New method for determining neutrino masses

Oscillation observed in radioactive ion decay with emission of unobserved neutrino.

noindent New and very interesting method for determining neutrino masses and mixing angles

When Paul Kienle called my attention to this experiment in December 2006 I said that it was nonsense, inconsistent with causality and quantum mechanics

How can a radioactive ion know about neutrino masses BEFORE it decays?

After some serious thinking and discussions with Paul and Walter Henning I began to think that there may be some sense in this experiment.

They told me to contact Fritz Bosch. This led to many discussions

With Paul Kienle, Walter Henning, Fritz Bosch, Yuri Litvinov and Andrei Ivanov

Many thanks!!!!!

My approach differs from their theoretical derivation; I start from the end

If we know the neutrino mass, there are no oscillations.

Why is this not a "missing mass" experiment?

No oscillations if conservation laws determine mass of unobserved neutrino

We start with a well-defined initial state of a radioactive ion

Why can't conservation laws for the decay to final state determine neutrino mass?

What prevents us from knowing the neutrino mass?

Coherence in neutrinos emitted in nuclear beta decay

Reactor ν 's oscillate with distance from reactor.

Weak decay produces final state wave packet with different energies and momenta

Stodolsky's theorem - No coherence between different energies

Coherence destroyed by neutrino detector in thermal equilibrium with environment

Oscillation results from interference only within coherent pair states with same energy

Between components with different neutrino masses and different momenta.

Momentum difference δp changes relative phase between two components

Change linear with distance from source produces oscillations observed in experiments

Measurement of oscillation wave length gives information about $\Delta(m^2)$

Where did this momentum difference δp come from?

Same ν emitted in GSI experiment oscillates in same way with distance from source

Momentum is conserved in the weak transition

Same momentum difference δp must be present in initial state of GSI experiment

Measurement of initial δp in GSI experiment gives same information about $\Delta(m^2)$

Without detecting neutrino !!!

But how can we find the relevant initial state momenta and measure δp ?

Can ν masses be measured without observing the ν?

K-capture decay of radioactive atom moving in storage ring

Decay in time with emission of ν_e not exponential. Ten second modulation

Related to the mass difference between ν eigenstates in ν_e wave function

How can a radioactive nucleus know about ν masses

BEFORE EMISSION OF AN UNOBSERVED ν?

Essential quantum mechanics

Initial state of radioactive "Mother" ion decays into another "Daughter" ion

Emission of ν_e - linear combination of several ν mass eigenstates.

If initial state has definite momentum and energy

Conservation of energy and momentum determine ν mass

In this "Missing mass" experiment; ν mass determined without ν observation

Interference between amplitudes from different ν mass states not observable

Interference is possible only if we can't know everything

Ignorance alone does not produce interference; Quantum mechanics must hide information

What do we know? what can we know? what can't we know?

Energy-time uncertainty - Violation of energy conservation is crucial

Enables nonconservation of energy in sufficiently short transition time

Why GSI experiment can be a major breakthrough in ν oscillations

Answers two principal difficulties of neutrino experiments

1. Ordinary neutrino oscillation experiments are difficult because
   (a) The neutrino absorption cross section is small.

The number of neutrino events actually used in ordinary experiments is many orders of magnitude smaller than the number events creating the neutrinos.

   (b) The oscillation wave lengths are so large that it is difficult to actually follow one oscillation period in any experiment.

2. This experiment opens up a new line for dealing with these difficulties
   (a) The oscillation is measured without detecting the neutrino.

Detection of every neutrino creation event avoids the losses from the low neutrino absorption cross section.

   (b) The long wave length problem is solved by having the radioactive source move a long distance circulating around in a storage ring.

At first seems rather peculiar that neutrino oscillations can be observed

in the state of a radioactive ion before decay into unobserved neutrino.

