Applications of QCD Sum Rules to Flavour Physics

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QCD Sum Rules

- Correlator of quark currents
  = hadronic sum (dispersion relation)

▶ two-point (SVZ) sum rule

$$\int d^4x\; e^{iqx}\langle 0|T\{j_1(x)j_2(0)\}|0\rangle = \sum_h \frac{\langle 0|j_1|h\rangle\langle h|j_2|0\rangle}{m_h^2 - q^2}$$

$|q^2| \gg \Lambda^2_{QCD}$ $\Downarrow$ $x \to 0$

$$\boxed{\sum_{d=0,3,4,..} C_d(q^2, m_q, \alpha_s)\langle 0|O_d(0)|0\rangle} \Leftarrow \text{OPE}$$

$d \neq 0$, $\langle 0|O_d|0\rangle$- vacuum condensates

- light-cone sum rule (LCSR)

$$\int d^4x\, e^{iqx}\langle 0|T\{j_1(x)j_2(0)\}|H(p)\rangle = \sum_h \frac{\langle 0|j_1|h\rangle\langle h|j_2|H\rangle}{m_h^2-(p-q)^2}$$

$$|q^2| \sim |(p-q)^2| \gg \Lambda^2_{QCD} \qquad \Downarrow \qquad x^2 \to 0$$

$$\boxed{\sum_t C_t(q^2,(p-q)^2,m_q,\alpha_s)\langle 0|O_t(x,0)|H(p)\rangle} \Leftarrow \text{OPE}$$

$\langle 0|O_t(x,0)|H(p)\rangle$ -
light-cone distribution amplitudes (DA's)

QCD Sum Rules

- twofold use :

  (I) hadronic sum from experiment
  $\Longrightarrow$ QCD/OPE parameters:
  m_q, condensates, DA's

  (II) correlator from OPE
  $\Longrightarrow$ hadronic matrix elements:
  $\langle 0|j_1|h\rangle$, $\langle h|j_2|H\rangle$

- Uses of QCD sum rules in flavour physics:

(I) determination of quark masses,

(II) meson decay constants,
semileptonic form factors $\Rightarrow |V_{CKM}|$

Outline

- Recent applications:

▶ quark masses: m_s with 5-loop accuracy

▶ SU(3)-asymmetry a_1^K in the kaon DA

▶ heavy-light form factors and $|V_{ub}|$

- new light-cone sum rules
  for $B \to D^{(*)}$ form factors

- f_{D_s}, is there a puzzle ?

- how accurate are QCD sum rules ?

Light quark masses

$$m_q \equiv \overline{m}_q(2\ \mathrm{GeV}),\ q = u, d, s$$

- less accurate than the other SM parameters: in PDG 2006 $\sim 25\%$ accuracy for m_s ; compared: $\sim 10\%$ for m_c and $\sim 2\%$ for m_b

- the reason: $\Lambda_{QCD} \sim m_s \gg m_u, m_d$: small influence of $m_{u,d,s}$ on hadronic observables (exception: $m_{\pi,K,\eta}$)

Light quark masses

- Chiral Perturbation Theory:

$$R = \frac{m_s}{\hat{m}} = 24.4 \pm 1.5, \quad Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} = (22.7 \pm 0.8)^2$$

[Leutwyler, 1996] $\hat{m} = \frac{1}{2}(m_u + m_d)$

▶ determine m_s, obtain $m_{u,d}$ "for free ":

$$m_d = \frac{m_s}{R}\left(1 + \frac{R-1}{4Q^2}\right), \quad m_u = \frac{m_s}{R}\left(1 - \frac{R-1}{4Q^2}\right)$$

“Nonlattice” methods

- based on quark-current correlators and OPE

▶ positivity bounds

▶ QCD (SVZ) sum rules

▶ Finite-energy sum rules (FESR)

▶ inclusive $\tau \longrightarrow s\bar{u}\nu_\tau$ decays

m_s from QCD sum rules

- Correlator with scalar (pseudoscalar) currents:

$$j_{S(P)} = \partial^\mu \bar{s}\gamma_\mu(\gamma_5)q = (m_s{}_{(\mp)} m_q)\bar{s}(\gamma_5)q, \qquad (q = u, d)$$

$$\boxed{\Pi^{(P)}(q^2) = i\int d^4x \; e^{iq\cdot x}\langle 0|T\left\{j_P(x)j_P^\dagger(0)\right\}|0\rangle}$$

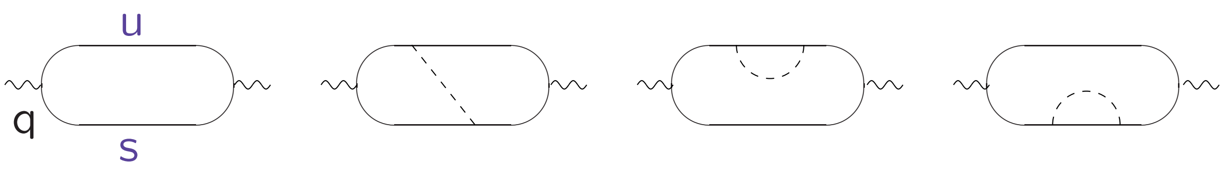
- Dispersion relation (doubly differentiated) for $\Pi^{(P)}(q^2)$ at $Q^2 = -q^2 \gg \Lambda^2_{QCD}$:

$$[\Pi^{(P)''}(q^2)]_{OPE} = 2\int_0^\infty ds \frac{\rho^{(P)}(s)}{(s-q^2)^3}\,,$$

