Myths of Collective Structures in Atomic Nuclei

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Lord Rayleigh, Proc. Roy. Soc. of London 29, 71 (1879) App. II, Equ. (40)

frequency of vibration of a spherical liquid drop

$$\omega^2 = \frac{(\lambda-1)\lambda(\lambda+2)\gamma}{\rho R^2}$$

WEIZÄCKER SEMI-EMPIRICAL MASS FORMULA Zeitschrift für Physik **96** (1935) 431

$$M_{nucleus}(Z,N) = \{ Zm_p + Nm_n \} - C_v A + C_s A^{2/3} + a_3 Z^2 A^{-1/3} + a_4 (N-Z)^2 A^{-1} \pm a_5 A^{-1}$$

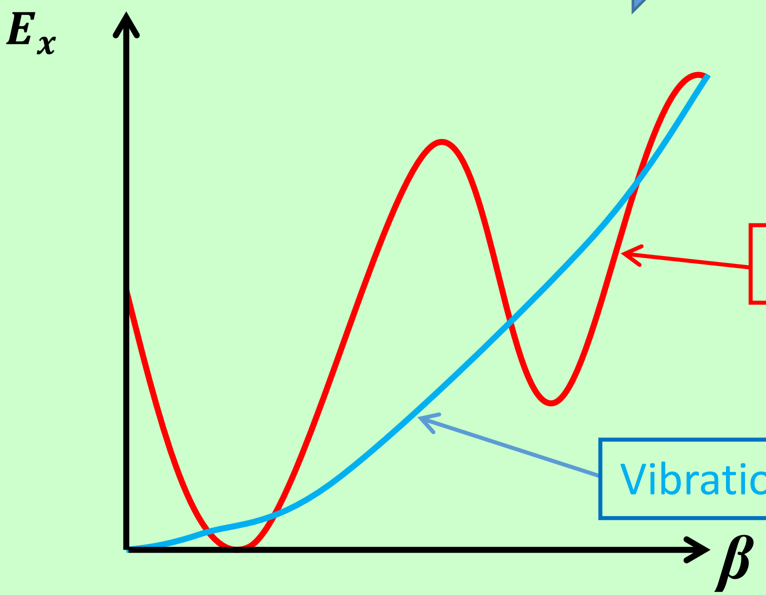
Leon van Dommelen, "*Quantum Mechanics for Engineers* "www.eng.fsu.edu/~dommelen/quantum/style_a/nt_liqdrop.html

$$\omega^2 = \frac{(\lambda-1)\lambda(\lambda+2)}{3}\frac{C_s}{R_A^2 m A} - \frac{2(\lambda-1)\lambda}{(2\lambda+1)}\frac{e^2 Z^2}{4\pi\epsilon_0 R_A^3 m_p A^2}$$

Put $R = R_A A^{1/3}$ $R_A = 1.3\ fm$ $C_s = 18MeV$

Then for A=150, $\lambda = 2$ and using $E_x = \hbar\omega$

$$\boldsymbol{E_x(\lambda = 2) = 2.4\ MeV}$$



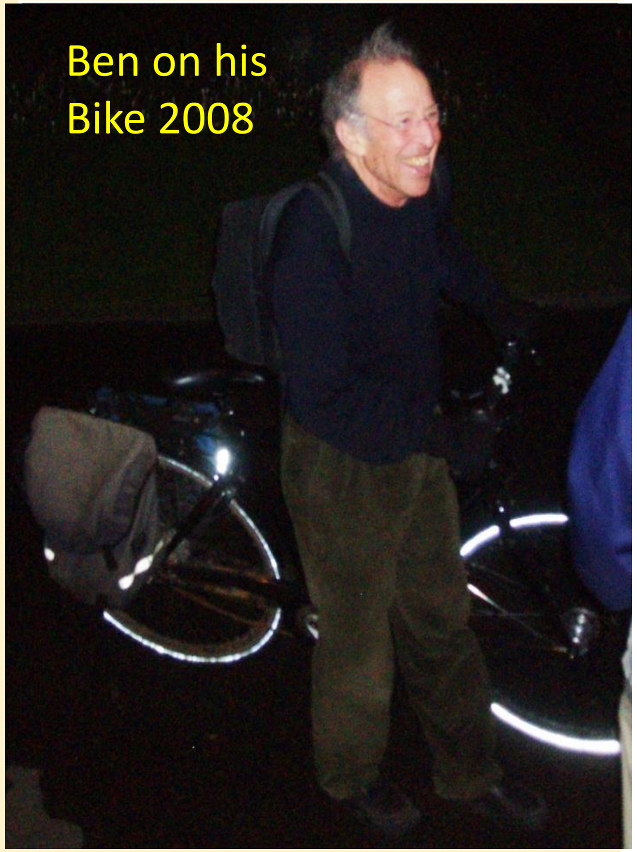
Shell corrections make the nucleus stiffer putting up the vibrational frequency

Moments-of-Inertia: $\mathscr{J}_{irr} < \mathscr{J}_{expt} < \mathscr{J}_{rigid}$

Rayleigh, B&M assume irrotational behaviour of the nuclear "fluid". We would expect the experimental value to make the shape stiffer.

Bohr & Mottelson Vol II

Page 363 !!



Ben on his Bike 2008

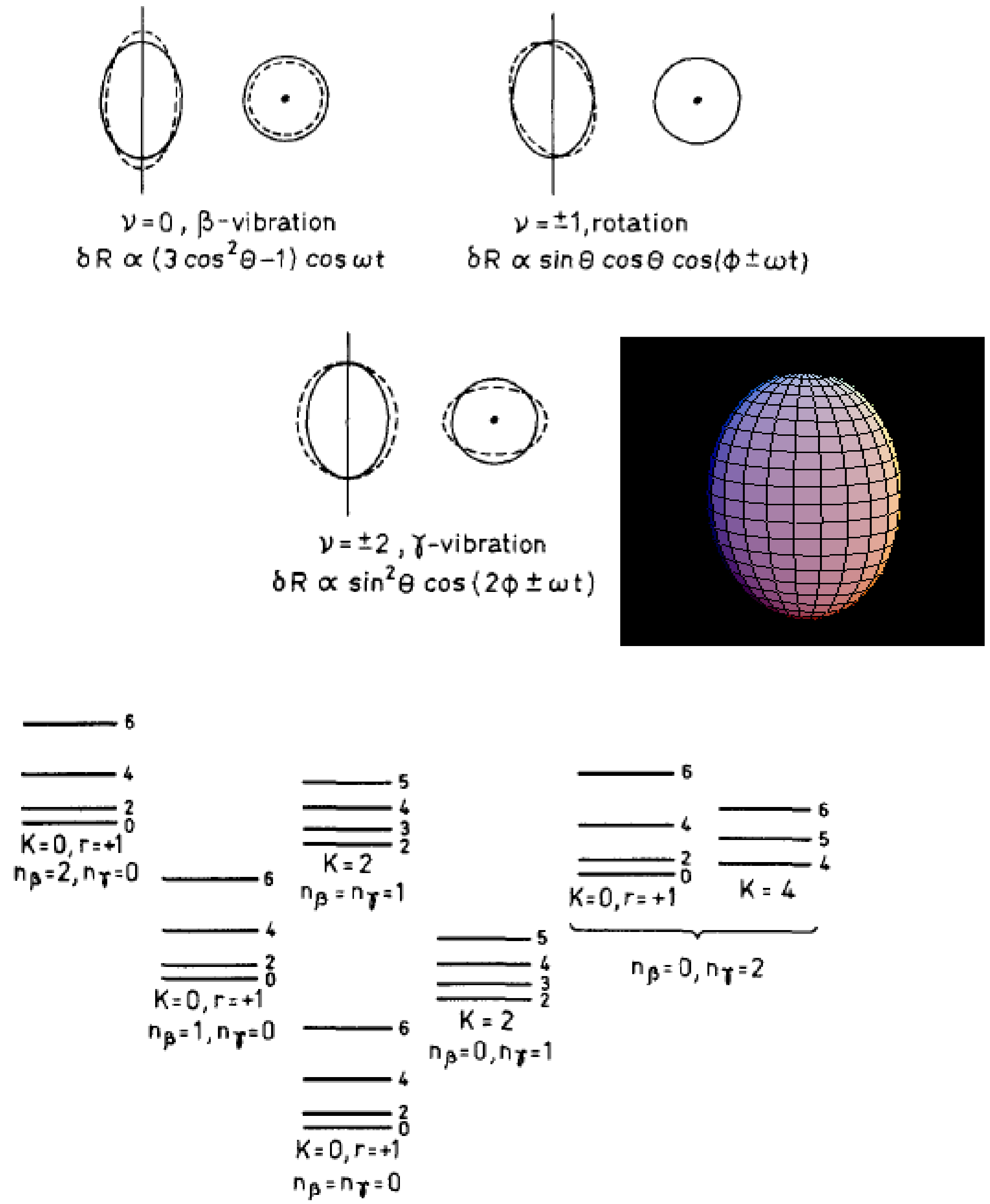
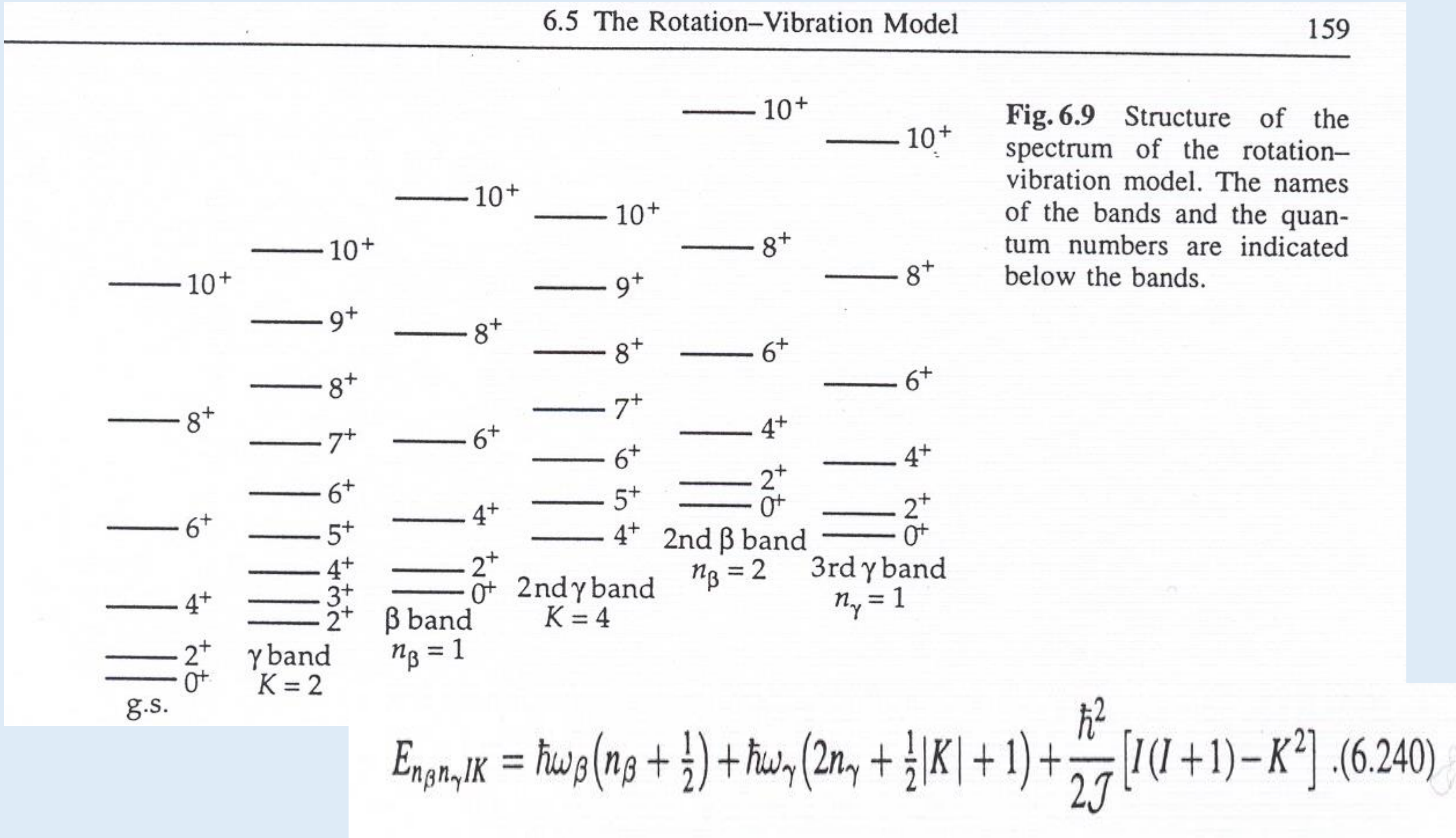


