Heavy quarks phenomenology from the lattice

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Outline

- B-Physics on the lattice:

  motivations, problems and approaches

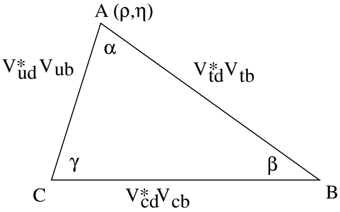
- Selection of recent results

  systematics effects due to pert. renorm., lattice artifacts, quenching

- HQET on the lattice [Rainer's talk]
- Conclusions

B-Physics on the lattice for

- Matrix elements relevant for CKM parameters:
  - B and D mesons decay constants $[V_{ub}, V_{cd}]$
  - $B_{B_{(s)}}$ and ξ $[V_{td}, V_{ts}]$
  - B semileptonic decays $(B \to \pi,\ B \to D)$ $[V_{ub}, V_{cb}]$



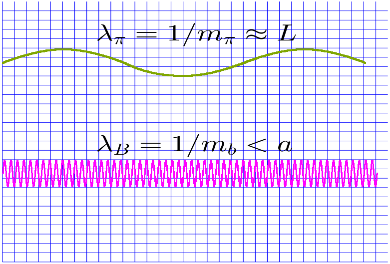
- b-quark mass
- Spectrum and lifetimes of b-hadrons.

The problems

Competition of two systematical effects that should be kept small:

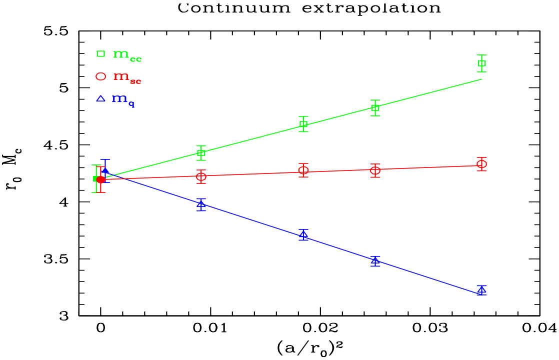
- finite volume effects are mainly triggered by the light degrees of freedom. The usual requirement is $m_{PS}L > 4$ and m_{PS} is typically around the kaon mass in real lattice simulations $\Rightarrow L \simeq 2$ fm.
- cutoff effects are tuned by the heavy quark mass. $a << 1/m_b \simeq 0.03$ fm .

$\Rightarrow L/a \simeq 100$ is needed to have those systematics under control !!



Charm is just doable, although cutoff effects might be large.

Example: Quenched charm quark mass from $a < 0.1$ fm in a O(a) improved regularization [Sint and Rolf, 02]. Three different lattice definitions:



Approaches

1) Extrapolations in $1/m_h$ from around the charm quark mass.
Continuum limit and b-mass limit should be taken in the correct order

$$\lim_{m_h \to m_b} \lim_{am_h \to 0} F(m_h, am_h)$$

2) Anisotropic lattices [Peardon, Ryan & co.]: $a_t \ll a_s$. Delicate (non-perturbative) fine tuning needed in taking the continuum limit (eg at fixed a_s/a_t)

3) Rome II (step-scaling) method [Petronzio & co.]. Idea: finite size effects should not depend strongly on the heavy mass. One defines

$$\sigma(L, s, m_h) = \frac{F(sL, m_h)}{F(L, m_h)}, \quad s > 1$$

starting at $L_0 \simeq 0.4$ fm . The extrapolation of σ to m_b is expected to be smooth, so far confirmed numerically. Result in large volume

$$F(4L_0) = \sigma(2L_0, 2, m_b)\sigma(L_0, 2, m_b)F(L_0, m_b)$$

where the last σ is extrapolated to m_b from around m_c.

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It relies on the assumption that the expansion converges up to $am_h \simeq 1$. Numerically it seems to be OK.

5) Effective theories:
HQET [Eichten and Hill '89]: formal expansion in $1/m_h$:

$$\mathcal{L} = \bar{\psi}_h D_0 \psi_h + O(1/m_h) \, ,$$

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Remarks

1. Non-perturbatively NRQCD is non-renormalizable even at the lowest order, as the Lagrangean includes dimension 5 operators. The lattice theory is defined at finite cutoff $a \simeq 1/m_h$ only.
   On the contrary the LO HQET is non-perturbatively renormalizable and higher orders (in $1/m_h$) can be treated as insertions in correlation functions.