Toy model shows how to preserve causality

Observe and measure ν oscillations by looking only at radioactive source

Basic assumptions for understanding GSI experiment

Decay amplitude coherent sum of amplitudes

From all allowed paths in energy-momentum space

Coherence not introduced by ignorance of which path

Coherence only from QM uncertainty

What we do know

1. Initial state is "mother" ion in momentum wave packet
2. Final state is recoil "daughter" ion and electron neutrino

What we can know

1. Energy and momentum of daughter ion E_D, P_D
2. Energy of final neutrino E_ν

What we can't know

1. Momentum of final neutrino p_ν
2. Which components in initial state with momentum P and energy E produced ν_e

Which-path experiment - Initial momentum not known - Energy not conserved

Two components in initial state with energy difference δE can produce same final state

With unique energy via paths differing by $\Delta(m^2)$ in squared neutrino mass

Spin analog - electron polarized in x - direction; Magnetic field in z-direction

Spin precesses around z-axis. Component in x-direction oscillates with frequency $(\delta E)t$

“Two-slit” or “which-path” experiment

Quantum Mechanics of “which-path” experiment

1. Energy nonconservation allows initial states with different energies to go to same final state
2. Dicke superradiance between initial states that go into same final state
3. Energy difference gives oscillations between superradiance and subradiance
4. Watched pot effect preserves initial state in passing through storage ring

Initial ion state - wave packet in energy and momentum

Two components with different energies and different momenta

Final state ν with different masses, same energy and different momenta

Neutrino mass not observed, created from electron as ν_e mixture

Two components in same initial state make same final ν_e

Transition via two paths in energy-momentum space

Which path is not known

The energy difference between the two interfering components of the incident beam

$$\frac{\delta E}{E} \approx -\frac{\Delta(m^2)}{2E^2} \approx \frac{0.8 \cdot 10^{-4}}{2 \cdot [1.43 \cdot 140 \cdot 10^9]^2} \approx (10^{-23})$$

Dicke Superradiance shows how to measure δp

1. Experiment starts with initial state containing pairs of components with just the right momentum difference δp needed to produce the final electron neutrino.
2. Dicke has shown that whenever several states can decay into the same final state a particular linear combination of these states can be called a "superradiant" state.
3. In this state all the components interfere constructively and enhance the transition.
4. The other states called "subradiant" have suppressed transitions.
5. In the GSI experiment the initial state of the radioactive ion is a linear combination of pairs of states in which each component can produce the same final state.
6. In each pair one can define superradiant and subradiant linear combinations having enhanced and suppressed decays.
7. Pairs with momentum difference δp have energy difference δE
8. Two components with different energies δE travel around a storage ring with a relative phase difference $\delta E \cdot t$ which varies linearly with time.
9. States with this time dependent phase difference oscillate between superradiant and subradiant states which have well defined relative phases.
10. Periodic oscillations arise and produce observed periodic modulation in decay rate.

How initial states with different energies produce final state with same energy

Energy-time uncertainty

1. A pair of components with different momenta in an initial state of an ion with definite mass must also have different energies.
2. If both energy and momentum are conserved in the transition, a missing mass calculation gives the unobserved neutrino mass and there are no oscillations.
3. Violation of energy conservation is crucial.
4. Components in the initial state with different energies can produce a final state with the same energy because momentum is conserved in the transition but energy is not
5. Time of the transition is sufficiently short so that nonconservation of energy is allowed by the energy-time uncertainty principle.
6. Related to well-known broadening of a line when decay is observed in time short compared to its natural line width.
7. Here the decay to two final states is described by two Breit-Wigner energy distributions which are separated at long times.
8. In GSI experiment decay time is sufficiently short to make separation negligible in comparison with broadened widths.
9. Transition occurs coherently from two components of inital state with different energies to final state components with different momenta but the same energy.

Kinematics of simplified two component model

Simplified initial "mother" ion state - Equal mixture of two momentum eigenstates $|P\rangle$ and $|P + \delta P\rangle$ with energies E and $E + \delta E$ and relative phase ϕ.