Figure 6-3 Quadrupole shape oscillations in a spheroidal nucleus. The upper part of the figure shows projections of the nuclear shape in directions perpendicular and parallel to

40th Anniversary Of the Bohr & Mottelson Nobel Prize (1975)

Greiner & Maruhn “Nuclear Models”



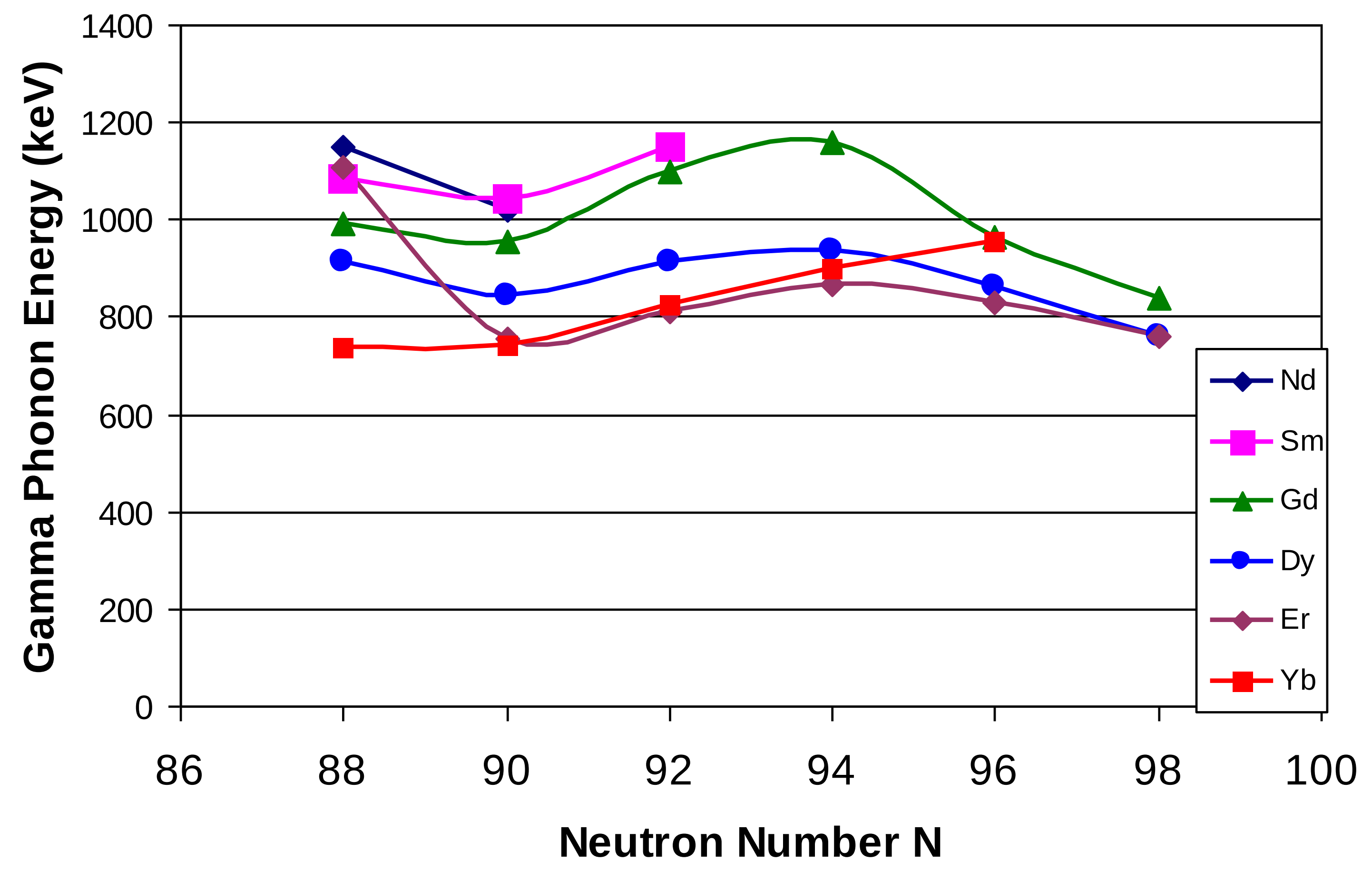
6.5 The Rotation–Vibration Model 159

Fig. 6.9 Structure of the spectrum of the rotation–vibration model. The names of the bands and the quantum numbers are indicated below the bands.

$$E_{n_\beta n_\gamma I K} = \hbar\omega_\beta\left(n_\beta + \tfrac{1}{2}\right) + \hbar\omega_\gamma\left(2n_\gamma + \tfrac{1}{2}|K| + 1\right) + \frac{\hbar^2}{2\mathcal{J}}\left[I(I+1) - K^2\right] \quad .(6.240)$$

$\hbar\omega_\gamma$

keV



$$E_{n_\beta n_\gamma IK} = \hbar\omega_\beta\left(n_\beta + \tfrac{1}{2}\right) + \hbar\omega_\gamma\left(2n_\gamma + \boxed{\tfrac{1}{2}|K|} + 1\right) + \frac{\hbar^2}{2\mathcal{J}}\left[I(I+1) - K^2\right] \quad .(6.240)$$

$E_{gnd\ state} = \tfrac{1}{2}\,\hbar\omega_\beta + \hbar\omega_\gamma$

$E_x\,(0,0,2,2) = \hbar\omega_\gamma + \hbar^2/\mathcal{J} \longleftarrow$ **K = 2 Gamma Vibration Band Head Energy**

Bohr & Mottelson Vol II

“A vibrational mode of excitation is characterized by the property that it can be repeated a large number of times. The *n*th excited state of a specified mode can thus be viewed as consisting of *n* individual quanta. The quanta obey Bose statistics…” ***Page 330***

“… but it might be expected that the zero-point oscillations in the γ direction would be of similar magnitude as those in the β direction. The experimentally observed E2-matrix elements for exciting the β vibrations are comparable to those exciting the $K^{\pi}=2^{+}$ bands….” ***Page 166***

“In view of the systematic occurrence of excited $K^{\pi}=2^{+}$ bands in the spectra of strongly deformed even-even nuclei, one may consider the possibility of describing these spectra in terms of a rotor deviating slightly from axial symmetry.” ***Page 186***

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS

J. Phys. G: Nucl. Part. Phys. **27** (2001) R1–R22 www.iop.org/Journals/jg PII: S0954-3899(01)18337-4

TOPICAL REVIEW

Characterization of the β vibration and 0_2^+ states in deformed nuclei

So, WHAT are the 0_2^+ states ??

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Received 26 October 2000, in final form 7 November 2000

Abstract

A summary of the experimental properties of the first excited 0^+ states in deformed rare-earth nuclei is presented. By appealing to the original definition of a β vibration laid down in the Bohr–Mottelson picture, it is re-emphasized that most of the 0_2^+ states are not β vibrations. A consideration of all available data, especially that from transfer reactions, and of microscopic calculations of 0^+ states underscores the need to consider the role of pairing in the description, and labelling, of these states.

Two Neutron Transfer to ^{154}Gd (N=90)