2. Effective theories contain power law-divergences due to the mixings of operators of different dimensions. The dimensionful mixing coefficients c_k need to be computed non-perturbatively to take the continuum limit (if it exists)

$$\Delta c_k \simeq \frac{g_0^{2(l+1)}}{a} \propto \frac{1}{a[\ln(a\Lambda)]^{l+1}} \to \infty \;\; \text{as } a \to 0 \,,$$

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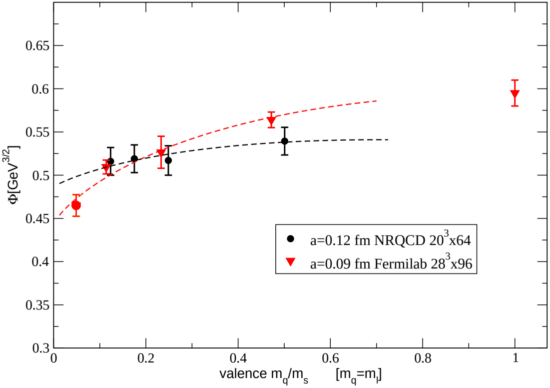
B and D mesons decay constants

$\langle 0|A_\mu|P\rangle = F_P p_\mu$ describes leptonic decays of the pseudoscalar P

Experimentally

- $\mathcal{B}(B^- \to \tau^- \bar{\nu}_\tau) = (1.36 \pm 0.48) \times 10^{-4}$ av. Belle and Babar [Faccini, 2006]
  $|V_{ub}|_{excl} = (3.47 \pm 0.29 \pm 0.03) \times 10^{-3}$ [J. Flynn and J. Nieves, after HPQCD revision]
  $\Rightarrow F_B = 254(50)$ MeV
- B_s leptonic decays not yet observed. $F_{B_s} = 229 \pm 9$ MeV $\pm$ granum salis from UT_{angles} fits.
- $F_{D_s} = 274 \pm 10$ MeV and $F_{D_s}/F_D = 1.23 \pm 0.10$ [Rosner, Stone for PDG 2008]



$N_f = 3$

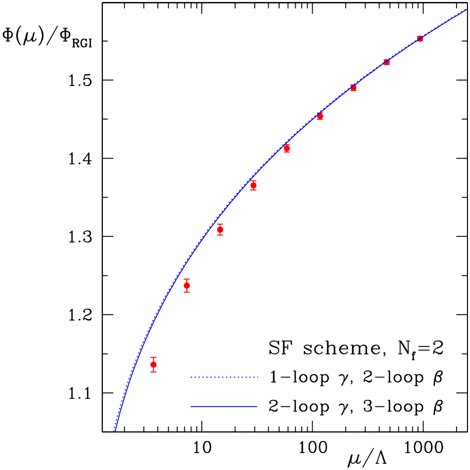
Fermilab, HPQCD

F_B=216(9)(19)$_m$(4)(6) MeV

$F_{B_s}/F_B = 1.20(3)_{stat+\chi}(1)$

$= 1.27(2)(6)_\chi$

- m_{sea} down to about $m_s/10 \Rightarrow$ great improvement in chiral behavior compared to few years ago (some sensitivity to logs in Fermilab data)
- same $S\chi PT$ formulae used $\Rightarrow$ cutoff effects visible
- perturbative renormalization only also for power divergent subtractions in NRQCD
- Fermilab result updated with two additional coarser lattice spacings: $F_B = 191(5)(8)$ MeV and $F_{B_s}/F_B = 1.30(3)(4)$, [Simone LAT07]



$N_f = 2$

NP ren. of static axial current in SF

[MDM, P. Fritzsch, J. Heitger '07]

PT applicable for $\mu \geq 4$ GeV !!
but no sign within PT !!

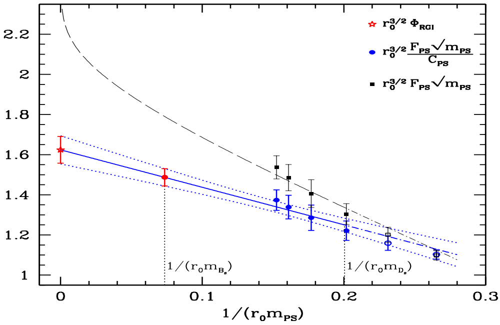
PT off by $\simeq 5\%$ at the hadronic scale

Preliminary result: $F_{B_s}^{stat} = 297(14)$ MeV

@$a = 0.08$ fm and $m_{sea} = m_s$

more work to be done, ongoing ALPHA project

Comparing three $N_f = 0$ determinations beyond the static approximation



$F_{B_s} = 193(6)$ MeV [ALPHA '07]