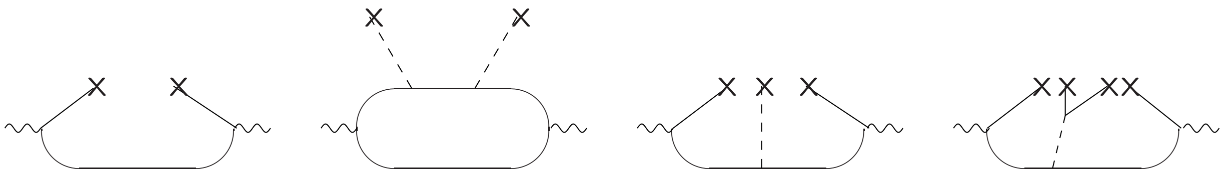
$$\rho^{(P)}(s) = \sum_K \langle 0|j_P|K(q)\rangle\langle K(q)|j_P|0\rangle$$

$\sum_K = \{\text{kaon} \oplus \text{excitations}\} \oplus$ quark-hadron duality

OPE: diagrams



$\oplus O(\alpha_s^2) \oplus O(\alpha_s^3) \oplus O(\alpha_s^4)$



$\langle \bar{q}q \rangle$ $\langle GG \rangle$ $\langle \bar{q}Gq \rangle$ $\langle \bar{q}q \rangle^2$

$O(\alpha_s^4)$ calculated

[Baikov, Chetyrkin, Kühn (2005)]

OPE for the pseudoscalar correlator

expansion in $1/(Q)^{d+2}$, $d = 0, 2, 4, 6$

$$[\Pi^{(P)''}(Q^2)]_{OPE} = \frac{3(m_s + m_u)^2}{8\pi^2 Q^2} \left\{ 1 + \sum_{i=1}^{4} C_{0,i} \left(\frac{\alpha_s}{\pi}\right)^i \right.$$

$$\left. -2\frac{m_s^2}{Q^2} \left(1 + \sum_{i=1,2} C_{2,i} \left(\frac{\alpha_s}{\pi}\right)^i \right) + \frac{\{d=4\}}{Q^4} + \frac{\{d=6\}}{Q^6} \right\}$$

$$\{d=4\} \sim \{m_s\langle\bar{q}q\rangle, \langle G^2\rangle, O(m_s^4)\ \}\ (1 \oplus O(\alpha_s))$$

$$\{d=6\} \sim m_s\langle\bar{q}Gq\rangle, \langle\bar{q}q\rangle^2$$

Coefficients multiplying $(\alpha_s/\pi)^n$ in $d=0$ part:
$(l_Q = \log Q^2/\mu^2)$

$$C_{0,1} = \frac{11}{3} - 2\,l_Q\,, \quad C_{0,2} = \frac{5071}{144} - \frac{35}{2}\,\zeta_3 - \frac{139}{6}\,l_Q + \frac{17}{4}\,l_Q^2,$$

$$C_{0,3} = \frac{1995097}{5184} - \frac{\pi^4}{36} - \frac{65869}{216}\,\zeta_3 + \frac{715}{12}\,\zeta_5 - \frac{2720}{9}\,l_Q + \frac{475}{4}\,\zeta_3\,l_Q + \frac{695}{8}\,l_Q^2 - \frac{221}{24}l_Q^3,$$

recent:

$$\begin{aligned} C_{0,4} = \quad & \frac{2361295759}{497664} - \frac{2915}{10368}\pi^4 - \frac{25214831}{5184}\,\zeta_3 + \frac{192155}{216}\,\zeta_3^2 + \frac{59875}{108}\,\zeta_5 - \frac{625}{48}\,\zeta_6 \\ - \; & \frac{52255}{256}\,\zeta_7 + l_Q\left[-\frac{43647875}{10368} + \frac{1}{18}\pi^4 + \frac{864685}{288}\,\zeta_3 - \frac{24025}{48}\,\zeta_5\right] \\ + \; & l_Q^2\left[\frac{1778273}{1152} - \frac{16785}{32}\,\zeta_3\right] + l_Q^3\left[-\frac{79333}{288}\right] + l_Q^4\left[\frac{7735}{384}\right], \end{aligned}$$

Hierarchy in α_s and d

- Relative contributions to $[\Pi^{(P)''}(M^2)]_{OPE}$
  (after Borel transformation $Q^2 \to M^2$)

$$r_n^{(d)}(M^2) = \frac{\{\Pi^{(P)''}(M^2)\}^{(d)}_{O(\alpha_s^n)}}{\Pi^{(P)''}(M^2)}$$

$$r_n^{(d=0,2)}(2.5\,\mathrm{GeV}^2) = 52.4\%,\ 28.3\%,\ 14.4\%,\ 4.0\%,\ -0.3\%$$

$$(n = 0, 1, 2, 3, 4)$$

$$r^{(d=4,6)}(2.5\ \mathrm{GeV}^2) = 1.2\%.$$

The hadronic sum

- $\{K, K2\pi, K^*\pi, \rho K, ...\}$
  $\rightarrow$ 3-resonance ansatz $\{K, K_1(1460), K_2(1830))\}$

m_{K_1}=1460 MeV, Γ_{K_1} = 260 MeV;
m_{K_2}=1830 MeV, Γ_{K_2}=250 MeV [PDG]

$$\rho^{(P)}_{hadr}(s) = \quad f_K^2 m_K^4 \delta(m_K^2 - s) + \sum_{i=1,2} f_{K_i}^2 m_{K_i}^4 \frac{1}{\pi}\left(\frac{\Gamma_{K_i} m_{K_i}}{(s-m_K^2)^2 + (\Gamma_{K_i} m_{K_i})^2}\right)$$

- decay constants: $\langle 0|j_P|K(q)\rangle = f_K m_K^2$,

f_K = 159.8 MeV , $f_{K_1,K_2} \ll f_K$ (ChPT)
(fitted from FESR [Kambor,Maltman '03])

