The MSSM with minimal flavour violations and its running

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Outline

Introduction

Minimal flavour violation

Running of MFV models

Summary and conclusions

Work done in collaboration with Emanuel Nikolidakis and Christopher Smith

Work on the same problem has recently appeared, Paradisi et al. `arXiv:0805.3989[hep-ph]`

SUSY and flavour

- the MSSM has many more sources of flavour violation than the SM

  Masses:

  $$-\tilde{Q}^\dagger \mathbf{m}_Q^2 \cdot \tilde{Q} - \tilde{U} \mathbf{m}_U^2 \tilde{U}^\dagger - \tilde{D} \mathbf{m}_D^2 \tilde{D}^\dagger - \tilde{L}^\dagger \mathbf{m}_L^2 \cdot \tilde{L} - \tilde{E} \mathbf{m}_E^2 \tilde{E}^\dagger$$

  Trilinear terms:

  $$-\tilde{U} \mathbf{A}_u (\tilde{Q})_a (H_u)^a + \tilde{D} \mathbf{A}_d (\tilde{Q})_a (H_d)^a + \tilde{E} \mathbf{A}_\ell (\tilde{L})_a (H_d)^a + h.c.$$

- precise measurements of flavour violations at low energy show agreement with the SM
- unless $M_{\mathrm{SUSY}} \geq 10^2$ TeV
  $\Rightarrow$ flavour violating couplings are strongly constrained...

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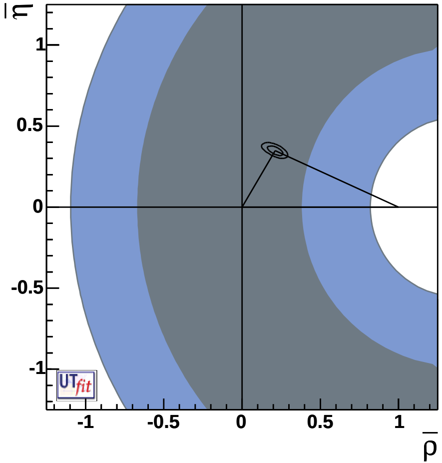
- unless $M_{\text{SUSY}} \geq 10^2$ TeV
  $\Rightarrow$ flavour violating couplings are strongly constrained...
- ...but there is still room for discovering new effects!

SUSY and flavour



Constraints from $K^+ \to \pi^+ \nu \bar{\nu}$

Minimal flavour violation

- Minimal flavour violation solves the flavour problem by transforming it into a symmetry principle
- it drastically reduces the number of free parameters in the MSSM $\Rightarrow$ although MFV defines a large class of models it is still predictive
- makes the agreement with the known phenomenology easy to achieve
- deviations from the SM rather mild
  $\Rightarrow$ look hard if you want to find them

SUSY and unification

- the MSSM provides a successful unification of the coupling constants $\Rightarrow$ strong point in favour of SUSY
- Interplay with the flavour problem?

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  - imposing that flavour violations be small at one scale can make them large at another scale
  - “solutions” of the flavour problem at one scale may break down at a different scale or appear fine tuned
  - in particular a symmetry solution like MFV should better be scale invariant
  - how does an MFV-model change as you change the scale? RGE’s for MFV models?

Flavour symmetry in the SM

Gauge interactions are flavour-blind. Flavour mixing in the SM occurs in the Yukawa terms:

$$\mathcal{L}_Y = \bar{U}_R\,\mathbf{Y}_u\,Q_L H + \bar{D}_R\,\mathbf{Y}_d\,Q_L H_c + E_R\,\mathbf{Y}_e\,L_L H_c + \mathrm{h.c.} \quad ,$$

if $\mathbf{Y}_u = \mathbf{Y}_d = \mathbf{Y}_e = 0$ the SM acquires a large global flavour symmetry:

$$G_F \equiv G_q \otimes G_\ell \otimes U(1)_B \otimes U(1)_L \otimes U(1)_Y \otimes U(1)_{PQ} \otimes U(1)_{E_R}$$

where

$$G_q \equiv SU(3)_{Q_L} \otimes SU(3)_{U_R} \otimes SU(3)_{D_R} \ , \qquad G_\ell \equiv SU(3)_{L_L} \otimes SU(3)_{E_R}$$

