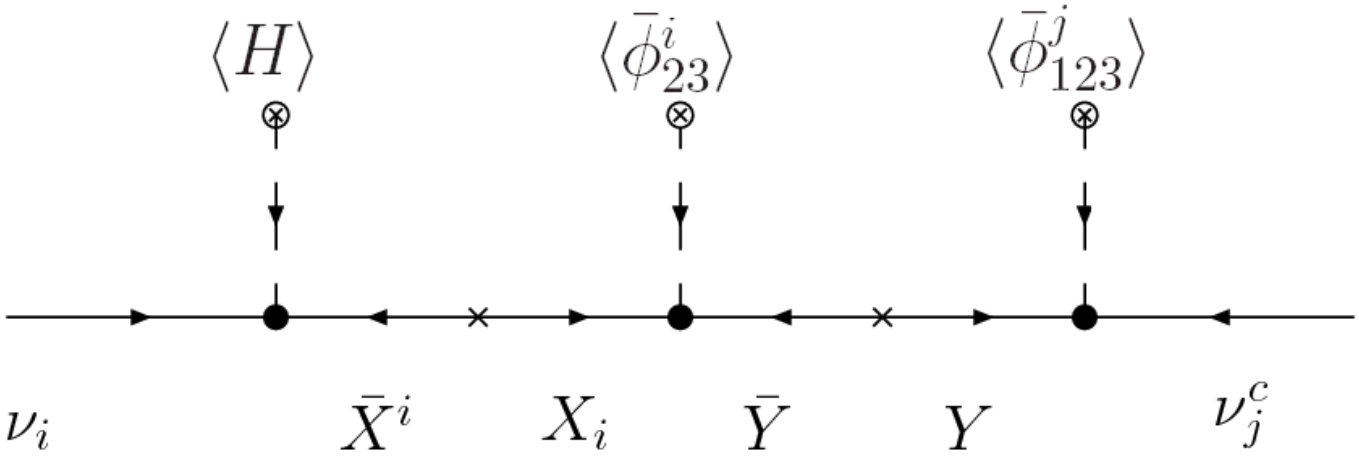
$$+(\bar{\phi}^i_{23}\nu^c_i)(\bar{\phi}^j_{23}\nu^c_j)(...) \rightarrow \odot$$

$$+M_3\nu^c_3\nu^c_3 \rightarrow decouples$$

Drawing TBM 1

Step by step

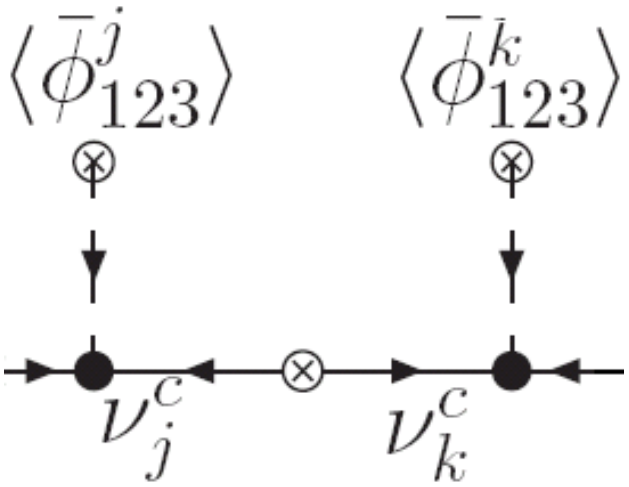
$$P_Y = (\bar{\phi}^i_{23}\nu_i)(\bar{\phi}^j_{123}\nu^c_j)H \to @$$



Drawing TBM 2

Step by step

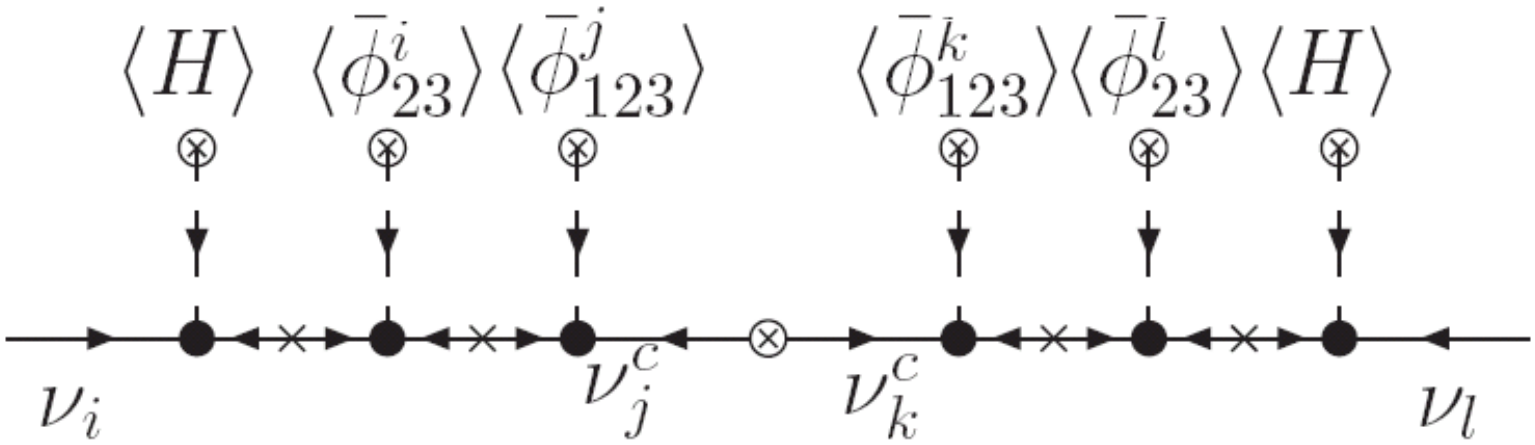
$$P_M = (\bar{\phi}^j_{123}\nu^c_j)(\bar{\phi}^k_{123}\nu^c_k)(\ldots) \rightarrow @$$



