



東京大学  
THE UNIVERSITY OF TOKYO



CENTER for  
NUCLEAR STUDY  
THE UNIVERSITY OF TOKYO



**KU LEUVEN**

*“Reflections on the atomic nucleus”,  
University of Liverpool  
July 30 (28-30), 2015*

*Dual quantum liquid picture of nuclei  
and  
its implication to reflection asymmetry*

Takaharu Otsuka

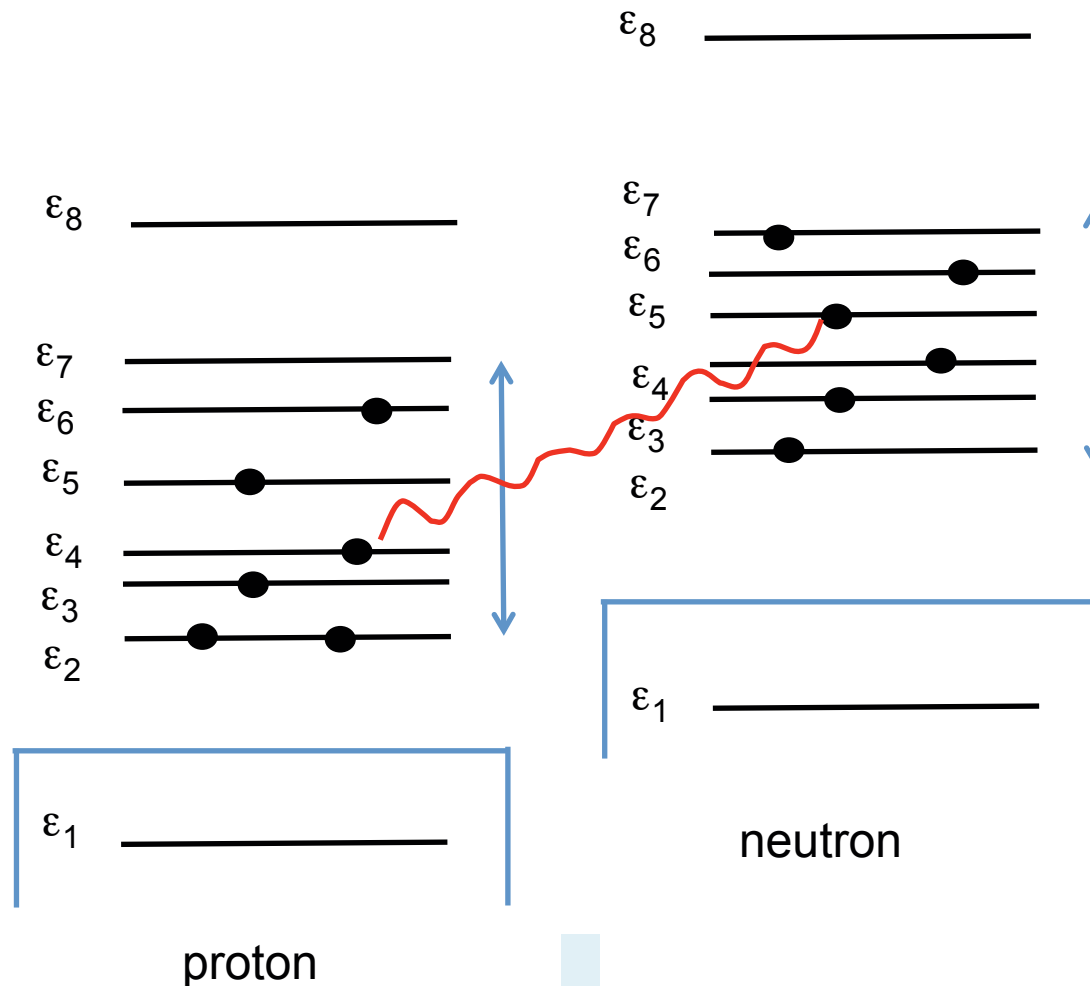
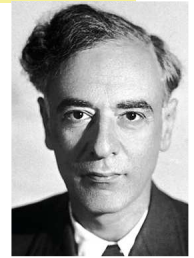
*University of Tokyo / MSU / KU Leuven*

HPCI Strategic Programs for Innovative  
Research (SPIRE)

Field 5 “The origin of matter and the universe”

# The atomic nucleus is a Quantum (Fermi) Liquid (of Landau)

described by  
interplay between single-particle energies and “residual” interaction  
- in a way like free particles -



For most of states, there may have been Ansatz that

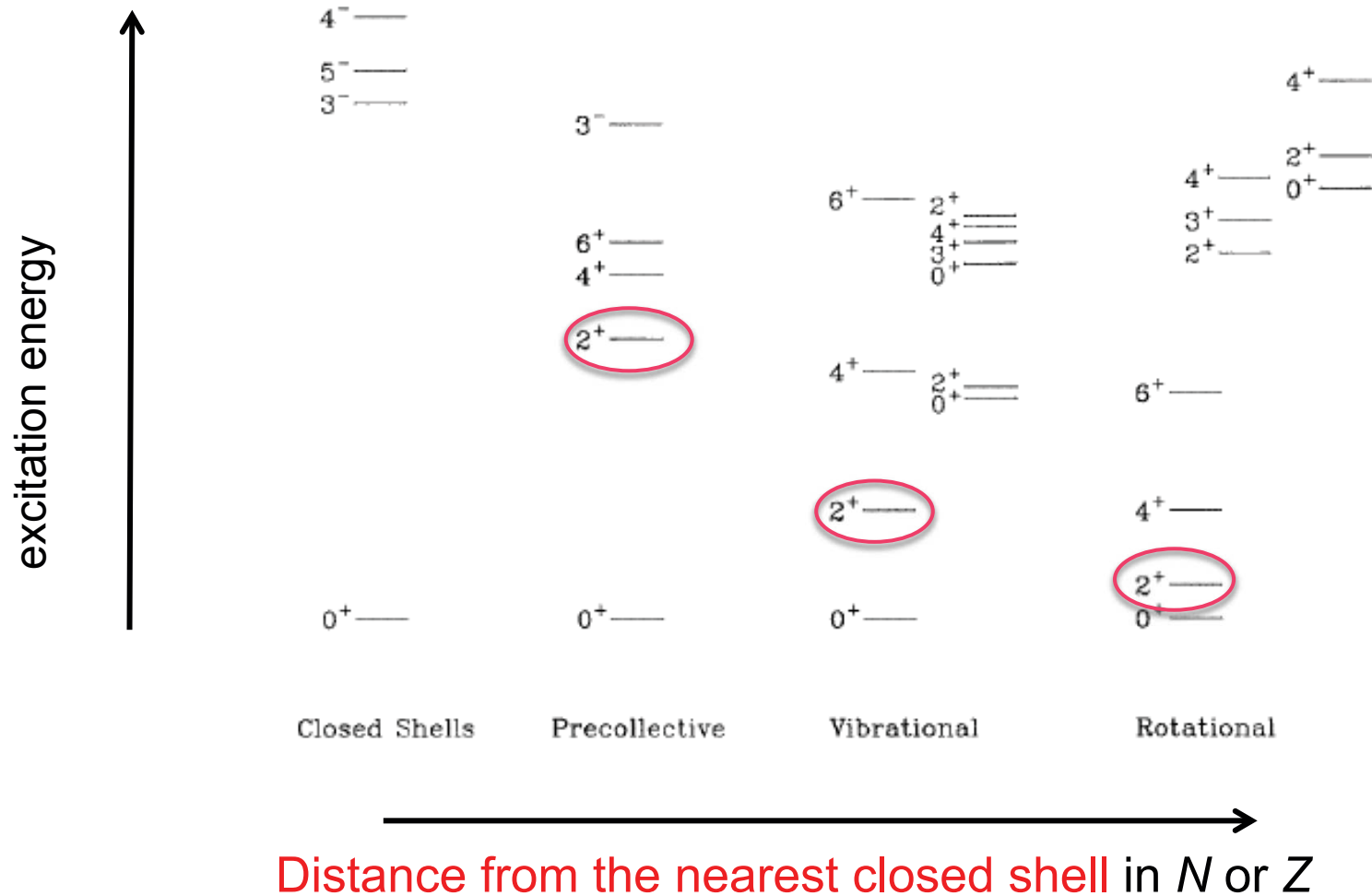
Spherical single particle energies remain basically unchanged.  
-> spherical part of *Nilsson model*

Correlations originating in nuclear forces (residual interaction) produce various features, including shape evolution and shape coexistence.

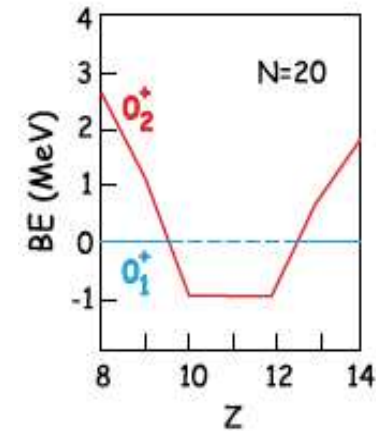
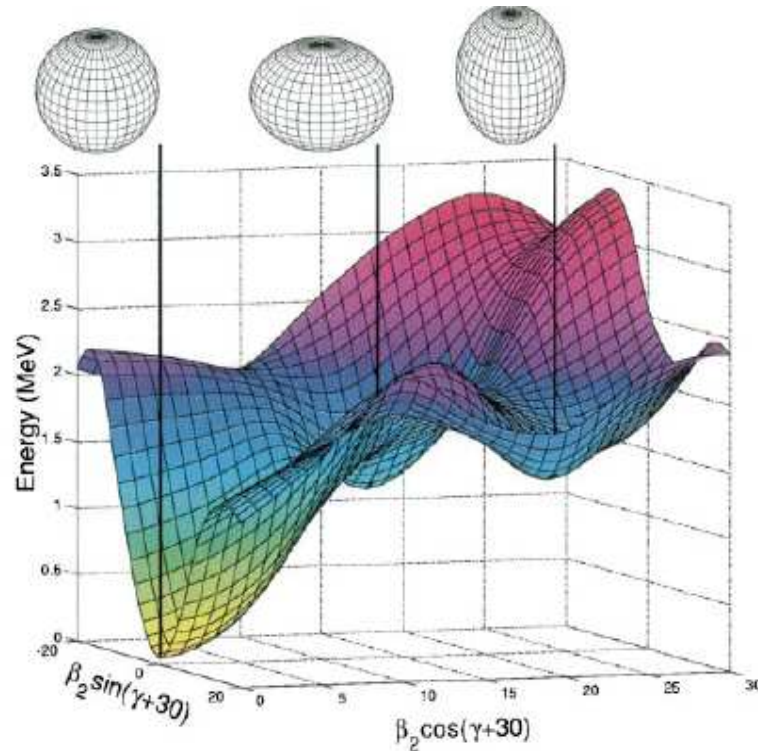
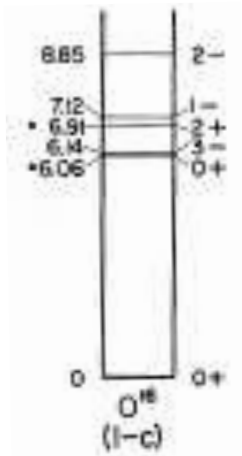
# Schematic picture of shape evolution (sphere to ellipsoid)

- monotonic pattern throughout the nuclear chart –

*one “shape” per one nucleus in many stable nuclei*



# shape coexistence



Island of Inversion  
( $Z=10\sim 12$ ,  $N=20$ )

$^{16}\text{O}$

H. Morinaga  
(1956)

$^{186}\text{Pb}$

A.N. Andreyev *et al.*,  
Nature **405**, 430 (2000)

REVIEWS OF MODERN PHYSICS, VOLUME 83,  
Shape coexistence in atomic nuclei  
Kris Heyde\* John L. Wood†