Explicit fully non-perturbative computation of the $1/m_b$ corrections in HQET, preliminary result $F_{B_s} = 185(21)$ MeV [Garron LAT07] and more later

Rome II SSF method with static constraints: $F_{B_s} = 191(6)$ MeV

[Guazzini, Sommer and Tantalo '07]

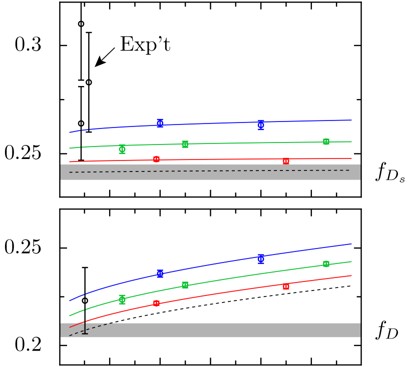
F_{D_s} and F_D

HPCQD + UKCQD, arXiv:0706.1726

- $N_f = 3$ HISQ m_l down to $m_s/10$

V	a	am_c
$16^3 \times 48$	0.15 fm	0.85
$20^3 \times 64$	0.12 fm	$\simeq 0.65$
$24^3 \times 64$	0.12 fm	$\simeq 0.65$
$28^3 \times 96$	0.09 fm	$\simeq 0.43$

- $F_{D_s} = 241(3)\text{MeV}, F_{D_s}/F_D = 1.162(9)$

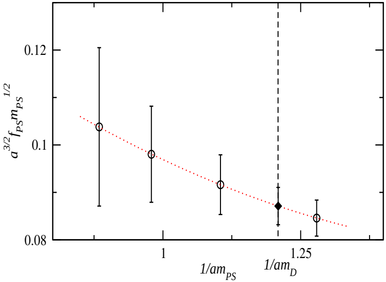


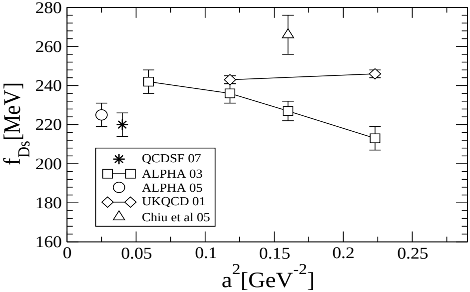
This could be state of the art if:

- effect of rooting completely clarified [Creutz LAT07, Kronfeld LAT07]
- Discussion of the errors based on more details, in particular on:
  - Bayesian fits
  - Chiral (and continuum limit) fits
  - Algorithmic details (missing for $m_{sea} < 0.2m_s$ and largest a)
  - [longer publication announced]

Preliminary $N_f = 2$, ETMC

- maximal twist: automatic O(a) improvement, no Z factors needed for F_{PS}
- m_{sea} down to $m_s/5$, $V = 24^3 \times 48$, $a \simeq 0.09$ fm $[32^3 \times 64, a \simeq 0.07$ fm$]$, LW gauge action
- $F_{D_s} = 271(6)(4)(5)_a$ MeV and $F_{D_s}/F_D = 1.35(4)(1)(7)_\chi$. [Blossier LAT07]

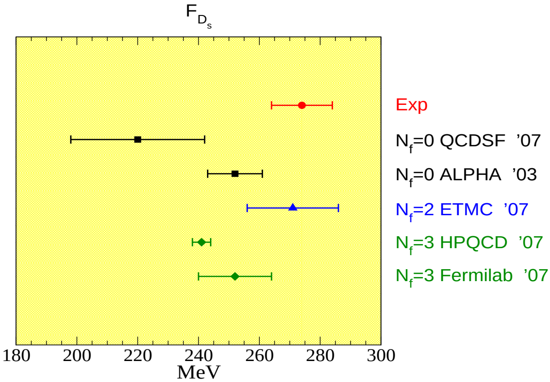




$N_f = 0$, QCDSF

Clover quarks, $a \simeq 0.04$ fm, $V = 40^3 \times 80$ linear chiral extrap from $m_\pi \simeq 500$ MeV: $F_{D_s} = 220(6)(5)(11)_a$ MeV and $F_{D_s}/F_D = 1.068(18)(20)$.
[Ali-Khan LAT07] including also preliminary results on $D \to \pi l \nu$ form factors.

