MSSM predictions for $B_s \to \mu^+\mu^-$ at low $\tan\beta$

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Work in progress with J. Rosiek and P. Tanedo.

See also lectures by A. Ali, Y. Okada, and B. Allanach

Literature

There are two approaches to calculate the dominant contributions

- Feynman Diagrammatic Approach + Ressumation
  Choudhury and Gaur 1999; Huang, Liao, Yan, Zhu, 2001;
  Chankowski and Slawianowska 2001;
  Bobeth, Ewerth, Krüger, Urban, 2001,2002;
  Dedes, Dreiner, Nierste, 2001 ;......;
  S. Heinemeyer, X. Miao, S. Su and G. Weiglein, 2008 .
- Effective Lagrangian Approach
  Babu and Kolda 2000; Isidori and Retico, 2001;
  Dedes and Pilaftsis 2003 ;
  Buras, Chankowski, Rosiek, Slawianowska, 2003 ;......;
  Foster, Okumura, Roszkowski, 2005;
  Carena, Menon, Noriega-Papaqui, Szynkman, Wagner, ’06
  J. Ellis, J. S. Lee and A. Pilaftsis, 2007.

Both have advantages and disadvantages

Introduction

Thanks to the Tevatron, the upper bound on $\mathcal{B}(B_s \to \mu^+\mu^-)$ is at most ~ 15 times away from its SM prediction : $(4.4 \pm 0.7) \times 10^{-9}$

If a possible enhancement is found it could arise from large-ish value of $\tan\beta$ and light-ish Higgs sector. This scenario is very popular and calculational details have been worked out.

However, the small $\tan\beta$ (or Higgs-penguin free) case needs to be fully completed and see if there are interesting effects.

Interesting effects could be partial or complete cancellation of the SM contribution due to SUSY box and Z-penguins.

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In every of those cases there is always a LL - squark mixing generated at the EW scale, i.e.,

$$(\delta^d_{LL})_{ij} \equiv \frac{(\mathcal{M}^d_{LL})^2_{ij}}{\tilde{M}^2} \neq 0 \,, \quad \text{with} \quad i \neq j \,.$$

In cases (A, B) this is small because it results from Renormalization Group running

In cases (C,D,E) this is of O(1) and triggers interesting flavour effects in $\mathcal{B}(B_s \to \mu^+\mu^-)$ at low $\tan\beta$.

Note that in cases (A,B), the δ^d_{LL} sign is fixed while in cases (C,D,E) could be both signs.

Effective Hamiltonian

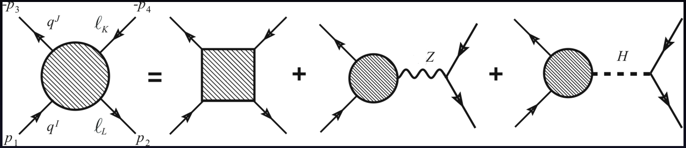
There are 10 effective operators for the amplitude : $b \longrightarrow s\,\ell\,\ell'$

$$\mathcal{H}_{\text{eff}} = \sum_{X,Y=L,R} \left(C^{V}_{XY}\, \mathcal{O}^{V}_{XY} \;+\; C^{S}_{XY}\, \mathcal{O}^{S}_{XY} \;+\; C^{T}_{XY}\, \mathcal{O}^{T}_{X} \right)$$

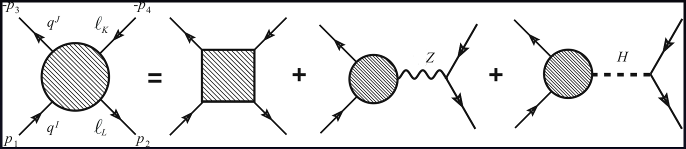
$$\begin{aligned} \mathcal{O}^{V}_{XY} &= \bar{b}\gamma^{\mu}P_X s \otimes \bar{\ell}\gamma_{\mu}P_Y \ell' \\ \mathcal{O}^{S}_{XY} &= \bar{b}P_X s \otimes \bar{\ell}P_Y \ell' \\ \mathcal{O}^{T}_{X} &= \bar{b}\sigma^{\mu\nu}P_X s \otimes \bar{\ell}\sigma_{\mu\nu}P_Y \ell' \end{aligned}$$

