Heavy Quark Masses

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Outline

- General Remarks on Heavy Quark Masses
- R-Evolution of Heavy Quark Masses
- Bottom Mass Determinations
- Charm Mass Determinations
- Top Quark Mass from the Tevatron
- What mass is determined at Tevatron?

Remarks on Quark Masses

- Important QCD input parameters for SM predictions
- Confinement $\Longrightarrow$ quark masses not physical observables

$$\mathcal{L}_{\rm QCD} = \frac{1}{4} G_{\mu\nu}^2 + \sum_i \bar{\psi}_i \left(\not{D} - M_i \right) \psi$$

  - ▷ defined as formal parameters in QCD action → $\delta m_Q < \Lambda_{\rm QCD}$ possible
  - ▷ (renormalization) scheme dependent
  - ▷ to be well defined: $m_q^{\rm schemeA} = m_q^{\rm schemeB} (1 + \alpha_s + \alpha_s^2 + \ldots)$
  - ▷ some schemes more appropriate than others

We only want to use short-distance mass scheme that do not suffer from the $\mathcal{O}(\Lambda_{\rm QCD})$ renormalon inherent to the pole scheme!

… the POLE MASS

What's wrong with it?

Pole mass based on a particular renormalization prescription:

$$\Gamma^{q\bar{q}}(q) = \not{q} - m_0 + \Sigma(q, m_0) \stackrel{!}{=} \not{q} - m_{\rm pole}$$

to all orders in perturbation theory

The partonic quark on-shell limit does not exist due to confinement!

So what? The strong coupling α_s isn't defined by a physical definition either.

Yes, but the partonic on-shell limit involves IR singularities.

The pole mass introduces, in an artificial, way a sensitivity to the Coulomb self energy of the heavy quark.

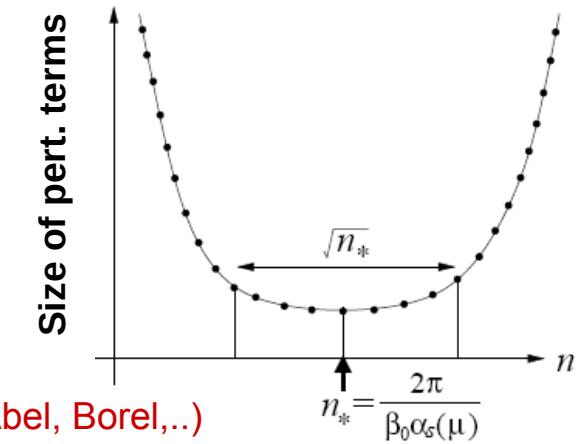
All mass-dependent quantities in the pole scheme develop a bad high order behavior.

$$A(m) \propto m \sim \mu \sum_{n \gg 1} \alpha_s^{n+1}(\mu)\, 2^n\, \beta_0^n\, n!$$

Perturbation theory sucks!

Perturbative series are divergent.

Fortunately it is useful physically because the series are still asymptotic.



Math for divergent series: (Bernoulli, Euler, Abel, Borel,..)

The Borel-process: (for $\sum_{n=0}^{\infty} a_n x^n$)

The series is called Borel-summable if the sum

$$B(t) = \sum_{n=0}^{\infty} \frac{a_n t^n}{n!}$$

is convergent in the original sense for $t > 0$ and if the integral

$$S(x) = \frac{1}{x} \int_0^{\infty} dt\, e^{-t/x}\, B(t)$$

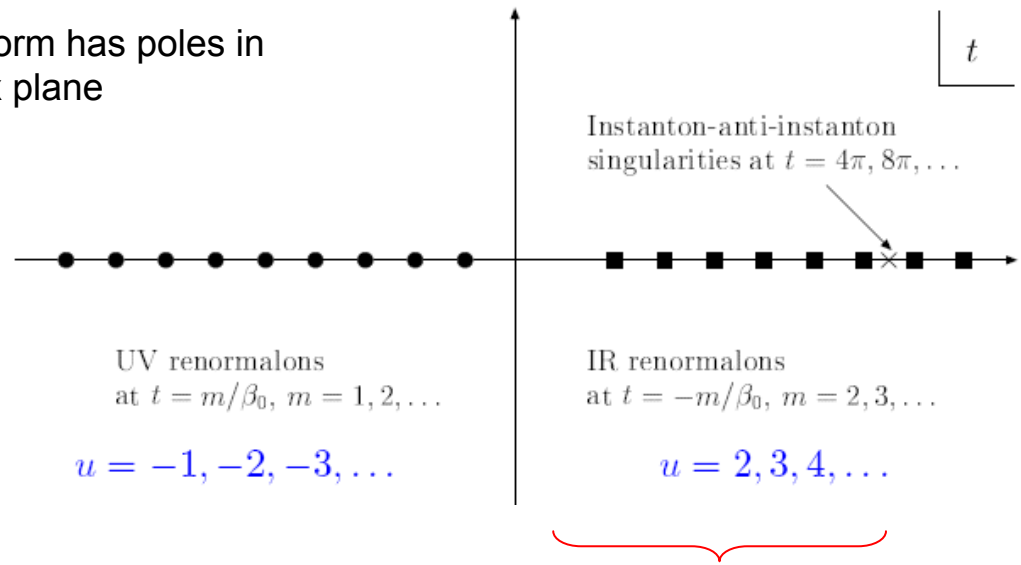
exists. The value of the series is $S(x)$.

$$\sum_n a_n = 1 - x + x^2 - x^3 + \ldots \stackrel{?}{=} \frac{1}{1-(-x)} = \frac{1}{1+x}$$

$$B(t) = e^{-t} \quad \rightarrow S(x) = \frac{1}{x} \int_0^{\infty} dt\, e^{-t(1+x)/x} = \frac{1}{1+x}$$

Perturbation theory sucks!

Borel transform has poles in the complex plane



IR renormalons lead to ambiguities in

$$S(x) = \frac{1}{x} \int_0^\infty dt \; e^{-t/x} \, B(t)$$

Perturbation theory & OPE

→Ambiguity from a pole at $u = k$:

$$\Delta\left[\int_0^\infty du\; e^{-\frac{4\pi u}{\beta_0 \alpha_s}}\,\frac{1}{u-k}\right] \sim e^{-(4\pi/\beta_0\alpha_s)\,k} \sim \left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)^k$$

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2_{\rm QCD})}$$

"Power-Ambiguity"

- smaller for $Q^2 \to \infty$ (asymptotic freedom)
- pole closest to origin most important
- signals non-perturbative effects, power corrections

  → Operator-Product-Expansion

Pole mass scheme: $A(m) \propto m \sim \mu \sum_{n\gg 1} \alpha_s^{n+1}(\mu)\, 2^n\, \beta_0^n\, n!$

$\frac{1}{u-1/2} \leftrightarrow$ Ambiguity of $\mathcal{O}(\Lambda_{\rm QCD})$, "Pole mass renormalon"

There is NO operator with the right dimension! Do not use the pole mass!

Short-Distance Masses

$$m_{\rm pole} = m_{\rm sd} + \delta m$$

short-distance mass without $\mathcal{O}(\Lambda_{\rm QCD})$ ambiguity

perturbation series that contains the pole mass ambiguity of $\mathcal{O}(\Lambda_{\rm QCD})$

$$\delta m \sim \mu \sum_{n \gg 1} \alpha_s^{n+1}(\mu)\, 2^n\, \beta_0^n\, n!$$

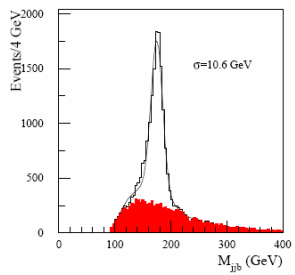
What's the best way to define δm ?

- **infinitely many possible schemes exist**
- **but only certain classes can be used for certain types of problems.**

How relevant it is to find a good scheme depends on the size of the uncertainties one has anyway or is willing to accept.