Summary of recent determinations of F_{D_s}



more than 3 sigmas discrepancy between the Experimental and the HPQCD results. That can be accomodated in some 2HD models or R-parity violating Supersymmetric models [Dobrescu, Kronfeld '08].

- Decay constants are now 'measured' at experiments and the precision will improve in the future.
- In lattice computations the quenched approximation has been almost removed.
- Also small quark masses have been reached and better agreement with NLO χPT formulae is found.
- In most cases continuum limit extrapolations are missing (in some cases, like for NRQCD, not even possible in theory).
- NP renormalization (when needed) done only in few cases.
- F_{D_s} is one of the quantities best measured experimentally and on the lattice. Quenching effects ($N_f = 3$ vs $N_f = 0$) do not appear to be large after continuum limit extrapolation. Still the lattice result lie at the lower end of the experimental ones from CLEO-c and BaBar.

$\overline{B}_{(s)} - B_{(s)}$ mixing

$$\Delta m_q = \frac{G_F^2 m_W^2}{6\pi^2} |V_{tq}^* V_{tb}|^2 \eta S_0(x_t) m_{B_q} F_{B_q}^2 B_{B_q}$$

$$\langle \overline{B}_q | O_{VV+AA} | B_q \rangle = \frac{8}{3} F_{B_q}^2 B_{B_q} m_{B_q}^2$$

Experiments: $\Delta m_d = 0.507 \pm 0.005 ps^{-1}$ [PDG]
$\Delta m_s = 17.35 \pm 0.25 ps^{-1}$ [CDF,D0]

Exp. errors here are at the percent level !

In Effective theories (eg HQET):

$$O_{VV+AA}^{QCD}(m_b) = C_L(m_b, \mu)\ O_{VV+AA}^{HQET}(\mu) + C_S(m_b, \mu)\ O_{SS+PP}^{HQET}(\mu) + O(1/m_b)$$

$N_f = 3$: AsqTad, $m_l/m_s = 0.5, 0.25$, NRQCD, $a \simeq 0.12$ fm, $V = 20^3 \times 64$.
No dep. on m_l visible: $F_{B_s}\sqrt{B_{B_s}^{RGI}} = 281(21)_{m+stat}$ MeV $\overset{2l}{\Rightarrow}$ $B_{B_s}(m_b) = 0.76(11)$
Results also for $\Delta\Gamma_s$ and preliminary estimates of B_B [HPQCD '07 and Davies LAT07]

- Operators of dim 7 are included in the matching between NRQCD and QCD $\Rightarrow$ power divergent contributions have to be subtracted
- the way this is done is critical, with the 1 loop coeff the subtraction is 10% of the final number for B_{B_s} !
- Staying in PT the problem will only get worse when decreasing a

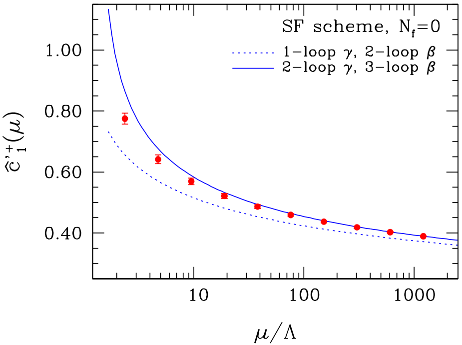
$N_f = 3$ by RBC-UKQCD: static approximation (HYP2 action) with light domain wall fermions [Wennekers LAT07]. $L \simeq 2$ fm, $L_5 = 16$ and $a \simeq 0.12$ fm

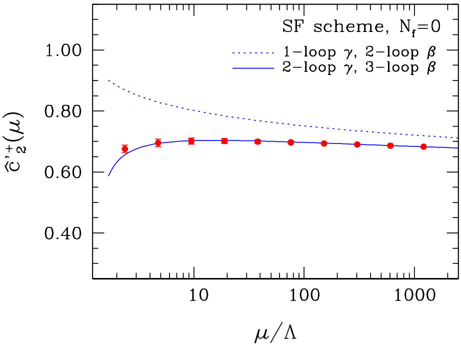
$$F_{B_s}^{stat} = 220(32)\ \mathrm{MeV}, \quad F_{B_s}^{stat}/F_B^{stat} = 1.10(^{+11}_{-5}),$$

$$B_{B_s}^{stat}(m_b) = 0.79(4) \quad \text{and} \quad B_B^{stat}(m_b) = 0.74(10)$$

Preliminary results obtained by using 1-loop renormalization and matching and by linearly extrapolating from "pions" of 400 MeV.