“Canonical” shell structure

Eigenvalues of HO potential

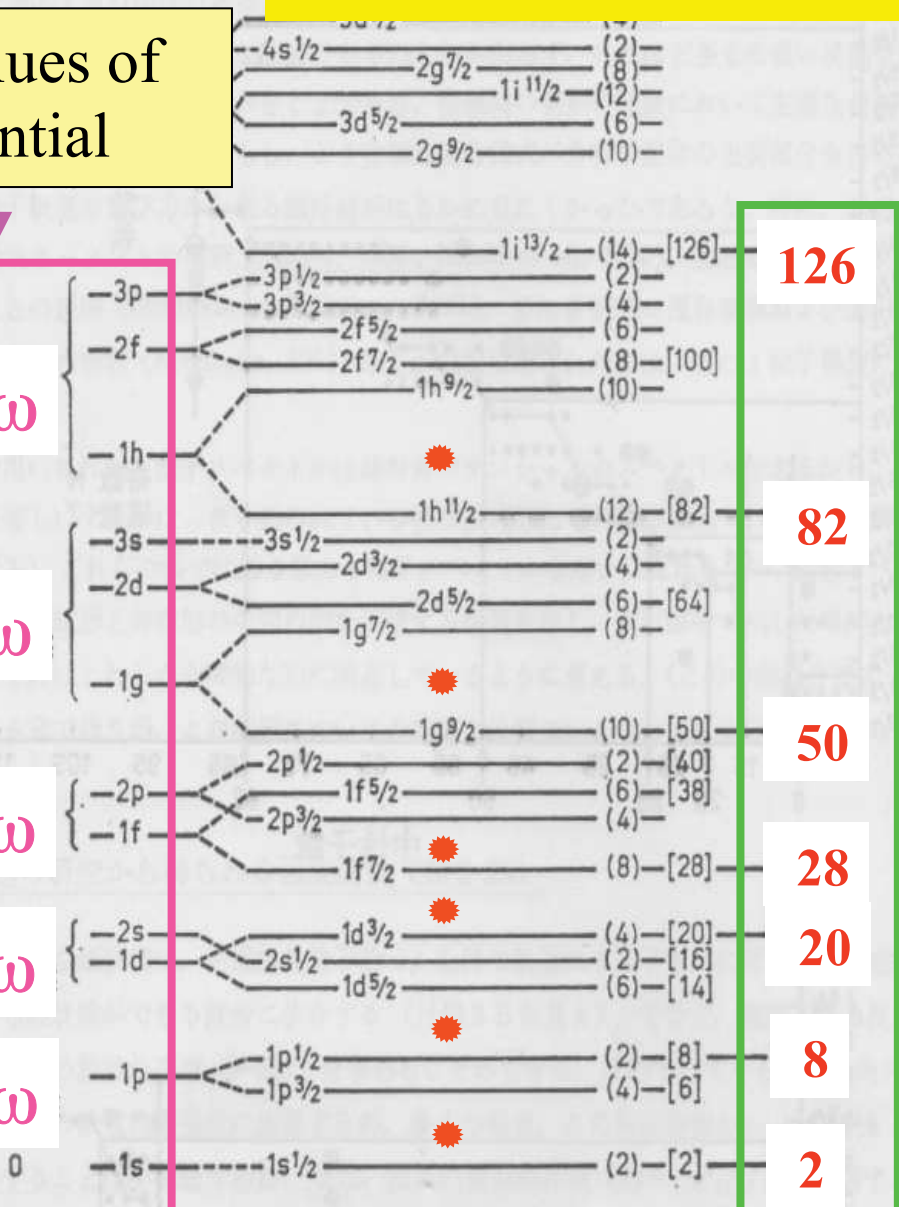
$5\hbar\omega$

$4\hbar\omega$

$3\hbar\omega$

$2\hbar\omega$

$1\hbar\omega$



Magic numbers by Mayer and Jensen (1949)

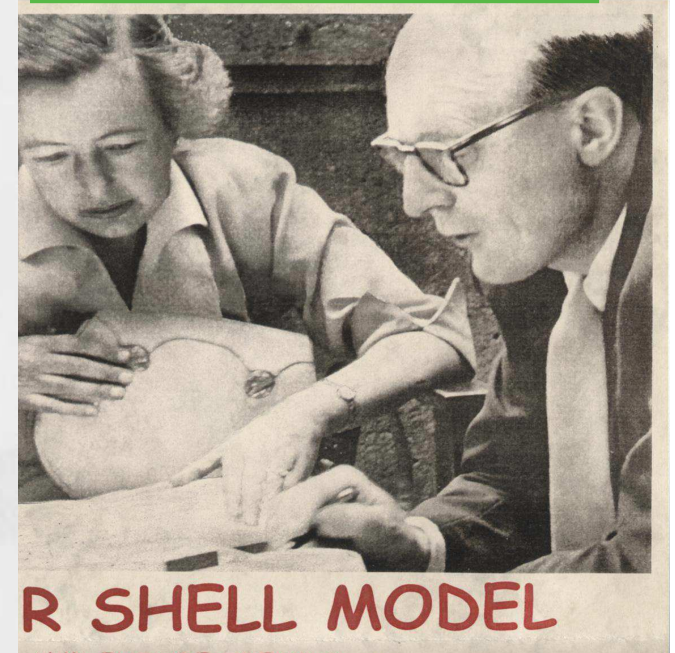


図2-23 1粒子軌道の順序。図は M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure*, p. 58, Wiley, New York, 1955 からとった。

As  $N$  or  $Z$  is changed to a large extent in exotic nuclei, the shell structure may be changed (evolved) .

- **Monopole component of the  $NN$  interaction**

$$v_{m;j,j'} = \frac{\sum_{k,k'} \langle jk j'k' | V | jk j'k' \rangle}{\sum_{k,k'} 1},$$

➔ Averaged over possible orientations

Linearity: Shift

$$\Delta \epsilon_j = v_{m;j,j'} n_{j'}$$

$n_{j'}$  : # of particles in  $j'$

For  $j' = 9/2$ , the multiplication by a factor of 10 !

*Poves and Zuker made a major contribution in initiating systematic use of the monopole interaction. (Poves and Zuker, Phys. Rep. 70, 235 (1981))*

What parts of nuclear forces are relevant ?

$$v_{m;jj'} = \frac{\sum_J (2J+1) \langle j_1, j_2, J | V | j_1, j_2, J \rangle}{\sum_J (2J+1)}$$



This becomes **larger generally**, if the overlap of radial wave functions of orbits  $j_1$  and  $j_2$  becomes larger.

The monopole interaction  $v_{m;jj'}$  becomes stronger for **central force** with a short range.

e.g., Federman-Pittel (1977)



The overlap of the radial wave functions are larger, if

- $j_1$  and  $j_2$  are spin-orbit partner, e.g.,  $d_{3/2}$  and  $d_{5/2}$
- $j_1$  and  $j_2$  are both high  $j$  orbits, e.g.,  $f_{7/2}$  and  $g_{9/2}$

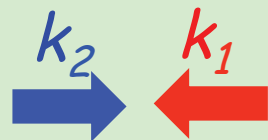
What else ?

# Monopole effect of tensor force

TO, Suzuki *et al.* PRL 95, 232502 (2005)  
 TO, Phys. Scr. T152, 014007 (2013)

## One-dimensional collision model

At collision point:  $\Psi \propto e^{ik_1x_1} e^{ik_2x_2} + e^{ik_2x_1} e^{ik_1x_2} = 2e^{iKX} \cos(kx)$



large relative momentum  $k$

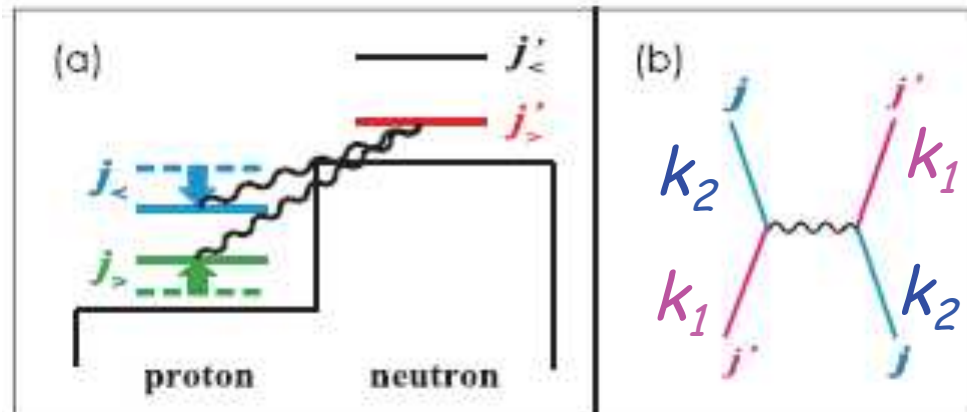
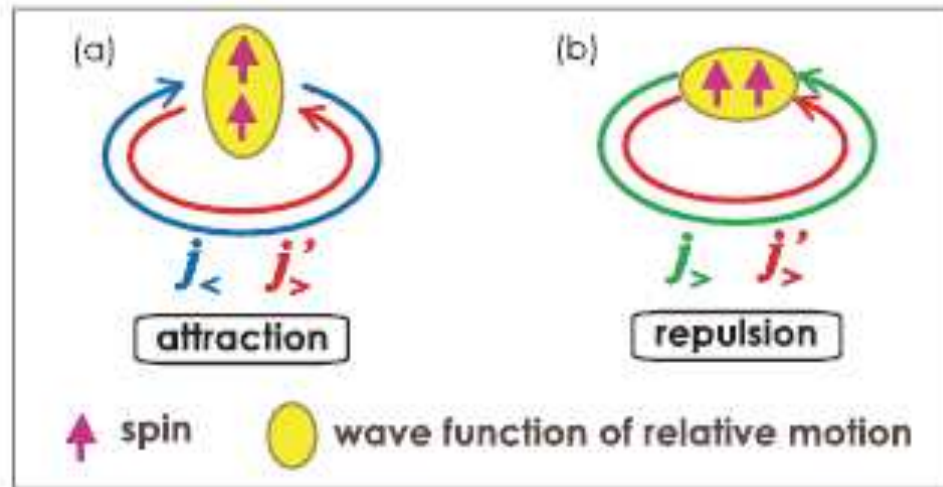


strong damping



wave function of relative coordinate

$$k = k_1 - k_2, \quad K = k_1 + k_2$$



small relative momentum  $k$



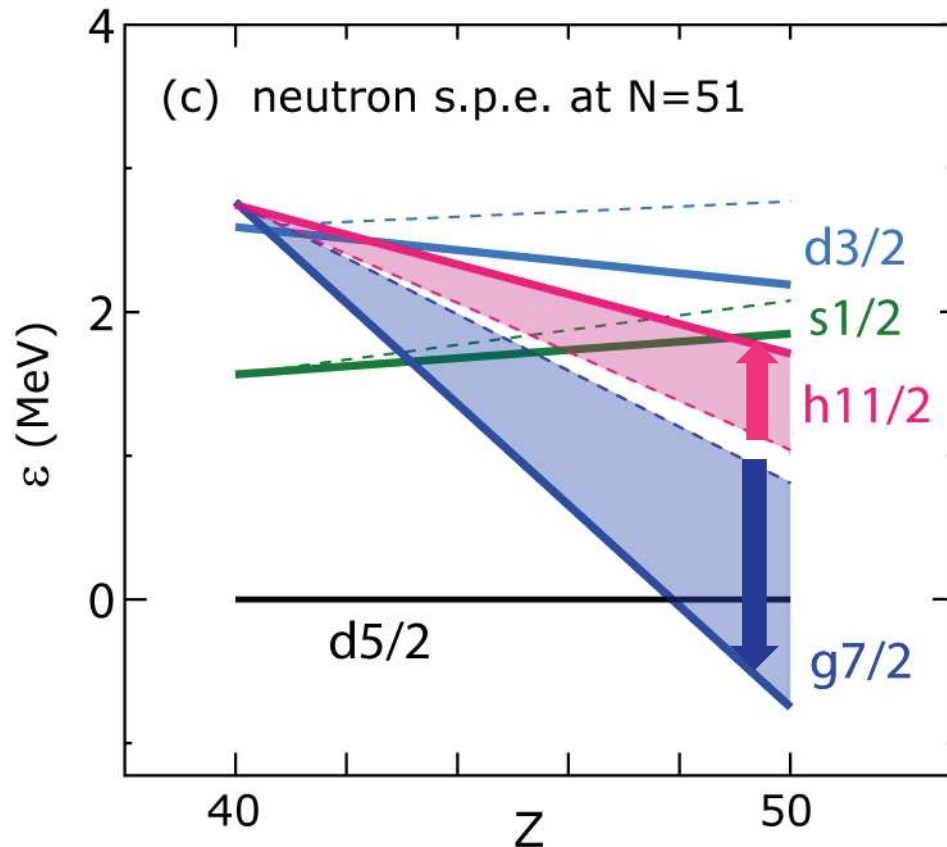
loose damping



wave function of relative coordinate



## Shell evolution from $^{90}\text{Zr}$ to $^{100}\text{Sn}$



solid line : full  
(central + tensor)

dashed line : central only

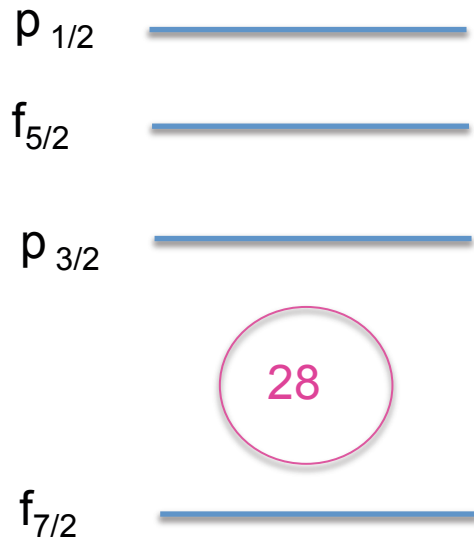
shaded area :  
effect of tensor force

Exp. d5/2 and g7/2 should be close  
Seweryniak et al.