Pseudoscalar sum rule to $O(\alpha_s^4)$

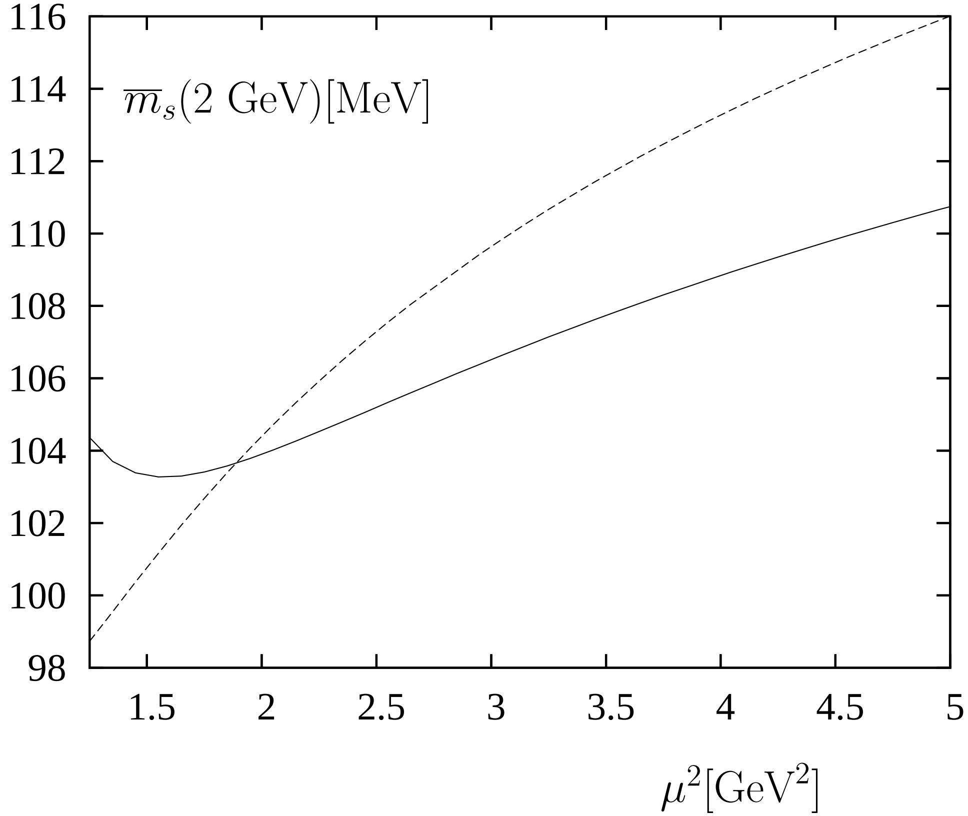
[Chetyrkin, A.K.,(2006)]

- The result:

$$\overline{m}_s(2\ \text{GeV}) = \left(105 \pm 6\Big|_{OPE} \pm 7\Big|_{hadr}\right)\text{MeV}\,,$$

▶ if the $O(\alpha_s^4)$ terms are removed:
$\simeq 2$ MeV increase of the central value

- The scale dependence: solid: $O(\alpha_s^4)$, dashed: $O(\alpha_s^3)$



Recent m_s determinations

Method accuracy	$m_s(2\mathrm{GeV})$ [MeV]	Reference
OPE bound $O(\alpha_s^4)$	> 76	Baikov,Chetyrkin, Kühn '05
QCD SR (P) $O(\alpha_s^4)$	$105 \pm 6 \pm 7$	A.K., Chetyrkin '05
QCD SR (P) $O(\alpha_s^4)$	97^{+11}_{-8}	Jamin, Oller, Pich '06
QCD SR (S) $O(\alpha_s^4)$	88^{+9}_{-7}	Jamin, Oller, Pich '06
FESR (P) $O(\alpha_s^4)$	102 ± 8	Dominguez et al.'08
Lattice QCD $2 \oplus 1$	87 ± 4 ± 4	HPQCD'06 Mason et al.
Lattice QCD $2 \oplus 1$	$90 \pm 5 \pm 4$	MILC '06 Bernard et al.
Lattice QCD $2 \oplus 1$	$91.1^{+14.6}_{-6.2}$	CP-PACS/JLQCD '07 Ishikawa et al.
Lattice QCD $2 \oplus 1$	107.3 ± 4.4 $\pm 9.7 \pm 4.9$	RBC /UKQCD '08 Allton et al,

- all $O(\alpha_s^4)$ sum rule intervals
and $2 \oplus 1$ lattice determinations agree,
uncertainty in m_s smaller than in PDG'06

- all m_s determinations obey the OPE bound,

- P, S sum rules:
no further need for improvement of OPE for $\Pi^{(P,S)}(q^2)$

- The hadronic sum in P -channel:
radially excited kaon $(J^P = 0^-)$ states
(resonances in $K\pi\pi$ and $K^*\pi, \rho K$)
accessible, e.g. in $\tau \to K_1\nu_\tau$, $D \to K_1 l \nu_l$,
$B \to K_{1,2}D^{(*)}$

- scalar sum rules, a more complicated hadronic
sum: nonres. $K\pi$ $(J^P = 0^+)$ states important

c, b-quark masses from sum rules

- recent determinations:

first moments of quarkonium sum rules (relativistic), $O(\alpha_s^3)$ accuracy achieved:

$\bar{m}_c(\bar{m}_c)$ [GeV]	$\bar{m}_b(\bar{m}_b)$ [GeV]	Reference
1.286± 0.013	4.164± 0.025	Kühn,Steinhauser, Sturm '06
1.295± 0.015	4.205± 0.058	Boughezal,Czakon, Schutzmeier '06

- more accurate values of quark masses ⇒ relations between quark/lepton masses in hypothetical flavour scenarios

Determination of a_1^K at NNLO level

[Chetyrkin, A.K., A.Pivovarov (2007)]

- does the valence s-quark in the kaon have a larger average momentum than the antiquark?
intuitively (quark model) → "yes"