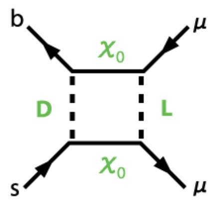
The operator $\mathcal{O}^{T}_{X}$ does not contribute to $\mathcal{B}(B_s \to \mu^+\mu^-)$

Diagrams



Diagrams





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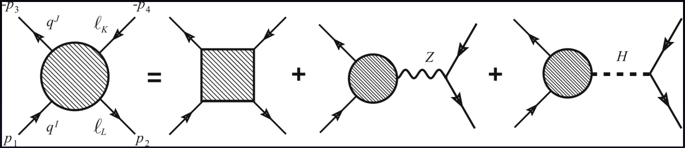

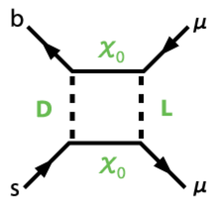

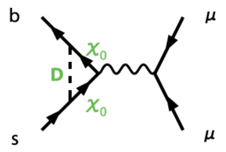

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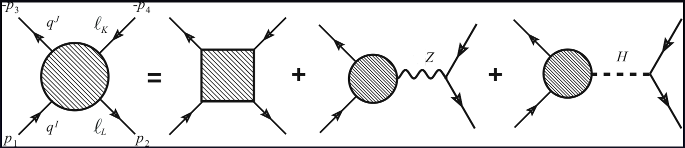
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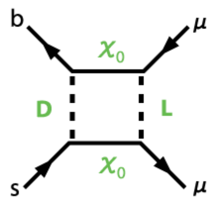
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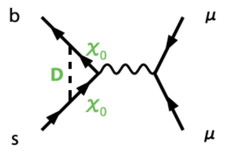
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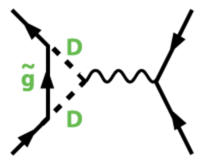
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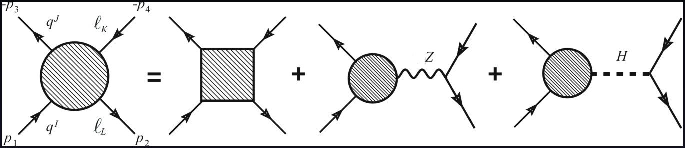
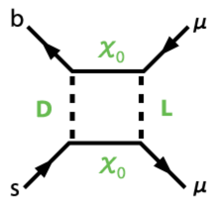
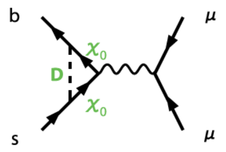
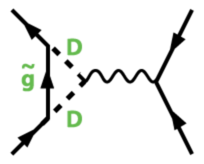
Diagrams









Note : we choose low $\tan\beta \leq 10$ and large M_A so the SUSY Higgs penguin is highly suppressed.

Br

We focus on lepton flavour conserving $B_q \to \mu^+\mu^-$. The squared amplitude goes approximately like :

$$|\mathcal{M}|^2 \approx 2M_{B_q}^2 \left(|F_S|^2 \; + \; |F_P + 2\, m_\ell \, F_A|^2 \right) \; ,$$

with various form-factors being :

$$\begin{aligned} F_S &= \frac{i}{4} \frac{M_{B_s}^2 f_{B_s}}{m_b + m_s} \left(C_{LL}^S + C_{LR}^S - C_{RR}^S - C_{RL}^S \right) , \\ F_P &= \frac{i}{4} \frac{M_{B_s}^2 f_{B_s}}{m_b + m_s} \left(-C_{LL}^S + C_{LR}^S - C_{RR}^S + C_{RL}^S \right) , \\ F_A &= -\frac{i}{4} f_{B_s} \left(-C_{LL}^V + C_{LR}^V - C_{RR}^V + C_{RL}^V \right) . \end{aligned}$$

Cancellation of the SM contribution is, in principle, possible : it must be $F_A \approx 0$. Thus $\tan\beta$ must be low since only then $F_{S,P} \approx 0$.