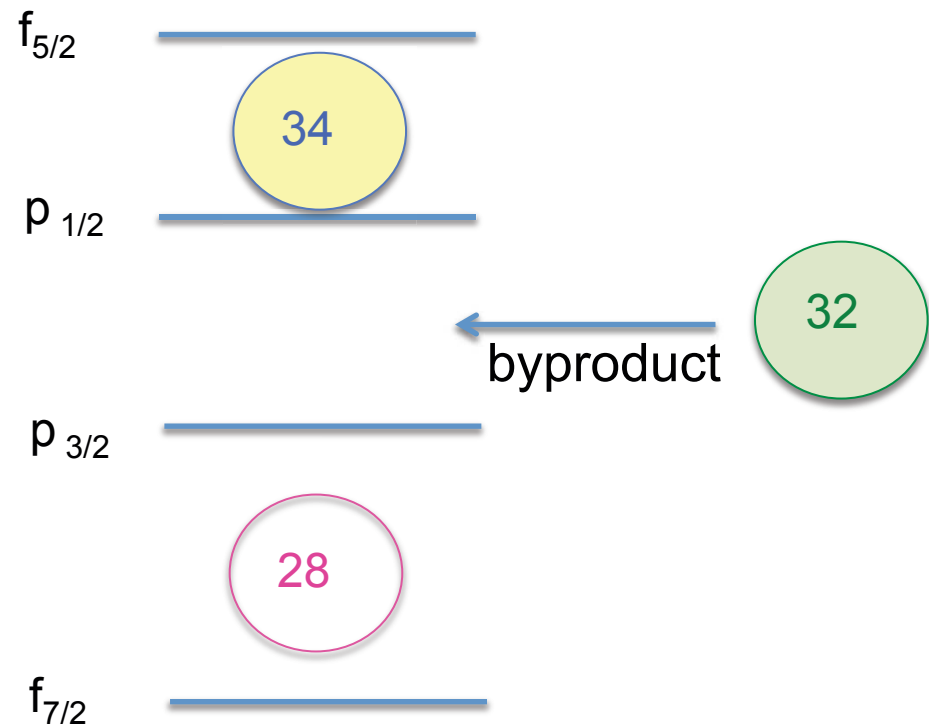
Phys. Rev. Lett. 99, 022504 (2007)  
Gryzywacz et al.

# Appearance of N= 32 and 34 magic structures

shell structure  
for **neutrons**  
in **Ni** isotopes  
( $f_{7/2}$  fully occupied)



N=34 magic number may appear  
if proton  $f_{7/2}$  becomes vacant (**Ca**)  
( $f_{5/2}$  becomes less bound)



Predicted by TO *et al*, PRL 87, 082502 (2001)

**N=34 magic number**  
and the shell evolution due to proton-neutron interaction

neutron  $f_{5/2} - p_{1/2}$  spacing increases by  $\sim 0.5$  MeV per one-proton removal from  $f_{7/2}$ , where tensor and central forces works coherently and almost equally.

note :  $f_{5/2} = j <$   $f_{7/2} = j >$

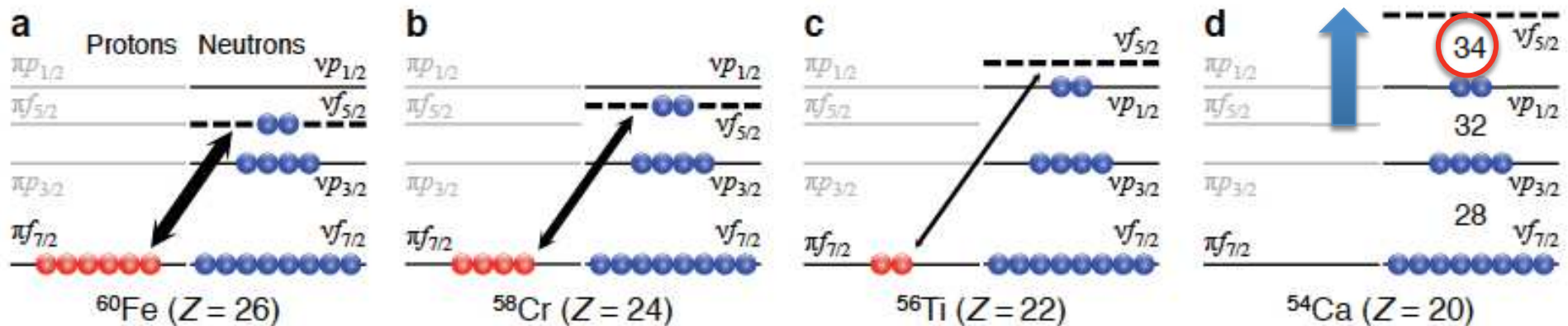
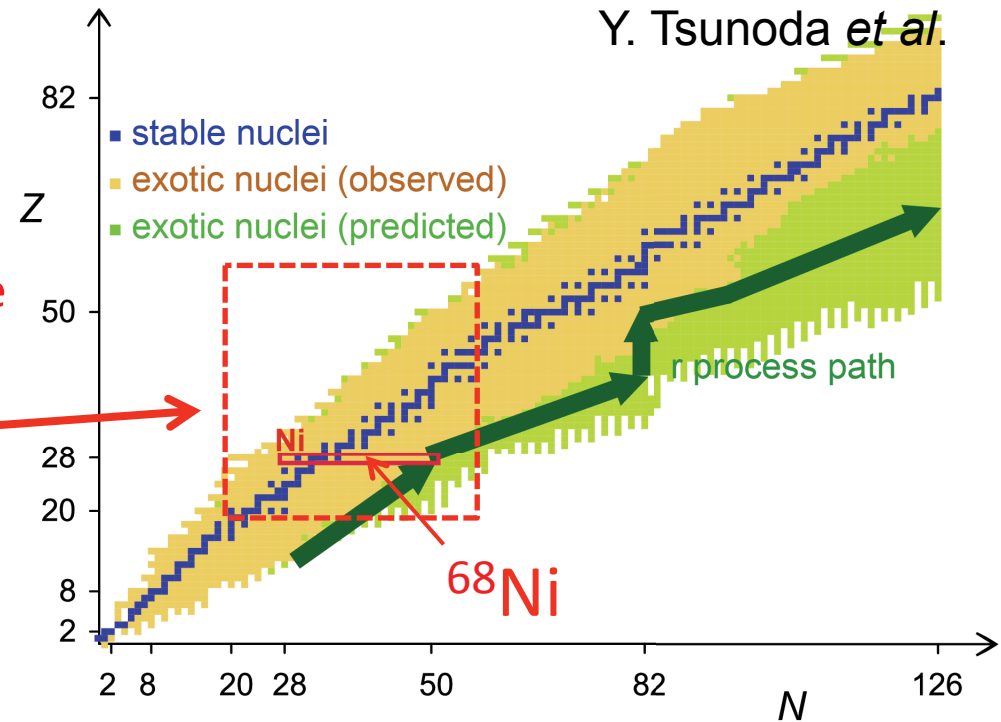
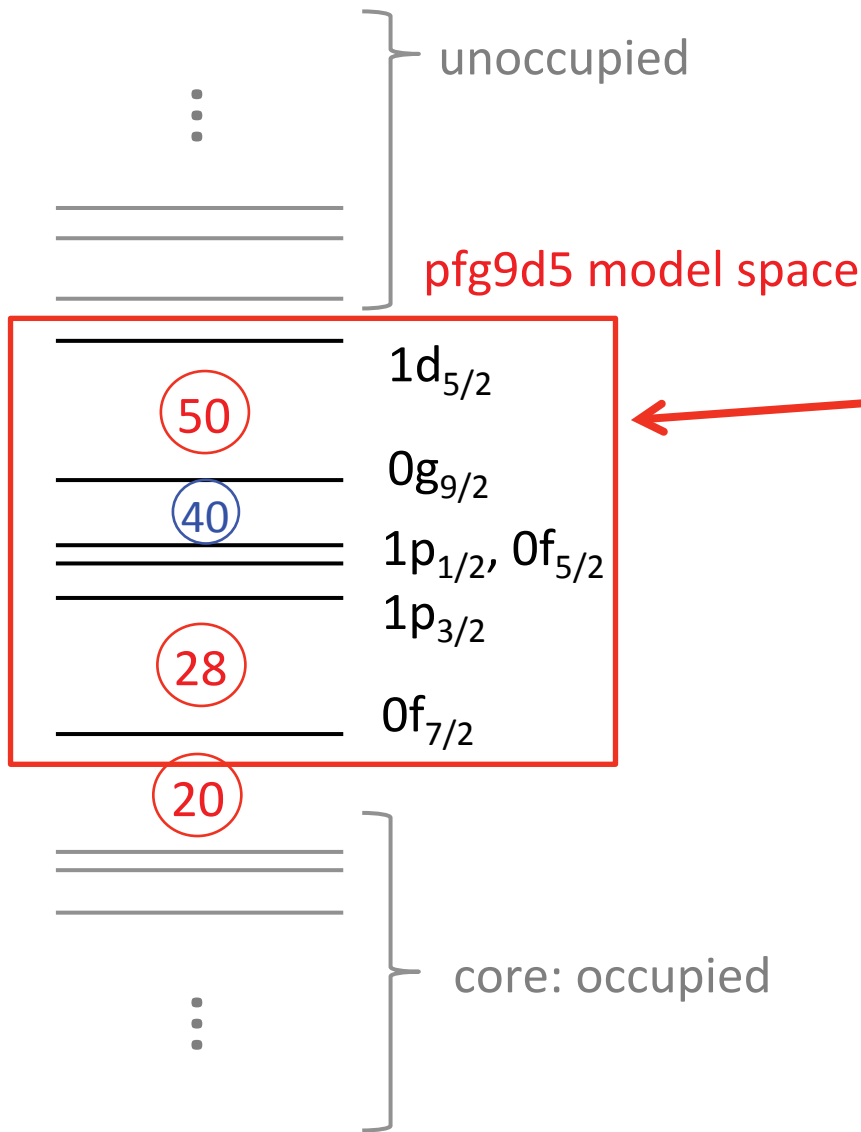


FIG. 1: Schematic illustration highlighting the attractive interaction between the proton  $\pi f_{7/2}$  and neutron  $\nu f_{5/2}$  single particle orbitals for  $N = 34$  isotones. a–c, As protons are removed from the  $\pi f_{7/2}$  orbital (from a,  $^{60}\text{Fe}$ , through b,  $^{58}\text{Cr}$  to c,  $^{56}\text{Ti}$ ), the strength of the  $\pi$ - $\nu$  interaction reduces, as represented by the decreasing width of the diagonal arrows, causing the  $\nu f_{5/2}$  orbital to shift up in energy relative to the  $\nu p_{3/2}$ - $\nu p_{1/2}$  spin-orbit partners. Consequently, a significant shell closure presents itself at  $N = 32$  in isotopes far from stability. d, The possibility of an additional shell closure at  $N = 34$  for  $^{54}\text{Ca}$  is presented. The  $\nu f_{5/2}$  SPO is indicated as a bold-dashed line to help guide the eye.