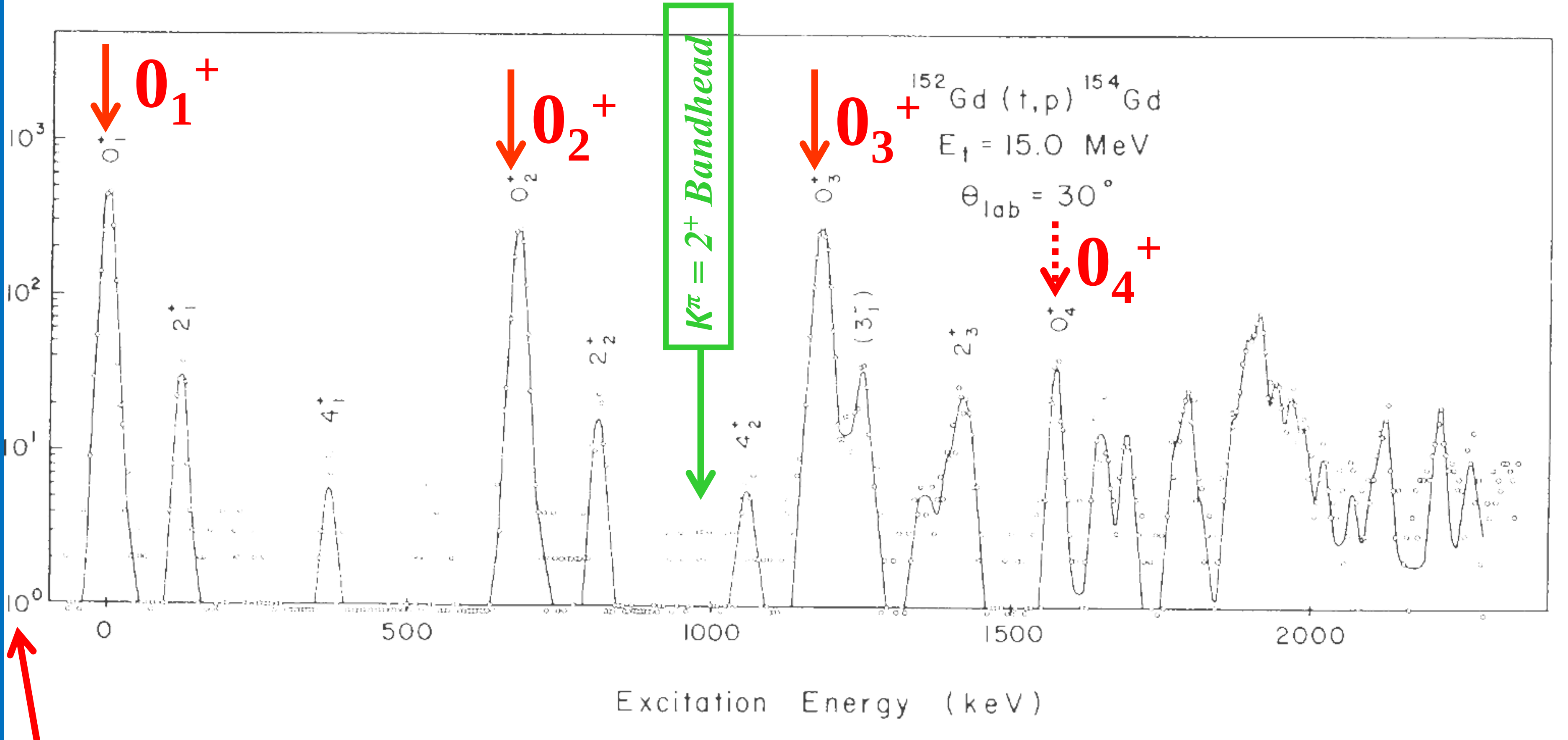


Fig. 1. A sample spectrum of the ^{152}Gd(t, p)^{154}Gd reaction at E_t = 15 MeV and θ_{lab} = 30°.

*Shahabuddin et al; NP **A340** (1980) 109*

N.B. Log$_{10}$ scale

Configuration Dependent or Quadrupole Pairing

R. E. Griffin, A. D. Jackson and A. B. Volkov, Phys. Lett. 36B, 281 (1971).

Suggested that $\Delta_{pp} \approx \Delta_{oo} >> \Delta_{op}$

for Actinide Nuclei where 0_2^+ states were observed in (p,t) that were not pairing- or β-vibrations.

Suppose there are *n prolate* and *n oblate* degenerate levels at the Fermi Surface;

Assume that each pairing matrix element is the same for the same type *-a*

BUT the *prolate-oblate* matrix elements are very weak $-\varepsilon a$

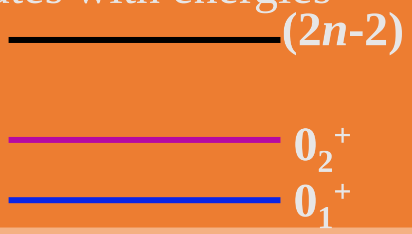
Then if the prolate $n*n$ matrix is A, the oblate matrix is also A

The matrix for the total system is;

$$\begin{vmatrix} A & \varepsilon A \\ \varepsilon A & A \end{vmatrix}$$

Then there are $(2n\text{-}2)$ states with ZERO energy and 2 states with energies

$$E_{1,2} = -(1 \pm \varepsilon)\, na$$



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W. I. van Rij and S. H. Kahana, Phys. Rev. Lett. 28, 50 (1972).

S. K. Abdulvagabova, S. P. Ivanova and N. I. Pyatov, Phys. Lett. 38B, 251 (1972).

D. R. Bès, R. A. Broglia and B. Nilsson, Phys. Lett. 40B, 338 (1972).

took up the suggestion

I. Ragnarsson and R. A. Broglia, Nucl. Phys. A263, 315 (1976).

coined the term "pairing isomers" for these 0^+ states

Single-Particle Quadrupole Moments in a deformed W-S potential

[505]11/2$^-$

Low Density of Oblate s-p States Below the Fermi Surface

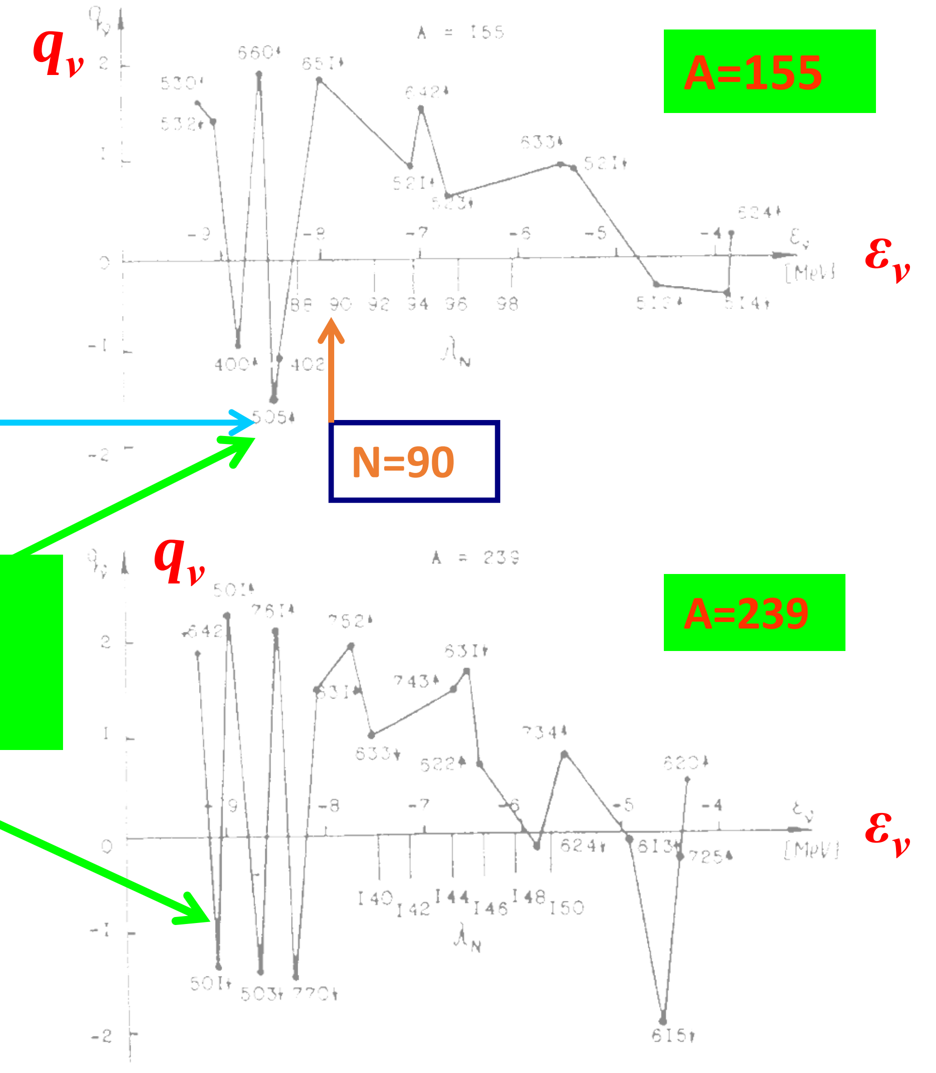


Fig.1. Distribution of the singe-particle quadrupole moments for the states in the Saxon-Woods potential well in two regions of deformed nuclei (q_ν are given in dimensionless units). The location of the chemical potential λ_N in nuclei with different number of neutrons is shown too.

Abdulvagabova, Ivanova & Pyatov

Phys. Lett. **28B** (1972) 215

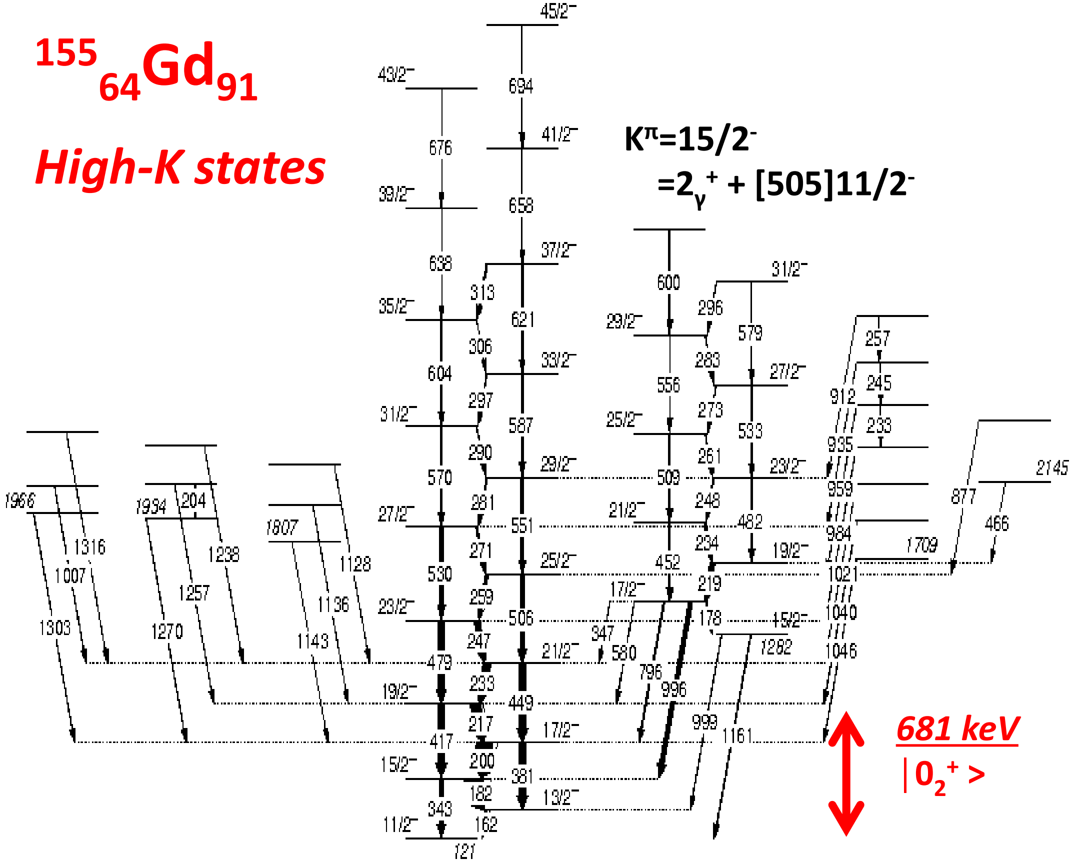
What is the $|0_2^+>$ Configuration ?

