$1/D$ Expansion of Quantum Gravity

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Abstract. *Non-trivial fixed points of quantum gravity are studied from the Asymptotic Safety picture with the use of non-perturbative renormalisation techniques, in a large number of dimensions via a 1/D expansion where D is the dimensionality of space-time.*

Motivation

The Large-N expansion leads to a very successful non-perturbative expansion scheme in quantum field theory. For the strong interactions in $SU(N)$ Yang-Mills theory, it has been shown by 't Hooft[1] that a large-N limit leads to a planar diagram limit where only the quarks at the edges of this planar diagram dominate, thus the theory simplifies in large-N. Because D is the dimensionality of the gauge group of gravity (i.e. $GL(D, R)$) just like N for $SU(N)$, we can expect a similar result where gravity simplifies at the large-D limit. It has been suggested by A. Strominger that gravity simplifies in general extra dimensions via a $1/D$ expansion. He has shown that disjoint bubble graphs are preferred over nested ones. [2]. $1/D$ expansion has also been studied from an effective field theory point of view by Bjerrum-Bohr [3] and it has been shown again that bubble graphs dominate over nested graphs. For general relativity, Emparan et. al. showed that the theory simplifies to a system of non interacting particles in the infinite dimensional limit.[4]

The mass dimensions of the gravitational constant is $2 - D$, which suggests that gravity is perturbatively non-renormalisable when $D > 2$ and 4 dimensions is not special for this case. Hence it is interesting to look at the behaviour of gravity in higher dimensions. Especially from a phenomenological point of view, models such as ADD [5] gives us inspiration to look at gravity in a large number of dimensions.

In addition to these we would like to perform a $1/D$ expansion from asymptotic safety point of view. It would be interesting to see if gravity simplifies in the large-D limit within asymptotic safety.

Renormalisation Group

The β-functions are obtained from a functional renormalisation group methods, by using the background field technique, with a background metric $\bar{g}_{\mu\nu}$ and a fluctuating dynamical part $h_{\mu\nu}$. So that we have the equations in terms of dimensionality D and the gauge fixing α. The action is the Einstein-Hilbert action with a cosmological constant, so we have two coupling constants, gravitational constant and the cosmological constant. Then the flow equations for this cutoff function are found to be,,[6, 7]

$$\partial_t \lambda \equiv \beta_\lambda = (-2+\eta)\lambda + g(a_1(\lambda) - \eta a_2(\lambda)),$$
$$\partial_t g \equiv \beta_g = (D-2+\eta)g, \quad \eta = \frac{g b_1(\lambda)}{1 + g b_2(\lambda)}.$$

Here η is the anomalous dimension. (*note.* We rescaled the coupling g to g/c_D. We can resotre c_D simply by multipying it with g again.)

Results

From this point we can search for fixed points by setting the β-functions to zero, by performing a $1/D$ expansion. For $\alpha = 1$ we get,

$$\lambda = \frac{1}{2} - \frac{6}{D} + \frac{90}{D^2} - \frac{678}{D^3} - \frac{2778}{D^4} + \frac{63690}{D^5} + \cdots$$
$$g = \frac{6}{D^3}\left(1 - \frac{28}{D} + \frac{525}{D^2} - \frac{6458}{D^3} + \frac{48739}{D^4} - \frac{396436}{D^5} + \cdots\right)$$