Steppenbeck *et al.* Nature, 502, 207 (2013)

# Monte Carlo Shell Model (MCSM) calculation on Ni isotopes

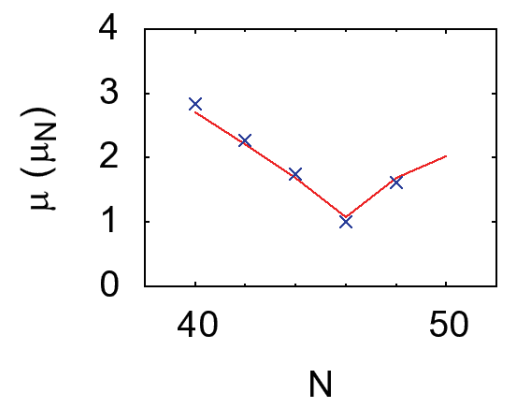
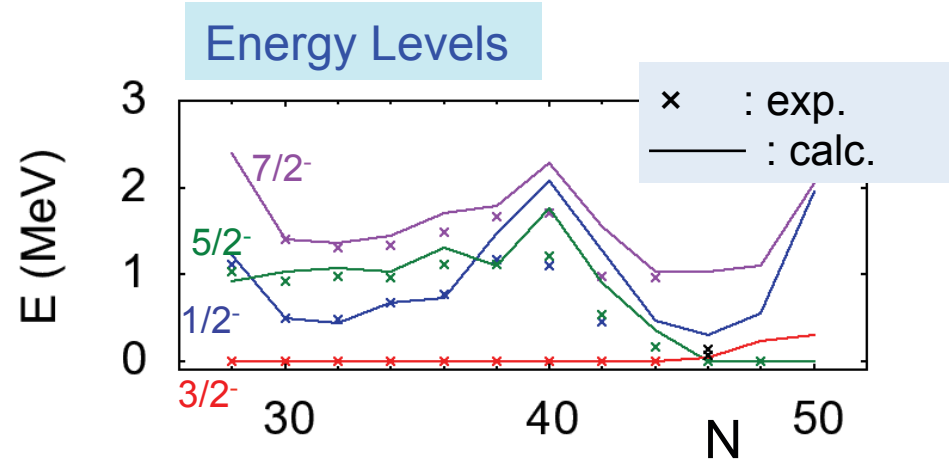
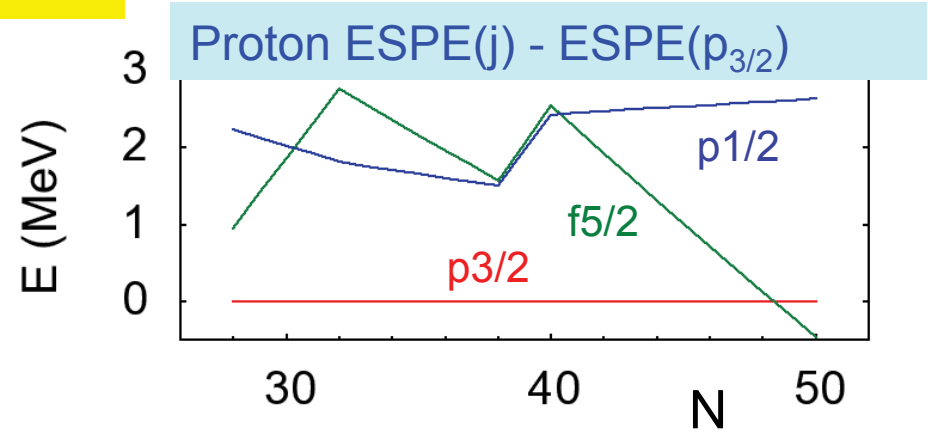
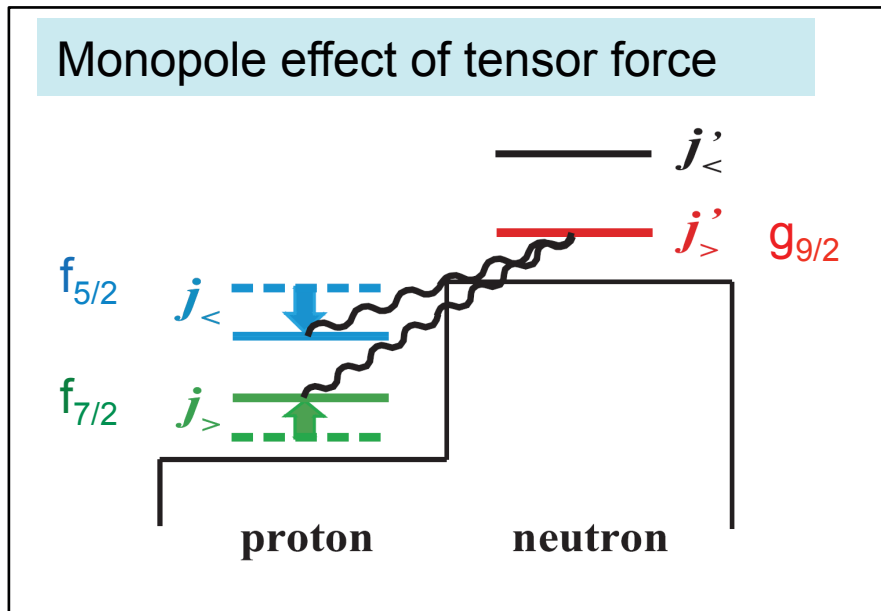


This model space is wide enough to discuss how **magic numbers 28, 50** and **semi-magic number 40** are visible or smeared out.

Interaction:  
A3DA interaction is used with minor corrections

# Cu isotopes

- proton  $p_{3/2}$ - $f_{5/2}$  level crossing from  $N = 40$  to  $N = 50$  (type I shell evolution)



Magnetic moment of ground state

exp. : Flanagan et al. (2009)



# Energy levels and B(E2) values of Ni isotopes

Description by the same Hamiltonian

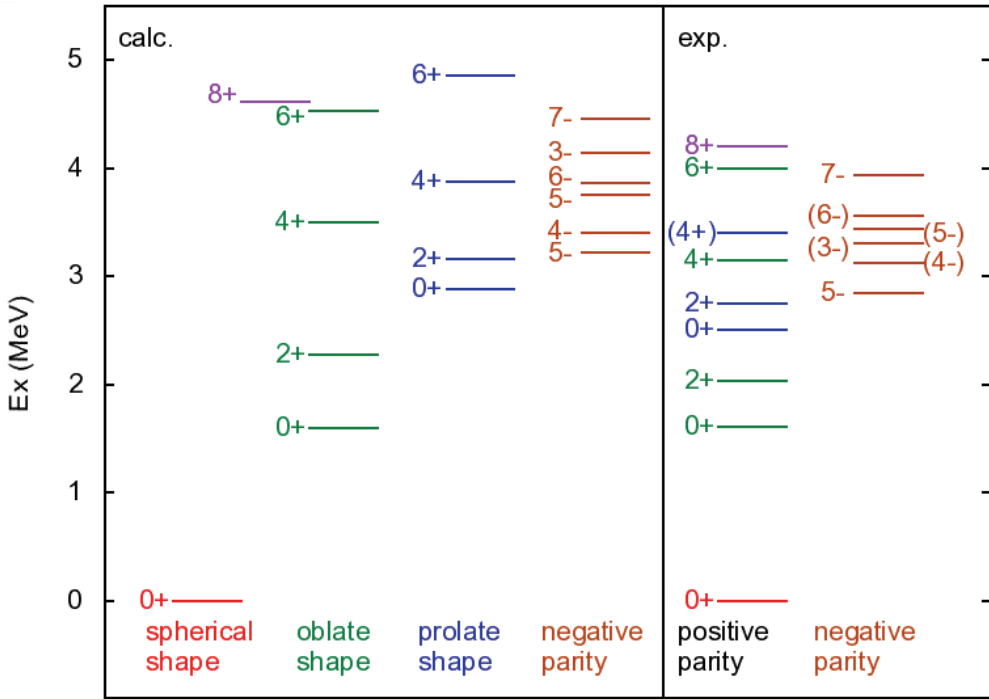
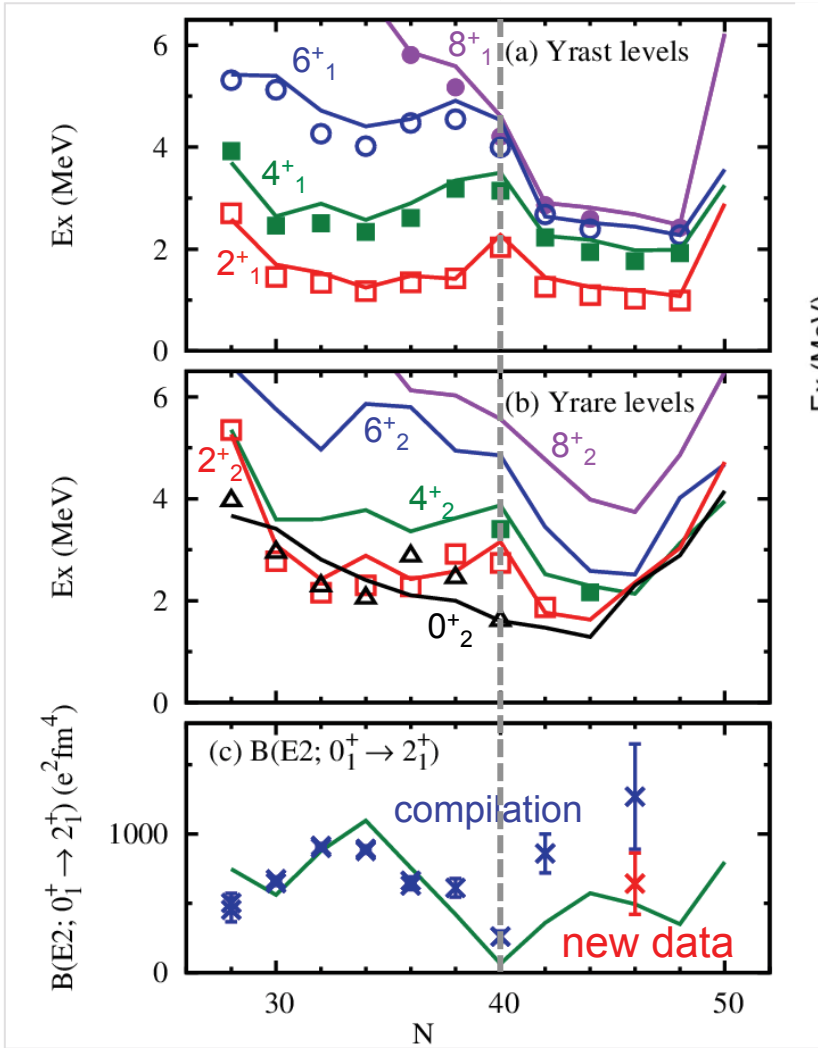
Shape coexistence in  $^{68}\text{Ni}$

× exp.  
— calc.

$^{68}\text{Ni}$

calc.

exp.

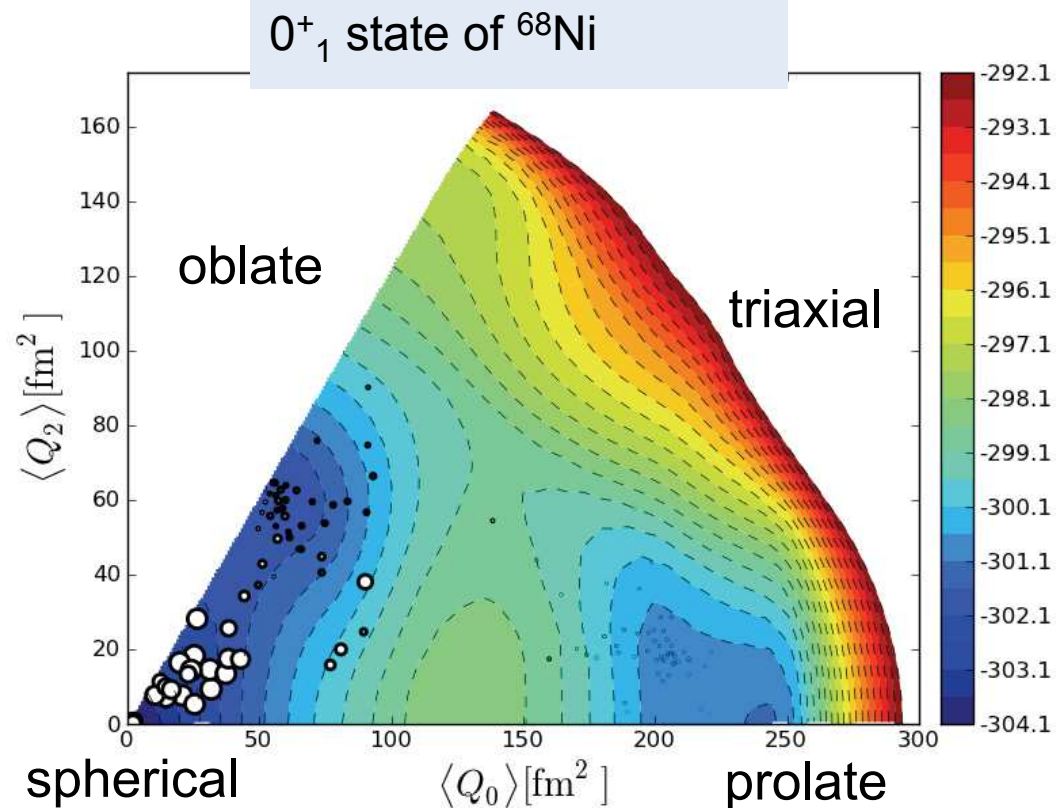


Y. Tsunoda, TO, Shimizu, Honma and Utsuno, PRC 89, 031301 (R) (2014)

# MCSM basis vectors on Potential Energy Surface

eigenstate  $\Psi = \sum_i c_i P[J^\pi] \Phi_i$  ← Slater determinant → intrinsic shape