- (t,p) & (p,t) → $|0_2^+>$ is $2p_n$- $2h_n$ **this gives J^π but nothing on the orbit.**
- **Single particle transfer would give l_n but does not populate $|0_2^+>$.**

→ **In { $|0_2^+>$ + neutron }, look to see which orbit does *NOT* couple to $|0_2^+>$.**

$^{155}_{64}\mathrm{Gd}_{91}$

High-K states



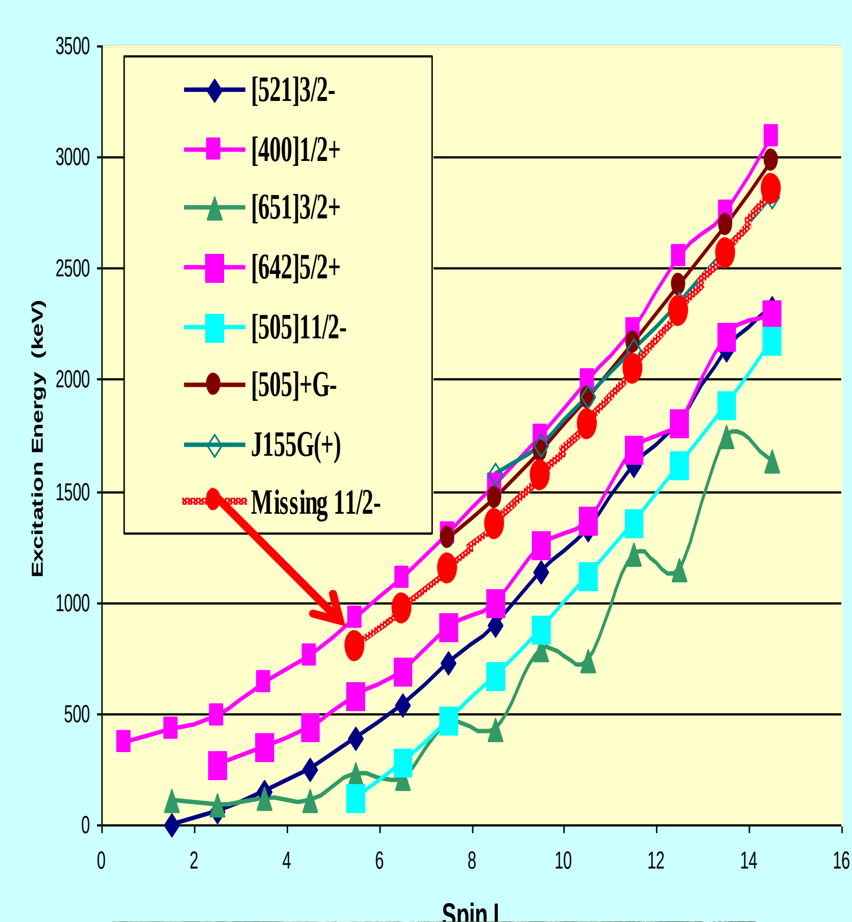
$K^\pi = 15/2^-$

$= 2_\gamma^+ + [505]11/2^-$

681 keV

$|0_2^+>$

[505]11/2⁻

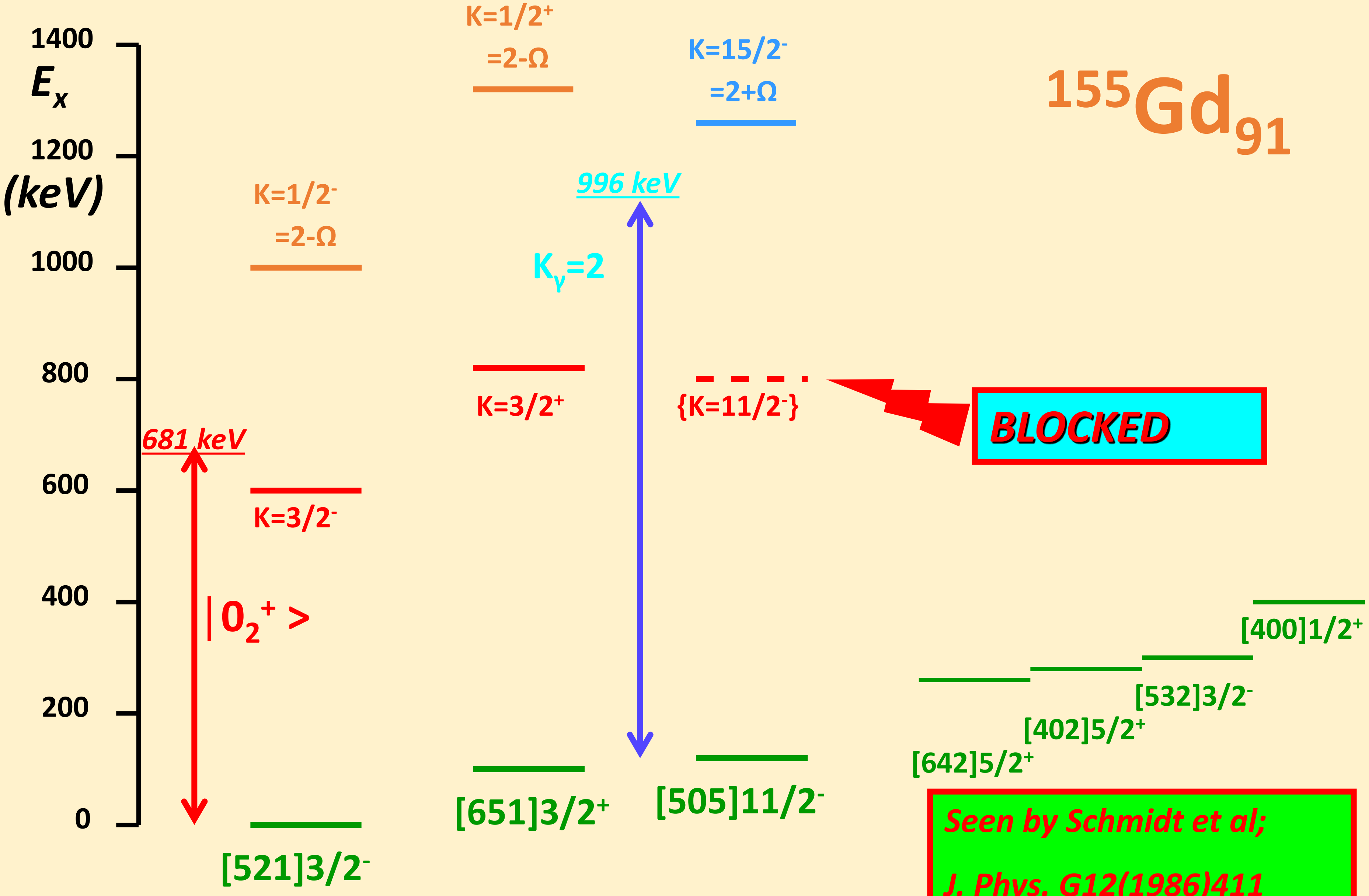


JFS-S et al. EPJ A47 (2011) 6

Tshifhwa Madiba
en familie

[505]11/2⁻ is BLOCKED from coupling to the core 0_2^+

$^{155}\mathrm{Gd}_{91}$



Seen by Schmidt et al;

J. Phys. G12(1986)411

in (n,γ) (d,p) & (d,t)

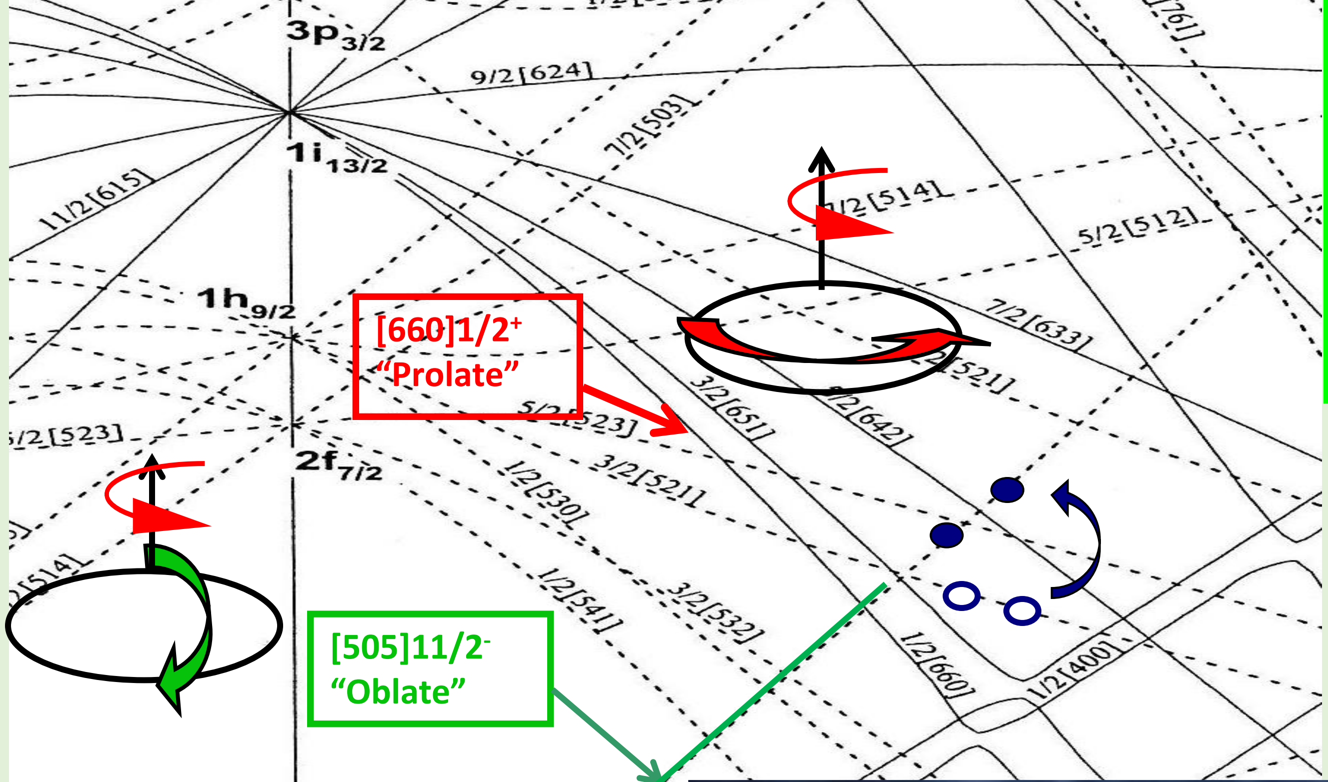
Configuration Dependent or Quadrupole Pairing; Assume