Decay after very short time into final state

$$|i(t=0)\rangle = \frac{1}{\sqrt{2}} \cdot [|P\rangle + e^{i\phi}\,|P\delta P\rangle]; \quad |i(t)\rangle = \frac{1}{\sqrt{2}} \cdot [|P\rangle + e^{i\delta Et} \cdot e^{i\phi}\,|P + \delta P\rangle]$$

$$|f(E_\nu)\rangle \equiv |P_D; \nu_e(E_\nu)\rangle = \cos\theta\,|P_D; \nu_1(E_\nu)\rangle + \sin\theta\,|P_D; \nu_2(E_\nu)\rangle$$

$\vec{P}_D$ and E_D denote the momentum and energy of a recoil "daughter" ion, ν_e final electron neutrino with energy E_ν, ν_1 and ν_2 neutrino mass eigenstates with masses m_1 and m_2 , $\cos\theta$ and $\sin\theta$ are elements of the neutrino mass mixing matrix.

Momentum conserving transition matrix elements between initial and final state

$$\langle f(E_\nu)|\,T\,|i(t)\rangle = \frac{1}{\sqrt{2}} \cdot [\langle P_D; \nu_1(E_\nu)|\,T\,|P\rangle\rangle \cos\theta + e^{i\delta Et} \cdot e^{i\phi} \cdot \langle P_D; \nu_2(E_\nu)|\,T\,|P + \delta P)\rangle \sin\theta]$$

Square of transition matrix element shows oscillations in time

$$|\,\langle f(E_\nu)|\,T\,|i(t)\rangle\,|^2 \approx \frac{1}{2} \cdot [|\,\langle P_D; \nu_1(E_\nu)|\,T\,|P)\rangle\,|^2 \cdot [1 + \sin 2\theta \cos(\delta Et + \phi)]$$

Analog with Dicke superradiance seen by defining superradiant and subradiant states.

$$|Sup(E_\nu)\rangle \equiv \cos\theta\,|P)\rangle + \sin\theta\,|P+\delta P)\rangle\,; \quad |Sub(E_\nu)\rangle \equiv -\sin\theta\,|P)\rangle + \cos\theta\,|P+\delta P)\rangle$$

The transition matrix elements for these two states are then

$$\langle f(E_\nu)|\,T\,|Sup(E_\nu)\rangle = [\cos\theta{+}\sin\theta]\,\langle f|\,T\,|P\rangle\,; \quad \langle f(E_\nu)|\,T\,|Sub(E_\nu)\rangle = [\cos\theta{-}\sin\theta]\,\langle f|\,T\,|P\rangle$$

where we have neglected the dependence of the transition operator T on the small change in the momentum P. The squares of the transition matrix elements are

$$|\,\langle f(E_\nu)|\,T\,|Sup(E_\nu)\rangle\,|^2 = [1{+}\sin 2\theta]|\,\langle f|\,T\,|P\rangle\,|^2; \quad |\,\langle f(E_\nu)|\,T\,|Sub(E_\nu)\rangle\,|^2 = [1{-}\sin 2\theta]|\,\langle f|\,T\,|P\rangle\,|^2$$

For maximum neutrino mass mixing, $\sin 2\theta = 1$ and

$$|\,\langle f(E_\nu)|\,T\,|Sup(E_\nu)\rangle\,|^2 = 2|\,\langle f|\,T\,|P\rangle\,|^2; \quad |\,\langle f(E_\nu)|\,T\,|Sub(E_\nu)\rangle\,|^2 = 0$$

This is the standard Dicke superradiance all the transition strength goes into the superradiant state and there is no transition from the subradiant state.

How can a weak decay not be exponential?

Non-exponential time-dependence observed in decay probability

Although counterintuitive, it follows naturally from being a "watched pot" experiment.