Calculation

We perform the calculation using analytical Feynman Diagramatic methods :

1. taking into account EW symmetry breaking effects
2. without resorting to “mass insertion” expansion
3. including all flavour effects and complex parameters
4. present a code for this calculation (and not only ...)

Example : $\tilde{g}$-contribution

The full $\tilde{g}$-contribution in $b \to s$ self energy is :

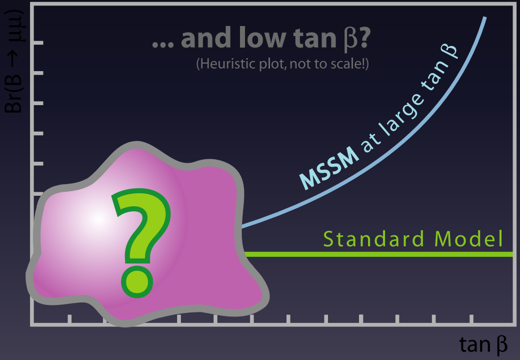
$$(4\pi)^2 (\Sigma^d_{VL})^{JI} \;=\; \frac{32\pi\alpha_s}{3} \sum_{k=1}^{6} Z_D^{Jk*}\, Z_D^{Ik}\, B_1(s, m^2_{\tilde{g}}, m^2_{D_k}) \;.$$

If $m^2_{D_k} \approx m_0^2$ then we can Taylor expand and using unitarity of Z-matrix we obtain

$$\begin{aligned}(4\pi)^2 (\Sigma^d_{VL})^{JI} \;&\approx\; \frac{32\pi\alpha_s}{3} (\delta^d_{LL})_{23} \left[B_1(s, m^2_{\tilde{g}}, m^2_{D_3}) - B_1(s, m^2_{\tilde{g}}, m^2_{D_2}) \right] \\ &+\; O[(\delta^d_{LL})^2_{23}]\end{aligned}$$

Results

The aim is to calculate all possible SUSY contributions to $\mathcal{B}(B_s \to \mu^+\mu^-)$ at low $\tan\beta$ and thus completing the picture.



Quantity	Current Measurement	Experimental Error
$m_{\chi_1^0}$	$>$ 46 GeV	
$m_{\chi_1^\pm}$	$>$ 94 GeV	
$m_{\tilde{b}}$	$>$ 89 GeV	
$m_{\tilde{t}}$	$>$ 95.7 GeV	
m_h	$>$ 92.8 GeV	
$\|\epsilon_K\|$	$2.232 \cdot 10^{-3}$	$0.007 \cdot 10^{-3}$
ΔM_d	$3.337 \cdot 10^{-13}$ GeV	$0.033 \cdot 10^{-13}$ GeV
ΔM_s	$116.96 \cdot 10^{-13}$ GeV	$0.79 \cdot 10^{-13}$ GeV
Br($B \to X_s \gamma$)	$3.34 \cdot 10^{-4}$	$0.38 \cdot 10^{-4}$
Br($K_L \to \pi^0 \nu \bar{\nu}$)	$< 1.5 \cdot 10^{-10}$	–