$N_f = 0, 2$: With Wilson fermions (in the static approximation) the mixings with operators of wrong chirality can be removed by using tmQCD [MDM '04, Palombi et al. '05]. NP renormalization for the relevant parity odd operators completed in the SF scheme





PT seems to work for $\mu \geq 1\,\text{GeV}$ for both $N_f = 0,\ 2$ [Papinutto and Pena LAT07].
For $N_f = 2$ the errors on the ren. factors are a bit large (up to 5%).

- Experimental numbers are very precise. Errors on CKM parameters extracted from Δm_q are dominated by uncertainties on the hadronic matrix elements. It is important to reduce them, although it seems difficult to do better than 10% on $F^2_{B_q} B_{B_q}$
- Not many new lattice results, especially for B_B
- Anyway the quenched approximation is being removed and rather small sea quark masses reached
- No results in the continuum limit
- In the static approximation the NP renormalization has been completed for the twisted mass approach and for $N_f = 0, 2$
- No clear expectation about 1/M corrections

Semileptonic B decays

prototype: $B \to \pi l \nu$, q = lepton pair momentum, $\Delta_{m^2} = (m_B^2 - m_\pi^2)/q^2$

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} [\kappa(q^2)]^{3/2} |f_+(q^2)|^2$$

$$\langle \pi(\vec{k}) | V^\mu | B(\vec{p}) \rangle = f_+(q^2)(p + k - q\Delta_{m^2})^\mu + f_0(q^2) q^\mu \Delta_{m^2}$$

1. for PS $\to$ V transitions 4 form factors.
2. In the heavy $\to$ heavy case, HQET gives relations among them valid up to O($1/M$). In the static limit the Isgur-Wise function $\xi(v \cdot v')$ describes all the form factors.
3. Experiments measure in the small q^2 region ($d\Gamma \propto p_\pi^3$), lattice can access the large q^2 one (a eff.). Also, HQET is applicable only there.
4. The kinematical factor in front of f_+ vanishes at $q_{max} = (m_B - m_\pi, \vec{0})$.
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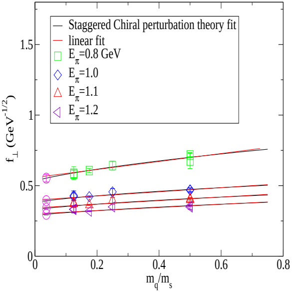
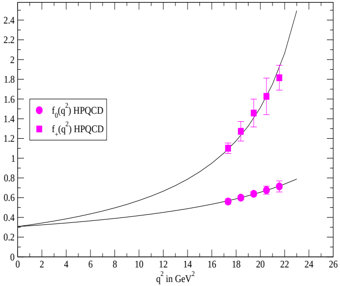
5. Lattice results cover a small region of q. Parameterization of the form factors are then used (which include kin. constraints, HQET scaling and disp. rel.) [Becirevic and Kaidalov '99]

$B \to \pi l \nu$

$N_f = 3$ HPQCD, same set as for B_{B_s} but m_l/m_s down to 0.125.
$\vec{p}_\pi = (000, 001, 011, 111) \times \frac{2\pi}{L}$





source of error	size of error (%)
statistics + chiral extrapolations	10
two-loop matching	9
discretization	3
relativistic	1
Total	14

- $g_{B^*B\pi}$ varies in the SχPT fits as a function of E_π (required for large E_π)
- Stat. errors grow at large q^2. Statistic is being accumulated

$$\frac{1}{|V_{ub}|^2}\int_{16GeV^2}^{q^2_{max}} \frac{d\Gamma}{dq^2}dq^2 = 2.07(41)(39)ps^{-1} \overset{HFAG}{\Longrightarrow} |V_{ub}| = 3.55(25)(50)\times 10^{-3}$$

the tension with the inclusive value $(4.49(33) \times 10^{-3}$ [Lubicz '07]) is still there (or maybe not, $|V_{ub}^{incl}| = 3.69(13)(31) \times 10^{-3}$ [Aglietti et al. '08] and [Ricciardi on Wed.])