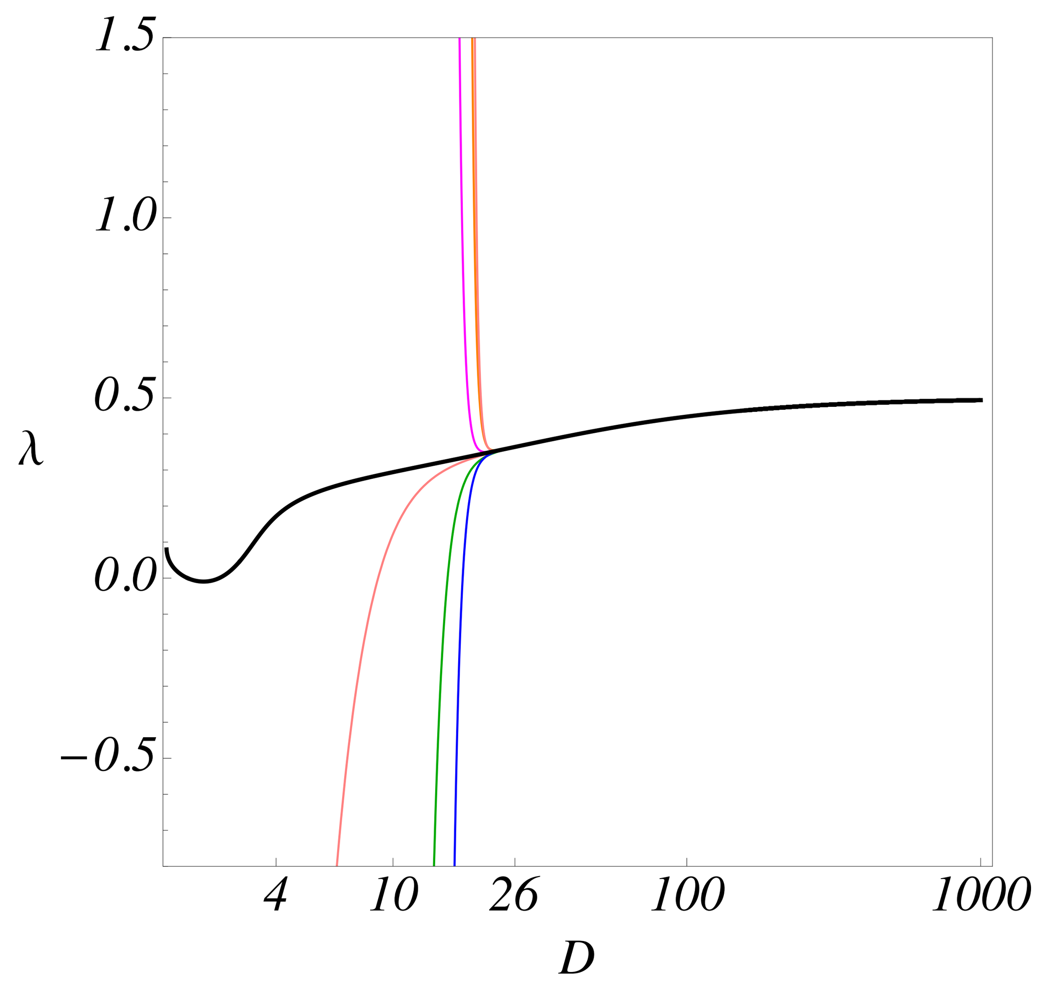
We also calculate the elements and the eigenvalues of the stability matrix (i.e the scaling exponents) by using this dynamical set up. We are interested in the behaviour of the flow in the vicinity of the fixed point. The scaling exponents are found to be,

$$\theta_1(D,1) = \frac{D^3}{156}\left(1 + \frac{1}{13D} + \frac{50506}{169D^2} - \frac{15138226}{2197D^3} + \cdots\right),$$
$$\theta_2(D,1) = 2D\left(1 + \frac{1}{D} + \frac{98}{D^2} - \frac{37104}{13D^3} + \cdots\right).$$