- **PES** is calculated by CHF for the shell-model Hamiltonian
- **Location of circle** : quadrupole deformation of unprojected MCSM basis vectors
- **Area of circle** : overlap probability between each projected basis and eigen wave function



Called ***T-plot*** in reference to

Y. Tsunoda, TO, Shimizu, Honma and Utsuno,  
PRC 89, 031301 (R) (2014)

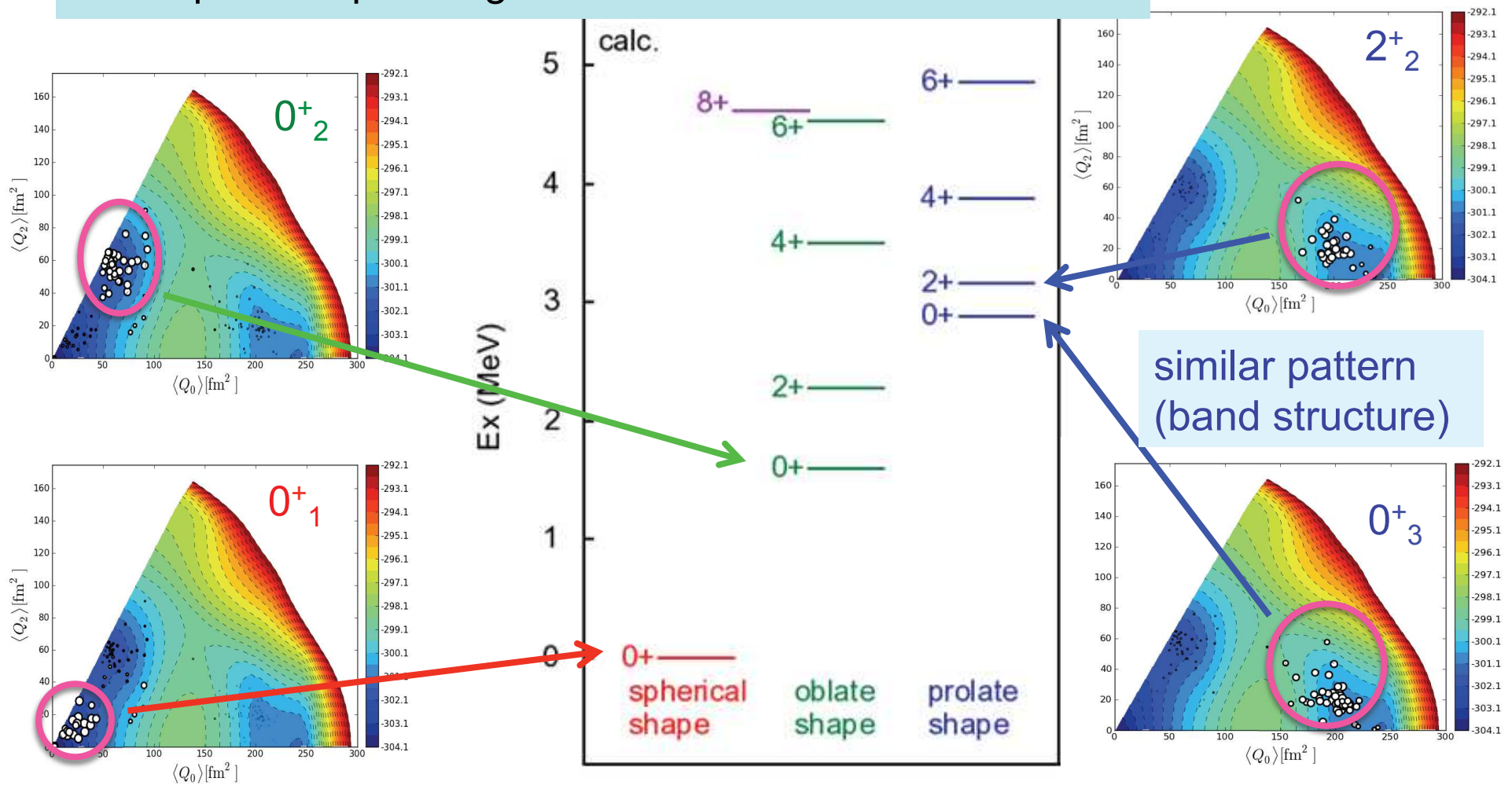
## General properties of T-plot :

Certain number of large circles in a small region of PES

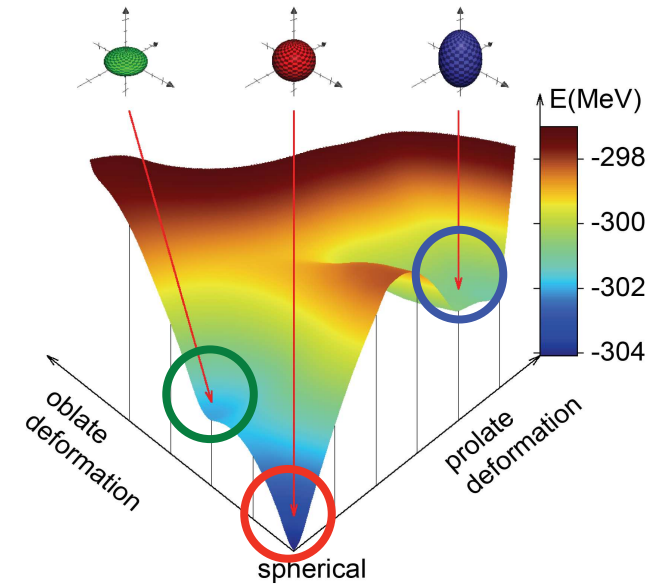
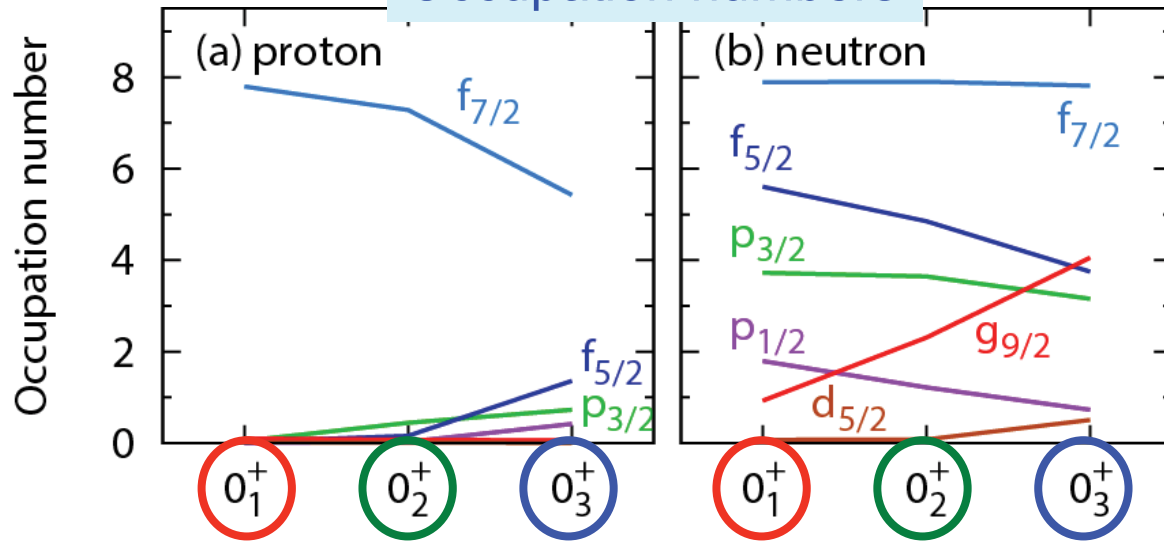
⇔ pairing correlations

Spreading beyond this can be due to shape fluctuation

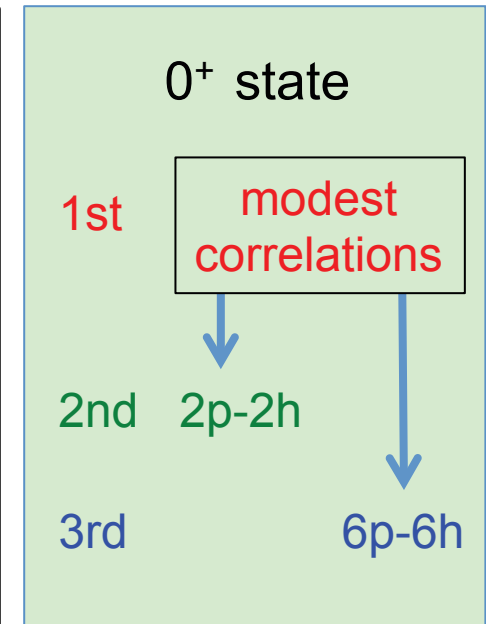
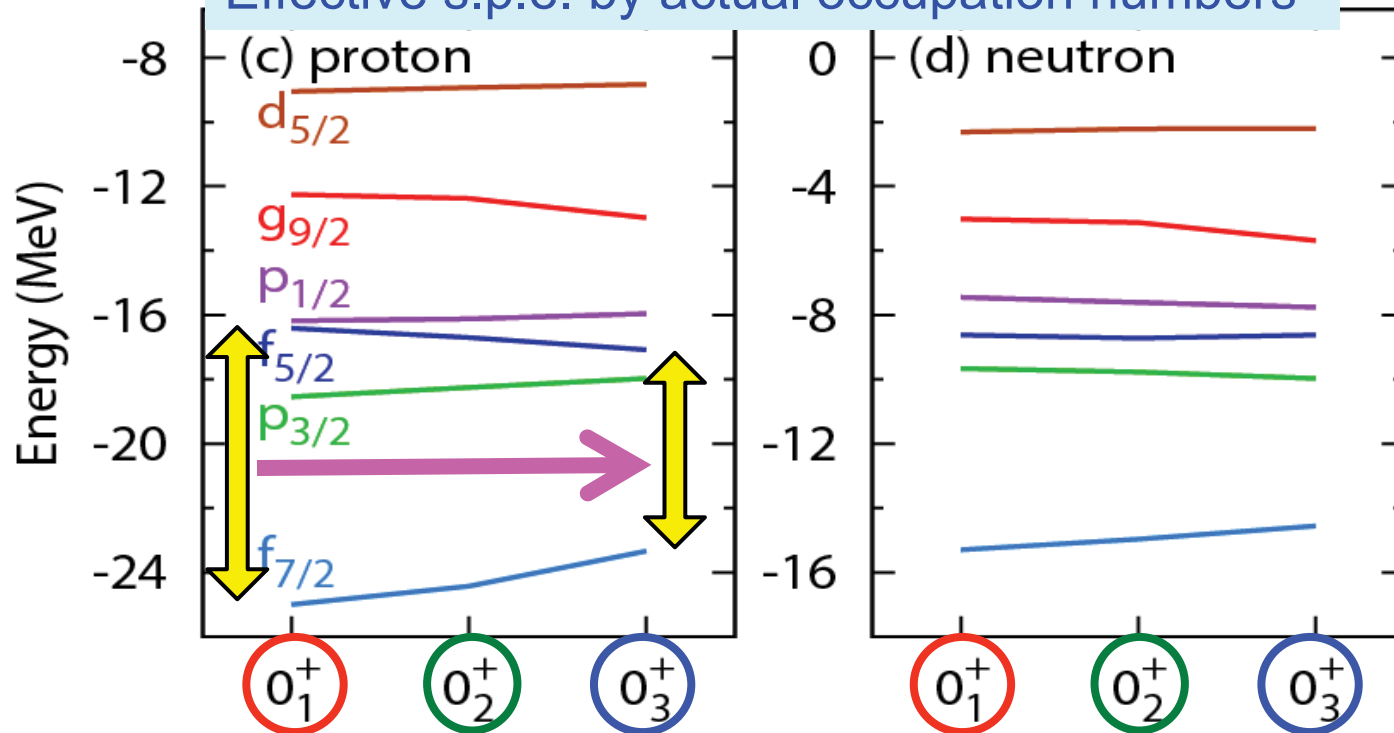
## Example : shape assignment to various $0^+$ states of $^{68}\text{Ni}$