Top Quark Mass Schemes

→ Different types of short-distance masses for different types of processes



Top Resonance → Jet/resonance mass schemes

$$m_J(R) - m^{\rm pole} = -R\,\frac{C_F\alpha_s(R)}{2\pi} + \ldots \qquad R = \Gamma_t \sim 1.5\ \text{GeV}$$

$$M^{\rm peak} = m_t + \Delta M_{\rm pert} + \Delta M_{\rm non-pert}$$

$$\sim \Gamma\left[\alpha_s(\Gamma) + \alpha_s^2(\Gamma) + \ldots\right] \qquad \sim \frac{Q}{m_t}\Lambda_{\rm QCD}$$

absorbes infrared contributions below R into $m_J(R)$

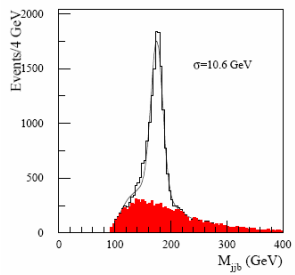
Fleming, Mantry, Stewart, AHH

$$\left(\frac{d^2\sigma}{dM_t^2\,dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

Top Quark Mass Schemes

→ Different types of short-distance masses for different types of processes

Top Resonance → Jet/resonance mass schemes



$$m_J(R) - m^{\rm pole} = -R\,\frac{C_F\alpha_s(R)}{2\pi} + \ldots \qquad R = \Gamma_t \sim 1.5\ \text{GeV}$$

Manohar,Stewart,Teubner,AH

Top Threshold → 1S, PS mass schemes

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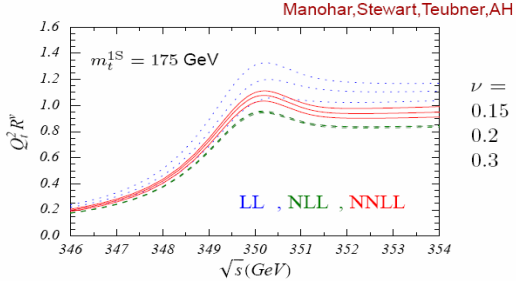
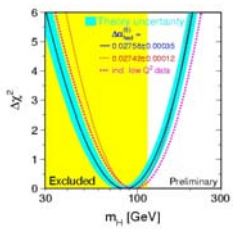
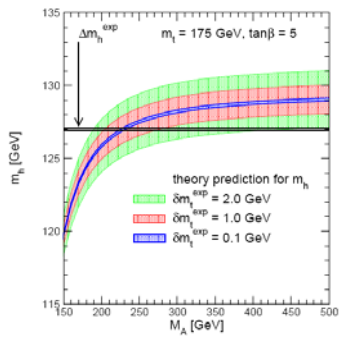


$$m_{1S} - m^{\rm pole} = -R\,\frac{C_F\alpha_s(R)}{8} + \ldots$$

$$R = \langle r_{t\bar t}^{-1}\rangle = C_F\alpha_s m_{1S} \approx 20\ \text{GeV}$$

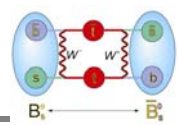


High Energy/Off-shell → MS/MSbar schemes



$$\overline{m}(\overline{m}) - m^{\rm pole} = -R\frac{C_F\alpha_s(R)}{\pi} + \ldots \qquad R = \overline{m}(\overline{m}) \approx 163\ \text{GeV}$$

$$\overline{m}(\mu) - m^{\rm pole} = -R\frac{C_F\alpha_s(R)}{\pi} + \ldots \qquad R = \overline{m}(\mu)$$



Bottom+Charm Mass Schemes

→ m_b and m_c are not very large.

→ Only two distinct classes need to be defined in practice.

Threshold Schemes:

- B/D physics (inclusive decays)
- Quarkonia: $b\bar{b}$, $c\bar{c}$
- non-relativistic sum rules

Kinetic mass:

- o from B meson form factor sum rules
- o cut-off dependent

Bigi, Uraltsev

$\mathcal{O}(\alpha_s^2), \mathcal{O}(\alpha_s^3\beta_0)$

$$m^{\rm kin}(\mu_f) = m^{\rm pole} - \left[\bar{\Lambda}(\mu_f)\right]_{\rm pert} - \left[\frac{\mu_\pi^2(\mu_f)}{2m_{\rm pole}}\right]_{\rm pert} + \ldots$$

1S mass:

- o from pert. $\Upsilon(1S)$ mass
- o scale independent

Ligeti, Manohar, AHH

$$m^{\rm 1S} = \frac{1}{2}\left[M_{\Upsilon_{Q\bar{Q}}(1\,^3S_1)}\right]_{\rm pert}$$

$\mathcal{O}(\alpha_s^3)$

Shape fct mass:

- o from moments of B-decay shape function
- o cut-off + renormalization scale dependent

Neubert

$\mathcal{O}(\alpha_s^2)$

Bottom+Charm Mass Schemes

→ m_b and m_c are not very large.

→ Only two distinct classes need to be defined in practice.

MSbar Mass Scheme:

- high-energy, inclusive processes

  e.g. $\sigma(e^+e^- \to b\bar{b})$, $\sigma(pp \to b\bar{b})$

  e.g. $\Gamma(Z \to b\bar{b})$

- off-shell, highly virtual b and c quarks

  e.g. $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$

  e.g. $\mathcal{B}(B \to X_s\gamma)$

What impact do schemes have?

Inclusive B decays: $m_b \gg \Lambda_{\rm QCD}$

$$\Gamma(B\,\text{meson}) = \Gamma(b\,\text{quark}) + \sum_{i,n} C(\alpha_s)\frac{\langle \mathcal{O}^{\rm HQET}_{i,n}\rangle}{m_b^n}$$

$\mathcal{O}(\Lambda^2_{\rm QCD})$: $\lambda_1,\ \lambda_2$ $\qquad$ $\mathcal{O}(\Lambda^3_{\rm QCD})$: $\rho_1,\ \rho_2,\ \tau_1,\ \tau_2,\ \tau_3,\ \tau_4$

$$\Gamma(B \to X_u e\bar{\nu}) = \frac{G_F^2|V_{ub}|^2}{192\pi^3} \times (m_b^{\rm pole})^5\,[1-0.17-0.10]$$

→ IR sensitive, $\mathcal{O}(\Lambda_{\rm QCD})$ renormalon

$$(\overline{m}_b(\overline{m}_b))^5\,[1+0.30+0.13]$$

→ for high-energy processes

$$\frac{\delta V_{\rm ub}}{V_{\rm ub}} \sim 2.5\frac{\delta m_b}{m_b}$$

$$(m_{b,1}^{\rm kin})^5\,[1-0.02-0.001]$$

$$(m_{b,2}^{\rm PS})^5\,[1-0.03-0.01]$$

$$(m_b^{\rm 1S})^5\,[1-0.12-0.03]$$

"threshold masses"

What impact do schemes have?

Top Threshold:

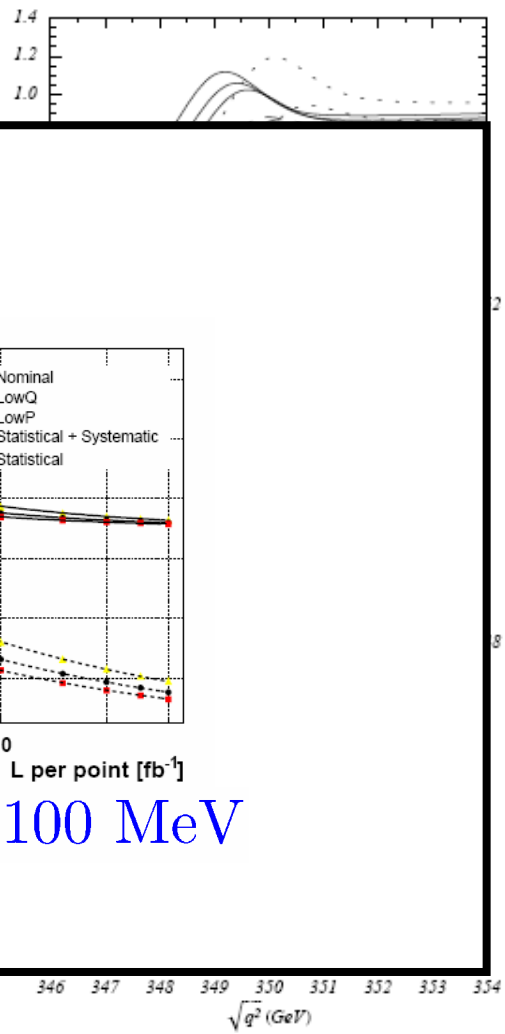
$\sigma_{\rm tot} \propto$

$\propto$

$\left(-\frac{\nabla^2}{m} - \frac{\nabla^4}{4m^3}\right.$

Theory S

- LL + N
- NNLL
- NLL +

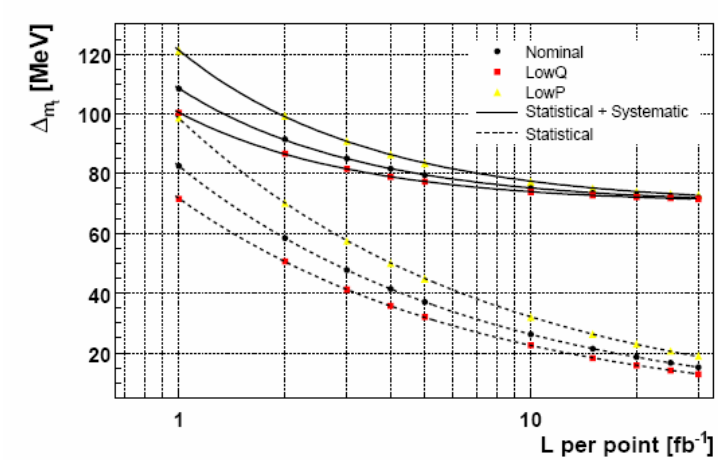


$m_t^{\rm pole}$

$\overline{m}(\overline{m})$

$m_t^{\rm 1S}$

ILC Prospect



$$(\delta m_t^{\rm 1S})_{\rm exp+theo} \leq 100 \text{ MeV}$$

Need for a precise Charm mass

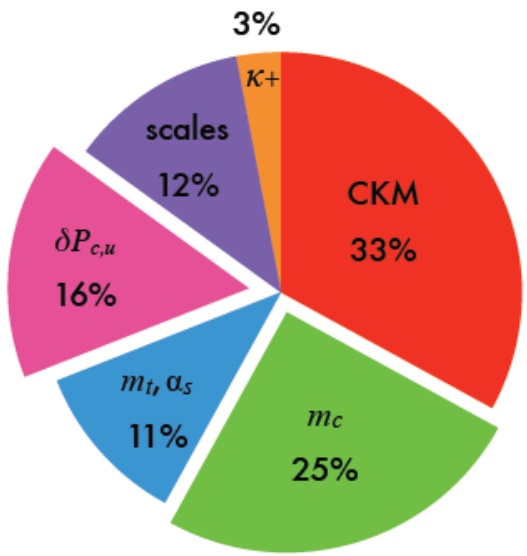
SM prediction(s) of $K^+ \to \pi^+ \nu\bar{\nu}$: error budget