Chivukula, Georgi (1987)

Spurions

The SM remains (formally) invariant under $G_q \otimes G_\ell$ if the Yukawas are promoted to spurion fields transforming as

$$\mathbf{Y}_u \sim (\bar{3}, 3, 1) \,, \quad \mathbf{Y}_d \sim (\bar{3}, 1, 3) \quad \text{under } G_q$$
$$\mathbf{Y}_e \sim (\bar{3}, 3) \quad \text{under } G_\ell \ .$$

Symmetry breaking occurs if the Yukawa's take a specific value

VEV of truly dynamical fields?
Dynamical explanation of flavour violations?

Minimal flavour violation

An extension of the SM is defined to respect minimal flavour violation (MFV) if it is symmetric under $G_q \otimes G_\ell$ in the presence of:

D'Ambrosio, Giudice, Isidori, Strumia (02)

- new matter fields transforming nontrivially under G_F
- the Yukawa spurions
- no other spurion fields

AND if coupling constants in front of the Yukawa's are $\mathcal{O}(1)$

In supersymmetric extensions of the SM the superpotential is automatically MFV. On the other hand a generic, nondiagonal soft SUSY-breaking term does not respect MFV unless one can appropriately express the matrices in terms of the Yukawas

MFV in the MSSM

D'Ambrosio et al. (02) wrote the soft SUSY-breaking terms in the MSSM (almost) as follows

$$
\begin{aligned}
\mathbf{m}_Q^2 &= m_0^2 \left(a_1 \mathbf{1} + b_1 \mathbf{Y}_u^\dagger \mathbf{Y}_u + b_2 \mathbf{Y}_d^\dagger \mathbf{Y}_d + b_3 (\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d) \right) \\
\mathbf{m}_U^2 &= m_0^2 \left(a_2 \mathbf{1} + b_5 \mathbf{Y}_u \mathbf{Y}_u^\dagger \right) \\
\mathbf{m}_D^2 &= m_0^2 \left(a_3 \mathbf{1} + b_6 \mathbf{Y}_d \mathbf{Y}_d^\dagger \right) \\
\mathbf{A}_u &= A_0 \mathbf{Y}_u (a_4 + b_7 \mathbf{Y}_d^\dagger \mathbf{Y}_d) \\
\mathbf{A}_d &= A_0 \mathbf{Y}_d (a_5 + b_8 \mathbf{Y}_u^\dagger \mathbf{Y}_u)
\end{aligned}
$$

- number of free parameters is drastically reduced
- flavour violations are kept under control

Revisiting the construction of the MSSM with MFV

Consider the term $\mathcal{L}_{\mathbf{m}_Q^2} \equiv -\tilde{Q}^\dagger \mathbf{m}_Q^2 \cdot \tilde{Q}$

this respects MFV if $\mathbf{m}_Q^2$ transforms like $(8, 1, 1)$:

$$\mathbf{m}_Q^2 = a_1 \mathbf{1} + b_1 \mathbf{Y}_u^\dagger \mathbf{Y}_u + b_2 \mathbf{Y}_d^\dagger \mathbf{Y}_d + b_3 \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u + c_1 \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d + \dots$$

with in principle an infinite sum of admissible terms.

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with in principle an infinite sum of admissible terms.
However, $\mathbf{m}_Q^2$ is a 3×3 hermitian matrix and Cayley-Hamilton identities

$$\mathbf{X}^3 - \langle \mathbf{X} \rangle \mathbf{X}^2 + \frac{1}{2} \mathbf{X} \left(\langle \mathbf{X} \rangle^2 - \left\langle \mathbf{X}^2 \right\rangle \right) - \det \mathbf{X} = 0$$

constrain the number of independent terms

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with in principle an infinite sum of admissible terms.