Alternative approach for large q^2 by Heavy flavor χPT

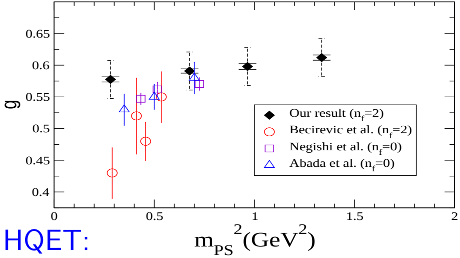
$$f_+(q^2) = -\frac{F_{B^*}}{2F_\pi}\left[g_{B^*B\pi}\left(\frac{1}{v\cdot k_\pi - m_{B^*} + m_B} - \frac{1}{m_B}\right) + \frac{F_B}{F_{B^*}}\right] \qquad [F_{B^*}] = 2$$

In the static approx. $\langle B^*(0)|A_\mu|B(0)\rangle = 2m_B\hat{g}\epsilon_\mu = g_{B^*B\pi}F_\pi\epsilon_\mu + O(1/M)$

-New $N_f = 0$ result using all to all propagators with 100 ev [J. Foley et al. '05] for 2 and 3 pt functions on 32 confs at $\beta = 6$, $16^3 \times 48$ (Clover, HYP1) and $m_\pi \geq 650$ MeV

$\hat{g} = 0.517(16)$ [Negishi, Matsufuru and Onogi '06]

Preliminary $N_f = 2$ result [Ohki LAT07]
$a \simeq 0.2\ fm$, HYP1, 200 ev, PT ren
$\hat{g} = 0.54(3)(3)_\chi(3)_{PT}(6)_{disc}$

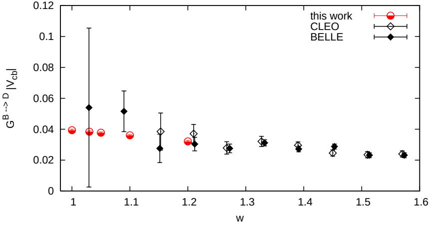


Towards a computation of $f_+(q^2)$ in HQET:

scale independent ratio $\frac{Z_V^{stat}}{Z_A^{stat}}$ computed NP for $N_f = 0$ and various static actions (EH, APE, HYP) using WI [Palombi '07]

Heavy$\rightarrow$ heavy transitions

$B \rightarrow Dl\nu$ [$\Rightarrow |V_{cb}|$] in the Rome II SSF approach [ROME II '07 and Tantalo LAT07]



1. The computation is done in quenched QCD starting in small volumes (0.4 fm, where b and c quark are accessible)
2. (3 times) Larger volumes are reached through 2 SSF:

$$\sigma^{i\rightarrow f}(w, L_0, L_1) = \frac{F^{i\rightarrow f}(w, L_1)}{F^{i\rightarrow f}(w, L_0)}$$

   idea: FSE might be large but depend mildly on m_{heavy} and can be extrapolated in $1/M$ from masses, in the last step, around the charm
3. Results in the continuum limit , although using two lattice resolutions for the ssf
4. Momenta injected by phases in the boundary conditions for fermions
5. Chiral limit by extrapolating from $m \geq m_s/4$

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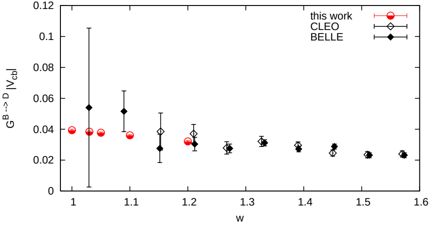

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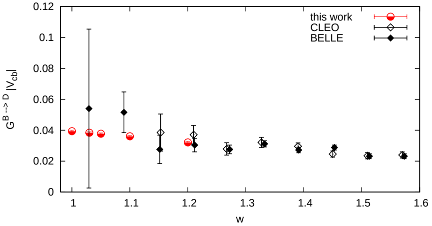

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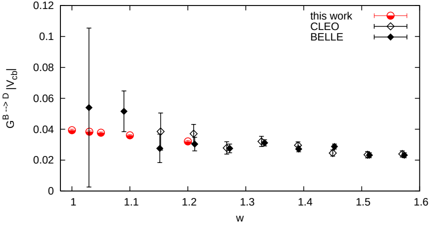

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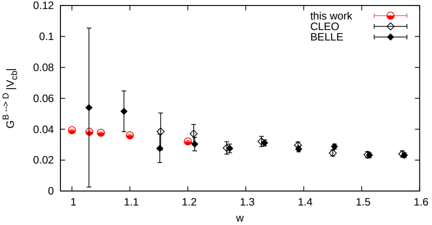

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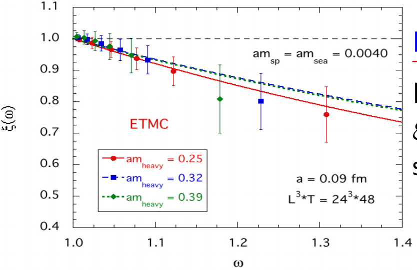