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}(\gamma)) = \{8.57 \pm 0.93, 8.51 \pm 0.73, 8.04 \pm 0.98\} \times 10^{-11}$$

$$m_c(m_c) = (1.30 \pm 0.05)\ \mathrm{GeV}$$

$$m_c(m_c) = (1.286 \pm 0.013)\ \mathrm{GeV}^{*}$$

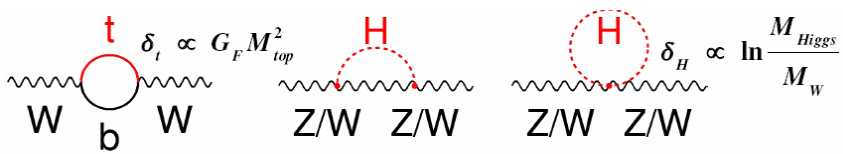
$$m_c(m_c) = (1.224 \pm 0.017 \pm 0.054)\ \mathrm{GeV}^{\dagger}$$



*Kühn et al. '07 †Hoang & Manohar '05

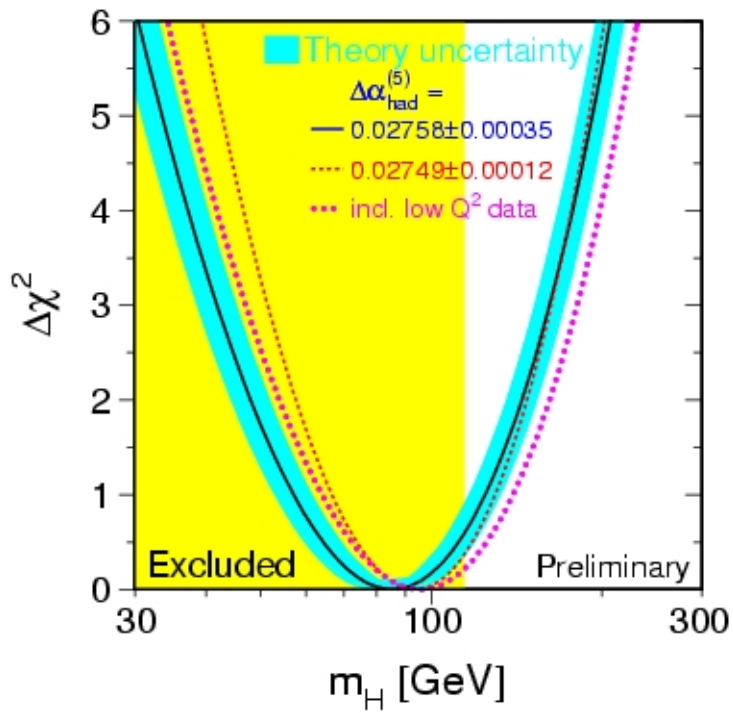
Need for a precise Top mass

Fit to electroweak precision observables



$$\sin\theta_W \times \Big(1 + \delta(m_t, m_H, \ldots)\Big)$$

$$= 1 - \frac{M_W^2}{M_Z^2}$$



$$m_H = 76^{+33}_{-24}\ \text{GeV}$$

$$m_H < 182\ \text{GeV}\ \ (95\%\text{CL})$$

$$m_t = 170.9 \pm 1.8\ \text{GeV}$$

2 GeV error: 15% change in m_H

Need for a precise Top mass

Mass of Lightest MSSM Higgs Boson

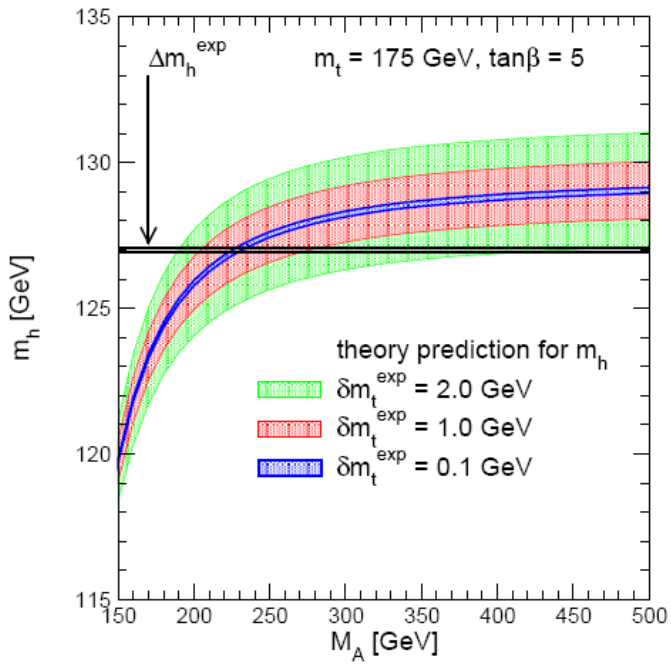
5 Higgs bosons:

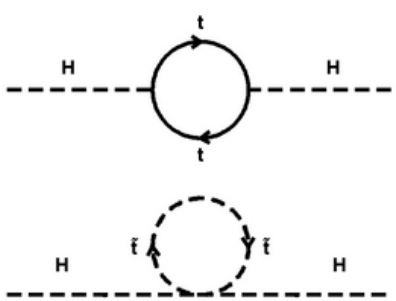
m_h (scalar, neutral)

m_H (scalar, neutral)

m_A (speudo-scalar, neutral))

$m_H^{\pm}$ (charged)





$$m_h^2 \simeq M_Z^2 + \frac{G_F\, m_t^4}{\pi^2 \sin^2\beta} \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

R-Evolution

Jain, Scimemi, Stewart, AH arXiv:0803.4214

- Perturbative relations between short-distance masses for widely separated R contain large logs that cannot be summed by usual UV renormalization group running.

$$m_{\rm pole} - m_J(R,\mu) \;=\; R\, e^{\gamma_E} \left\{ \left(\frac{\alpha_s(\mu)}{\pi}\right) \left[\frac{4}{3}\ln\frac{\mu}{R} + \frac{2}{3}\right] + \ldots \right\} \qquad (R \sim \Gamma_t)$$

$$m_{\rm pole} - \overline{m}(\mu) \;=\; \overline{m}(\mu) \left\{ \left(\frac{\alpha_s(\mu)}{\pi}\right) \left[2\ln\frac{\mu}{\overline{m}(\mu)} + \frac{4}{3}\right] + \ldots \right\} \qquad (R \sim \overline{m}(\mu))$$

- **<u>Golden Rule:</u>** The SAME renormalization scale MUST be used to cancel the renormalon. $\Delta m \sim \mu \sum_n \alpha_s^{n+1}(\mu)\, 2^n\, \beta_0^n\, n!$

$m_J(R,\mu) - \overline{m}(\mu)$ There is no choice of μ that avoids large logs!