## Occupation numbers



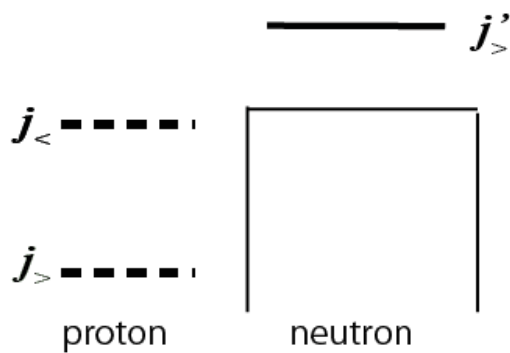
## Effective s.p.e. by actual occupation numbers



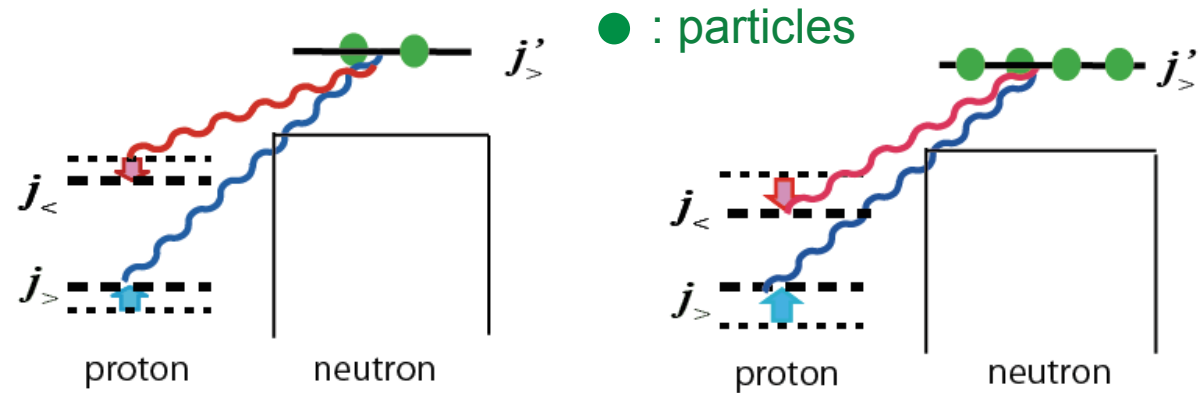
Underlying mechanism of the appearance of low-lying deformed states :  
**Type II Shell Evolution**

Monopole effects on the shell structure from the tensor interaction

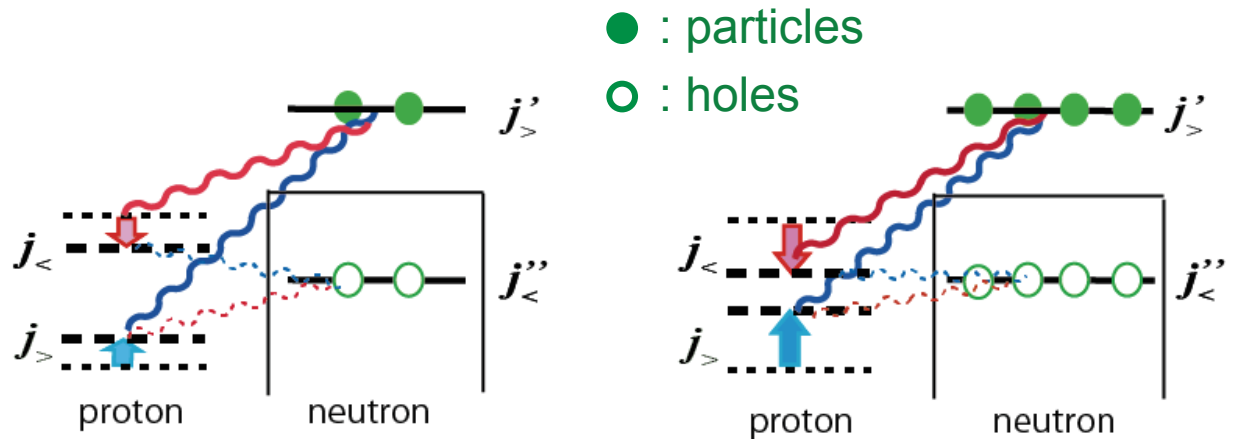
(a)



**Type I Shell Evolution : different isotopes**

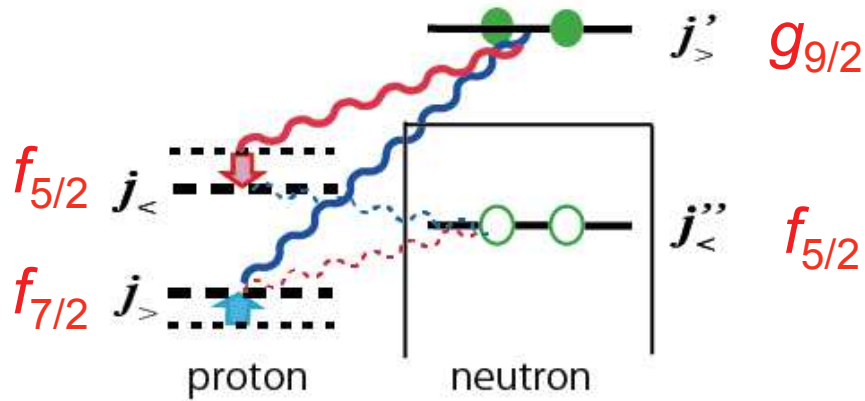


**Type II Shell Evolution : within the same nucleus**





# Type II Shell Evolution in $^{68}\text{Ni}$ ( $Z=28, N=40$ )



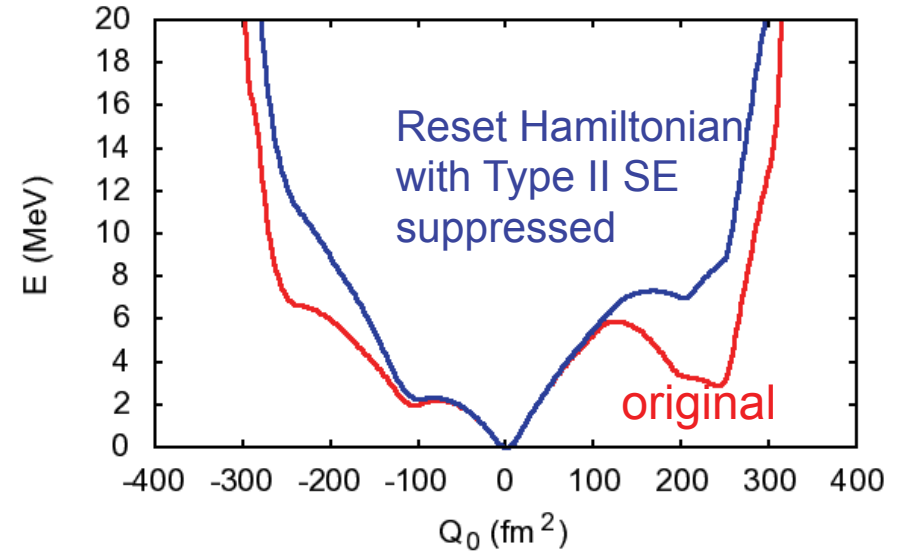
Spin-orbit splitting works against quadrupole deformation (cf. Elliott's  $SU(3)$ ).

weakening of spin-orbit splitting

**Type II shell evolution**

stronger deformation of protons  
 $\rightarrow$  more neutron p-h excitation

PES along axially symmetric shape



Type II shell evolution is suppressed by **resetting monopole interactions** as

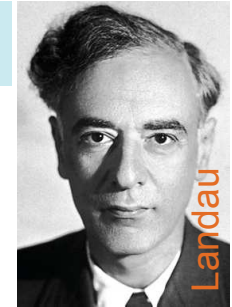
$$\pi f_{7/2} - \nu g_{9/2} = \pi f_{5/2} - \nu g_{9/2}$$

$$\pi f_{7/2} - \nu f_{5/2} = \pi f_{5/2} - \nu f_{5/2}$$

The local minima become much less pronounced.

**Shape coexistence** is enhanced by **type II shell evolution** as the same quadrupole interaction works more efficiently.

# Nucleus is a quantum liquid

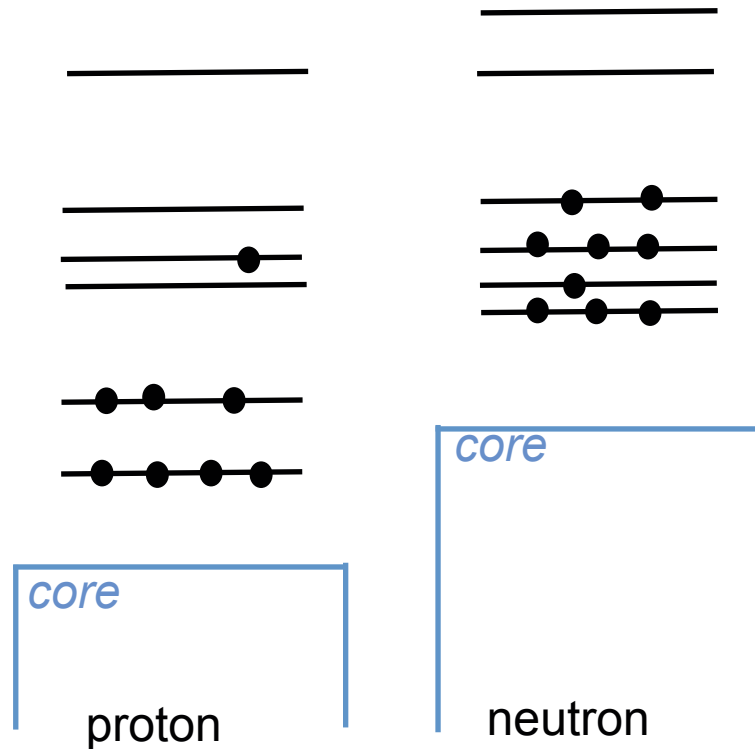


## Dual quantum liquids in the same nucleus

Certain configurations produce different shell structures owing to (i) tensor **force** and (ii) proton-neutron **compositions**

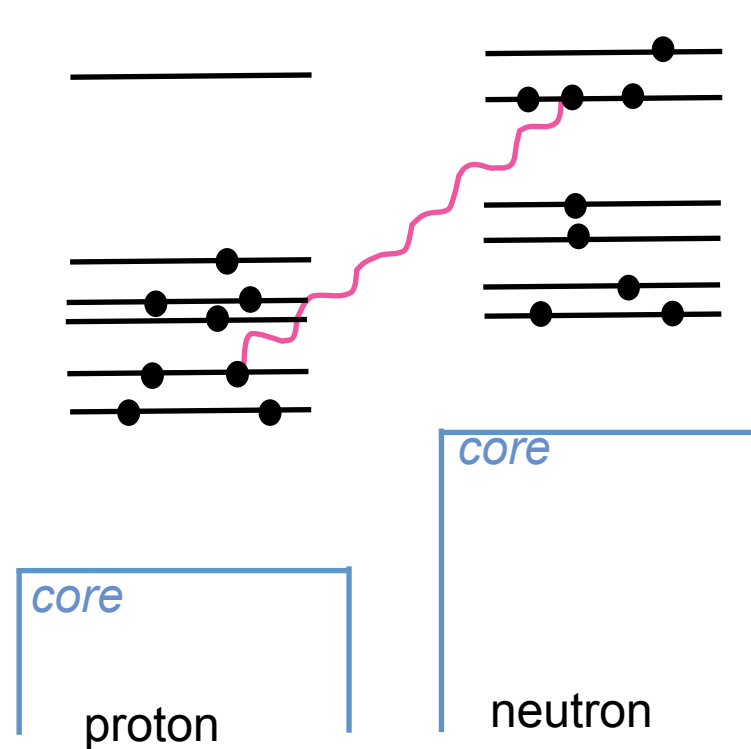
Liquid 1 (~**constant spherical SPE**)