$\Delta_{pp} \approx \Delta_{oo} >> \Delta_{op}$

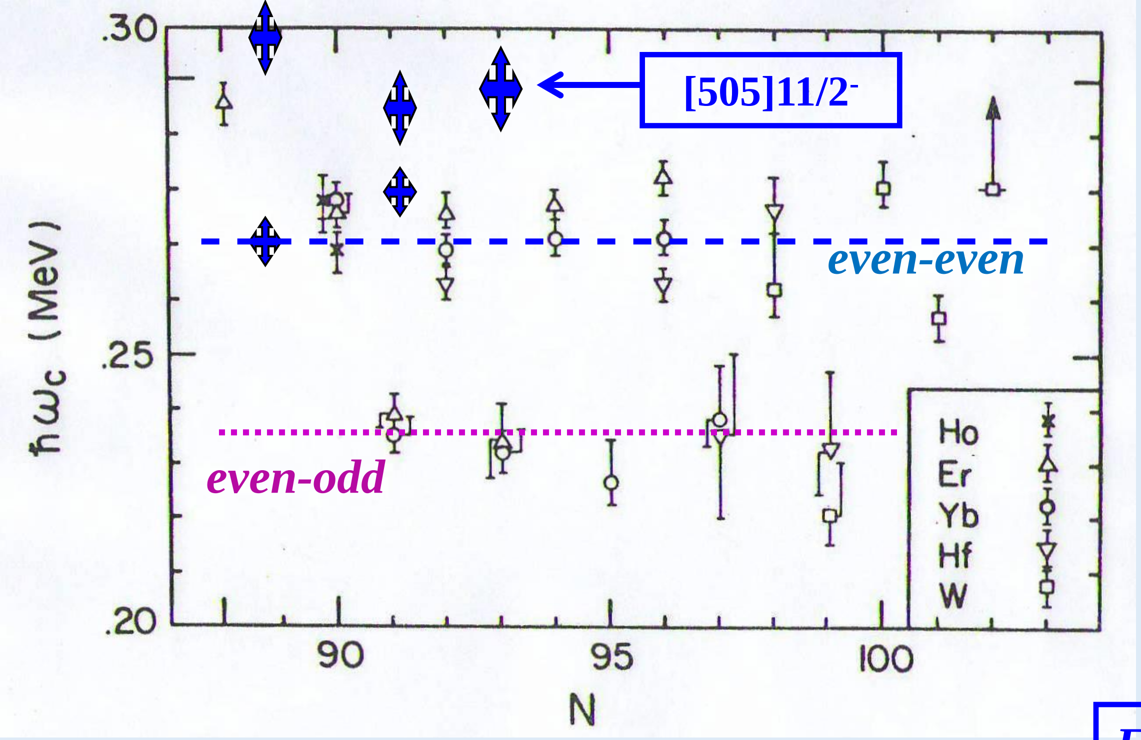
R. E. Griffin, A. D. Jackson and A. B. Volkov, Phys. Lett. **36B**, 281 (1971).

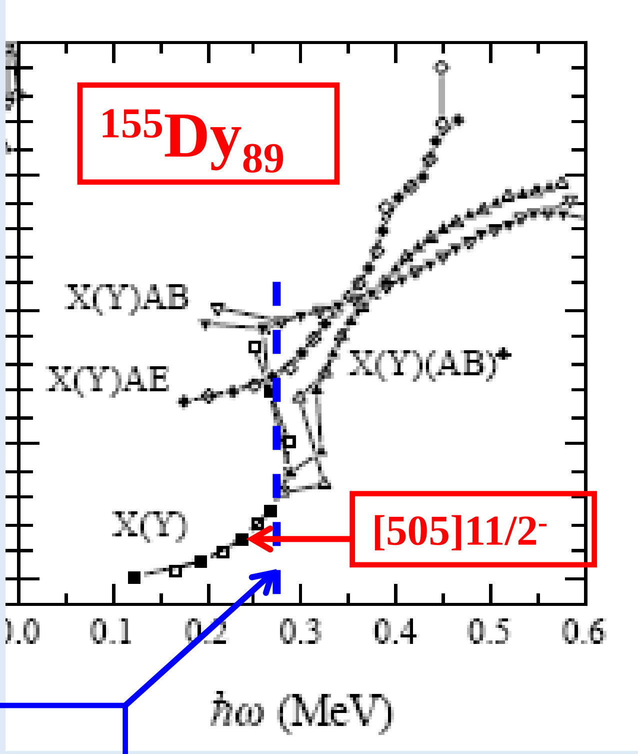
W. I. van Rij and S. H. Kahana, Phys. Rev. Lett. **28**, 50 (1972).

S. K. Abdulvagabova, S. P. Ivanova and N. I. Pyatov, Phys. Lett. **38B**, 251 (1972).

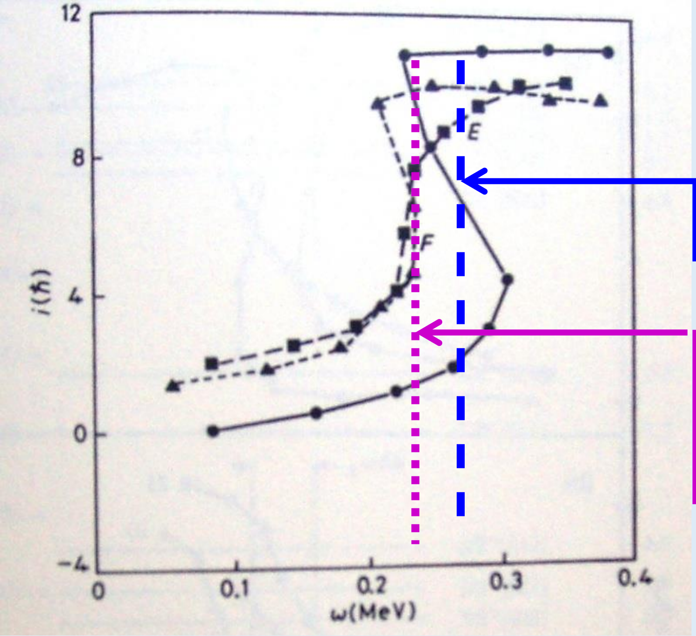


First Excited 0_2^+ states in In *N=88,90* nuclei





Full Unblocked Pairing



^{162}Yb$_{92}$ gsb

^{163}Yb$_{93}$ gsb

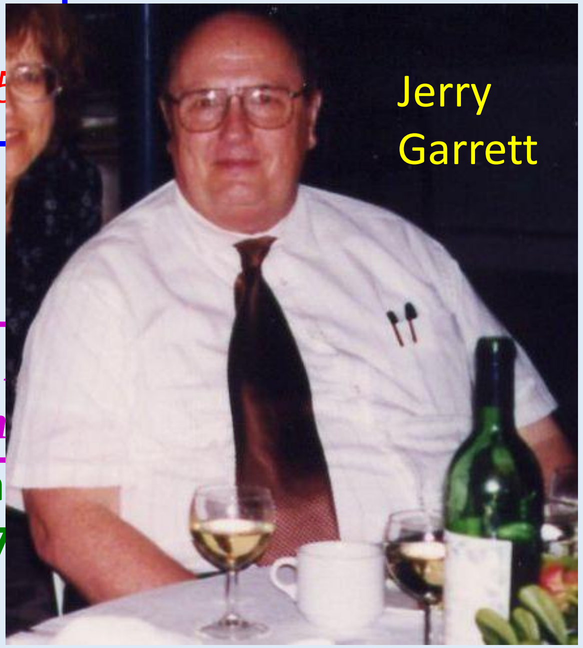
[521]3/2$^-$

Pairing

Neutron

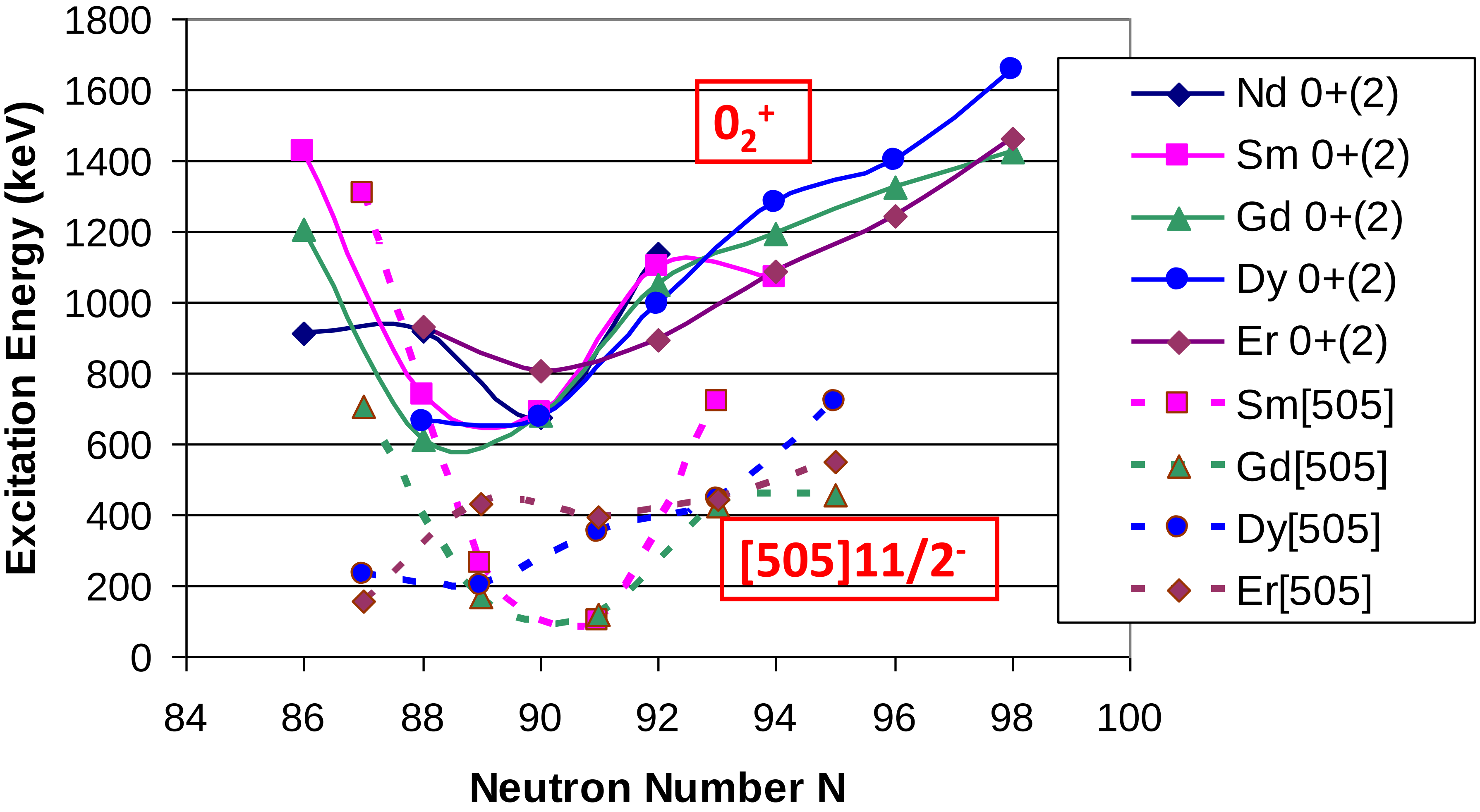
Jerry Ga

118, 297



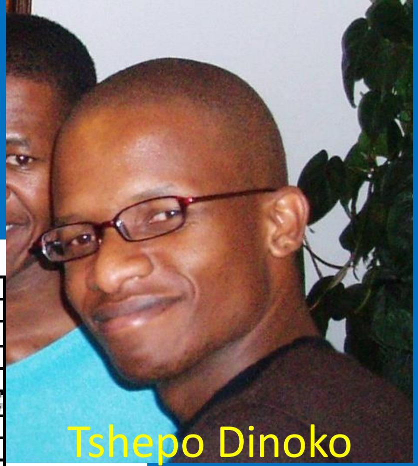
Systematics of Energies of 0_2^+ and [505]11/2$^-$ states