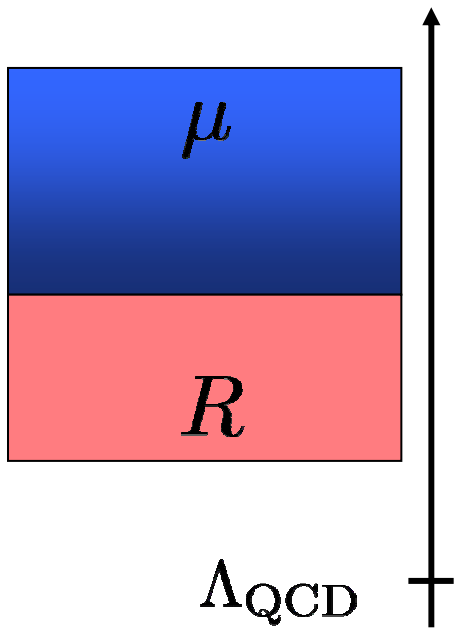
R-Evolution

Jain, Scimemi, Stewart, AH arXiv:0803.4214

In general, any short-distance mass depends on two scales:

$$m(R,\mu)$$

IR-cutoff UV-cutoff



$$m_{\rm pole} = m(R,\mu) + \delta m(R,\mu)$$

Consider R as a continuous parameter to describe an infrared renormalization group flow.
(Bigi, Uraltsev, Shifmann, Vainshtein)

(A) Resolves the problem of large logs for $(\mu = R)$

(B) Allows to study of renormalons without using bubble chains (large-nf)

R-Evolution: practical application

$$\delta m(R) = R \sum_{n=1}^{\infty} \left(\frac{\alpha_s(R)}{4\pi}\right)^n a_n$$

R-evolution equation

$$\boxed{R\frac{d}{dR}m(R) = -\frac{d}{d\ln R}\delta m(R) = R\sum_{n=0}^{\infty}\gamma_n^R\left[\frac{\alpha_s(R)}{4\pi}\right]^{n+1}}$$

renormalon-free !

$$\gamma_0^R = a_1, \qquad \gamma_1^R = a_2 - 2\beta_0 a_1\,, \ldots$$

LL Solution:

$$m(R_1) - m(R_0) = \frac{\Lambda_{\rm QCD}^{(0)}\,\gamma_0^R}{2\beta_0}\int_{t_1}^{t_0} dt\,\frac{e^{-t}}{t}$$

$$= \frac{\Lambda_{\rm QCD}^{(0)}\,\gamma_0^R}{2\beta_0}\left[\Gamma(0,t_1) - \Gamma(0,t_0)\right]$$

$$\Lambda_{\rm QCD}^{(0)}\Gamma = \frac{-\gamma_0^R R_1}{2\beta_0}\sum_{n=0}^{\infty}\left[\frac{\beta_0\alpha_1}{2\pi}\right]^{n+1} n!\left(1 - \frac{R_0}{R_1}\sum_{k=0}^{n}\frac{1}{k!}\ln^k\frac{R_1}{R_0}\right)$$

All logs are summed up properly.

Pole mass renormalon cancelled.

R-Evolution: practical application

$$\delta m(R) \;=\; R \sum_{n=1}^{\infty} \left(\frac{\alpha_s(R)}{4\pi}\right)^n a_n$$

R-evolution equation

$$\boxed{R\frac{d}{dR}m(R) = -\frac{d}{d\ln R}\delta m(R) \;=\; R \sum_{n=0}^{\infty} \gamma_n^R \left[\frac{\alpha_s(R)}{4\pi}\right]^{n+1}}$$

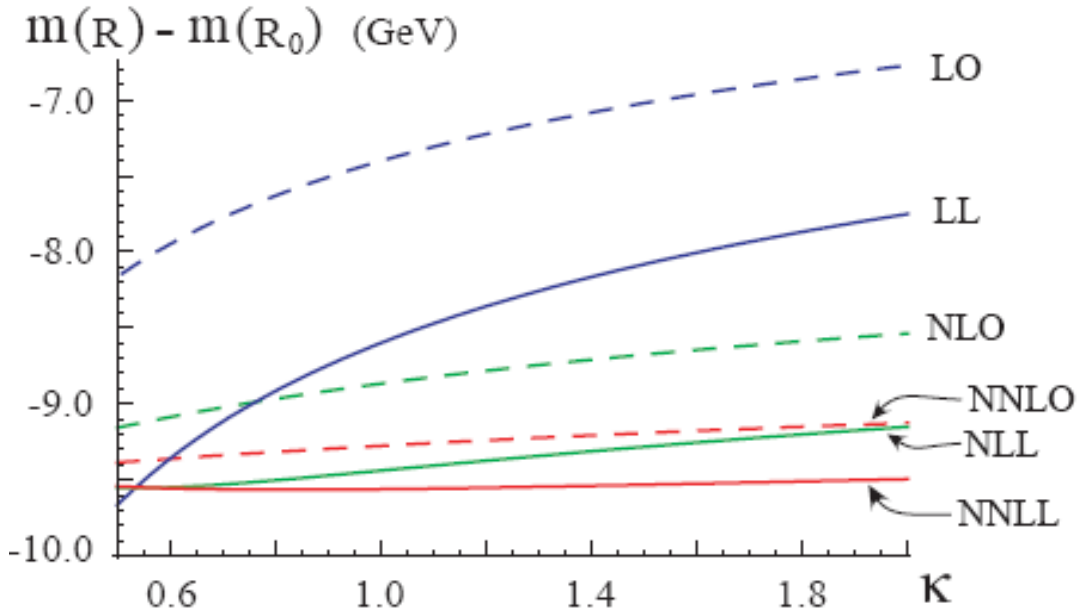
renormalon-free !

$$\gamma_0^R = a_1, \qquad \gamma_1^R = a_2 - 2\beta_0 a_1\,, \ldots$$

All order Solution:

$$\left[m(R_1) - m(R_0)\right]^{\mathrm{N}^k\mathrm{LL}} = \Lambda_{\mathrm{QCD}}^{(k)} \sum_{j=0}^{k} S_j\,(-1)^j$$
$$\times\, e^{i\pi\hat{b}_1}\left[\Gamma(-\hat{b}_1 - j, t_1) - \Gamma(-\hat{b}_1 - j, t_0)\right].$$

R-Evolution: practical application



$$R_0 = 3 \text{ GeV}$$

$$R = 163 \text{ GeV}$$

$$\ln \frac{R_1}{R_0} = \int_{\alpha_0}^{\alpha_1} \frac{d\alpha_R}{\beta[\alpha_R]} = \int_{t_1}^{t_0} dt\, \hat{b}(t) = G(t_0) - G(t_1)$$

$$\alpha_i \equiv \alpha_s(R_i) \qquad t_i \equiv -2\pi/(\beta_0 \alpha_i)$$

$$\Lambda_{\rm QCD} = R\, e^{G(t)} = R_0\, e^{G(t_0)}$$

R-Evolution: conceptual application

Study the pole mass renormalon without bubble chain approximation

$$\lim_{R_0 \to 0} m(R_0) = m_{\rm pole}$$

has all the bad properties of the pole mass

$$m(R_1) - m_{\rm pole} = \Lambda_{\rm QCD} \int_{t_1}^{\infty} dt\ \gamma_R(t)\ \frac{d}{dt}\, e^{-B(t)}$$

$$= \int_0^{\infty} du\ B(u)\ e^{-u \frac{4\pi}{\beta_0 \alpha_s(R)}}$$

$$B(u) = 2R \left[\sum_{\ell=0}^{\infty} g_\ell\, Q_\ell(u) - P_{1/2} \sum_{\ell=0}^{\infty} g_\ell\, \frac{\Gamma(1+\hat{b}_1-\ell)}{(1-2u)^{1+\hat{b}_1-\ell}} \right]$$

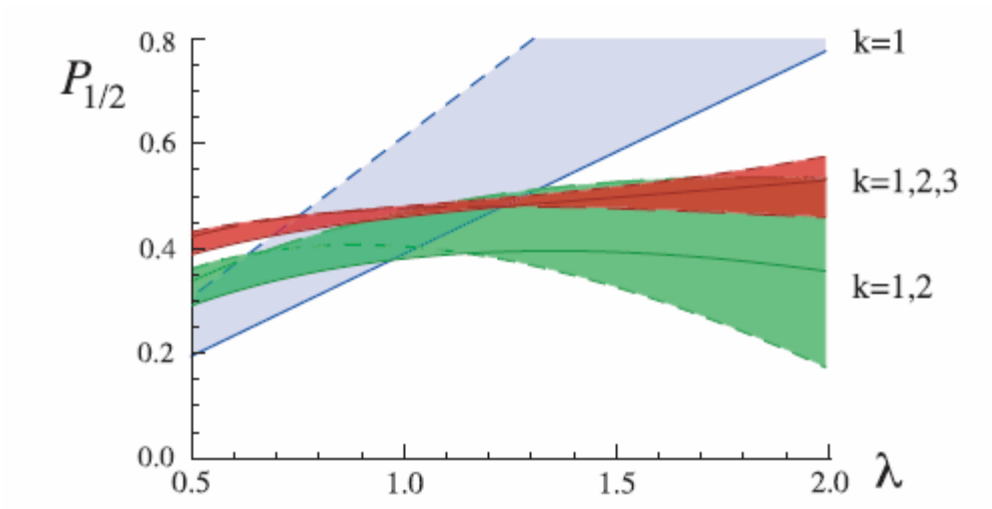
$$P_{1/2} = \sum_{k=0}^{\infty} \frac{S_k}{\Gamma(1+\hat{b}_1+k)}$$

Absolute convergent sum rule !

Sums to the same value for ALL short-distance masses!

R-Evolution: conceptual application

Convergence of the sum rule for masses known to 3-loop order



PS mass

static mass

MSR mass

$$P_{1/2} = 0.47 \pm 0.10$$

Bubble chain: $P_{1/2} = 0.80$

R-Evolution: spin-off

Jain, Scimemi, Stewart, AHH
arXiv:0803.4214

⟹ Can the MSbar mass RG-run below the mass ??