Cayley–Hamilton $\Rightarrow$

$$\begin{aligned}
\mathbf{m}_Q^2 &= z_1 \mathbf{1} + z_2 \mathbf{Y}_u^\dagger \mathbf{Y}_u + z_3 \mathbf{Y}_d^\dagger \mathbf{Y}_d + z_4 \left(\mathbf{Y}_u^\dagger \mathbf{Y}_u\right)^2 + z_5 \left(\mathbf{Y}_d^\dagger \mathbf{Y}_d\right)^2 \\
&+ z_6 \left(\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathrm{h.c.}\right) + z_7 \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u \\
&+ z_8 \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d + z_9 \left(\left(\mathbf{Y}_u^\dagger \mathbf{Y}_u\right)^2 \left(\mathbf{Y}_d^\dagger \mathbf{Y}_d\right)^2 + \mathrm{h.c.}\right)
\end{aligned}$$

MFV can be viewed as a reparametrization!

An almost singular parametrization

MFV provides a very special parametrization: the Yukawa's are very far from generic matrices. Choose, e.g.

$$\mathbf{Y}_u = \lambda_u V \ , \quad \mathbf{Y}_d = \lambda_d \ , \quad \mathbf{Y}_e = \lambda_e$$

where

$$\lambda_u = \mathrm{diag}(y_u, y_c, y_t) \ , \lambda_d = \mathrm{diag}(y_d, y_s, y_b) \ , \quad V = V_{\mathrm{CKM}}$$

then

$$\left(\mathbf{Y}_u^\dagger \mathbf{Y}_u\right)^2 - y_t^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u \sim \mathcal{O}(y_c^2) \qquad \left(\mathbf{Y}_d^\dagger \mathbf{Y}_d\right)^2 - y_b^2 \mathbf{Y}_d^\dagger \mathbf{Y}_d \sim \mathcal{O}(y_s^2)$$

$\Rightarrow$ if one assumes that the coefficients in front of each MFV monomial is $\mathcal{O}(1)$, one can dispose of terms containing squares of $\mathbf{Y}_u^\dagger \mathbf{Y}_u$ and $\mathbf{Y}_d^\dagger \mathbf{Y}_d$.

$\Rightarrow$ strong reduction of the number of free parameters

MSSM with MFV

- strong hierarchy of the Yukawa couplings ✓
- CKM almost diagonal ?

$$V = \begin{pmatrix} 1 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1-(\rho+i\eta)) & -A\lambda^2 & 1 \end{pmatrix}$$

where $\lambda = V_{us} \sim 0.23$

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Counting scheme:

$$\frac{m_u}{m_t} \sim \mathcal{O}(\lambda^7), \quad \frac{m_c}{m_t} \sim \mathcal{O}(\lambda^4), \quad y_t \sim \mathcal{O}(1)$$
$$\frac{m_d}{m_t} \sim \mathcal{O}(\lambda^7), \quad \frac{m_s}{m_t} \sim \mathcal{O}(\lambda^5), \quad \frac{m_b}{m_t} \sim \mathcal{O}(\lambda^3),$$

MSSM with MFV

Systematic analysis:

- write down all independent terms
- drop terms of absolute order $\mathcal{O}(\lambda^6)$ or which provide only corrections of relative order $\mathcal{O}(\lambda^2)$

$$
\begin{aligned}
\mathbf{m}_Q^2 &= m_0^2\left[a_1 + b_1\mathbf{Y}_u^\dagger\mathbf{Y}_u + b_2\mathbf{Y}_d^\dagger\mathbf{Y}_d + b_3(\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u + \mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d)\right] \\
\mathbf{m}_U^2 &= m_0^2\left[a_2 + b_4\mathbf{Y}_u\mathbf{Y}_u^\dagger\right] \\
\mathbf{m}_D^2 &= m_0^2\left[a_3 + \mathbf{Y}_d\left(b_5 + b_6\mathbf{Y}_u^\dagger\mathbf{Y}_u\right)\mathbf{Y}_d^\dagger\right] \\
\mathbf{A}^U &= A_0\mathbf{Y}_u\left[a_4 + b_7\mathbf{Y}_u^\dagger\mathbf{Y}_u + b_8\mathbf{Y}_d^\dagger\mathbf{Y}_d\right] \\
\mathbf{A}^D &= A_0\mathbf{Y}_d\left[a_5 + b_9\mathbf{Y}_u^\dagger\mathbf{Y}_u + b_{10}\mathbf{Y}_d^\dagger\mathbf{Y}_d + b_{11}\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\right]
\end{aligned}
$$

This is valid also in the case of large $\tan\beta$. If $\tan\beta \sim 1$ all terms containing $\mathbf{Y}_d^\dagger\mathbf{Y}_d$ can be dropped

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\end{aligned}$$

Remark: invariant terms can be built also with ϵ tensors – they are not relevant in this context, but very interesting in theories without R-parity

Nikolidakis & Smith (07)

MFV as a substitute for *R* parity?