Drawing TBM 3

Step by step

$$P_Y = (\bar{\phi}^k_{123}\nu^c_k)(\bar{\phi}^l_{23}\nu_l)H \to @$$



Getting the effective symmetry

Unwanted terms

A pity if this term contributes:

$$P_T = (\bar{\phi}^i_{23}\nu_i)(\bar{\phi}^j_{23}\nu^c_j)H$$

"Seesaws" into $P_{\nu_{mix}}$, spoils TBM:

$$\rightarrow P_{\nu_{mix}} = (\bar{\phi}^i_{23}\nu_i)(\bar{\phi}^j_{123}\nu_j)$$

- Added symmetry blocks terms like P_T
- Effectively producing the required Z_2

Added symmetry (reduced)

Field	$U(1)$
ψ	**0**
ψ^c	**0**
H	**0**
$\bar{\phi}_3$	**0**
$\bar{\phi}_{23}$	$-$**1**
$\bar{\phi}_{123}$	**1**

$\Delta(27)$ family symmetry

Why is it interesting?

- small subgroup of $SU(3)_f$
- distinct "triplets" and "anti-triplets"
- forbids the "quadratic" invariant $\psi_i \psi_i^c$
- added invariants useful for vacuum alignment
- discrete family symmetries don't have associated D-terms

$\Delta(27)$ invariants

Transformation properties

Field (ϕ_i)	Z_3	Z_3'
ϕ_1	ϕ_1	ϕ_3
ϕ_2	$\alpha\phi_2$	ϕ_1
ϕ_3	$(\alpha)^2\,\phi_3$	ϕ_2

- Allowed: all $SU(3)_f$ invariants (e.g. $\bar{\phi}^i_A\psi_i$)
- Disallowed: some $SO(3)$ invariants (e.g. $\psi_i\psi_i^c$)
- Allowed: higher order discrete invariants (e.g. $\bar{\phi}^i\phi_i\bar{\phi}^i\phi_i$)

Soft radiative breaking

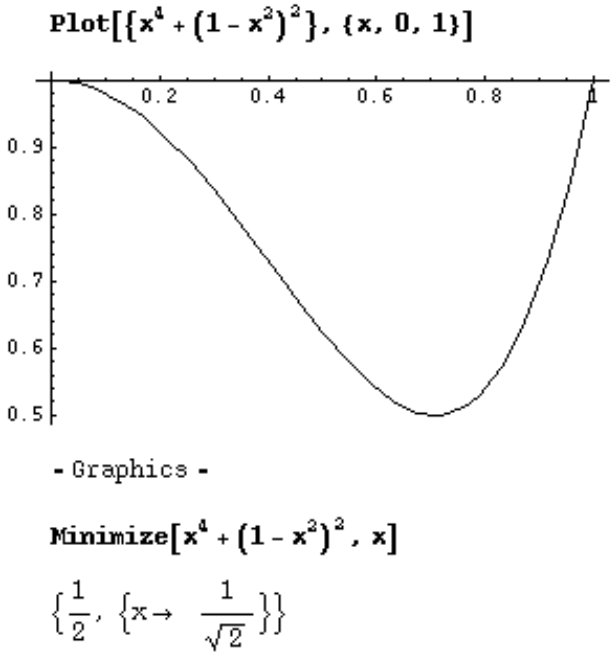
Continuous Breaks

- $V_{cr} = -m^2(\varphi^i \varphi_i^\dagger)$
- Continuous symmetry, continuum of vacuum
- No specified $\langle \varphi \rangle$ direction

Soft alignment

2 generation

- Discrete symmetry, discrete vacuum
- $V = V_{cr} \pm m_{3/2}^2(\varphi^i\varphi_i^\dagger\varphi^i\varphi_i^\dagger)$
- Extremise $|\varphi_1|^4 + |\varphi_2|^4$ (with set magnitude)
- + , VEV $\propto (1,1) \rightarrow V \sim +2v^4/4$
- - , VEV $\propto (0,1) \rightarrow V \sim -v^4$



$\bar{\phi}_3$ and $\bar{\phi}_{123}$

Quartic term minimisation

- $V = V_{cr} \pm m_{3/2}^2 (\varphi^i \varphi_i^\dagger \varphi^i \varphi_i^\dagger)$
- For $\varphi = \bar{\phi}_{123}$, $+ \rightarrow \langle \bar{\phi}_{123} \rangle \propto (1, 1, 1)$
- For $\varphi = \bar{\phi}_3$, $- \rightarrow \langle \bar{\phi}_3 \rangle \propto (0, 0, 1)$

Mixing angles

PMNS angles

- $s_{23}^2 \simeq (\frac{1}{\sqrt{2}} + (...))^2$
- $s_{12}^2 \simeq (\frac{1}{\sqrt{3}} + \frac{\theta_C}{3\sqrt{2}} \cos(\delta))^2$
- $s_{13}^2 \simeq (0 + \frac{\theta_C}{3\sqrt{2}})^2$

Measured angles (3 σ)

- $s_{12}^2 = 0.32 \pm 0.08$
- $s_{23}^2 = 0.50 \pm 0.16$
- $s_{13}^2 < 0.050$ (BFG 0.007)

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Summary

F.S. & F.M.M.

- Lepton (ν) mixing may be a powerful hint of a symmetry
- The seesaw and alignment of vevs may play key roles
- Ambitious, viable, and unifiable model(s) available

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