<u>Answer:</u> Not quite, but we can construct a mass from it than can.

<u>MSbar Scheme:</u> $(\mu > \overline{m}(\overline{m}))$

$$\overline{m}(\overline{m}) - m^{\rm pole} = -\overline{m}(\overline{m})\,[\,0.42441\,\alpha_s(\overline{m}) + 0.8345\,\alpha_s^2(\overline{m}) + 2.368\,\alpha_s^3(\overline{m}) + \ldots]$$

⇓

<u>MSR Scheme:</u> $(\mu < \overline{m}(\overline{m}))$

$$m_{\rm MSR}(R) - m^{\rm pole} = -R\,[\,0.42441\,\alpha_s(R) + 0.8345\,\alpha_s^2(R) + 2.368\,\alpha_s^3(R) + \ldots]$$

$$m_{\rm MSR}(m_{\rm MSR}) = \overline{m}(\overline{m})$$

⟹ $m_{\rm MSR}(R)$ runs with the R-evolution equation
can be used as jet/resonance or threshold mass

Bottom Quark Mass Determinations

- **Spectral Moments of Inclusive B Decays**
- **Υ Sum Rules** → **relativistic**
  → **non-relativistic**

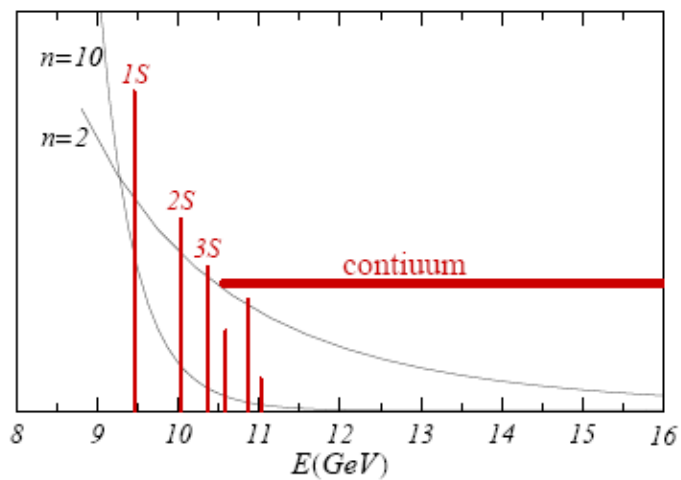
Υ Sum Rules

- Moments:

$$P_n = \int_{M_{\Upsilon(1S)}}^{\infty} dE \, \frac{R_b(E)}{E^{2n+1}} \sim \frac{1}{m_b^{2n}}$$

$$= \left(\frac{d}{dq^2}\right)^n \int d^4 e^{iqx} \langle 0 | T j(x) J(0) | 0 \rangle \Big|_{q^2=0}$$

duality bound $n \lesssim 10$



→ Generic form: $P_n^{\text{theory}} \sim \frac{1}{m_b^{2n}} \sum_i \left(\alpha_s(m_b/n)\sqrt{n}\right)^i$

A) relativistic: $n \lesssim 4$ ⇒ $\overline{\text{MS}}$ mass

→ usual loop expansion in powers of α_s ,

B) non-relativistic: $4 \lesssim n \lesssim 10$ ⇒ threshold masses

→ $(\alpha_s \sqrt{n})$ terms must be summed using NRQCD

nonperturbative power corrections very small

Υ Sum Rules

Relativistic:

exp. data:

- $\Upsilon(1S)$,..., $\Upsilon(6S)$: Masses, $\Gamma(e^+e^-)$ (ARGUS, CUSB, CLEO)
- no data in the R_b continuum, $\sqrt{s} > 11.1$ GeV $\rightarrow$ models (!)

A) use $\mathcal{O}(\alpha_s^2)$ prediction ("theory driven") Kühn, Steinhauser 2001

$$\overline{m}_b(\overline{m}_b)^{n=3} = 4.20 \pm 0.04\,(\Upsilon) \pm 0.004\,(\text{cont}) \pm 0.01\,(\alpha_s) \pm 0.01\,(\mu) \qquad \boxed{4.19 \pm 0.05}$$

B) use $\mathcal{O}(\alpha_s^2)$ prediction & assign 10% uncertainty Corcella, AH 2002

$$\overline{m}_b(\overline{m}_b)^{n=3} = 4.20 \pm 0.04\,(\Upsilon) \pm 0.04\,(\text{cont}) \pm 0.01\,(\alpha_s) \pm 0.01\,(\mu) \qquad \boxed{4.20 \pm 0.07}$$

Kühn, Steinhauser, Sturm (2006) $\longrightarrow$ 3-loop for n=1 (update)

$$\boxed{4.164 \pm 0.025}$$

Boughezal, Czakon, Schutzmeier (2006) $\longrightarrow$ 3-loop for n=1 (update)

$$\boxed{4.205 \pm 0.058}$$

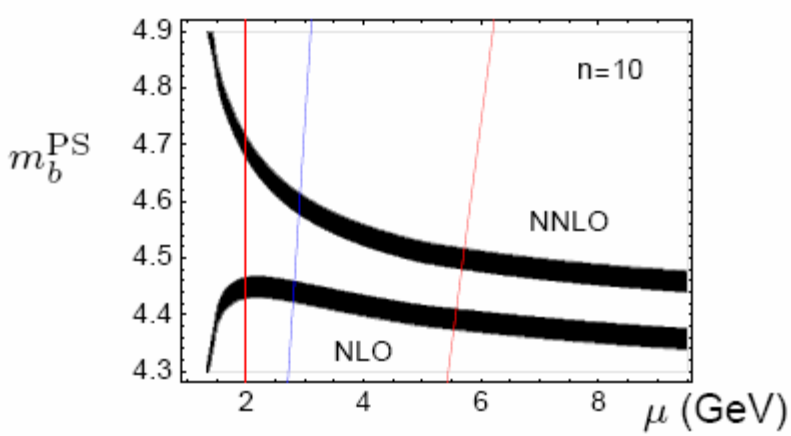
Υ Sum Rules

Non-Relativistic:

exp. data:

- $\Upsilon(1S)$, $\Upsilon(2S)$ dominate
- very rough model for R_b continuum suffi cient

$\longrightarrow$ pert. expansion marginal (large coeffi cients)



Melnikov: alternating series assumed

$\longrightarrow m_b^{1S} = 4.71 \pm 0.06$ GeV

Beneke: only NNLO considered

$\longrightarrow m_b^{1S} = 4.77 \pm 0.10$ GeV

Hoang: χ^2-fi t for several moments

$m_b^{1S} = 4.69 \pm 0.03$ GeV
(with nonzero charm mass)

m^{1S}	4.69	4.71	4.77
$\overline{m}(\overline{m})$	4.17	4.19	4.24

$\longrightarrow$ NNLL RG-improved analyis (w.i.p.)

partial result: Pineda, Signer

Global Fits

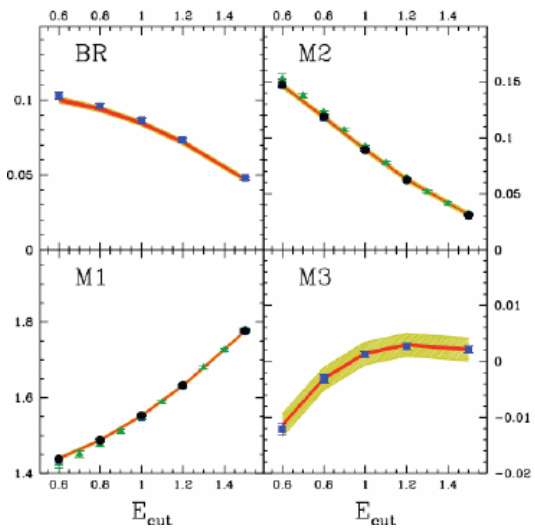
Experimental Data:

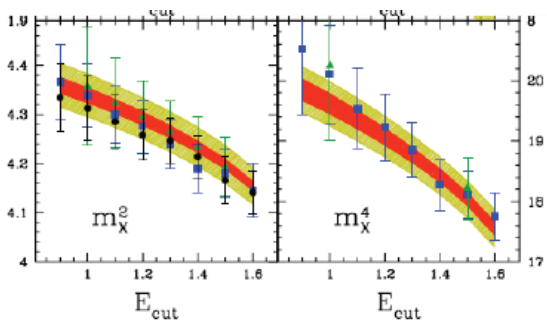
moments in semileptonic decays

E_l : lepton energy spectrum in $B \to X_c l \nu$ (BaBar Belle CLEO DELPHI)

M_X^2: hadronic mass spectrum in $B \to X_c l \nu$ (BaBar CDF CLEO DELPHI)

E_γ : photon energy spectrum in $B \to X_s \gamma$ (Babar Belle CLEO)





Global Fits

Theoretical Moments:

- **theory input:** **perturbative QCD** $\mathcal{O}(\alpha_s, \alpha_s^2\beta_0)$

  power corrections $\mathcal{O}(\Lambda_{\rm QCD}^2/m^2, \Lambda_{\rm QCD}^3/m^3)$