- The soft SUSY breaking terms discussed so far are those relevant in the case of an *R*-parity respecting theory
- Nikolidakis and Smith have analyzed how MFV constrains the *R*-parity violating terms
- They have shown that, surprisingly:
  MFV alone is sufficient to forbid a too fast proton decay

RGE's of MFV parameters?

- viewing MFV as a reparametrization, it is clear that one can derive the RGE for the MFV parameters exactly
- MFV is useful only because one can throw away some terms and reduce the number of free parameters
- the scheme is RGE invariant only if terms which can be neglected at one scale do not become important at another scale (and viceversa)
- $\Rightarrow$ apply systematically our counting rule also to the β-functions

RGE's for the Yukawa's

In MFV the basis is expressed in terms of the Yukawa's, which, however, also run, according to:

$$\beta_{\mathbf{Y}_u} = \mathbf{Y}_u \left[3\mathrm{Tr}\left(\mathbf{Y}_u \mathbf{Y}_u^\dagger\right) + 3\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_d^\dagger \mathbf{Y}_d - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{13}{15} g_1^2 \right]$$

where

$$\frac{d}{dt}\mathbf{Y}_u = \frac{1}{16\pi^2}\beta^{(1)}_{\mathbf{Y}_u} + \dots$$

Use the running Yukawa's to define a “running” MFV?

A new basis

Keep the CKM matrix at $\mu = M_Z$ fixed and express everything – including the Yukawa's – in terms of this.

$$[\mathbf{Y}_u(M_Z)]_{ij} = y_c \delta_{2i} V_{2j} + y_t \delta_{3i} V_{3j} \ , \quad [\mathbf{Y}_d(M_Z)]_{ij} = y_s \delta_{2i} \delta_{2j} + y_b \delta_{3i} \delta_{3j}$$

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At a different scale, the Yukawa's will look different and contain new structures. A complete list (up to this order) is the following:

$$\begin{array}{llll}
X_1 = \delta_{3i}\delta_{3j} & X_5 = \delta_{3i}V_{3j} & X_9 = V^*_{3i}\delta_{3j} & X_{13} = V^*_{3i}V_{3j} \\
X_2 = \delta_{2i}\delta_{2j} & X_6 = \delta_{2i}V_{2j} & X_{10} = V^*_{2i}\delta_{2j} & X_{14} = V^*_{2i}V_{2j} \\
X_3 = \delta_{3i}\delta_{2j} & X_7 = \delta_{3i}V_{2j} & X_{11} = V^*_{3i}\delta_{2j} & X_{15} = V^*_{3i}V_{2j} \\
X_4 = \delta_{2i}\delta_{3j} & X_8 = \delta_{2i}V_{3j} & X_{12} = V^*_{2i}\delta_{3j} & X_{16} = V^*_{2i}V_{3j}
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If we apply our counting rule also to the Yukawa's, however, only one new structure is generated by the running:

$$\begin{aligned} \mathbf{Y}_u(\mu) &= y_c(\mu) X_6 + y_t(\mu) X_5 + c_t(\mu) X_1 \\ \mathbf{Y}_d(\mu) &= y_s(\mu) X_2 + y_b(\mu) X_1 + c_b(\mu) X_5 \end{aligned}$$

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and the β-functions of the coefficients read

$$\begin{aligned} \beta_{y_c} &= y_c \left(3y_t^2 - K_u\right) \\ \beta_{y_t} &= y_t \left(6y_t^2 - K_u\right) \\ \beta_{c_t} &= y_t y_b^2 + c_t \left(6y_t^2 + y_b^2 - K_u\right) \end{aligned}$$