- in QCD ⇒ kaon light-cone DA (twist-2):

$$\langle K^-(q)|\bar{s}(z)\gamma_\mu\gamma_5\,[z,-z]\,u(-z)|0\rangle_{z^2\to 0}$$
$$= -iq_\mu f_K \int_0^1 du\; e^{iuq\cdot z - i\bar{u}q\cdot z}\varphi_K(u,\mu)\,,$$

quark-antiquark Fock state

- Gegenbauer (polynomial) expansion

$$\varphi_K(u,\mu) = 6u\bar{u}\Big\{1 + a_1^K(\mu)C_1^{3/2}(u-\bar{u}) + a_2^K(\mu)C_2^{3/2}(u-\bar{u}) + ...\Big\},$$

μ -factorization scale $\sim 1/|z^2|$

- pion DA (isospin symmetry)

$$\varphi_\pi(u,\mu) = 6u\bar{u}\Big\{1 + a_2^\pi(\mu)C_2^{3/2}(u-\bar{u}) + ...\Big\},$$

- $a_1^K \neq 0$ -$SU(3)_{fl}$ breaking parameter

  $(f_K/f_\pi\ ,\ a_2^K/a_2^\pi)$

- $a_1^K \neq 0$ influences $SU(3)_{fl}$ relations between form factors and/or charmless B decay amplitudes

  (QCDF, LCSR)

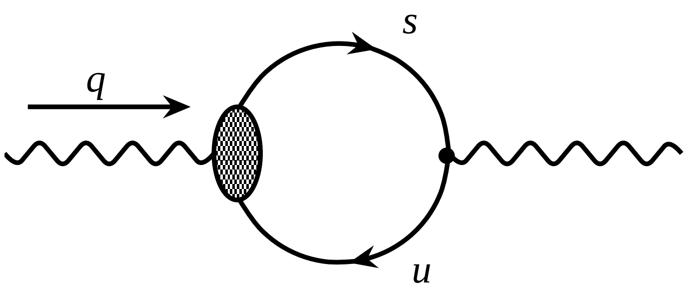
- related to the local hadronic matrix element

$$\langle K^-(q)|\bar{s}\gamma_\nu\gamma_5 i \overleftrightarrow{D}_\lambda u|0\rangle = -iq_\nu q_\lambda f_K \frac{3}{5} a_1^K ,$$

- multiplicative renormalization

$$a_1^K(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{\gamma_0}{\beta_0}} \left(\frac{\beta_0 + \beta_1(\alpha_s(\mu_0)/\pi)}{\beta_0 + \beta_1(\alpha_s(\mu)/\pi)}\right)^{\left(\frac{\gamma_0}{\beta_0} - \frac{\gamma_1}{\beta_1}\right)} a_1^K(\mu_0) ,$$

QCD sum rule for a_1^K



- The correlator

$$\Pi_{\mu\nu\lambda}(q) = i\int d^4x\; e^{iq\cdot x}\langle 0|T\left\{\bar{u}(x)\gamma_\mu\gamma_5 s(x), \bar{s}(0)\gamma_\nu\gamma_5 i\overleftrightarrow{D}_\lambda u(0)\right\}|0\rangle$$
$$= q_\mu q_\nu q_\lambda \Pi(q^2) + \ldots, \quad (1)$$

- Expansion in powers of $1/Q^2$

$$\Pi(Q^2,\mu) = \frac{\mathcal{A}_2(Q^2,\mu)}{Q^2} + \frac{\mathcal{A}_4(Q^2,\mu)}{Q^4} + \frac{\mathcal{A}_6(Q^2,\mu)}{Q^6} + \ldots,$$

- Expansion in α_s, m_s^2/Q^2

$$\mathcal{A}_2(Q^2,\mu) = \frac{m_s^2}{4\pi^2}\Bigg(1+\frac{\alpha_s}{\pi}\left[\frac{26}{9}+\frac{10}{9}l_Q\right] + \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{366659}{11664}-\frac{29}{9}\zeta(3)+\frac{14449}{972}l_Q+\frac{605}{324}l_Q^2\right]+3\frac{m_s^2}{Q^2}\left(\frac{5}{2}+l_Q\right)\Bigg).$$

$$\mathcal{A}_4(Q^2,\mu) = -m_s\langle\bar{s}s\rangle\Bigg(1-\frac{\alpha_s}{\pi}\left[\frac{112}{27}+\frac{8}{9}l_Q\right] - \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{28135}{1458}-4\zeta(3)+\frac{218}{27}l_Q+\frac{49}{81}l_Q^2\right]+2\frac{m_s^2}{Q^2}\Bigg) - m_s\langle\bar{u}u\rangle\Bigg(\frac{4\alpha_s}{9\pi}+\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{59}{54}+\frac{49}{81}l_Q\right]\Bigg),$$

$\langle\bar{q}q\rangle \equiv \langle 0|\bar{q}q|0\rangle$, $(q=s,u)$ quark-condensate,
$\mathcal{A}_6$ contains $d=4,5,6$ condensates

- the result $a_1^K(1\text{ GeV}) = 0.10 \pm 0.04$

  with evolution: $a_1^K(2\text{ GeV}) = 0.08 \pm 0.04$,

  - NLO corrections reshuffle OPE,
    $O(m_s^2)$ loop becomes important

  - old sum rule estimates: ($\langle \bar{q}q \rangle$ terms dominant)
    $a_1^K(1\text{GeV}) = 0.06 \pm 0.03$

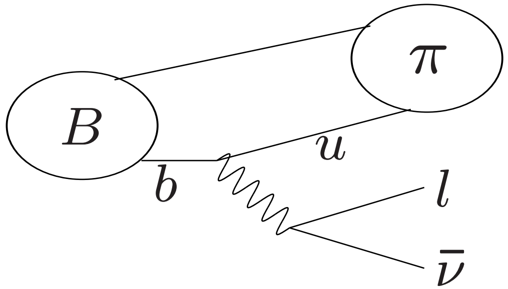
- lattice QCD :

  $a_1^K(2\text{ GeV}) = 0.0453 \pm 0.0009 \pm 0.0029$,
  [V.M.Braun et al., [QCDSF/UKQCD] (2006)]

  $a_1^K(2\text{ GeV}) = 0.048 \pm 0.003$,
  [M. A. Donnellan et al. (2007)]

  with a linear extrapolation in m_s , $O(m_s^2)$ effect?