for Z = 60-68 and N = 86-98



^{150}Sm(^{12}C,4n)^{158}Er 65 MeV

Tshepo Dinoko, PhD Thesis, UWC



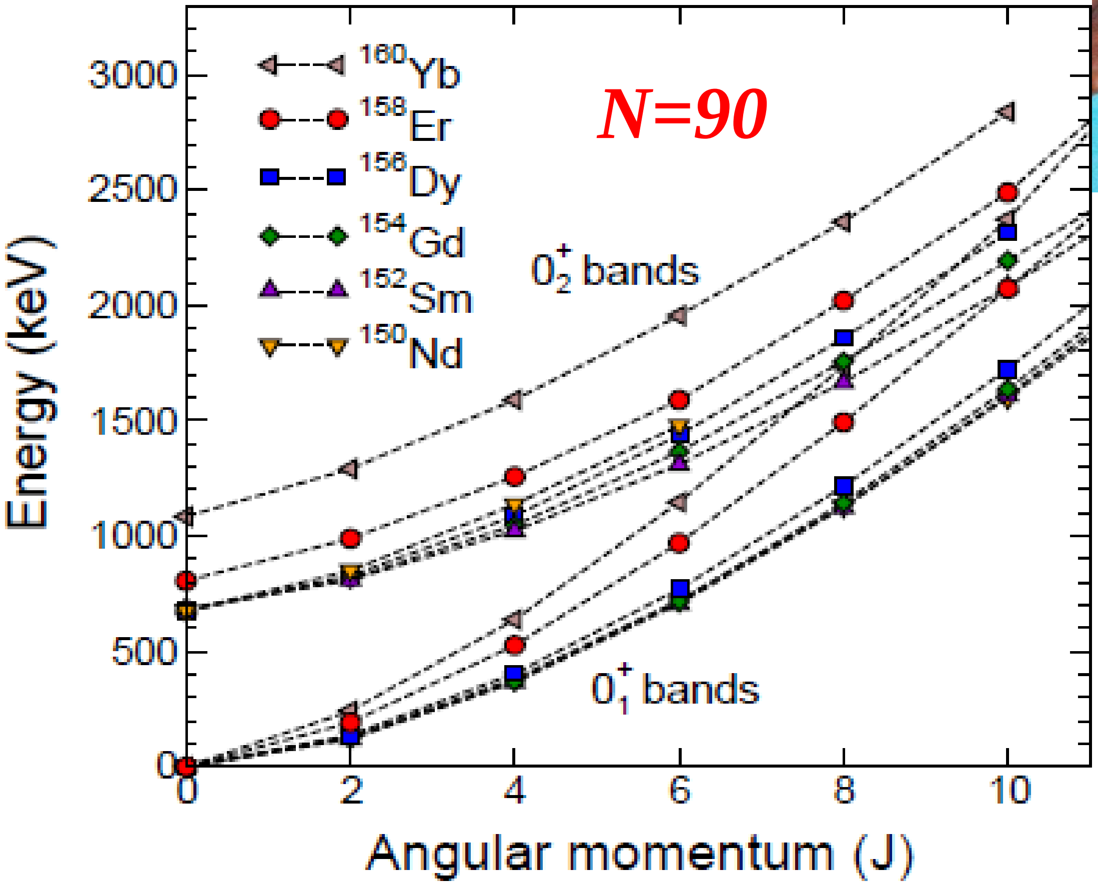
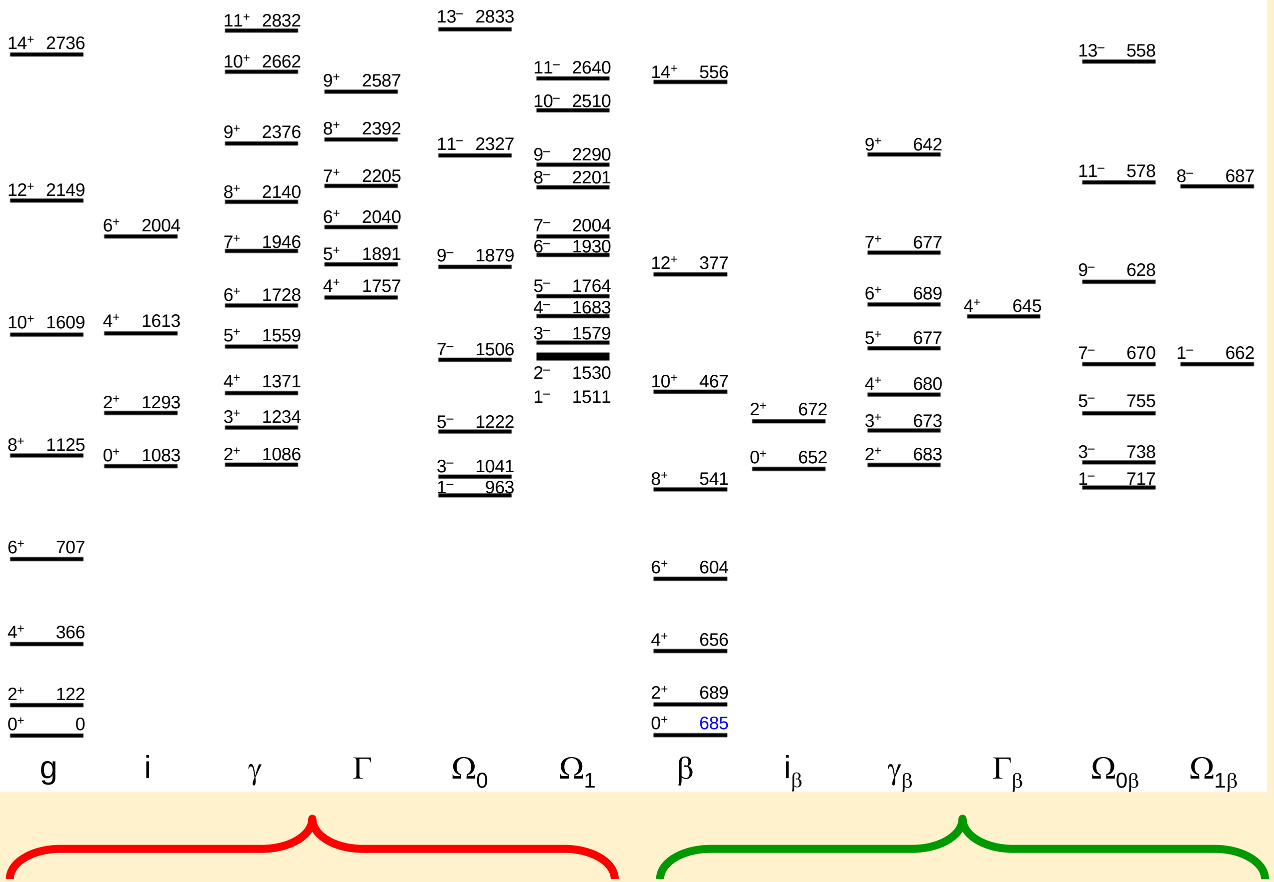


Figure 3: Excitation energy as a function of angular momentum for members of the 0_1^+ and 0_2^+ bands in the $N = 90$ isotones.

Coexisting (nearly) identical bands in ^{152}Sm

Dave Kulp, John Wood, Paul Garrett et al;

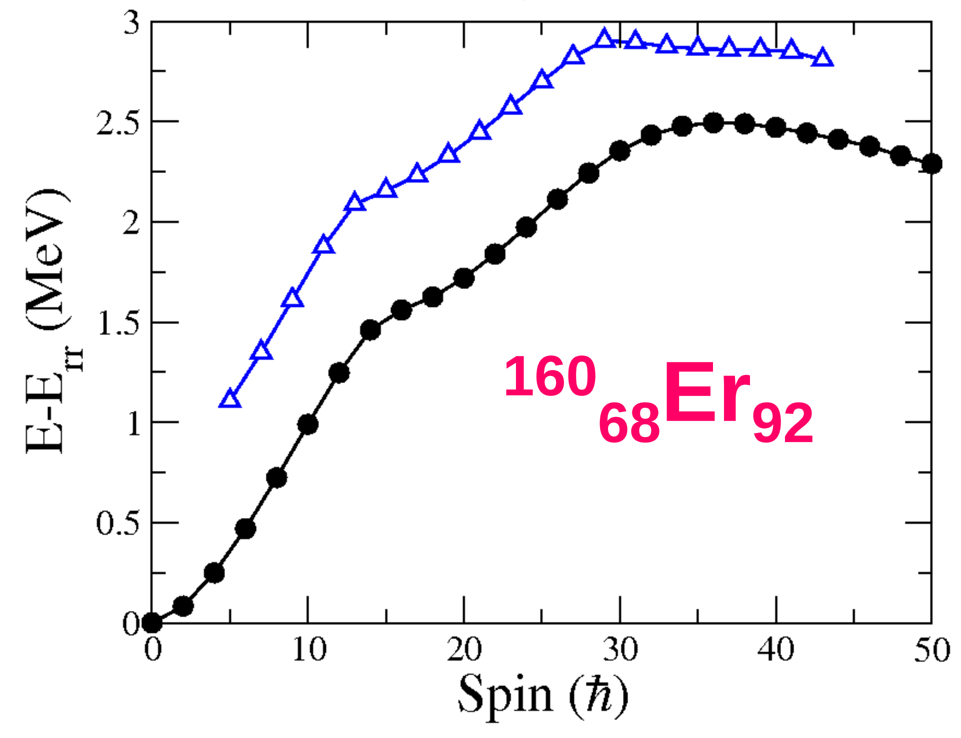
To be published.

