The EOS of Dense Matter and Neutron Star Structure

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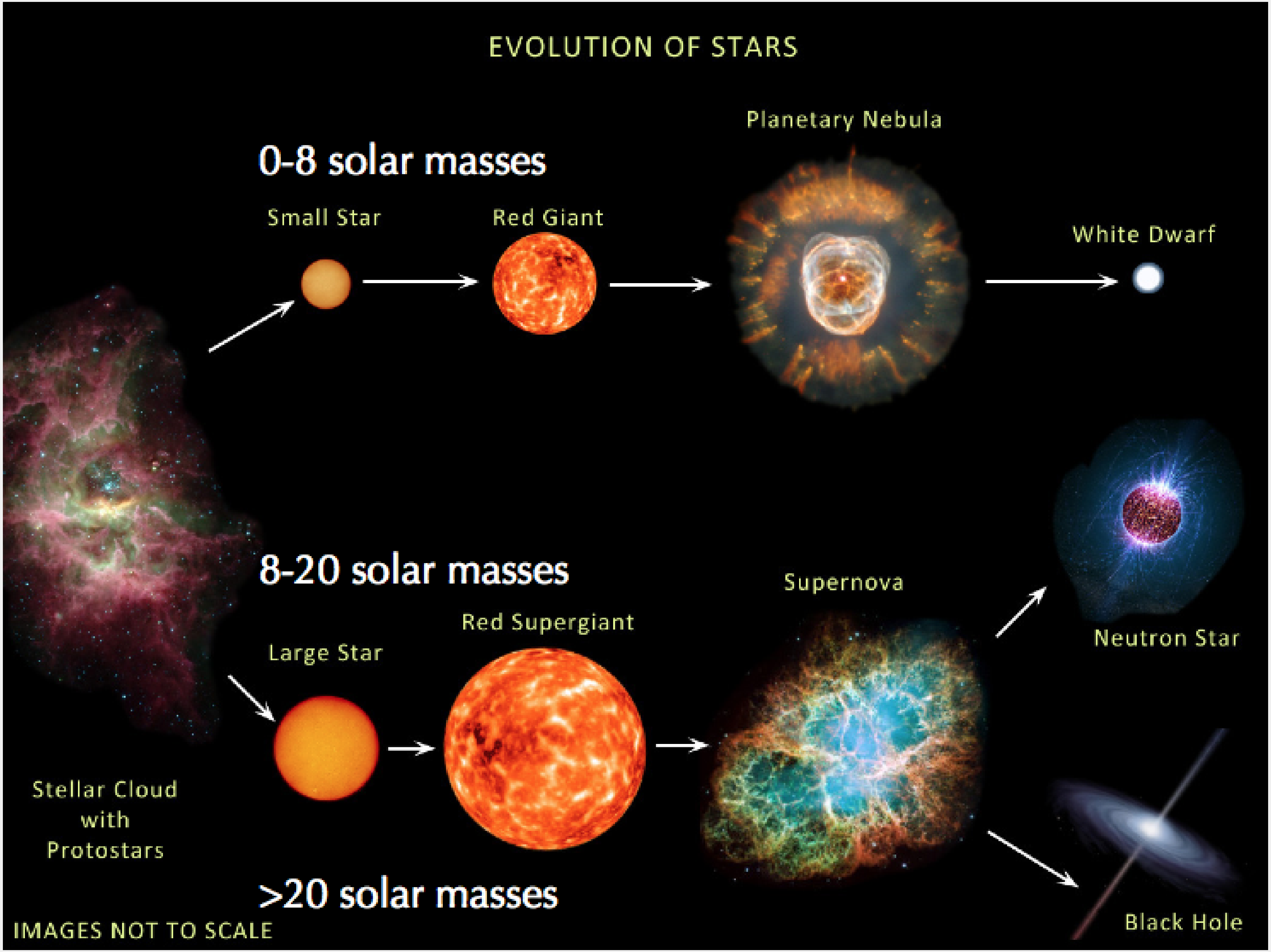
Outline

- Neutron stars
- Nuclei and the nucleon-nucleon interaction
- Neutron star matter
- Mass-radius curves
- Data analysis