The initial state of the ion is monitored during its passage around a storage ring, affirming that the ion has not yet decayed

"Schroedinger cat" experiment with door always open

Continuous measurement of whether the cat is still alive.

Initial state - free ion moving with continuous detection in fields of apparatus

Time interval t between entry into apparatus and last observation before decay

Time t' between last monitoring and decay negligible

$$t' \ll t$$

Initial state $|i(t)\rangle$ - Time dependent wave packet with components of different energies.

Relative phases determined by localization in space at point of entry into apparatus.

Hamiltonian H_o describes the motion of a free initial ion

In electromagnetic fields constraining its orbit in storage ring.

Evolution of initial state and transition matrix element for decay to final state $|f\rangle$

$$|i(t)\rangle = e^{iH_o t}\,|i(0)\rangle\,; \quad \langle f|\,T\,|i(t)\rangle = \langle f|\,Te^{iH_o t}\,|i(0)\rangle$$

The transition probability per unit time at time t is given by Fermi's Golden Rule,

$$W(t) = \frac{2\pi}{\hbar} | \langle f | T | i(t) \rangle |^2 \rho(E_f) = \frac{2\pi}{\hbar} | \langle f | T e^{iH_o t} | i(0) \rangle |^2 \rho(E_f)$$

Time dependence of decay not exponential. The probability P_i that the ion is still in its initial state and has not yet decayed satisfies the differential equation

$$\frac{d}{dt} P_i = -W(t) P_i; \qquad \frac{d}{dt} log(P_i) = -W(t); \qquad P_i = e^{-\int W(t) dt}$$

Exponential decay only for time-independent transition matrix element

Here the transition probabilty depends upon propagation of the initial state during time t between the entry of the ion into the apparatus and the time of the decay.

Although time-dependent perturbation theory might suggest that a decay amplitude can be present before the decay, the continued observation of the initial ion before the decay rules out any influence of any final state amplitude on the decay process.

Time dependence depends only on propagation of the initial state independent of final state created only at the decay point.

No violation of causality; no information about final state before decay.

Essential features of the "watched pot" experiment

Transition restricted to tiny region in space-time

1. The interval in space-time $(\delta x, \delta t)$ between two points in space-time
   (a) Where the initial state was last observed not to have decayed
   (b) where the decay was observed

known to be small with sufficient precision to be approximated by a point.

2. The transition matrix element $\langle f,t|\,T\,|i,t\rangle$ is between the values of the initial and final state wave functions at this point.
3. The final state wave function includes pairs of neutrino mass eigenstates with the same energy but different masses and momenta.
4. If both energy and momentum of the final state neutrino are observable there is no coherence and no oscillations.
5. Energy of final neutrino state observable; neutrino momentum not observable
6. The transitions to final neutrino states which have the same energy and different momenta are coherent. The final state wave function includes pairs of such states with well defined momentum differences and relative amplitudes and relative phases.
7. We assume that momentum is conserved in the transition. The momentum difference between components of the initial state that can interfere coherently is equal to the momentum difference between the neutrino mass eigenstates.

8. The transition matrix element therefore is determined by the overlap of pairs of components in the initial and final state wave functions with a well defined momentum difference; i.e. by their amplitudes and relative phases.
9. The relative amplitudes and phases of these pairs of components in the final state wave function at the decay point is known from the parameters of the neutrino mass matrix and the requirement that the final state is an electron neutrino. These are independent of the point in space-time where the decay occurs.
10. The relative amplitudes and phases of these pairs of components in the initial state wave function at known at the entry into the apparatus. The changes in these values between the entry and decay points are determined by their propagation through the magnetic fields in the storage ring over the space-time interval between these points.

    Only observable parameter in experiment which can produce oscillations.
11. Since the pairs of initial wave functions with different momenta have different energies, a phase difference between these components occurs which depends linearly on the energy difference. The propagation of these waves around the storage ring can introduce additional phase differences which remain to be calculated using the precise parameters describing the motion through the storage ring.