$B \to \pi$ form factor



$\Rightarrow |V_{ub}|$

- Hadronic matrix element:

$$\langle \pi^+(p)|\bar{u}\gamma_\mu b|\bar{B}^0(p+q)\rangle = f^+_{B\pi}(q^2)(2p_\mu+q_\mu)+\ldots$$

- differential decay distribution (neglecting m_l)

$$\frac{d\Gamma(B^0 \to \pi^- l^+ \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3}(E_\pi^2 - m_\pi^2)^{3/2}|f^+_{B\pi}(q^2)|^2$$

- $B \to \pi\mu\nu_\mu, \pi e\nu_e$:

$0 < q^2 < (m_B - m_\pi)^2 = q^2_{max} \simeq 26.4\ \text{GeV}^2$

Fixing the shape of $f^+_{B\pi}(q^2)$

- analytical properties of $f^+_{B\pi}(q^2)$:

▶ dispersion relation:

$$f^+_{B\pi}(q^2) = \frac{f_{B^*} g_{B^*B\pi}}{m^2_{B^*} - q^2} + \frac{1}{\pi} \int\limits_{(m_B+m_\pi)^2}^{\infty} ds \frac{\mathrm{Im} f^+_{B\pi}(s)}{s - q^2}$$

$\langle 0|\bar{u}\gamma_\mu b|B^*\rangle \sim f^*_B$, $\langle B^*|B\pi\rangle \sim g_{B^*B\pi}$,

integral convergent, perturbative QCD:

$\lim_{q^2\to\infty} f_{B\pi}(q^2) \sim 1/q^2$

Parametrization of $f^+_{B\pi}(q^2)$

- dispersion relation as a starting point,

replace the integral by effective pole(s):

- BK parametrization *[D.Becirevic, A.Kaidalov, 2000]*

(3 parameters → 2 , motivated by HQ limit)

$$[f^+_{B\pi}(q^2)]_{BK} = \frac{f_{B\pi}(0)}{(1 - q^2/m^2_{B^*})(1 - \alpha_{BK} q^2/m^2_{B^*})}$$

- modifications: 3-parameter*[P.Ball, R.Zwicky, '04]*

N-pole parametrization: *[R.Hill,'05]*

- Use of conformal mapping and dispersive bounds

BGL- parametrization,

[C.Boyd, B.Grinstein, R.Lebed '95,....]

- AFHNV parametrization, based on Omnes-representation

[Albertus, Flynn, Hernandez, Nieves, Verde-Velasco,05']

Extracting $|V_{ub} f^+_{B\pi}(0)|$ from $B \to \pi l \nu_l$ data

- q^2-distribution, many bins *[BaBar '06]*

- several parametrization (BK, BZ, BGLa, BGLb, AFHNV) fitted to the data *[P. Ball '06]*

$$|V_{ub} f^+_{B\pi}(0)| = (0.91 \pm [0.06]_{shape} \pm [0.03]_{BR}) \times 10^{-3}$$

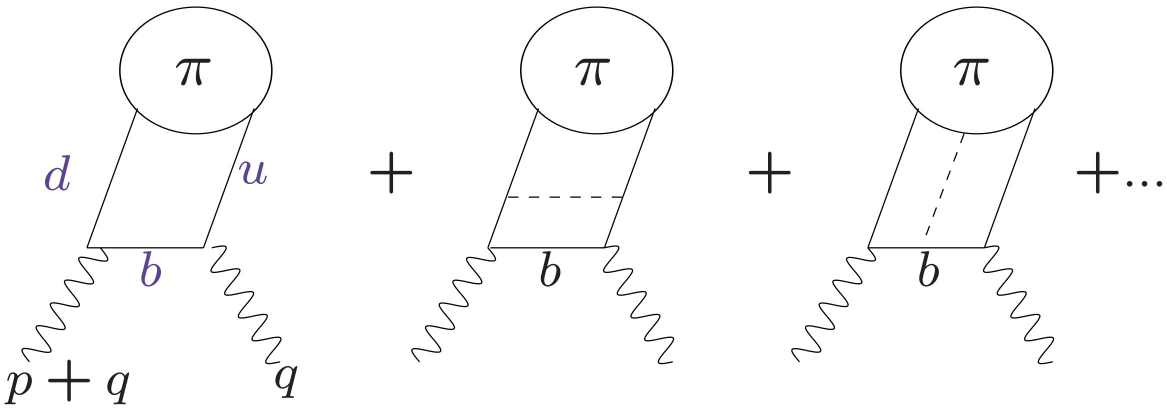
- remains to calculate the form factor normalization

Calculation of $f^+_{B\pi}(q^2)$ from LCSR

- small q^2 (large E_π) accessible
- full QCD with finite (large !) m_b,
- both factorizable (hard-scattering) and nonfactorizable (soft, "end-point") contributions

▶ price:
indirect access to the form factors via hadronic dispersion relation, quark-hadron duality approximation introduces "systematic uncertainty".