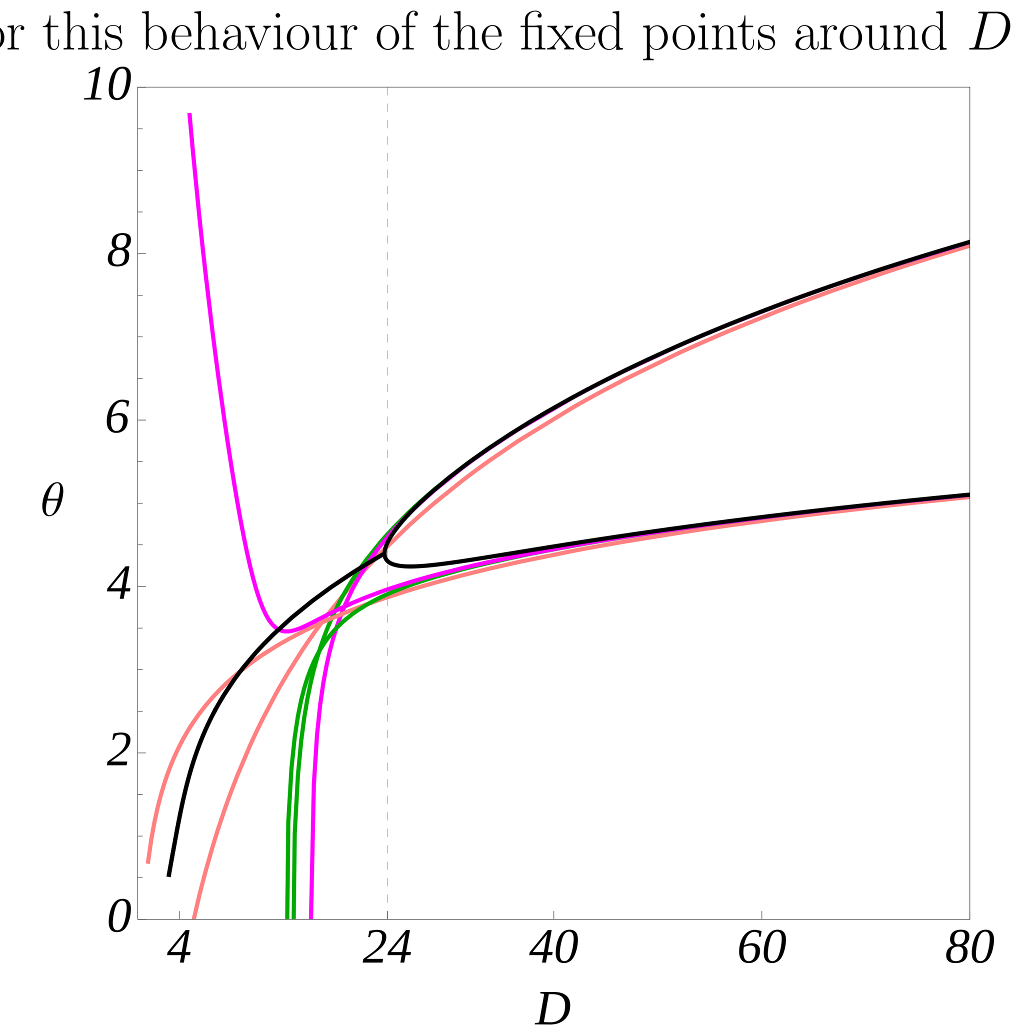
It is important to note that they are both real and positive as opposed to complex conjugate eigenvalues.

Discussion

Our result for θ_2 is consistent with the results in the references [8, 9]. For a review of the results from asymptotic safety with various approximations see ref.[10]. θ_1 is the contribution from the cosmological constant. If we do not include the cosmological constant in the Einstein-Hilbert action, we do not get this exponent we only observe the exponent with a leading behaviour as $2D$. We have also computed the expansions with different gauge fixing constants and we observed that this does not change the leading order behaviour. Thus we can say that our expansion is gauge independent to a certain degree. The radius of convergence of the series expansion has been calculated and is found around $D = 22$. Thus our approximation is not good enough in lower dimensions such as $D = 4$.



We have also observed that scaling exponents show a bifurcation point around $D = 24$ where the two complex conjugate exponents become real ones which is shown by the black curve on the plot below. The bifurcation point is very close to the convergence limit of the series. This shows that this behaviour might be linked and some subtlety of the dynamics is responsible for this behaviour of the fixed points around $D \sim 22 - 24$



On both figures, black curves are exact and coloured curves represent the series with different order in $1/D$. In conclusion, we have shown that the fixed points exist in very large dimensions within Einstein-Hilbert truncation, showing a certain leading order behaviour.

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