A tiny energy scale

The experimental result sets a scale in time of seven seconds
Tiny energy scale for difference between two waves beating with period of seven seconds.

$$\Delta E \approx 2\pi \cdot \frac{\hbar}{7} = 2\pi \cdot \frac{6.6 \cdot 10^{-16}}{7} \approx 0.6 \cdot 10^{-15}\text{eV}$$

Tiny energy scale must be predictable from standard quantum mechanics with scale from another input.

Only other input available - propagation of the initial state through the storage ring during the time interval between the entry into the apparatus and the decay.

One tiny scale available in the parameters that describe this experiment is the mass-squared difference between two neutrino mass eigenstates.

Tiny mass scale when this mass-squared is divided by the energy of the ion.

$$\frac{\Delta(m^2)}{E} \approx \frac{0.8 \cdot 10{-}4}{3 \cdot 10^{11}} \approx 2.7 \cdot 10^{-15}\text{eV}$$

where the value of $\Delta(m^2)$ is obtained from neutrino oscillation experiments.

Assumptions used in toy model

Sudden transition from initial ion state to final state of ion and ν_e

$$\left|I_i(\vec{P};E)\right\rangle \rightarrow \left|I_f(\vec{q});\nu_e(\vec{p}_\nu;E_\nu)\right\rangle$$

Interference between two amplitudes.

$$\left|I_i(\vec{P};E)\right\rangle \rightarrow \left|I_f(\vec{q});\nu_e(\vec{p}_\nu;E_\nu)\right\rangle; \quad \left|I_i(\vec{P}+\delta\vec{P};E+\delta E)\right\rangle \rightarrow \left|I_f(\vec{q});\nu_e(\vec{p}_\nu+\delta\vec{P};E_\nu)\right\rangle$$

Spin analog - electron polarized in x - direction; Magnetic field in z-direction

Spin precesses around z-axis. Component in x-direction oscillates with frequency $(\delta E)t$

From time dependent perturbation theory and energy-time uncertainty

Momentum conserved in transition, $\delta p_\nu = \delta P$; $\delta E = \frac{P}{E}\cdot\delta P$

The two transitions go via two mass eigenstates at time t,

$$\left\langle I_f(\vec{q});\nu_e\right| T_1 \left|I(E)\right\rangle \equiv \left\langle\nu_e\right|\nu(m_\nu)\rangle \left\langle I_f(\vec{q});\nu(m_\nu;E_\nu)\right| T \left|I_i(E)\right\rangle$$

$$\left\langle I_f(\vec{q});\nu_e\right| T_2 \left|I(E+\delta E)\right\rangle \equiv \left\langle\nu_e\right|\nu(m_\nu+\delta m)\rangle \left\langle I_f(\vec{q});\nu(m_\nu+\delta m;E_\nu)\right| T \left|I_i(E+\delta E)\right\rangle$$

$$\left\langle I_f(\vec{q});\nu_e\right| T \left|I(t)\right\rangle \equiv \left\langle I_f(\vec{q});\nu_e\right| T_1 \left|I(E)\right\rangle + e^{i\delta Et} \left\langle I_f(\vec{q});\nu_e\right| T_2 \left|I(E)\right\rangle$$

The kinematics of the transition

Momentum conserved for each wave packet component with momentum $\vec{P}$ and energy E

Final state "daughter" ion with $(\vec{P}_D; E_D)$ and a neutrino with $(\vec{p}_\nu; E_\nu)$

$$\vec{P}_D = \vec{P} - \vec{p}_\nu; \quad E_D = E - E_\nu$$

$$M^2 + m^2 - M_D^2 = Q \cdot [2M - Q] + m^2 = 2EE_\nu - 2\vec{P} \cdot \vec{p}_\nu$$