  A) Gambino, Uraltsev (2004)

  B) Bauer, Ligeti, Manohar, Trott (2004)

- **expansion scheme:** **A)** $1/m_b$ **expansion** $\Rightarrow |V_{cb}|, m_b, m_c, \lambda_{1,2}, \rho_{1,2}$

  B) $1/m_b$ & $1/m_c$ **expansion** $\Rightarrow |V_{cb}|, m_b, \lambda_1, \rho_1, \tau^*_{1,2,3}$

  input: meson mass difference

- **mass scheme:** m_b : **threshold mass ((A) kinetic, (B) 1S)**

  m_c : **A) short-distance mass (MSbar)**

  B) eliminated

Global Fits

Fit Results: (2007)

1S	V_{cb} (10^{-3})	m_b (GeV)
	41.78±0.30±0. 08	4.70±0.03
no b→sγ	41.56±0.39±0.08	4.751±0.058

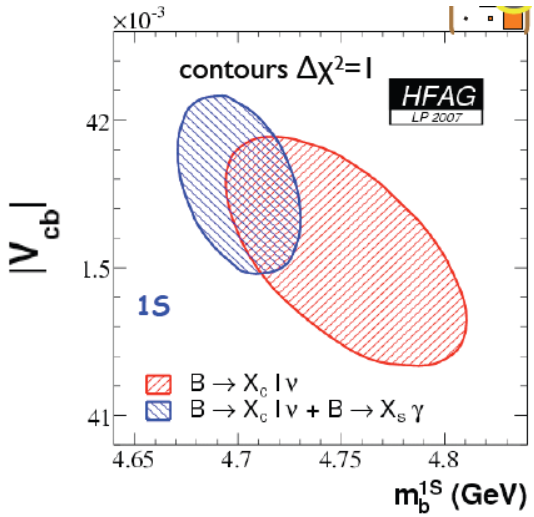
Kinetic	V_{cb} (10^{-3})	m_b (GeV)
	41.91±0.19±0.28±0.59	4.613±0.022+0.027
no b→sγ	41.68±0.39±0.58	4.677±0.053

$\rightarrow m^{1S} = 4.76$

$\rightarrow m^{1S} = 4.83$

?

E. Barberio



m^{1S}	4.70	4.75	4.76	4.83
$\overline{m}(\overline{m})$	4.18	4.22	4.23	4.30

→ **expansion scheme?**

→ **mass scheme ?**

→ **uncertainties underestimated ?**

Number for kinetic mass without $b \to s\gamma$ seem to be incompatible with Υ sum rules.

Global Fits

Fit Results:

(2007)

1S	V_{cb} (10^{-3})	m_b (GeV)
	41.78±0.30±0. 08	4.70±0.03
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$\rightarrow m^{1S} = 4.76$

$\rightarrow m^{1S} = 4.83$

?

E. Barberio

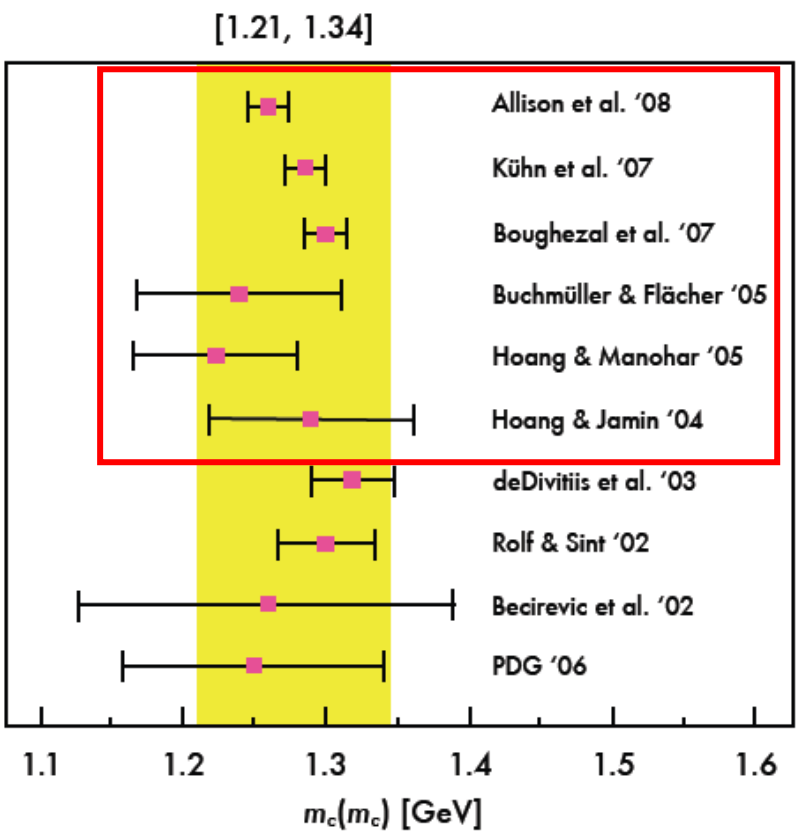
(2007, only BELLE)

	m_1^{kin} GeV	m^{1S}
no b→sγ	4.57 ± 0.13	4.72 ± 0.12
	4.54 ± 0.075	4.72 ± 0.06

Charm Quark Mass

- **Spectral Moments of Inclusive B Decays**
- **Υ Sum Rules** → **relativistic**

Recent determinations of charm mass



$m_c(m_c)$ [GeV]	method
1.266 ± 0.014	lattice, unquenched, staggered
1.286 ± 0.013	low-momentum sum rules, N^3LO
1.295 ± 0.015	low-momentum sum rules, N^3LO
1.24 ± 0.07	fit to B-decay distribution, $\alpha_s^2\beta_0$
1.224 ± 0.017 ± 0.054	fit to B-decay data, $\alpha_s^2\beta_0$
1.29 ± 0.07	NNLO moments
1.319 ± 0.028	lattice, quenched
1.301 ± 0.034	lattice, quenched
1.26 ± 0.04 ± 0.12	lattice, quenched
1.25 ± 0.09	PDG 2006

Charm mass from B decays

semileptonic moment fi ts: Bauer, Ligeti, Luke, Manohar, Trott

$$\Downarrow$$

$$m_b^{1S} = 4.681 \pm 0.034 \text{ GeV}, \quad \lambda_1 = -0.243 \pm 0.055 \text{ GeV}^2, \quad \rho_1, \tau_1, \tau_3$$

& correlation matrix

$$\Downarrow$$

Manohar, AH 2005

$$\overline{m}_B - \overline{m}_D = m_b^{\text{pole}} - m_c^{\text{pole}} - \frac{\lambda_1}{2}\left(\frac{1}{m_b} - \frac{1}{m_c}\right) + \frac{\rho_1 - \tau_1 - \tau_3}{4}\left(\frac{1}{m_b^2} - \frac{1}{m_c^2}\right)$$

$\longrightarrow$ perturbative error estimate tricky !

(A): extract $\overline{m}_c(4.2 \text{ GeV}) \xrightarrow{\text{RGE}} \overline{m}_c(\overline{m}_c) = 1521 - 203 - 72 - 15 = 1231 \pm 10$ MeV

(B): extract $\overline{m}_c(\overline{m}_c) = 1279 - 66 - 18 - 6 = 1201 \pm 4$ MeV

$$\overline{m}_c(\overline{m}_c) = 1.224 \pm 0.017\,(\text{exp}) \pm 0.031\,(B) \pm 0.023\,(m_c) \text{ GeV}$$

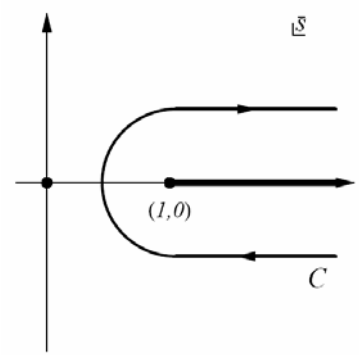
$$\boxed{1.22 \pm 0.06}$$

Buchmüller etal.
$\overline{m}_c(\overline{m}_c) = 1.24 \pm 0.07$
(based on Gambino,Uraltsev)

Charm Mass from $c\bar{c}$ Sum Rules

$$P_n = \int \frac{ds}{s^{n+1}} R_{qq}(s)$$

$$= \frac{1}{2\pi i} \int_{\mathcal{C}} \frac{ds}{s^{n+1}} \Pi_{cc}\left(\frac{s}{4\overline{m}_c^2}\right)$$



AH,Jamin, 2004

usual fixed order: $\mu^2 =$ fixed

RGI-A : $\mu^2 = s$

RGI-B : $\mu^2 = -s\beta^2\,, \quad \beta = \sqrt{1 - 4\frac{\overline{m}_c}{s}}$

pert. Behavior of moments: $n = 3, \overline{m}_c = 1.3$ GeV (2-loops)

$$8.77 + 1.44 + 0.75 = 10.95$$

$$8.77 + 1.23 + 0.72 = 10.72$$

$$8.77 + 3.13 + 0.44 = 12.35$$

1.63

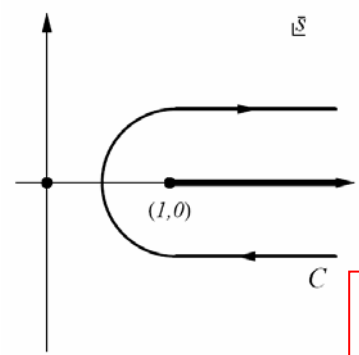
$$\boxed{1.29 \pm 0.07}$$

$$\overline{m}_c(\overline{m}_c)^{n=3} = 1.290 \pm 0.020\,(\text{exp}) \pm 0.015\,(\alpha_s) \pm 0.010\,(\langle G^2 \rangle) \pm 0.040\,(\text{th})\ \text{GeV}$$

Charm Mass from $c\bar{c}$ Sum Rules

$$P_n = \int \frac{ds}{s^{n+1}} R_{qq}(s)$$

$$= \frac{1}{2\pi i} \int_{\mathcal{C}} \frac{ds}{s^{n+1}} \Pi_{cc}\left(\frac{s}{4\overline{m}_c^2}\right)$$



Contour analysis should be done (w.i.p.)