Preliminary $N_f = 2$ results from tmQCD

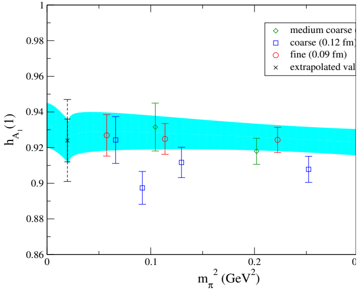
PS to PS form factors around the charm, $\xi(\omega)$ agress with $N_f = 0$, Rome II with larger stat errors, different systematics [Simula LAT07]

$B \to D^* l\nu$ at 0 recoil, Fermilab Collab. [Laiho LAT07]

- Rate larger than $B \to D$, preferred for $|V_{cb}|$
- At 0 recoil only 1 (h_{A_1}) of the 4 form factors. Matrix element of the axial current
- 1 'double ratio' (where most of the ren. constants cancel) instead of considering heavy mass dependence of 3 double ratios

$$h_{A_1}(1) = 0.924(12)(20)$$

- Same lattices as for $F_{D_{(s)}}$ [$N_f = 3$]



- Increasing efforts in recent times, especially in leaving the quenched approximation
- Still other systematics (mainly continuum limit for $N_f > 0$) are poorly studied
- Many different heavy-light and heavy-heavy processes considered and with different approaches
- Rather satisfactory overlap with experiments concerning the choice of processes and the accessible q^2 region. Improving on the latter requires considering very small lattice spacings.

HQET on the lattice at O($1/m_b$)

[m_b and F_{B_s}]

ALPHA Collaboration

In collaboration with B. Blosssier, P. Fritzsch, N. Garron, J. Heitger, M. Papinutto and R. Sommer

Why do we like HQET[Eichten and Hill '89] ?

- Theoretically very sound
- Can be treated non-perturbatively including renormalization (and O(1/M)) [Heitger and Sommer '03]
- Subleading corrections can be computed systematically or estimated by combining with relativistic quarks around the charm
- The continuum limit is well defined and can be reached numerically [ALPHA '03]
- Unquenching can be included now
- Can be used together with other methods, eg the Rome II method [Guazzini, Sommer and Tantalo '08]

still it might be a little involved ....

A bit of notation

Field content: ψ_h s.t. $P_+\psi_h = \psi_h$ with $P_+ = \frac{1+\gamma_0}{2}$

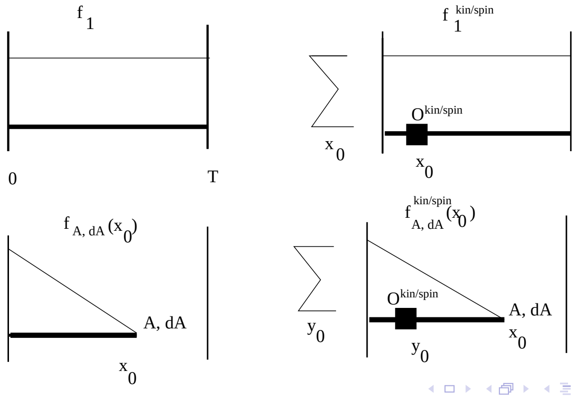
$$S_{HQET} = a^4 \sum_x \left\{ \bar{\psi}_h(D_0 + \delta m)\psi_h + \omega_{spin}\bar{\psi}_h(-\sigma\mathbf{B})\psi_h + \omega_{kin}\bar{\psi}_h\left(-\frac{1}{2}\mathbf{D}^2\right)\psi_h \right.$$

- 3 parameters (we'll get rid of one through spin-average) to be set in order to reproduce QCD up to $O(1/m_b^2)$.
- ω_{spin} and ω_{kin} formally $O(1/m_b)$.
- Renormalization and matching !
- The two steps could be performed separately. In particular at *leading order in* $1/m_b$ matching can be done in perturbation theory. Here we are interested in $1/m_b$ corrections and do the two things at the same time and non-perturbatively.