Light-Cone Sum Rule



- The correlation function:
  $q^2, (p+q)^2 \ll m_b^2$, b-quark highly virtual

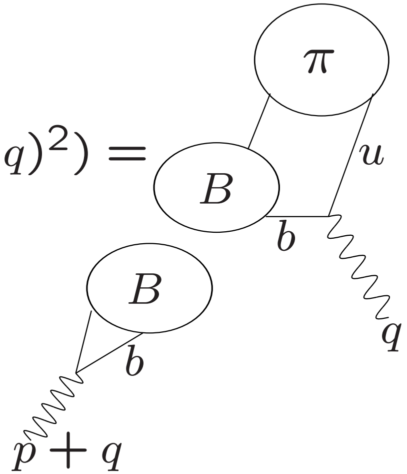
$$F((p+q)^2, q^2) = \sum_{t=2,3,4} \int Du_i \sum_{k=0,1} \left(\frac{\alpha_s}{\pi}\right)^k T_k^{(t)}((p+q)^2, q^2, u_i, m_b, \mu)\varphi_\pi^{(t)}(u_i, \mu)\,.$$

hard scattering amplitudes ⊗ pion DA's

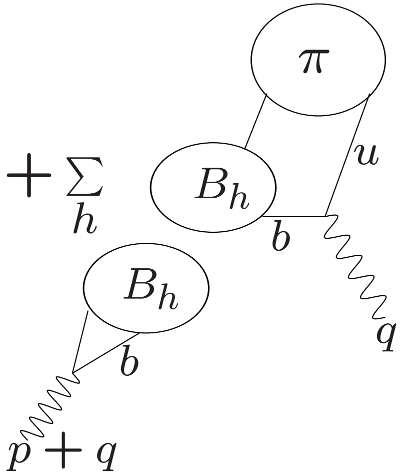
- universal input: $\langle \pi(p)|\bar{u}(x)....d(0)|0\rangle \sim \varphi_\pi^{(t)}(u_i)$
  pion DA's of twist $t = 2, 3, 4..$

Hadronic sum

$F(q^2,(p+q)^2) =$



$+\sum_h$



$f_B f^+_{B\pi}(q^2)$ $\qquad$ $\sum_{B_h} \rightarrow$ *duality* (s_0^B)

- matching at $\langle -(p+q)^2 \rangle \sim \mu^2 \sim m_b\Lambda$,
  using quark-hadron duality

$$[F((p+q)^2,q^2)]_{OPE} = \frac{m_B^2 f_B f^+_{B\pi}(q^2)}{m_B^2-(p+q)^2} + \int\limits_{s_0^B}^{\infty} ds \frac{[\mathrm{Im}F(s,q^2)]_{OPE}}{s-(p+q)^2}$$

Recent update of LCSR for $f^+(q^2)$

[G. Duplancić, A.K., Th. Mannel, B. Melić, N. Offen, arXiv:0801.1796 [hep-ph]]

- $\overline{MS}$ mass used: :

$m_b(m_b) = 4.164 \pm 0.025$ GeV
[J. Kühn,M. Steinhauser, C. Sturm]

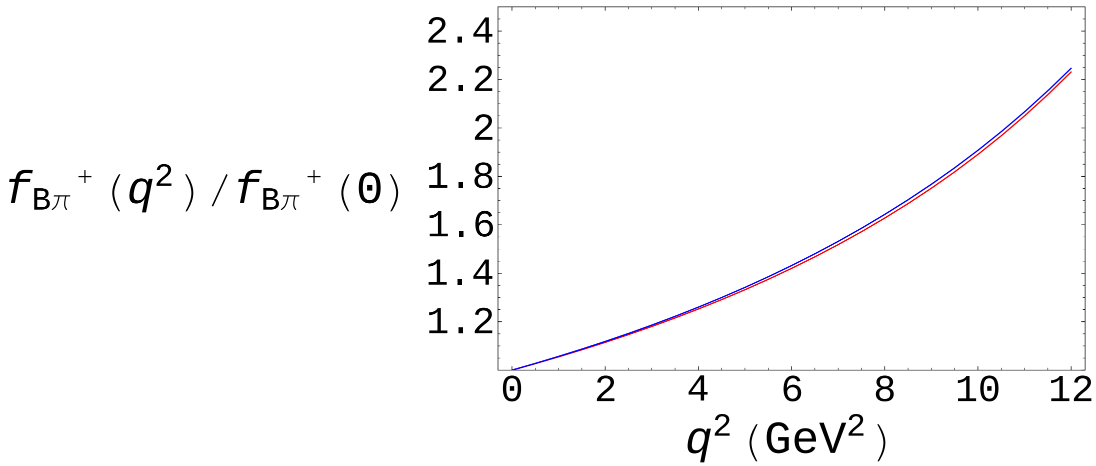
- s_0^B, M^2 fixed by calculating m_B directly from LCSR,
  control over duality approximation

- f_B determined from two-point sum rule in $\overline{MS}$
  $f_B = 210 \pm 19$ MeV

[M. Jamin, B. Lange (2001)]

- $\varphi_\pi(u)$, Gegenbauer moments at low scale 1 GeV: fitting the q^2 dependence to the measured slope:

▶ $a_2^\pi(1\text{GeV}) = 0.16 \pm 0.01$; $a_4^\pi(1\text{GeV}) = 0.04 \mp 0.01$



plot: LCSR vs BK parametrization of the BABAR data (almost indistinguishable):

we "trade" the $q^2 \neq 0$ LCSR calculation for the accuracy of $q^2 = 0$ result

- $a_2 = 0.25 \pm 0.15$ (aver. of recent determinations)
  $a_2 + a_4 = 0.1 \pm 0.1$ (pion-photon FF)

Result

$f^+_{B\pi}(0) \;=\; 0.262 \pm [0.005]_{fit} \pm [0.002]_{m_b} \left[{}^{+0.03}_{-0.02}\right]_{m_q} \pm$
$[0.002]_M \pm [0.001]_\mu \pm ...$
combining all individual uncertainties in quadrature:

$$\boxed{f^+_{B\pi}(0) = 0.26^{+0.04}_{-0.03}}$$

- most recent LCSR result:
  (with one-loop pole mass $m_b = 4.8 \pm 0.1$ GeV)

$$f^+_{B\pi}(0) = 0.258 \pm 0.031 \quad \text{[P.Ball, R.Zwicky(2004)]}$$

- using $|V_{ub} f^+(0)|$ from P. Ball's fit, our *new* result:

$$|V_{ub}| = \left(3.5[\pm 0.4]_{th} \pm [0.2]_{shape} \pm [0.1]_{BR}\right) \times 10^{-3}\,,$$