relevant to **normal** states in general



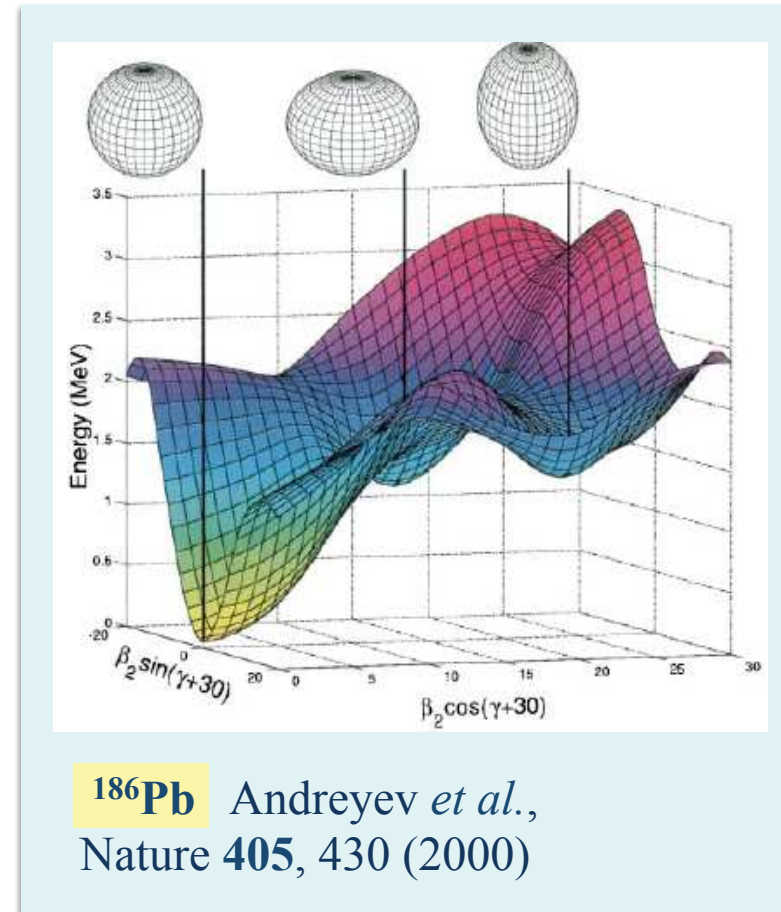
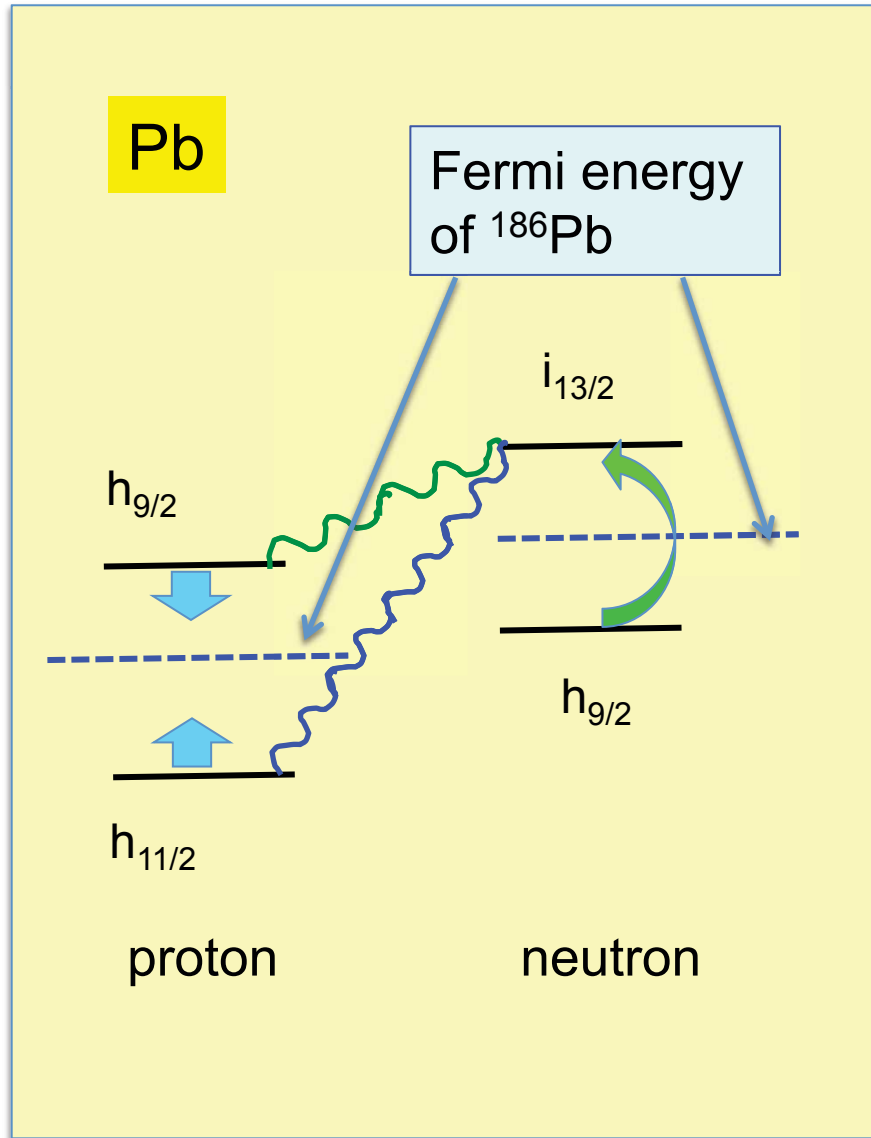
Liquid 2 (**varying spherical SPE**)

relevant to **specific intruder** states



*Note : Despite almost the same density, different single-particle energies appear*

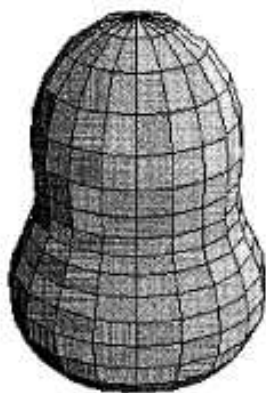
Other cases ..... just an example



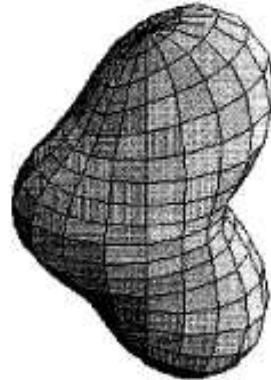
# Reflection asymmetric shapes

## Quadrupole-octupole shapes

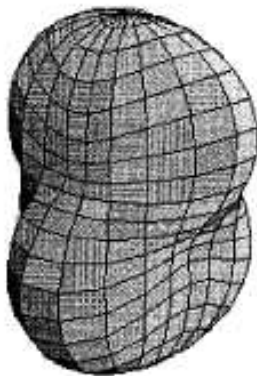
$$\beta_2=0.6, \beta_{3\mu}=0.35$$



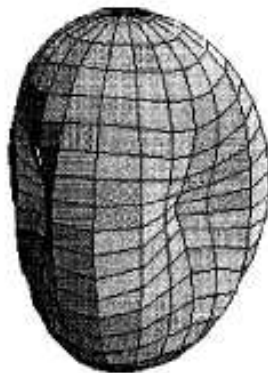
$\mu=0$



$\mu=1$



$\mu=2$



$\mu=3$

## Intrinsic reflection asymmetry in atomic nuclei

P. A. Butler

*Department of Physics, Oliver Lodge Laboratory, University of Liverpool,  
Liverpool L69 3BX, United Kingdom*

W. Nazarewicz

Reviews of Modern Physics, Vol. 68, No. 2, April 1996

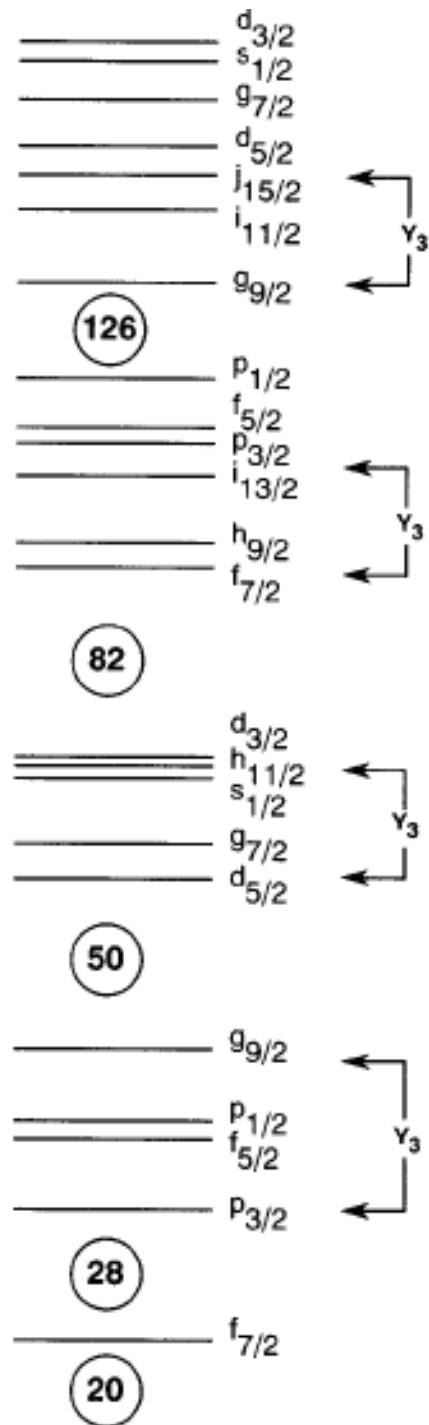
# Zr isotopes

Core :  $^{68}\text{Ni}$

Proton orbits :  $f_{5/2}$ ,  $p$ ,  $gds$

Neutron orbits :  $gds$ ,  $h_{11/2}$ ,  $f_{7/2}$ ,  $p_{3/2}$

Effective interaction :  
 $\text{JUN45} + \text{jSNBG3} + \text{VMU}$



neutrons

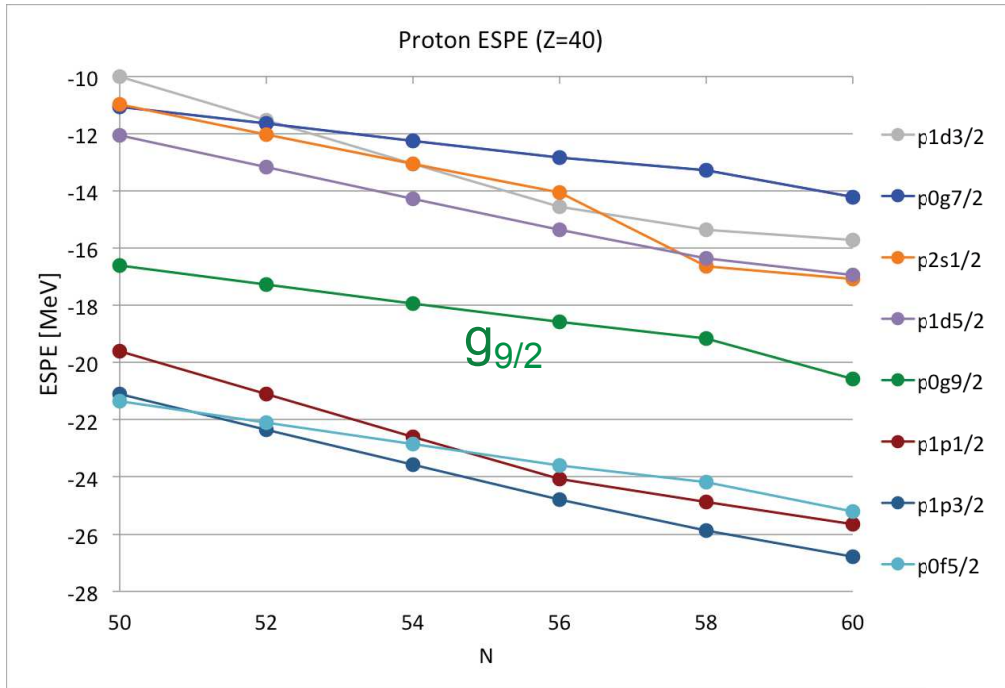
protons

FIG. 4. Nuclear spherical single-particle levels. The most important octupole couplings are indicated.