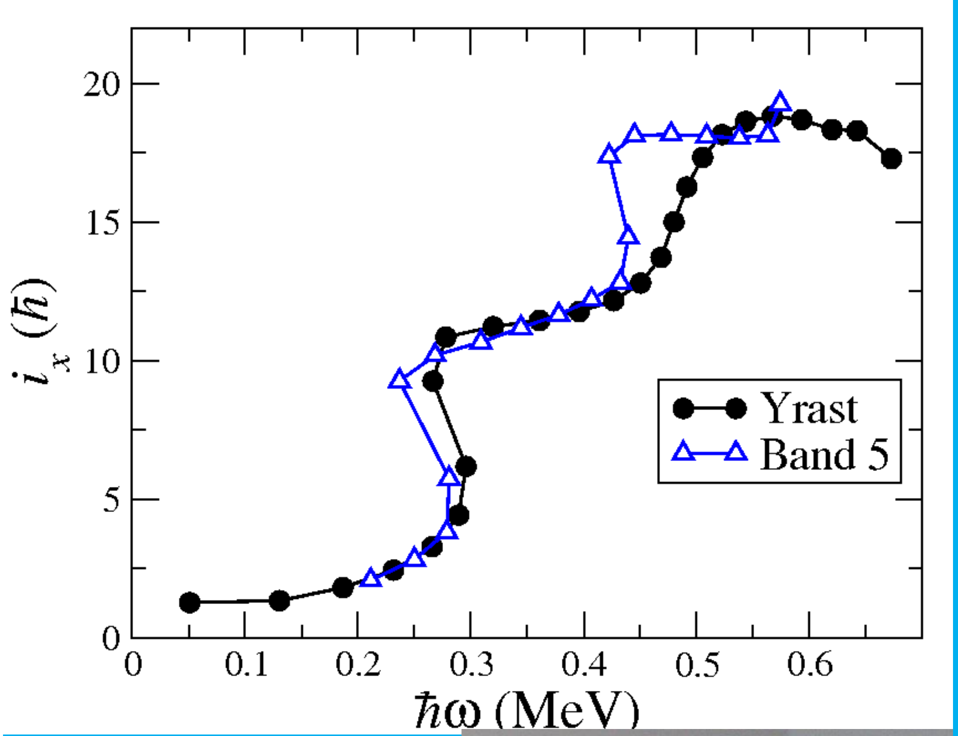


$|0_1^+>$

$|0_2^+>$

Congruent Structures in ^{152}Sm

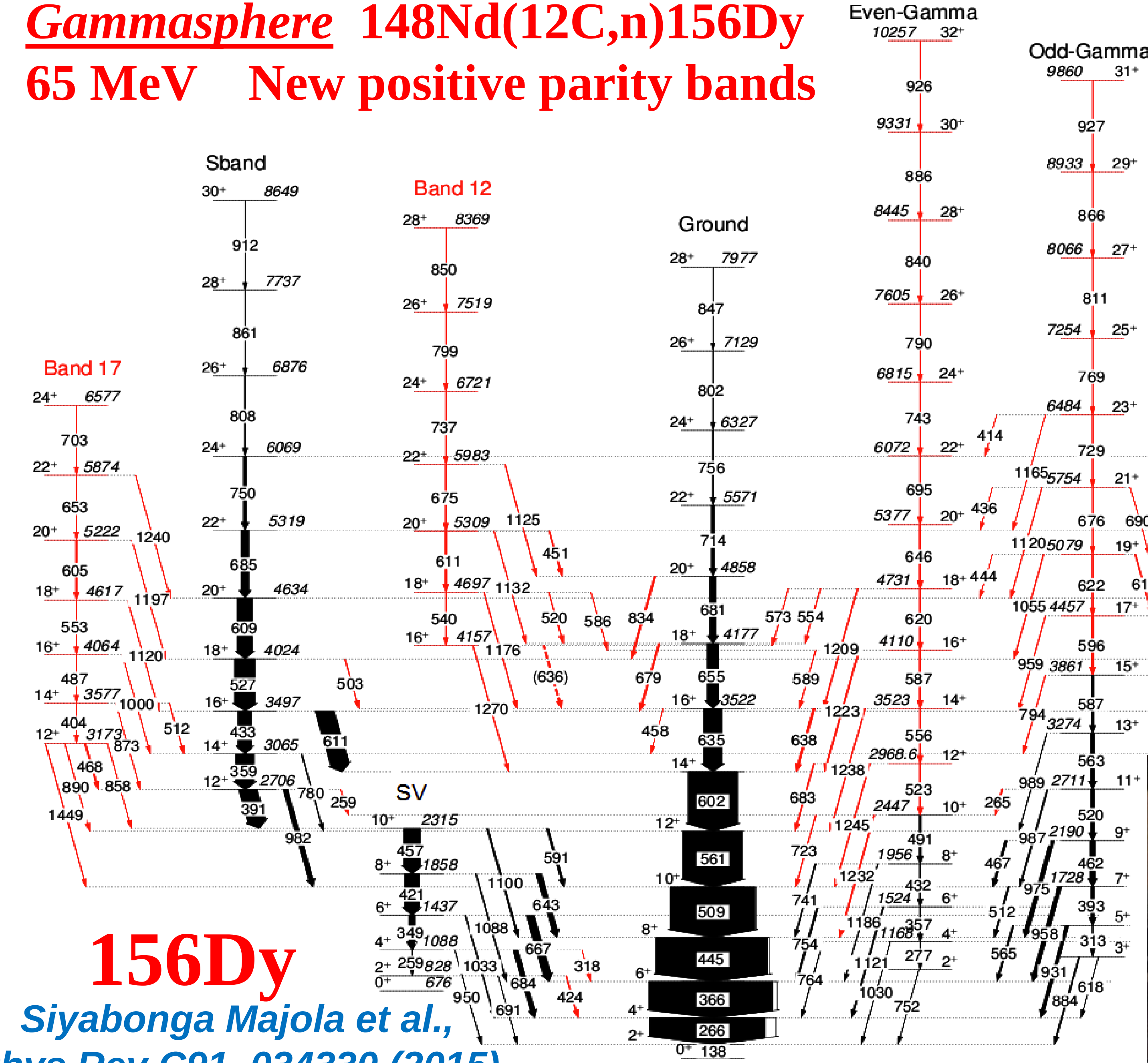




and in ^{160}Er

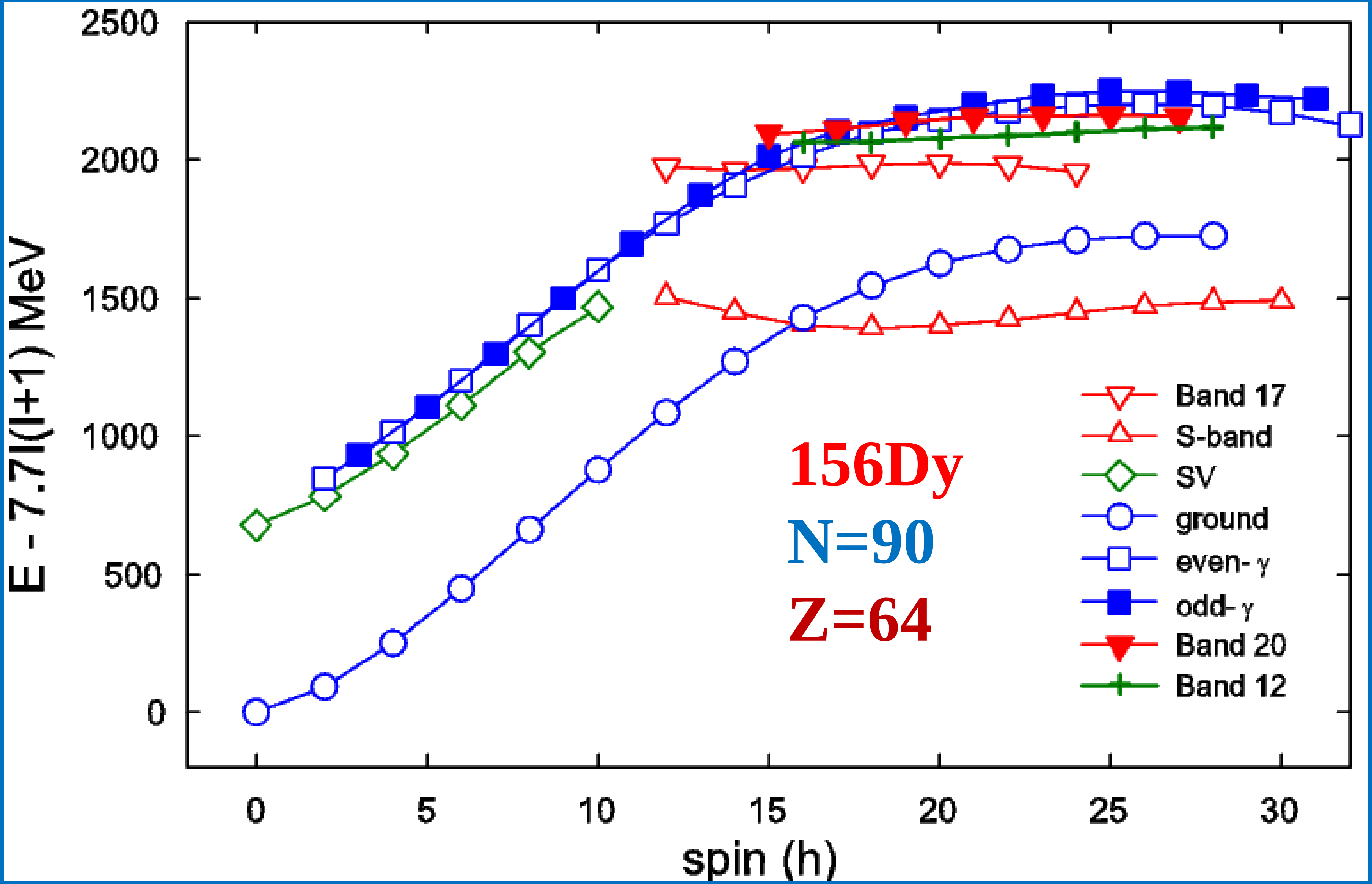
data; Ollier et al., Phys. Rev. C83

Gammasphere 148Nd(12C,n)156Dy
65 MeV New positive parity bands



156Dy

Siyabonga Majola et al.,
Phys Rev C91, 034330 (2015)