Recent $|V_{ub}|$ determinations from $B \to \pi l \nu_l$

[ref.]	$f^+_{B\pi}(q^2)$ calculation	$f^+_{B\pi}(q^2)$ input	$\lvert V_{ub}\rvert \times 10^3$
Okamoto et al.	lattice ($n_f = 3$)	-	3.78±0.25±0.52
HPQCD	lattice ($n_f = 3$)	-	3.55±0.25±0.50
Arnesen et al.	-	lattice⊕SCET	$3.54 \pm 0.17 \pm 0.44$
BecherHill	-	lattice	$3.7 \pm 0.2 \pm 0.1$
Flynn et al	-	lattice ⊕ LCSR	$3.47 \pm 0.29 \pm 0.03$
Ball, Zwicky	LCSR	-	$3.5 \pm 0.4 \pm 0.1$
this work	LCSR	-	$3.5 \pm 0.4 \pm 0.2 \pm 0.1$

$B_{(s)} \to K$ form factors: an update

- including m_s in OPE $\to$ kaon DA's

[G. Duplancić, B. Melić , paper in preparation]

$$f^+_{BK}(0) = 0.36^{+0.05}_{-0.04}\,,\; f^+_{B_sK}(0) = 0.30^{+0.04}_{-0.03}\,,$$

- ratios (some uncertainties cancel)

$$\frac{f^+_{BK}(0)}{f^+_{B\pi}(0)} = 1.38^{+0.11}_{-0.10} \qquad \frac{f^+_{B_sK}(0)}{f^+_{B\pi}(0)} = 1.15^{+0.17}_{-0.09}$$

▶ relevant for $B \to Kll$,
$SU(3)_{fl}$ relations for $B \to hh$ amplitudes:

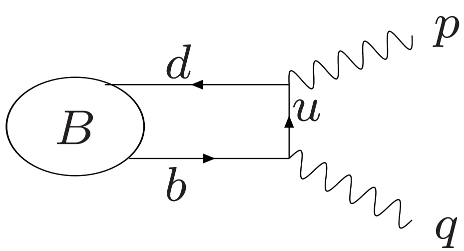
$$\xi = \frac{f_K}{f_\pi}\frac{f^+_{B\pi}(m_K^2)}{f^+_{B_sK}(m_\pi^2)}\frac{m_B^2 - m_\pi^2}{m_{B_s}^2 - m_K^2} = 1.01^{+0.07}_{-0.15}.$$

▶ close to previous LCSR estimates
of $SU(3)_{fl}$ violation

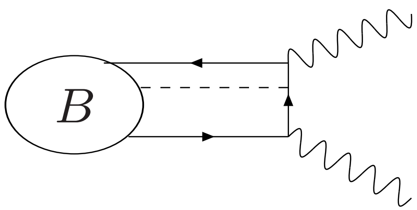
LCSR with B meson distribution amplitudes

[A.K., Th. Mannel, N. Offen, '06]

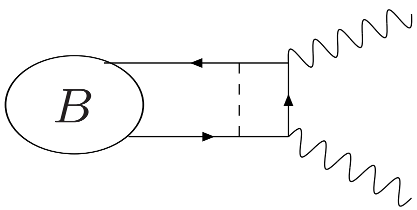
- use of "inverted" correlator: $\pi \to B$, $j_B \to j_\pi$,
easy to calculate any heavy-light form factor



(a)



(b)



(c)

- new universal input B-meson DA's (defined in HQET)

$$\langle 0|\bar{q}_{2\alpha}(x)[x,0]h_{v\beta}(0)|\bar{B}_v\rangle$$
$$= -\frac{if_B m_B}{4}\int\limits_0^\infty d\omega e^{-i\omega v\cdot x}\left[(1+\not{v})\left\{\phi_+^B(\omega) - \frac{\phi_+^B(\omega)-\phi_-^B(\omega)}{2v\cdot x}\not{x}\right\}\gamma_5\right]_{\beta\alpha}$$

- kee input parameter: the inverse moment

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty d\omega \frac{\phi_+^B(\omega,\mu)}{\omega}$$

- $\lambda_B(1\ \mathrm{GeV}) = 460 \pm 110\ \mathrm{MeV}$

[V.Braun, D.Ivanov, G.Korchemsky,'04]

- so far only tree-level calculations, 2,3-particle DA's:

$B \to \pi, K^{(*)}, \rho$: reasonable agreement with "old" LCSR

- a similar approach in SCET:

[Th. Feldmann, F.de Fazio, T.Hurth '06]