AH,Jamin, 2004

1.29 ± 0.070 (2-loops, accounting for countour analysis)

Kühn, Steinhauser, (Sturm) (2001/2006)

1.304 ± 0.027 (2-loops fixed o.)

1.286 ± 0.013 (3-loops fixed o.) + updated data

Boughezal, Czakon, Schutzmeier (2006)

1.295 ± 0.015 (3-loops fixed o.) + updated data

Most recent Analysis

High-Precision Charm-Quark Mass from Current-Current Correlators in Lattice and Continuum QCD

I. Allison,[1] E. Dalgic,[2] C. T. H. Davies,[3] E. Follana,[4] R. R. Horgan,[5] K. Hornbostel,[6] G. P. Lepage,[7,*] J. Shigemitsu,[4] H. Trottier,[2] and R. M. Woloshyn[1]
(HPQCD Collaboration)

K. G. Chetyrkin,[8] J. H. Kühn,[8] M. Steinhauser,[8] and C. Sturm[9]

arXiv:0805.2999v1

- Lattice results for $c\bar{c}$ moments of the pseudoscalar & vector current-current correlators ("substitute for experimental data")
  - "substitute" for experimental data
  - only continuum limit $a \longrightarrow 0$ needs to be taken
  - no further matching needed
  - very small errors O(1%)

I. Allison,[1] E. Dalgic,[2] C. T. H. Davies,[3] E. Follana,[4] R. R. Horgan,[5] K. Hornbostel,[6] G. P. Lepage,[7,*] J. Shigemitsu,[4] H. Trottier,[2] and R. M. Woloshyn[1]
(HPQCD Collaboration)

&

- 3-loop perturbative computations of the moments (fixed order method)

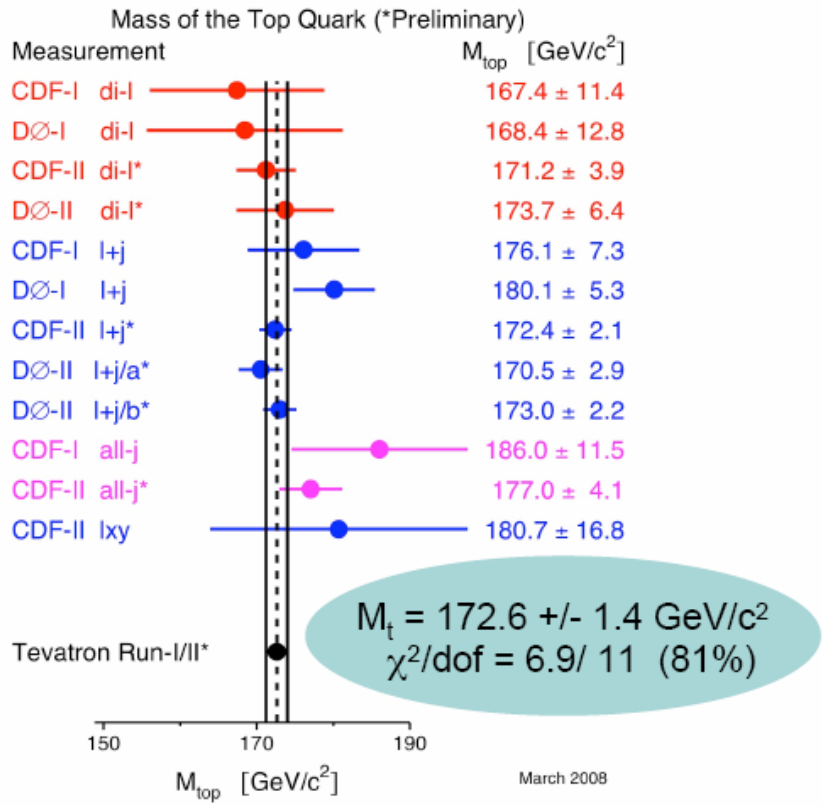
K. G. Chetyrkin,[8] J. H. Kühn,[8] M. Steinhauser,[8] and C. Sturm[9]

Top Quark Mass

- **Tevatron Mass Measurement**

Top Mass at Tevatron

Combination Results



- Relative uncertainty: 0.8%

Methods at Tevatron

Template Method

- Principle: perform kinematic fit and reconstruct top mass event

$$\chi^2 = \sum_{i=\ell,4jets} \frac{(p_T^{i,fit} -$$

$$+ \frac{(M_{\ell\nu} - M_W)^2}{\Gamma_W^2}$$

Usually pick

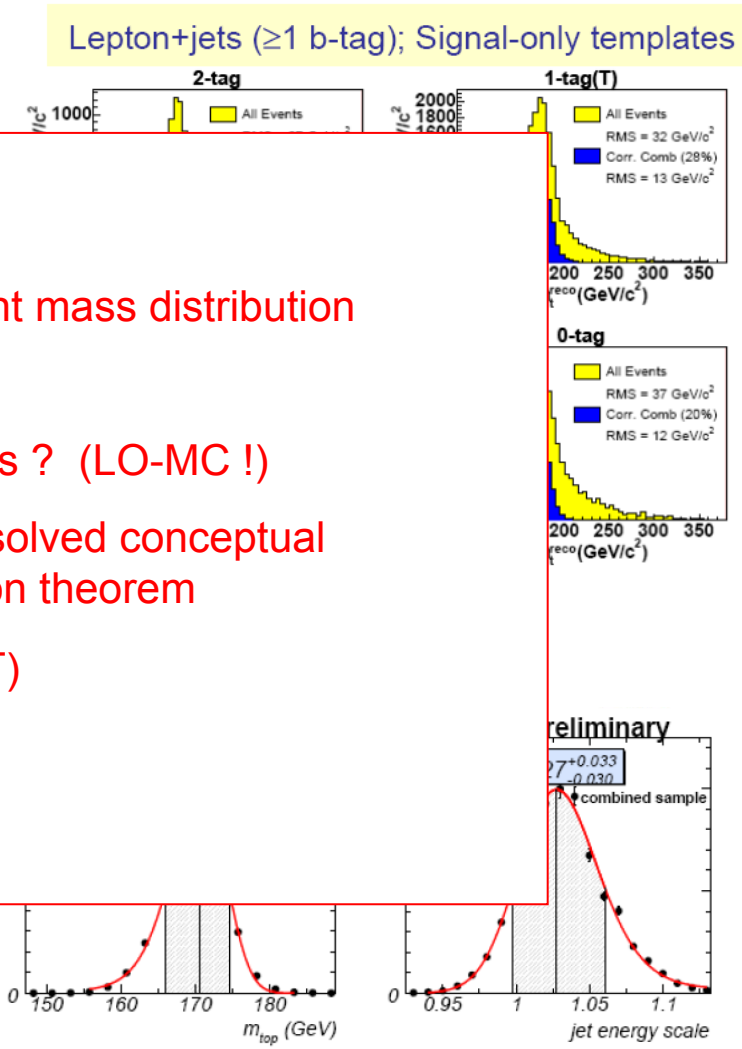
Dynamic

- Principle: c
a function c
objects in t
Maximize s

$$P(x;m_t) = \frac{1}{\sigma} \int d^n\sigma(y;m_t)\, dq_1 dq_2\, f(q_1) f(q_2)\, W(x \mid y)$$

differential cross section (LO matrix element)

transfer function: mapping from parton-level variables (y) to reconstructed-level variables (x)