Stellar Evolution



Neutron Star Composition

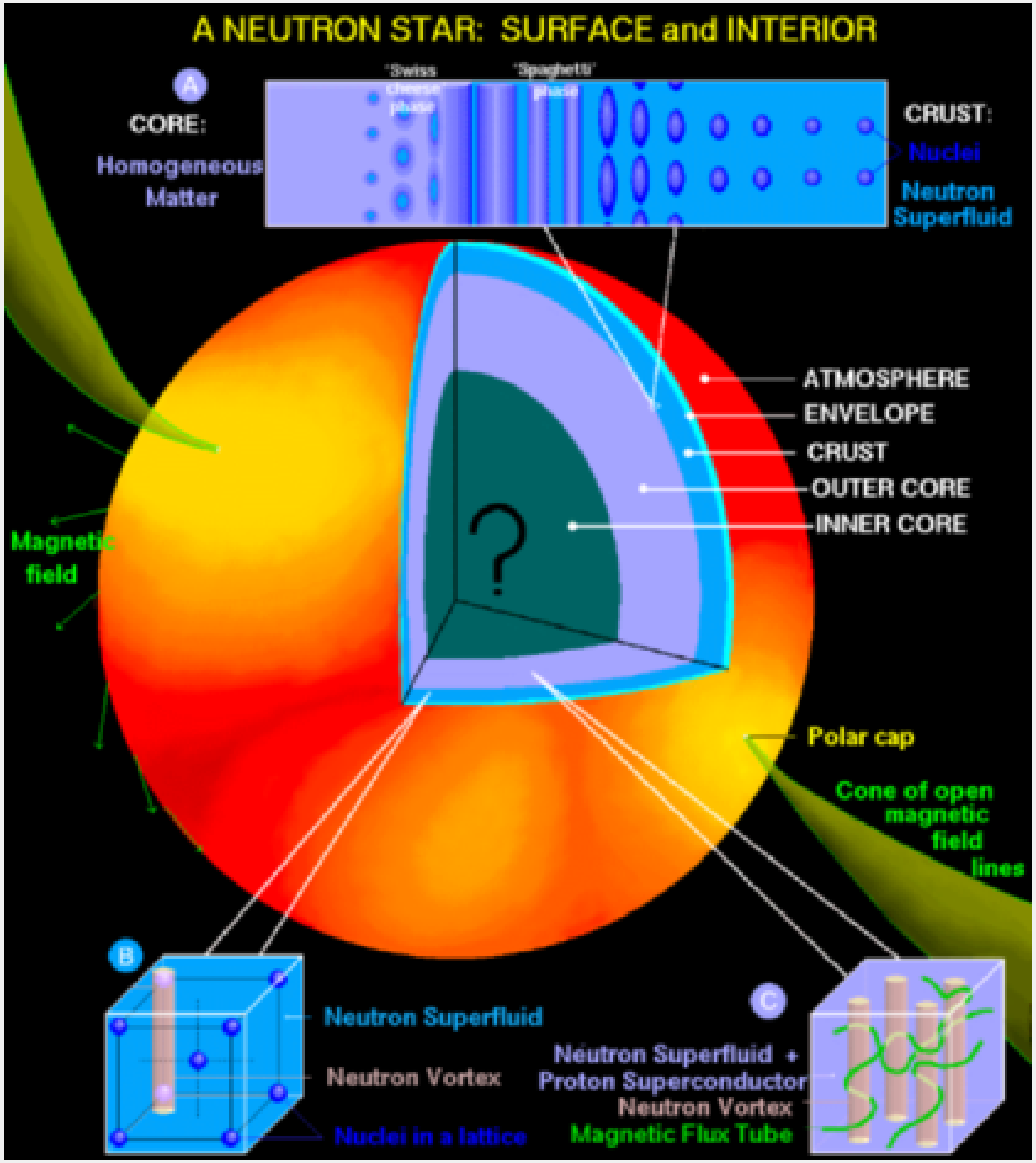


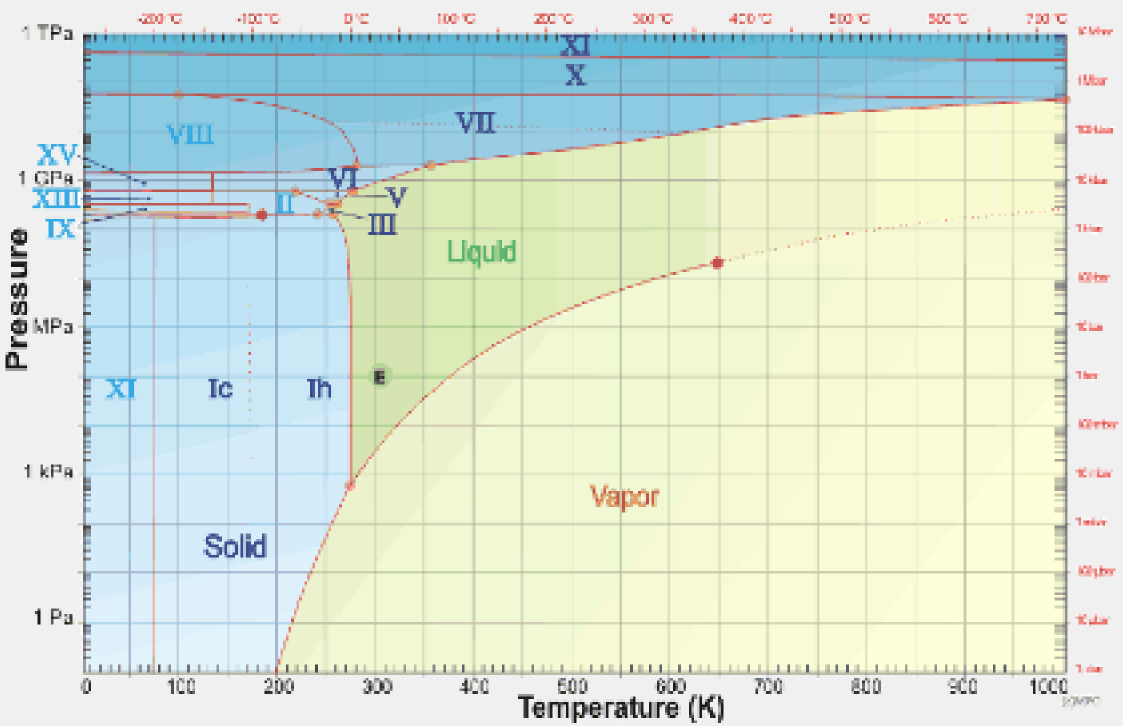
Figure by Dany Page

- Outer crust: of neutron-rich nuclei
- Inner crust: neutron-rich nuclei embedded in a sea of quasi-free superfluid neutrons
- Outer core: fluid of neutrons, protons, and electrons
- Inner core: hyperons, Bose condensates, deconfined quark matter

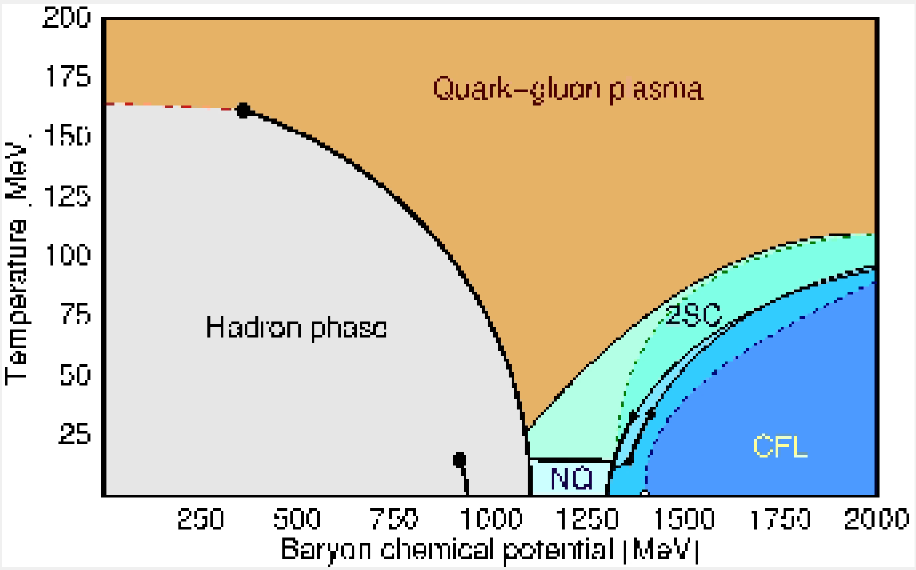
What are the correct degrees of freedom for the effective field theory which describes dense matter?

The QCD phase diagram

- QCD: The theory which describes the interactions of nucleons and quarks



http://www.lsbu.ac.uk/water/phase.html



Rüster et al. (2006)

- Heavy-ion collisions and lattice QCD sensitive primarily to high T, low μ regions

- Electromagnetic and gravitational wave observations of neutron star-related phenomena are the **best** probe of cold, dense (and non-perturbative) QCD