A small change $\Delta(m^2)$ in m^2 results from changes δp_ν, δP, δE_ν and δE

$$\Delta(m^2) = 2E(\delta E_\nu) + 2(\delta E)E_\nu - 2P(\delta p_\nu) - 2(\delta P)p_\nu$$

Interference between ν states with same energy and different momenta

No $\vec{P}_D$ change; $\delta E_\nu = 0$; Momentum conserved in transition, $\delta p_\nu = \delta P$; $\delta E = \frac{P}{E} \cdot \delta P$

$$\frac{\Delta(m^2)}{2} = (\delta E)\left[E_\nu - E - E \cdot \frac{p_\nu}{P}\right] = -E\delta E \cdot \left[1 + \left\{\frac{p_\nu}{P} - \frac{E_\nu}{E}\right\}\right] \approx -E\delta E$$

The phase difference $\delta\phi = -2\pi$ at a time t when

$$\delta\phi_{HJL} \approx -\delta E \cdot t = -\frac{\Delta(m^2)_{HJL}}{2E} \cdot t = -\frac{\Delta(m^2)_{HJL}}{2\gamma M} \cdot t \approx -2\pi; \quad \delta t \approx \frac{4\pi\gamma M}{\Delta(m^2)_{HJL}}$$

where γ denotes the Lorentz factor E/M.

The values of $\Delta(m^2)$ obtained from the observed period δt

$$\Delta(m^2)_{HJL} = \frac{4\pi\gamma M}{\delta t} \approx 2.75\Delta(m^2)_{exp}; \quad \Delta(m^2)_{exp} = \frac{\Delta(m^2)_{HJL}}{2.75} = 2E\delta E \cdot \left[1 - \frac{1.75}{2.75}\right]$$

This suggests that there is an additional phase in the traversal over the storage ring, Difference between straight line motion and storage ring orbit may give answer. For a storage ring with a straight section of length L and semicircle radius R

$$\delta\phi_{add}^{SL} = \delta P \cdot X = \frac{\delta P}{\delta E} \cdot \delta E \cdot Vt = \frac{P \cdot \delta P}{E \cdot \delta E} \delta E t = -\delta\phi_{HJL}; \quad \delta\phi_{HJL} + \delta\phi_{add}^{SL} = 0$$

$$\delta\phi_{add}^{stor} = -\delta\phi_{HJL} \cdot \left[\frac{1.75}{2.75}\right] = -0.64 \cdot \delta\phi_{HJL} = 0.64 \cdot \delta\phi_{add}^{SL} = \frac{L}{L + \pi R} \cdot \delta\phi_{add}^{SL}$$

The energy difference between the two interfering components of the incident beam

$$\frac{\delta E}{E} \approx -\frac{\Delta(m^2)}{2E^2} \approx \frac{0.8 \cdot 10^{-4}}{2 \cdot [1.43 \cdot 140 \cdot 10^9]^2} \approx (10^{-23})$$

The quantum mechanics of real ν experiments

In realistic experiment Heisenberg uncertainty prevents knowing initial state momentum with sufficient precision to determine ν mass

GSI observed periods of modulation ≈ 7 seconds

In single period of oscillation ions traveling at 0.71c travel $\approx 10^6$ kilometers

Uncertainty in position of experiment within laboratory tiny in comparison with enormous oscillation wave length.

Momentum uncertainty required to produce oscillations equally tiny compared to fluctuations required by confining experiment within the laboratory.

$$\frac{\delta x}{\lambda_{osc}} \approx \frac{\delta p_{osc}}{\delta p_{loc}} \ll 1$$

δx uncertainty in position of experiment; λ_{osc} oscillation wave length

δp_{osc} denotes momentum difference required for oscillations

δp_{loc} Initial momentum spread required by localization in laboratory.

Thus this is not a missing-mass experiment.