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\end{aligned}$$

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$$\begin{aligned}
\beta_{y_s} &= y_s \left(3y_b^2 + y_\tau^2 - K_d \right) \\
\beta_{y_b} &= y_b \left(6y_b^2 + y_\tau^2 - K_d \right) \\
\beta_{c_b} &= y_b y_t^2 + c_b \left(6y_b^2 + y_t^2 + y_\tau^2 - K_d \right)
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and the β-functions of the coefficients read

where $\quad K_u = \frac{16}{3} g_3^2 + 3 g_2^2 + \frac{13}{15} g_1^2 \ , \qquad K_d = K_u - \frac{2}{5} g_1^2$

MFV in the new basis

$$
\begin{aligned}
\mathbf{m}_Q^2 &= m_0^2\,[a_1 + x_1 X_{13} + y_1 X_1 + y_2 (X_5 + X_9)] \\
\mathbf{m}_U^2 &= m_0^2\,[a_2 + x_2 X_1] \\
\mathbf{m}_D^2 &= m_0^2\,[a_3 + y_3 X_1 + w_1 (X_3 + X_4)] \\
\mathbf{A}^U &= A_0\,[\tilde{a}_4 X_5 + y_4 X_1 + w_2 X_6] \\
\mathbf{A}^D &= A_0\,[\tilde{a}_5 X_1 + y_5 X_5 + w_3 X_2 + w_4 X_8]
\end{aligned}
$$

In this new basis all matrices are $\mathcal{O}(1)$ and whether a term is small or large can be seen in the coefficients:

$$
\begin{aligned}
x_i &\sim \mathcal{O}(1) \\
y_i &\sim \mathcal{O}\left(t_\beta^2 \lambda^6\right) \quad i = 1,2,3,4 \\
\tilde{a}_5 &\sim y_5 \sim \mathcal{O}\left(t_\beta \lambda^3\right) \\
w_1 &\sim \mathcal{O}(t_\beta^2 \lambda^{10})\,, \quad w_2 \sim \mathcal{O}(\lambda^4) \\
w_3 &\sim \mathcal{O}(t_\beta \lambda^5)\,, \quad w_4 \sim \mathcal{O}(t_\beta \lambda^7)
\end{aligned}
$$

... and the running of the coefficients

Applying the same counting rules as above (*i.e.* dropping any correction of order λ^2 in the β-functions) we get

$$\beta_{a_1} = -2\bar{K}_u + \frac{1}{5}g_1^2\left(S - \frac{28}{3}\frac{M_1^2}{m_0^2}\right)$$

$$\beta_{x_1} = 2y_t^2\left(\frac{m_{H_u}^2}{m_0^2} + a_1 + a_2 + x_1 + x_2 + y_2\right) + 2r_A(\tilde{a}_4^2 + {y_5}^2)$$

$$\beta_{y_1} = 2{y_b}^2\left(\frac{m_{H_d}^2}{m_0^2} + a_1 + a_3 + y_1 + y_2 + y_3\right) + 2r_A({\tilde{a}_5}^2 + {y_4}^2)$$

$$\beta_{y_2} = y_t^2(y_1 + y_2) + {y_b}^2(x_1 + y_2) + 2r_A(\tilde{a}_4 y_4 + \tilde{a}_5 y_5)$$

... and the running of the coefficients

Applying the same counting rules as above (*i.e.* dropping any correction of order λ^2 in the β-functions) we get

$$
\begin{aligned}
\beta_{a_2} &= -\frac{32}{3} g_3^2 \frac{M_3^2}{m_0^2} - \frac{4}{5} g_1^2 \left(S + \frac{8}{3} g_1^2 \right) \frac{M_1^2}{m_0^2} \\
\beta_{x_2} &= 4 y_t^2 \left(\frac{m_{H_u}^2}{m_0^2} + a_1 + a_2 + x_1 + x_2 + y_1 + y_2 \right) + 4 r_A (\tilde{a}_4 + y_4)^2 \\
\beta_{a_3} &= -\frac{32}{3} g_3^2 \frac{M_3^2}{m_0^2} + \frac{2}{5} g_1^2 \left(S - \frac{4}{3} g_1^2 \right) \frac{M_1^2}{m_0^2} \\
\beta_{y_3} &= 4 {y_b}^2 \left(\frac{m_{H_d}^2}{m_0^2} + a_1 + a_3 + x_1 + y_1 + 2 y_2 + y_3 \right) + 4 r_A (\tilde{a}_5 + y_5)^2 \\
\beta_{w_1} &= 2 w_1 {y_b}^2 - 4 V_{cb} y_s y_b (x_1 + y_2) + 4 r_A y_5 (w_4 - V_{cb} w_3) \\
&\dots
\end{aligned}
$$