We don't include the next to leading terms of the $1/m_b$ expansion in the action, the theory would be non renormalizable. We treat them as insertions into correlation functions and consider the static action only.

$$e^{-(S_{rel}+S_{HQET})} = e^{-(S_{rel}+S_{stat})} \times [1 - a^4 \sum_x \mathcal{L}^{(1)}(x, \omega_{spin}, \omega_{kin}) + \ldots]$$

and $S_{stat} = a^4 \sum_x \bar{\psi}_h(x) D_0^{HYP} \psi_h(x)$ [spin-flavor symmetric]

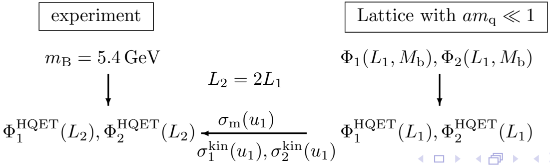


Overview of the approach

- We will use a finite volume scheme (Schrödinger functional). The volume L_1 should be small enough to simulate relativistic b-quarks ($a \ll 1/m_b$) but also such that $\frac{1}{L_1 m_b} \simeq \frac{\Lambda_{\rm QCD}}{m_b}$ (in the end $L_1 \simeq 0.4$ fm.)
- Considering spin-averaged quantities, we are left with two coefficients. Strategy: define two (sensible) quantities Φ_k and require (in small volume)

$$\Phi_k^{HQET} = \Phi_k^{QCD} \quad k = 1, 2$$

- Evolve these quantities in the effective theory to large volumes (through Step Scaling Functions σ). There the B-meson mass expressed in terms of Φ_k and large volume HQET quantities can be used to fix the b-quark mass.



The B_s meson decay constant

Operators have an expansion in $1/m_b$ too.

$$A_0^{HQET} = Z_A^{HQET} \left(A_0^{stat} + (O(a) + O(1/m_b)) \times c_A^{HQET} A_0^{(1)} \right) ,$$

$$A_0^{(1)}(x) = (\bar{\psi}_l(x)\gamma_j D_j)\psi_h(x)$$

In our notation Z_A^{HQET} includes the matching coefficient.

For the decay constant 4 Φ_i's are needed in the small volume matching to QCD. The SSF also becomes a 4×4 matrix [ALPHA LAT07]

- 12 matching conditions. All results agree, indicating very small $O(1/m_b^2)$.

θ_0	$r_0\,M_\mathrm{b}^{(0)}$	$r_0\,M_\mathrm{b} = r_0\,(M_\mathrm{b}^{(0)} + M_\mathrm{b}^{(1a)} + M_\mathrm{b}^{(1b)})$		
		$\theta_1 = 0$ $\theta_2 = 1/2$	$\theta_1 = 1/2$ $\theta_2 = 1$	$\theta_1 = 1$ $\theta_2 = 0$
0	17.25(20)	17.12(22)	17.12(22)	17.12(22)
0	17.05(25)	17.25(28)	17.23(27)	17.24(27)
1/2	17.01(22)	17.23(28)	17.21(27)	17.22(28)
1	16.78(28)	17.17(32)	17.14(30)	17.15(30)

- For F_{B_s} as well (Preliminary !!)

	$F_{B_s}^\mathrm{stat}$ [MeV]	$F_{B_s}^\mathrm{stat} + F_{B_s}^{(1)}$[MeV]		
θ_0		$\theta_1 = 0$ $\theta_2 = 0.5$	$\theta_1 = 0.5$ $\theta_2 = 1$	$\theta_1 = 1$ $\theta_2 = 0$
0	224 ± 5	185 ± 21	186 ± 22	189 ± 22
0.5	220 ± 5	185 ± 21	187 ± 22	189 ± 22
1	209 ± 5	184 ± 21	185 ± 21	188 ± 22

Results are more consistent than suggested by the errors, as eg

$$F_{\mathrm{B_s}}^{\mathrm{stat}+(1)}(\theta_0 = 0, \theta_1 = 1, \theta_2 = 0) - F_{\mathrm{B_s}}^{\mathrm{stat}+(1)}(\theta_0 = 1, \theta_1 = 0, \theta_2 = 0.5) = 4 \pm 2\ \mathrm{MeV}\,.$$

Conclusions

1. To keep the pace with forth-coming experiments and really help in the quest for New Physics, lattice results in Heavy Flavor Physics must aim at high precision.
2. To this end all the systematics must be kept under control. Unquenching, renormalization, continuum limit, chiral extrapolations, each of them can easily have a $5-10\%$ uncertainty associated.
3. A great effort has been put in recent years in removing the quenched approximation, with great success.
4. In my view it is now time to tackle also the other systematics.
5. I've given an example how this can be done discussing the b-quark mass in HQET. Almost done, it was quenched. Unquenching is ongoing [Fritzsch and Heitger LAT07]
    1. The approach can be extended to other quantities, eg for F_{B_s} or $B_{B_{(s)}}$
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