Coherence and decoherence; different from observed ν oscillatons

In neutrino oscillation experiments - Time not measured, momentum uncertainty

Detector is in thermal equilibrium with environment

Coherence only between pairs of ν states with same energy different momenta

GSI experiment completely different - Time measured

Time dependence may be crucial new ingredient

Preparation of the initial state is complicated

Radioactive ion trapped in storage ring

Circumference 108.3 m; Revolution frequency $\approx$ 2 MHz.

Time oscillations with a period of about 10 seconds

Seen in radioactive decay by unobserved ν emission

Questions requiring considerable thought

Can coherence be preserved over time span of ten seconds?

What is effect of continuous monitoring of state of the ion?

Learn from condensed matter and mesoscopic physics

Coherence and decoherence examined in many contexts.

Concepts of “preselection” and “postselection” introduced and extensively explored by Yakir Aharonov and collaborators.

The quantum mechanics of missing mass experiments

No coherence in a missing mass experiment

A radioactive ion that decays by K-capture emits a linear combination of ν mass eigenstates. If the energy and momentum of the initial nucleus and also the recoil momentum of the final atom are known the energy and momentum of the emitted ν are determined and therefore its mass. This "missing mass" experiment suggests no observable interference the ν mass eigenstates and no oscillations.

To see that this argument misses the exciting observable physics from the beta-decay experiment examine the $\pi \to \nu e$ decay of a pion at rest.

Energies E_e, E_ν and momenta $\vec{p}_e, \vec{p}_\nu$ of the electron and neutrino can all be known. This is then just a "Missing Mass" experiment. The neutrino mass M_ν is uniquely determined by $M_\nu^2 = (M_\pi - E_e)^2 - p_e^2$. So how can there be coherence and interference between states of different mass? We are guided to the resolution of this paradox by experience in condensed matter physics discussing which amplitudes are coherent in quantum mechanics

How can neutrinos with different masses be coherent?

Consider $\pi \rightarrow \mu\nu$; $\pi \rightarrow e\nu$ at rest

Two different stable neutrino mass eigenstates

"Missing Mass" experiments.

$M_{\nu_\mu}^2 = (M_\pi - E_\mu)^2 - p_\mu^2$; $M_{\nu_e}^2 = (M_\pi - E_e)^2 - p_e^2$.

In initial Lederman-Schwartz-Steinberger experiment

Neutrinos emitted in $\pi \rightarrow \mu\nu$ produced no e, only μ. Mass eigenstate ν incident on detector can produce both

Amplitudes for electrons at detector from both mass eigenstates coherent and exactly cancel.

Missing mass experiment was not performed

Sufficient information not available to determine ν mass from energy and momentum conservation.

Missing information was not simple ignorance.

Ignorance alone cannot provide coherence.

Simple quantum mechanics explains missing information

ν oscillations observable only if source position is known

With error much smaller than oscillation wave length.

Uncertainty principle prevents knowing momentum well enough to determine ν mass.

Why it is not a missing mass experiment

The original Lederman-SchwartzLederman-Steinberger experiment found that the neutrinos emitted in a $\pi - \mu$ decay produced only muons and no electrons. Experiments now show that at least two neutrino mass eigenstates are emitted in $\pi - \mu$ decay and that at least one of them can produce an electron in a neutrino detector. The experimentally observed absence of electrons can be explained only if the electron amplitudes received at the detector from different neutrino mass eigenstates are coherent and exactly cancel. This implies that sufficient information was not available to determine the neutrino mass from energy and momentum conservation. A missing mass experiment was not performed.

Coherence or interference between different neutrino mass eigenstates cannot be observed in a "missing mass" experiment where the mass of an unobserved neutrino is uniquely determined by other measurements and momentum and energy conservation.