... and the running of the coefficients

where

$$r_A \equiv A_0^2/m_0^2$$

$$S = \frac{m_{H_u}^2 - m_{H_d}^2}{m_0^2} + 3(a_1 - 2a_2 + a_3) + x_1 - 2x_2 + y_1 + 2y_2 + y_3$$
$$-3(a_6 - a_7) - x_{12} + x_{13}$$

$$\bar{K}_u = \frac{16}{3} g_3^2 \frac{M_3^2}{m_0^2} + 3g_2^2 \frac{M_2^2}{m_0^2} + \frac{13}{15} g_1^2 \frac{M_1^2}{m_0^2}$$

Remarks

The RGE's for the soft-SUSY breaking terms written in this form are simple and transparent

- it is easy to implement and solve them numerically
- the analytical form allows one to see who influences who
- MFV is explicitly scale invariant (but see below!):
  - no new terms are generated in the running (according to our counting rules)
  - the β-functions of the coefficients are of the same order (or lower) as the coefficients themselves

Numerical examples: MSUGRA and perturbations

Benchmark point SPS-1a:

$m_0 = 100\text{GeV},\ m_{1/2} = 250\text{GeV},\ A_0 = -100\text{GeV},\ t_\beta = 10,\ \mu > 0$

	$M_{\rm GUT}$	$M_{\rm SUSY}$
a_1/M_3^2	0.16	0.85
a_2/M_3^2	0.16	0.78
a_3/M_3^2	0.16	0.77
$\tilde{a}_4/(-y_t M_3)$	0.4	0.84
$\tilde{a}_5/(-y_b M_3)$	0.4	1.4
x_1/a_1	δ_1	$-0.17 + 0.026\delta_1 - 0.003\delta_2$
y_1/a_1	0	$-5.8 \cdot 10^{-3}$
y_2/a_1	0	$2.6 \cdot 10^{-4}$
x_2/a_2	δ_2	$-0.37 - 0.007\delta_1 + 0.024\delta_2$
y_3/a_2	0	$-1.1 \cdot 10^{-2} - 1.7 \cdot 10^{-4}\delta_1$

Numerical examples: MSUGRA and perturbations

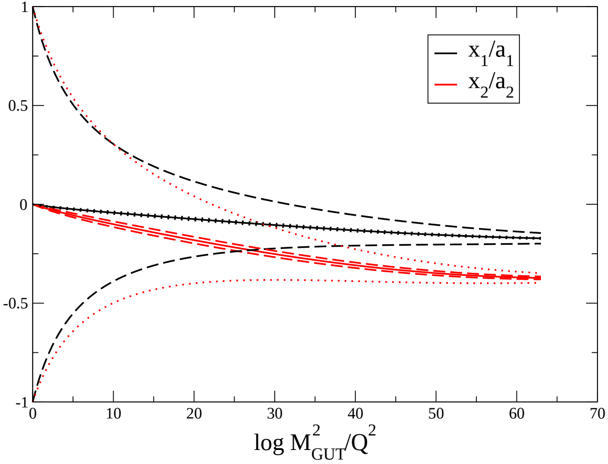
Benchmark point SPS-4:

$m_0 = 400\text{GeV},\ m_{1/2} = 300\text{GeV},\ A_0 = 0,\ t_\beta = 50,\ \mu > 0$