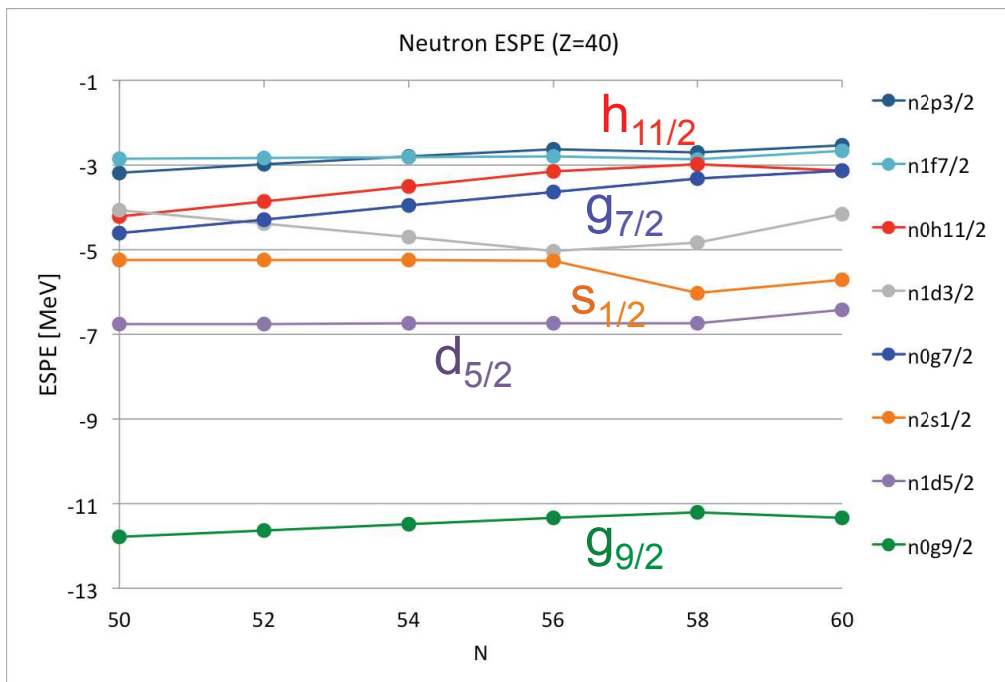


# Effective single-particle energies in filling scheme

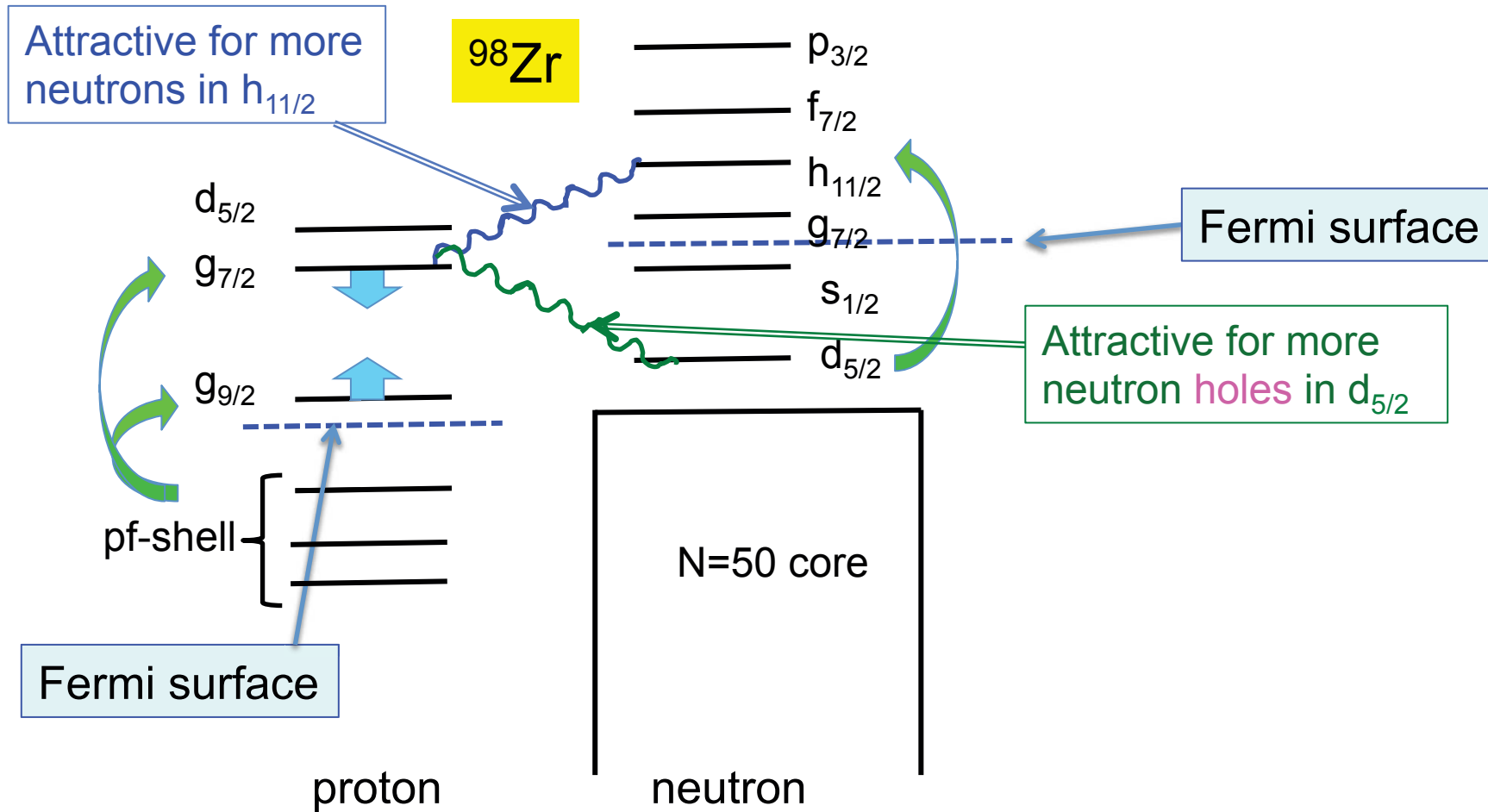
protons



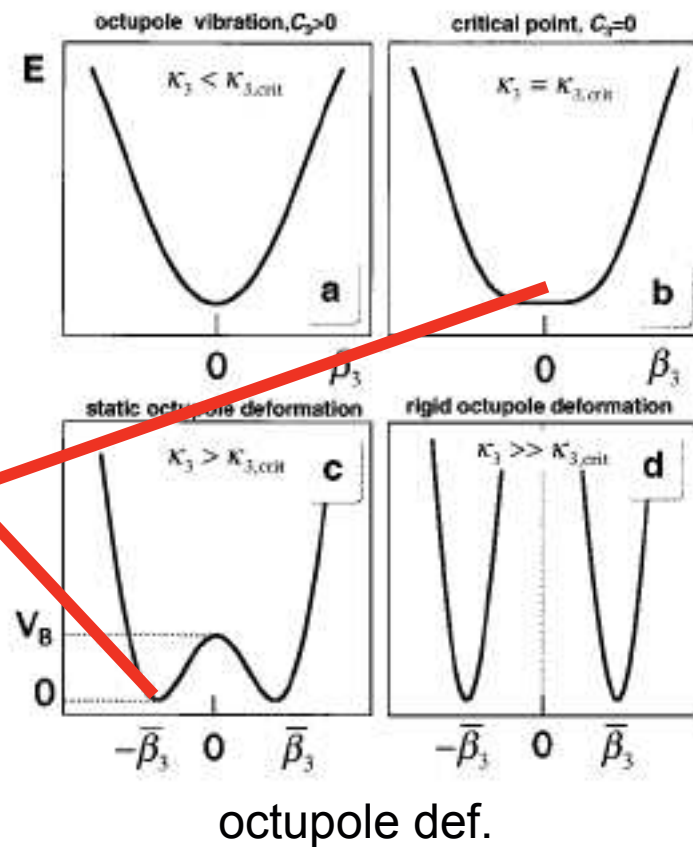
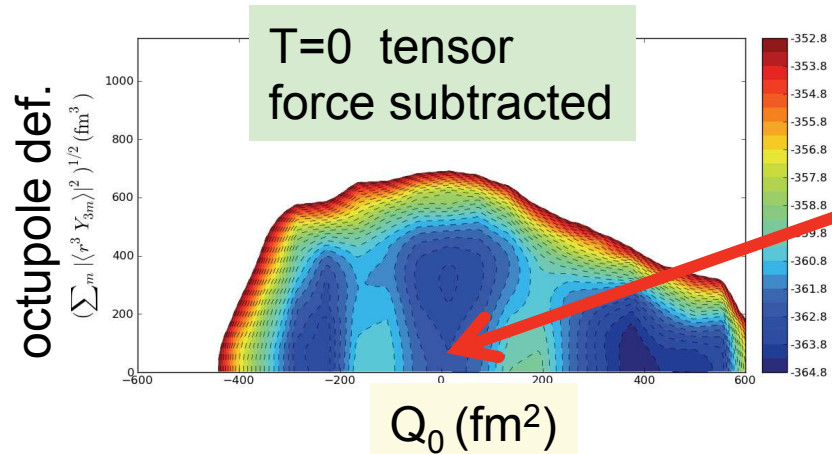
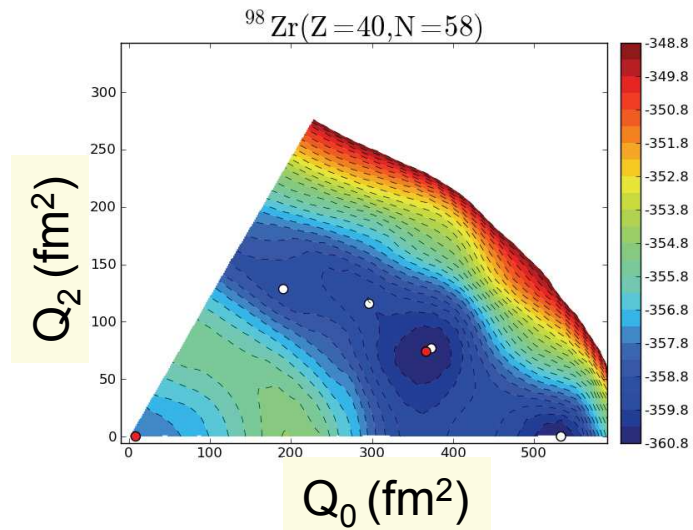
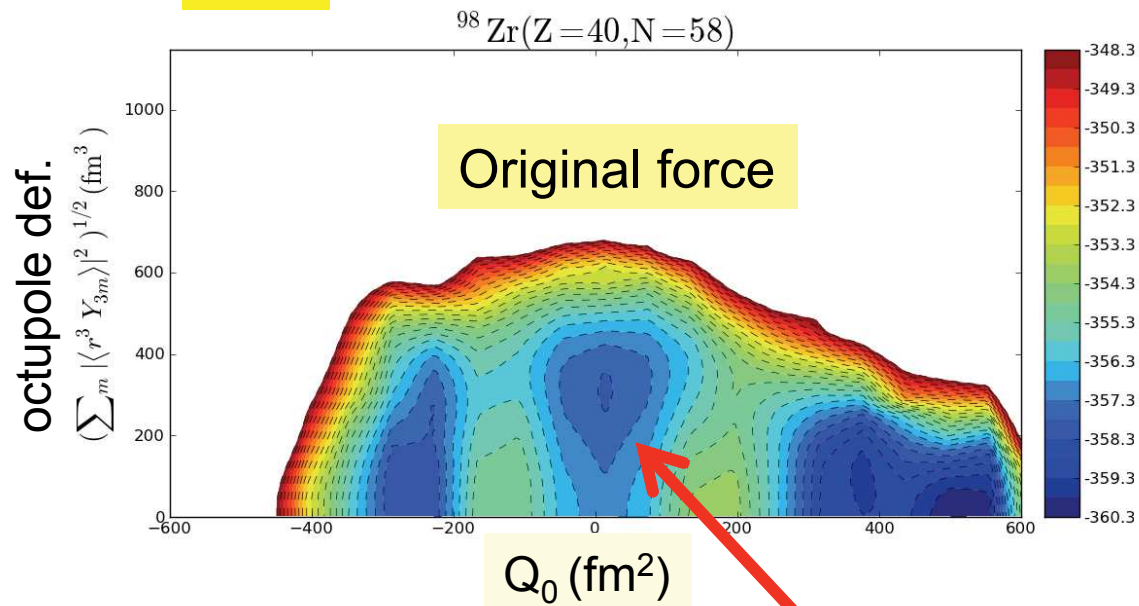
neutrons



Type II shell evolution produces substantial effects with massive particle-hole excitations between two shells of both parities. In certain cases, this enhances and/or is enhanced by octupole deformation, soft or static.



**$^{98}\text{Zr}$**



## Summary

Type I Shell Evolution, driven by central and tensor forces, can be found in many parts of the nuclear chart.

The shell evolution can be extended to Type II, resulting in Dual Quantum Liquid : shell structure can be (i) stable or (ii) dynamical

- spin-orbit splitting reduction due to the nuclear force
- proton-neutron contents of quantum liquid

This effect produces (low local minimum) and stabilize (high barrier) shape coexistence in various cases through a non-linear mechanism.

Correlations towards reflection asymmetric shapes appear in certain cases, enhancing and being enhanced by Type II shell evolution.

# Collaborators

- Y. Tsunoda (CNS, Tokyo)
- T. Togashi (CNS, Tokyo)
- Noritaka Shimizu (CNS, Tokyo)
- Yutaka Utsuno (JAEA)
- Michio Honma (Aizu)