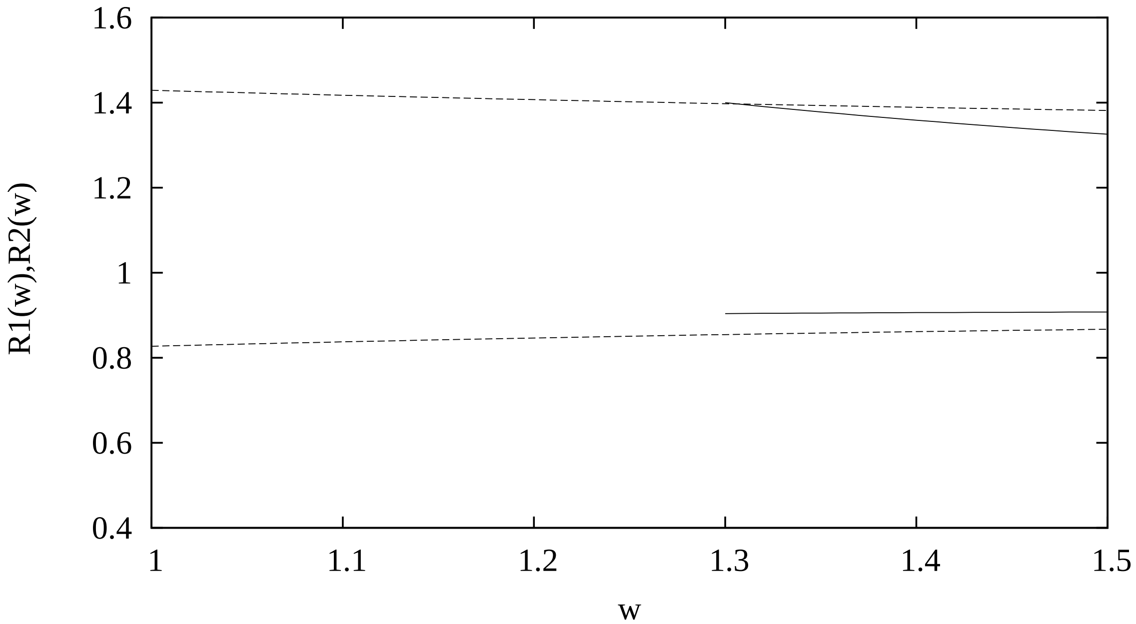
LCSR for $B \to D^{(*)}$ form factor

[S.Faller, A.K.,Ch.Klein, Th.Mannel, in preparation]

- virtual c quark in the correlator with B-meson DA

- $B \to D^{(*)}$ form factors near maximal recoil
- form factor ratios predicted

very preliminary: LCSR (solid) ,
BABAR $B \to D^*$ fitted to CLN param.(dashed)



$$R_1(w) = \frac{h_V(w)}{h_{A_1}(w)} \qquad R_2(w) = \frac{r^* h_{A_2}(w) + h_{A_3}(w)}{h_{A_1}(w)}$$

Last minute comment on f_{D_s}

(see talk by A.Kronfeld)

- $f_D = 223 \pm 17$ MeV, $f_{D_s} = 275 \pm 10$ MeV
(exp.average *[J.Rosner, S.Stone , for PDG08]*)

- QCD sum rule predictions:

$f_D = 195 \pm 20$ MeV *[A.Penin,M.Steinhauser '01]*
($O(\alpha_s^2)$ QCD SR in HQET)

no prediction *[M.Jamin, B.Lange '01]*
($O(\alpha_s^2)$ full QCD, MS)

$f_D = 203 \pm 20$ MeV , $f_{D_s} = 235 \pm 24$ MeV *[Narison '02]*

$f_{D_s} > f_D$ but lower than experiment

uncertainties are larger than in
the latest lattice QCD results (HPQCD)

- a rigorous upper bound for $f_{D_{(s)}}$
  from the same correlator/OPE :

$$f_D^2 m_D^4 e^{-m_D^2/M^2} + ... = \Pi(M^2; m_c, m_s, \alpha_s, \text{cond.}, \ \mu,)$$

- the correlator has a positive definite spectral density

▶ $f_D < \sqrt{\Pi(M^2)/(m_D^4 e^{-m_D^2/M^2})}$

▶ preliminary result (without $O(\alpha_s^2)$), taking M and μ optimally small ($\sim$ 1 GeV)

$$f_D < 230 \text{ MeV} \ , \ f_{D_s} < 260 \text{ MeV}$$

▶ exp. result looks unexpected

- missed $D_{(s)} \to l\nu\gamma$ admixture at low E_γ?
LCSR estimates at $(p_l + p_\nu)^2 \sim 0$ (large E_γ)
[A.K. G. Stoll, D. Wyler, '95],
pole model at large $(p_l + p_\nu)^2$ (small E_γ)
[G.Burdman, T.Goldman,D.Wyler '94],
has to be investigated more carefully

How accurate are the QCD sum rules?

- suggested almost 30 years ago

[M.Shifman, A.Vainshtein and V.Zakharov (1979)]

a method to parametrize QCD vacuum effects,

e.g., $\sim 90\%$ of $m_{nucleon}$ is due to quark condensate

- hadrons treated indirectly

(poles in the correlation function)

- initially not intended for precision calculations

- nowadays a popular working tool:

many applications, new modified approach (LCSR)

- applications to flavour physics demand

high accuracy

▶ uncertainties have to be identified and estimated

How accurate are the QCD sum rules?

(see also the talk by D. Melikhov)

- two main sources of uncertainties:
  (I) OPE: truncated, inputs uncertain
  (II) hadronic sum approximated
  with quark-hadron duality

- (I): a reasonable accuracy achieved in 2-point correlators, due to progress in multiloop calculations,
- (I): α_s, quark masses, quark/gluon condensates, DA's: accuracy slowly improving
- (I): in LCSR only NLO $t \leq 4$ available, twist expansion demands additional studies

- (II) more difficult, the most safe predictions are bounds from OPE
- (II) not easy to estimate the "systematic" error related with effective threshold s_0: (fitting s_0 by adjusting the hadron mass)
- (II) a better solution: experimental information on excited states $\Rightarrow$ the hadronic spectral function

- potential models used to illustrate how the method works, first discussed in
A.Vainshtein, V.Zakharov, V.Novikov, M.Shifman Sov.J.Nucl.Phys.32:840,1980

- however, too far from QCD to help to assess errors

- try more realistic models of hadronic spectrum?
(Veneziano model, emerging AdS QCD models?)

- a common prejudice: “Borel stability”,
stability is not a criterion !

- “Borel window”:
  M^2_{min} (subleading power terms small)
  M^2_{max} (suppression of excited states)
sometimes does not exist !

- M^2 variation included in the error budget
but not necessarily reflects the whole error
(e.g. scale dependence in a truncated pert. QCD)

- LCSR calculated at finite heavy masses and/or Q^2,
twist 2,3,4 terms manifest uniform behaviour
at $m \to \infty$ and/or $Q^2 \to \infty$, well known
($s_0 \to u_0, \; u_0 \to 1$, end-point region)
however not relevant at finite masses and/or Q^2 where
$u_0 < 1$ and higher twists are usually suppressed

$* * *$