	M_{GUT}	M_{SUSY}
a_1/M_3^2	1.8	1.13
a_2/M_3^2	1.8	1.06
a_3/M_3^2	1.8	1.05
$\tilde{a}_4/(-y_t M_3)$	0	0.77
$\tilde{a}_5/(-y_b M_3)$	0	0.98
x_1/a_1	δ_1	$-0.20 + 0.25\delta_1 - 0.03\delta_2 + 0.005\epsilon_1 + \ldots$
y_1/a_1	$\epsilon_1 t_\beta^2 \lambda^6$	$-0.11 + 0.075\epsilon_1 + 0.003\delta_1 + \ldots$
y_2/a_1	$\epsilon_2 t_\beta^2 \lambda^6$	$2.9 \cdot 10^{-3} + 0.07\epsilon_2 - 9 \cdot 10^{-3}\delta_1 + \ldots$
x_2/a_2	δ_2	$-0.42 - 0.06\delta_1 + 0.23\delta_2 - 0.03\epsilon_2 + \ldots$
y_3/a_2	$\epsilon_3 t_\beta^2 \lambda^6$	$-0.22 - 0.033\delta_1 + 0.08\epsilon_3 - 0.019\epsilon_2 + \ldots$

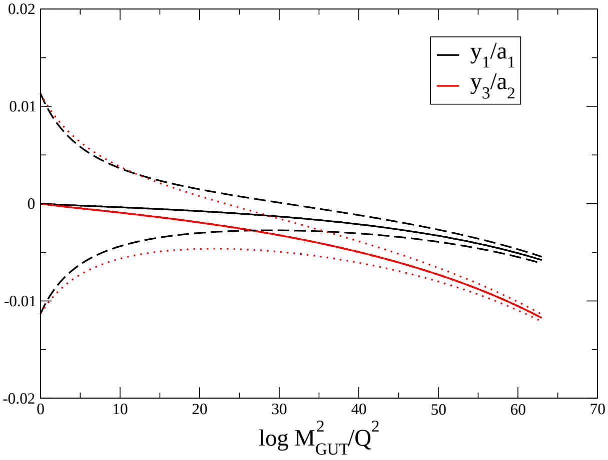
Running of x_1 and x_2

SPS-1a point



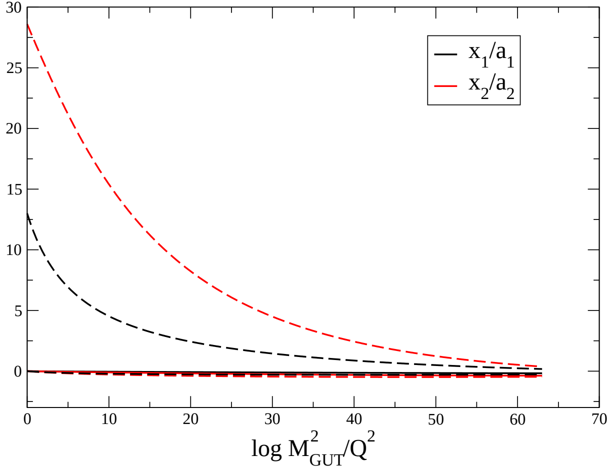
Running of x_1 and x_2

SPS-1a point



Running of x_1 and x_2

SPS-1a point



Flipping the sign of $\frac{x_1}{a_1}$ and $\frac{x_2}{a_2}$ at $Q = M_{\mathrm{SUSY}}$

Comparison with Paradisi et al.

In `arXiv:0805.3989[hep-ph]`, Paradisi et al. have studied the running of the MSSM with MFV

- they mainly rely on SOFTSUSY to do this analysis: input MFV boundary conditions at $Q = M_{\mathrm{GUT}}$ and fit the outcome at the low scale
- they use the original parametrization of MFV and do not apply a counting
- announce that they will study the large-t_β case later
- we confirm their finding about the existence of “fixed points”
- with our parametrization and counting, we are able to discuss the large-t_β case

Summary and conclusions

- I have reviewed the definition of minimal flavour violation in SUSY extensions of the standard model and bound it to power counting rules
- I have shown how the application of the same counting rules leads to a scale-invariant definition of MFV
- we have derived simple RGE's for the MFV parameters and shown the first results of our numerical analysis
- nontrivial consequences of the running of the MFV parameters start to emerge:
  if MFV originates at a higher scale, at low energy it is a lot more constraining than imagined so far