E - 7.7I(I+1) MeV
spin (h)
0
500
1000
1500
2000
2500
0
5
10
15
20
25
30
Band 17
S-band
SV
ground
even- γ
odd- γ
Band 20
Band 12
156Dy
N=90
Z=64

156Dy
N=90
Z=64

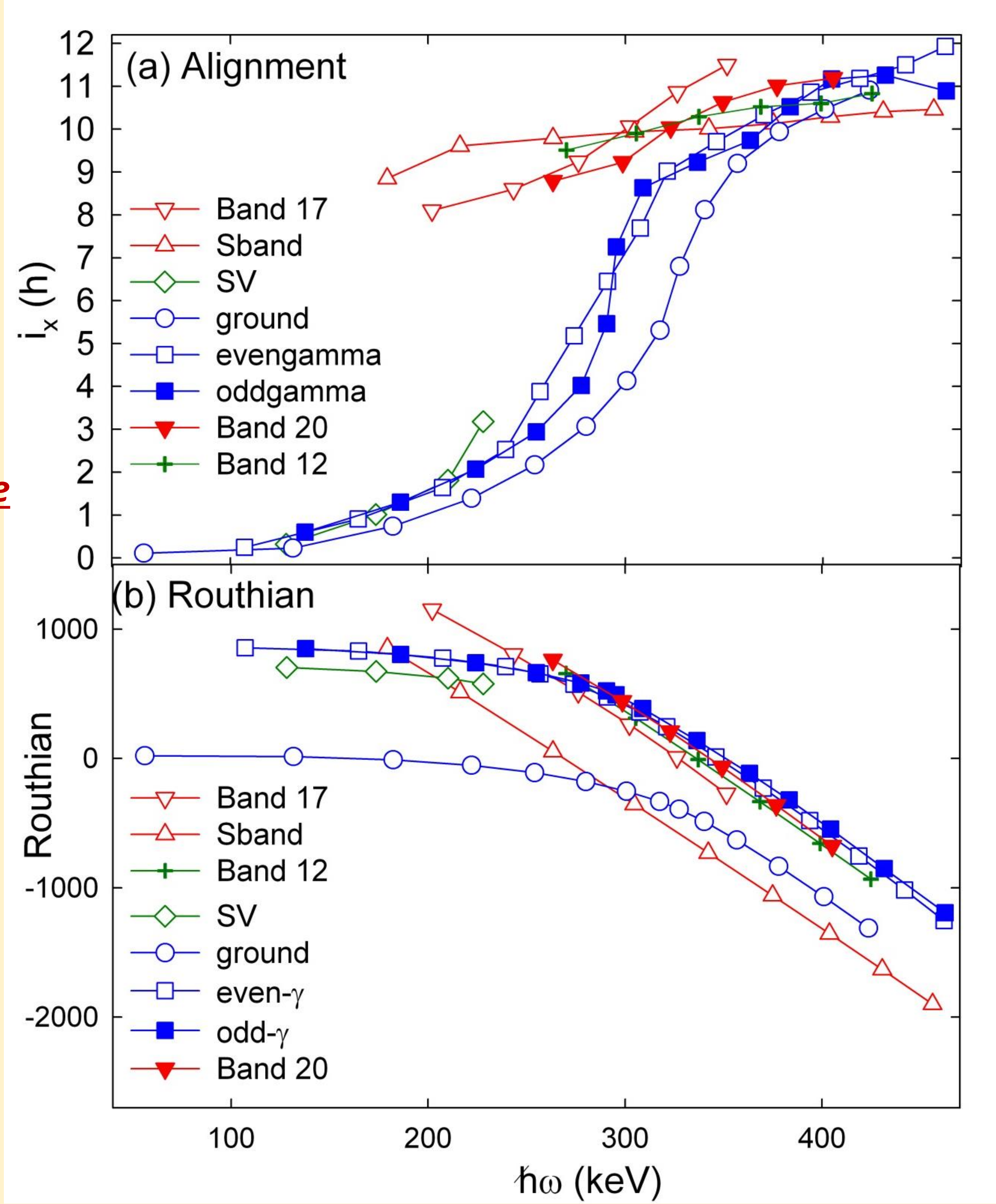
“The Sympletic Shell-Model Theory of Collective States”

Cavalho, Le Blanc, Vassanji, Rowe and McGrory

Nucl Phys A452 (1986) 240

“….the β and γ degrees of freedom are not algebraically linked to the other degrees of freedom… in any simple microscopically recognizeable way.”

“We choose…to think of K=2 bands as triaxial rotor states, rather than γ vibrations,….”



$^{165}_{67}\mathrm{Ho}_{98}$

Coulex

Thick

target

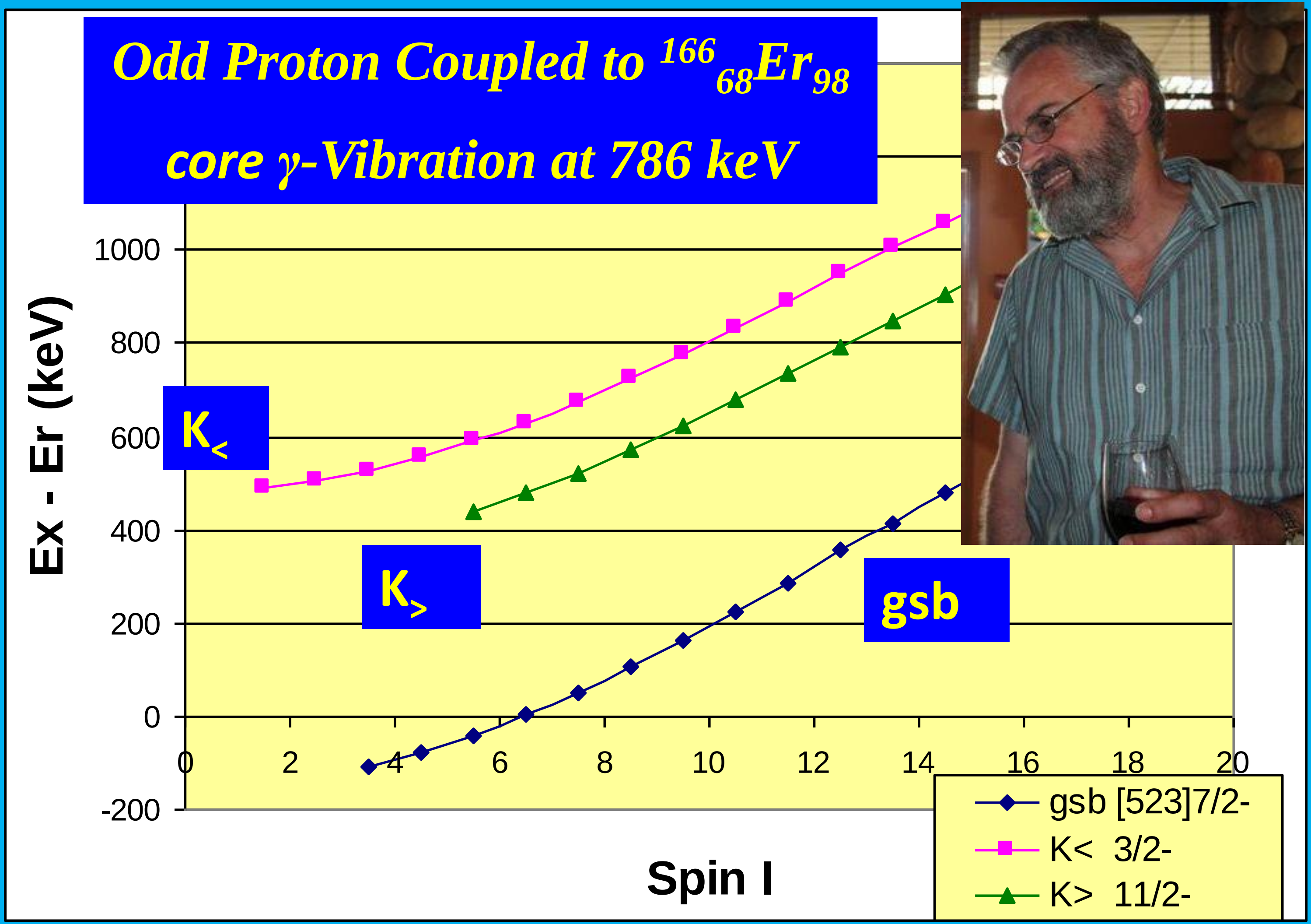
^{209}Bi beam

5.4 Mev/u

Chalk River

8π

Odd Proton Coupled to $^{166}_{68}\mathrm{Er}_{98}$ core γ-Vibration at 786 keV



The MEAN is lowered by 184 keV !!??

Dave Radford et al. Nucl Phys A624 (1997) 257

γ=2$^+$

gsb=7/2$^-$

$K_>$=11/2$^-$

Band head at 689 keV

Splitting ΔE = 174 keV

$K_<$=3/2$^-$

Band head at 515 keV

Deformed Rare Earths;

Zawischa, Speth & Pal, Nucl Phys A311(1978)445

QRPA, Deformed Woods-Saxon single particle Basis,

Zero-range density dependent residual interaction.

⇒"…it is more reasonable to identify the high-lying *$K^\pi = 0^+$ and 2^+* giant quadrupole resonances with the classical β- and γ-vibrations."

⇒ "This is due to the fact that the energies of the low-lying states are mainly given by the details of the single-particle structure at the Fermi surface whereas the high-lying states are of real collective nature."

⇒ "…the microscopic wave vector of the low-lying *$K^\pi = 0^+$* states is predominantly of the pairing-vibrational type in agreement with the enhanced two-particle transfer cross sections."

Conclusions

1. **If you break SPHERICAL symmetry, you get rotational bands**

2. **If you break AXIAL symmetry, you get $K^{\pi}=2^{+}$ bands**

Many thanks to all my collaborators:

Rob Bark, ***iThemba LABS***
Suzan Bvumbi, ***University of Johannesburg***
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Kati Juhász, ***University of Debrecen***
Elena Lawrie, ***iThemba LABS***
Kobus Lawrie, ***iThemba LABS***
Kevin Li, ***University of Stellenbosch***
Tshifhwa Madiba, ***University of Western Cape***
Siyabonga Majola, ***University of Cape Town***
Ani Minkova, ***University of Sofia***
Simon Mullins, ***iThemba LABS***
Barna Nyakó, ***ATOMKI Debrecen***
Paul Papka, ***University of Stellenbosch***
David Roux, ***University of Western Cape***
Maciej Stankiewicz, ***University of Cape Town***
Preston Vymers, ***University of Johannesburg***
Mathis Wiedeking, ***iThemba LABS***
And many others