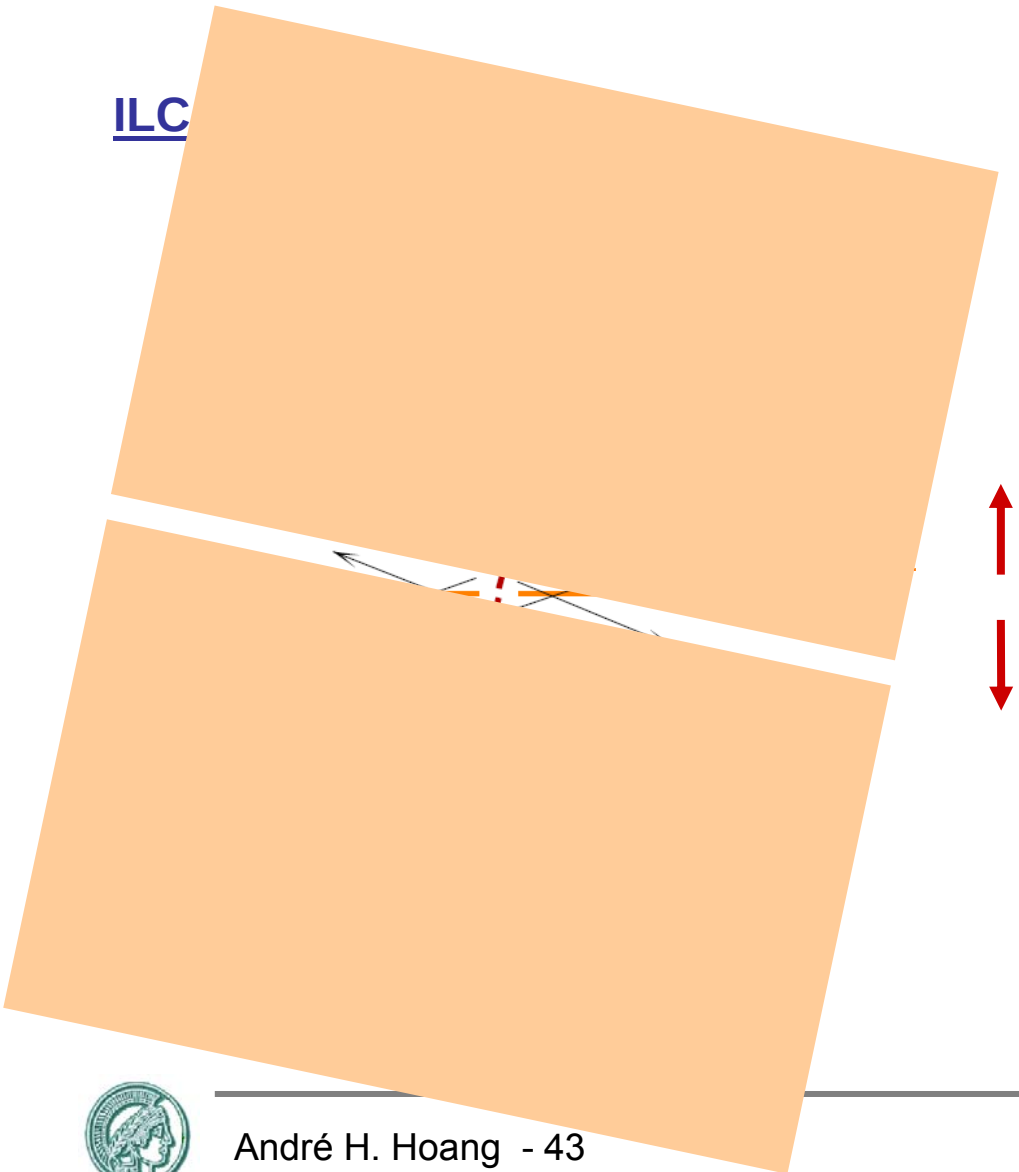


- dominated by the top invariant mass distribution
- systematics partly unclear
- effects of radiative corrections ? (LO-MC !)
- kinematics involves yet unresolved conceptual problems to proof a factorization theorem

(top quarks are slow, low p_T)

Top Production at the ILC

ILC



- No beam remnant : all soft radiation can be assigned to top or anti-top

→ k_T jet algorithm

→ hemisphere prescription

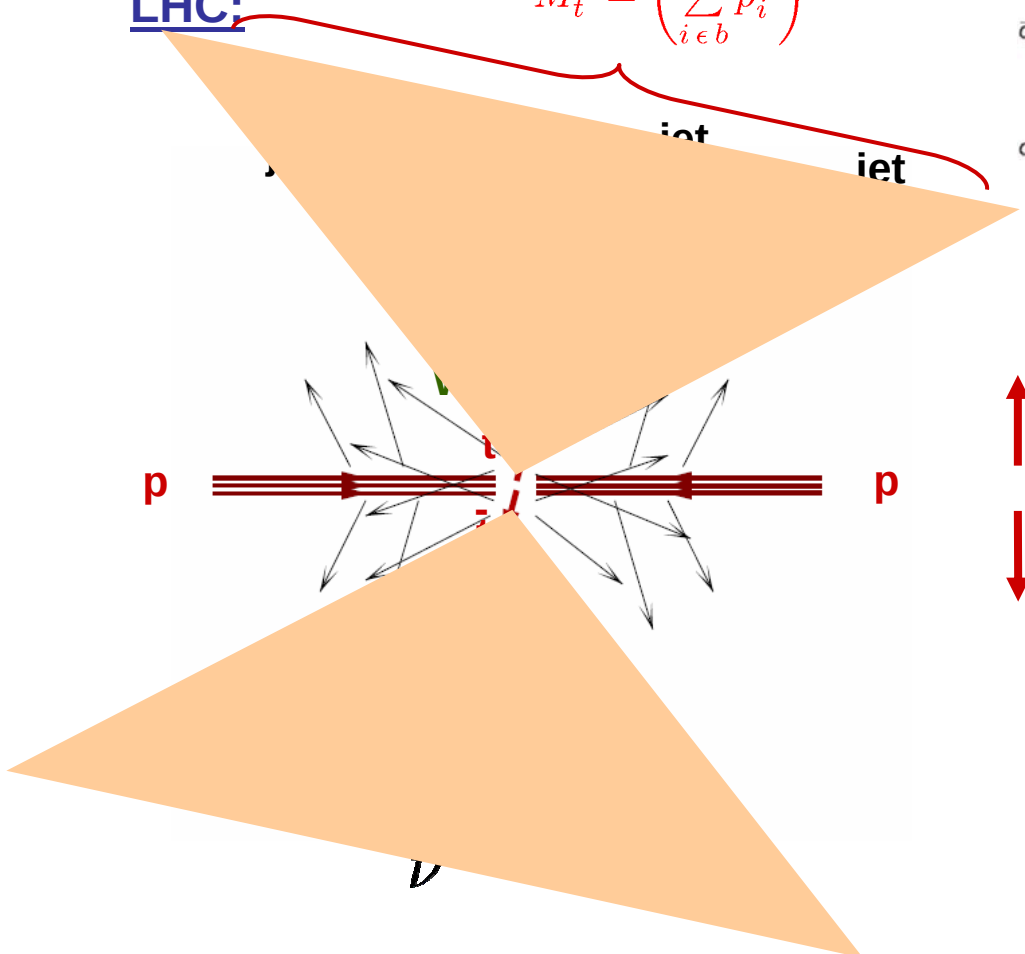
$$Q \gg m_t$$

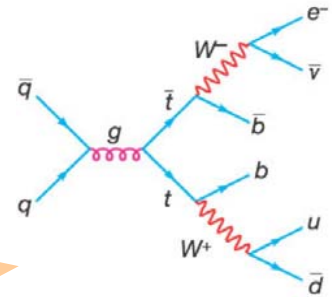
- Avoids : Underying Events & Inititial State Radiation
- Event-shape: complete event characterized/controlled by a few IR-safe variables

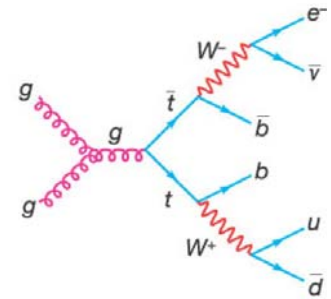
Large- p_T Method at the LHC

LHC:

$$M_t^2 = \left(\sum_{i \in b} p_i^\mu\right)^2$$







$$p_T \gg m_t$$

Tevatron: no large p_T events !

What is the MC-mass at Tevatron?

- Strictly speaking at this time it is a model parameter like the quark mass in a potential model for mesons. Top is treated like a physical T-meson with color charge. Nowhere is an on-shell condition implemented.

- To make more definite statements one needs to make a connection between MC and field theory.

- Facts: The Tevatron method(s) are dominated by the reconstructed top invariant mass spectrum & Peak of invariant mass distr. is close to top mass.

⟹ Tevatron measures very likely a resonance-like mass, but it is not known which one OR at which value for R exactly due to unknown higher order corrections.

$$m_{\rm Tev} = m_{\rm MSR}(R = 3^{+6}_{-2}\ {\rm GeV}) = 172.6 \pm 1.4\ {\rm GeV}$$

⇓ **R-RGE**

$$\overline{m}(\overline{m})_{\rm Tev} = m_{\rm MSR}(3^{+6}_{-2}\ {\rm GeV}) = 163.0 \pm 1.3 \pm 0.6 \pm 0.05\ {\rm GeV}$$

(ex) (R) (NNLL R-RGE)