The momentum difference between the different neutrino mass states is thus much smaller than the momentum uncertainly required by Heisenberg from knowing that the experiment takes place within the laboratory. The initial state is a wave packet in momentum space containing the different momenta required to produce decays to neutrino mass eigenstates with different masses. The transition to the final ν_e state can therefore go via different neutrino mass eigenstates with no record of which mass eigenstate produced the final ν_e. The contributions via different neutrino mass eigenstates define different paths in momentum space which are not observed in the experiment. The contributions to the final state amplitude via these different paths are therefore coherent and interference between them can be observed producing oscillations.

Decay to two Breit-Wigner energy distributions separated at long times

In GSI experiment short decay time broadens widths and makes separation unobservable.

Transition occurs coherently from two components of the inital state different energies

To final state containing components with different momenta but the same energy.

Initial state has component pairs with momentum difference needed to produce final ν_e

Oscillations produced when two components with different energies

Travel between point of entry of the beam into storage ring and decay

With relative phase difference varying linearly with time

K-capture experiment as "Two-slit" experiment

Basic quantum mechanics in unrealistic toy model

Crucial ingredient - Unknown momentum of initial state

Knowledge of position requires momentum uncertainty.

Initial state: Radioactive ion in wave packet confined to definite region of configuration space

Final state: Recoiling ion and linear combination of ν mass eigenstates produced when electron disappears

Location in space of the initial and final states well defined

Direct analog to "two-slit" or "which-path" experiment.

Transition can go via any ν mass eigenstates

Different paths between the initial and final states.

Initial state has neither sharp momentum nor sharp energy.

Waves on two paths overlap in momentum and energy.

Coherence observable only if we cannot know which path

Final state ν mass eigenstates have same observable energy

Different momenta which Heisenberg says is unobservable.

Momentum conservation in the transition requires different momenta for the radioactive ion in the initial state.

Momentum spread in the initial wave function sufficient to suppress all information on "which path"

Final ν_e state linear combination of mass eigenstates with same energy, different momenta and well defined phase.

During passage of the radioactive nucleus between point of entry to the apparatus and the point of decay relative phases between different momentum eigenstate components of initial wave function change linearly with the distance

Probability of decay to final ν_e state oscillates

Oscillations observed as function of distance from source

Wave length of oscillation depends upon momentum difference which in turn depends upon mass differences between mass eigenstates.

Experimental observation of these oscillations provides a new experimental method to obtain information about the neutrino mass differences and the mixing angles of the neutrino mass matrix.

Ion travels large distance in straight line before decay

Toy model highly unrealistic with very large distances

Precise analysis in realistic experiment more complicated

Conclusions

A new oscillation phenomenon providing information about neutrino mixing is obtained by following the nucleus before and during the decay. Coherence between amplitudes produced by the weak decay of a radioactive nucleus by the emission of neutrinos with different momenta has been shown in a toy model to follow from the localization of the initial radioactive nucleus within a space interval much smaller than the size of the initial nuclear state.

This coherence is observable in following the motion of the initial radioactive nucleus from its entry into the apparatus to its decay. The amplitude for production of an electron neutrino from several mass eigenstates depends upon the relative phases of the individual eigenstate contributions. These relative phases increase linearly with the distance of the decay point from the point of entry into the apparatus, The phase change produce oscillations with distance in the probability for the decay.

Observing these oscillations and their wave length gives information about the neutrino mass differences

Comparison with neutrino oscillation experiments

Most neutrino oscillation experiments also observe an oscillation in space coming from a "which path" experiment with interference observed between two paths having two different neutrino mass eigenstates with the same energy and different momenta. The relative phase between the two paths is with a well defined dependence upon the neutrino energy or momentum. Dependence on the same neutrino eigenstate masses occurs in reactor experiments which also produce the ν_e neutrinos in nuclear beta decay. The wave lengths of the oscillations in space should be related by comparing the result of for the neutrino experiments with result for this experiment in this paper for the oscillations of the decay point in the K-emission experiment discussed here. The same information about neutrino masses and mixing angles is obtainable from both types of experiments. Better values with smaller statistical errors should be obtainable by combining results from both.

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