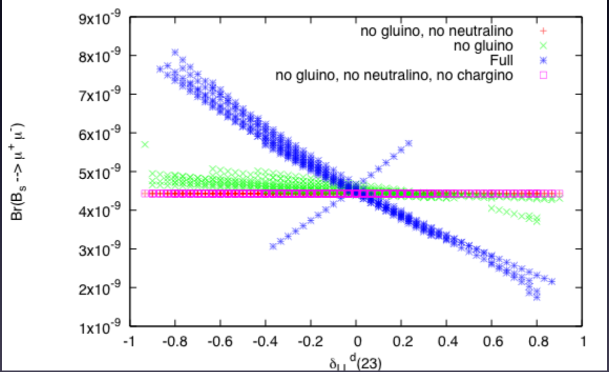
Bounds : for every observable we require :

$$|Q^{exp} \; - \; Q^{th}| \leq 3\, \Delta Q^{exp} \; + \; q|Q^{th}| \,, \qquad q = 0.5 \,.$$

[Similar to Buras, Ewerth, Jäger, Rosiek, 0408142]

Various SUSY contributions

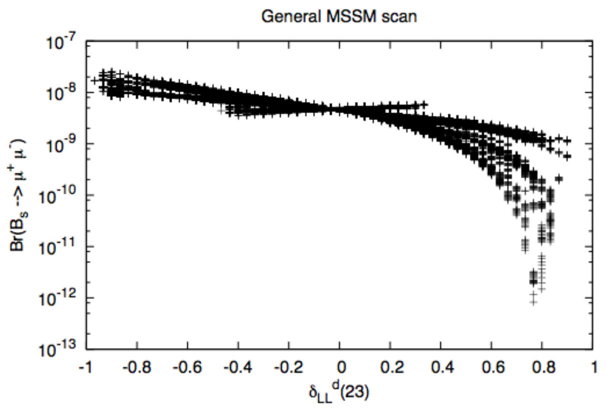
$$\begin{aligned} \tan\beta &= 5\,,\quad \mu = -450, -150, 150, 450~\mathrm{GeV}\,,\quad m_{\tilde g} = 300, 900, 1500~\mathrm{GeV}\,,\\ M_{SUSY} &= 1~\mathrm{TeV}\,,\quad M_{\tilde t_L} = M_{\tilde b_L} = 200~\mathrm{GeV}\,,\quad M_{\tilde t_R} = 150~\mathrm{GeV}\,,\ \gamma = 70^o\,. \end{aligned}$$



Parameter	Symbol	Min	Max	Step
Ratio of Higgs vevs	$\tan\beta$	2	10	1
CKM phase	γ	$-\pi$	π	$\pi/25$
CP-odd Higgs mass	M_A	100	500	200
SUSY Higgs mixing	μ	-450	450	300
$SU(2)$ gaugino mass	M_2	100	500	200
Gluino mass	M_3	$3M_2$	$3M_2$	0
SUSY scale	$M_{\rm SUSY}$	200	1000	200
Slepton Masses	$M_{\tilde{\ell}}$	$M_{\rm SUSY}/3$	$M_{\rm SUSY}/3$	0
Left top squark mass	$M_{\tilde{Q}_L}$	200	1000	200
Right bottom squark mass	$M_{\tilde{b}_R}$	200	1000	200
Right top squark mass	$M_{\tilde{t}_R}$	150	450	150
Mass insertion	δ^{23}_{dLL}	-1	1	1/30

Table: Range of input parameters. By “SUSY” scale here we mean the common mass parameter of squarks of the first two generations. All mass parameters are in GeV.

(Preliminary) [see parameters in the previous slide]



Cancellations seem to be possible and consistent with other observables. However, this needs some amount of accidental cancellations in ΔM_s and $b \to s\gamma$ observables.

Conclusions

- Full calculation of $B_q \to l^+l^-$ in the MSSM at low $\tan\beta$
- Corrections are important when squark mixing is significant i.e, $\delta^d_{LL} \sim O(1)$.
- Enhancement of the Branching ratio up to an order of magnitude
- but also, reduction of the SM by order of magnitude(s)
- The code will be available soon.