Weisacker-Bethe semi-empirical mass formula

$$E(Z,N) = -BA + E_{\text{surf}}A^{2/3} + CZ^2A^{-1/3} + S\frac{(N-Z)^2}{A}$$

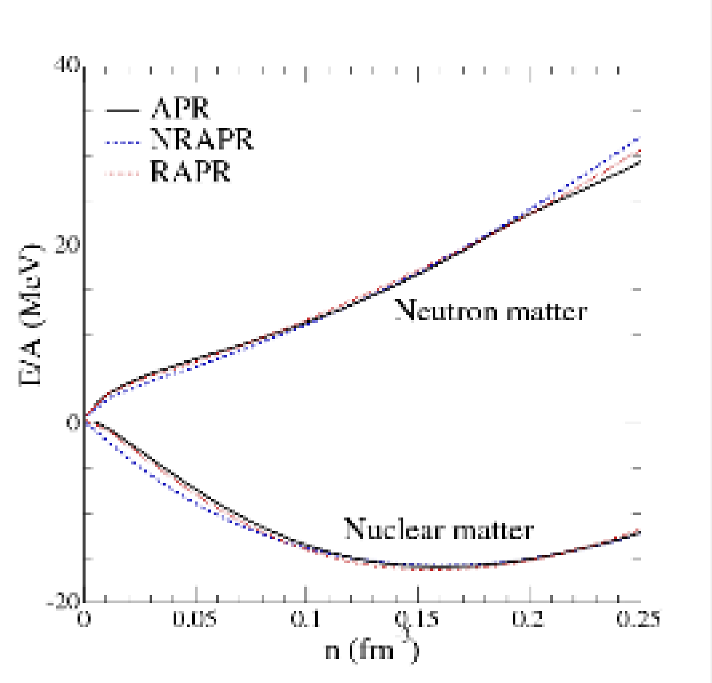
$$+E_{\text{pair}}\begin{cases} +1 & \text{N and Z odd} \\ -1 & \text{N and Z even} \\ 0 & \text{otherwise} \end{cases}$$

von Weisäcker (1935)

- Radius $\sim A^{1/3}$ - this is saturation
- Surface energy $\sim R^2 \sim A^{2/3}$; curvature energy $\sim R \sim A^{1/3}$
- Expansion in $1/R$
- Coulomb length scale = Debye screening length
- Important for the quark-hadron phase transition later
- Can add "shell effects" via Strutinsky method

Phenomenological equation of state

- Nucleonic matter near the saturation density
- n_n, n_p are number densities of neutrons and protons
- $n_B \equiv n_n + n_p$; $x \equiv n_p/n_B$; $E/A = \frac{\epsilon(n_n,n_p)}{n_B}$



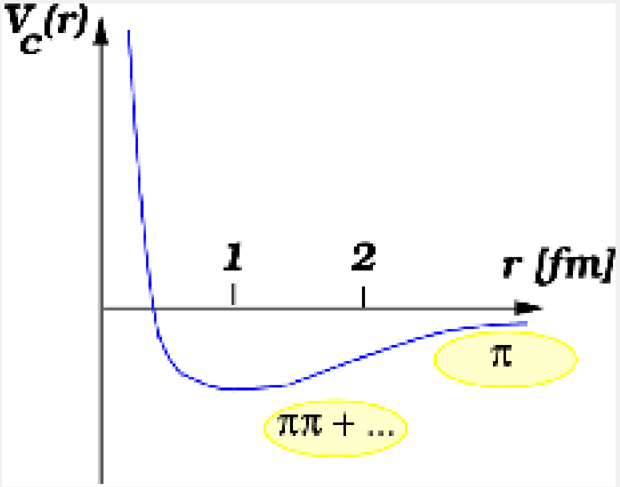
$$E/A = -B + \frac{K}{18n_0^2}(n_B - n_0)^2 + S(n_B)(1-2x)^2$$

- Binding energy, saturation density: nuclear masses
- Compressbility: giant monopole resonances
- Symmetry energy: many experiments and observations
- Particularly important is $L \equiv 3n_0 S'(n_0)$

Nucleonic matter

- "Nuclear matter", $n_n = n_p$
- "Neutron matter", $n_p = 0$
- "Neutron-star matter", beta-equilibrium, $\mu_n = \mu_p + \mu_e$; $n_n > n_p$
    - Only two conserved charges: baryon number and charge
    - $\mu_i = B_i \mu_n - Q_i \mu_e$
    - Charge neutrality: $n_Q = 0$

Nucleon-nucleon interaction



- One-pion exchange (attractive) at large distances, repulsion at short distance
- Phenomenological coordinate-space potentials: Argonne-potential

$$\mathcal{H} = \sum_i \frac{-\hbar^2}{2M_i}\nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$$

$$V_{ij}(r) = \sum_{p=1,8} v^p(r) O^p$$

$$O^p = (1, \sigma_1 \cdot \sigma_2, S_{12}, L \cdot S) \times (1, \tau_1 \cdot \tau_2)$$

Wiringa et al. (1995), Gandolfi et al. (2015)

- Three-nucleon force
  - Required for saturation

Quantum Monte Carlo

- Large class of methods; unmatched for describing light nuclei

- Diffusion Monte Carlo, project out ground state

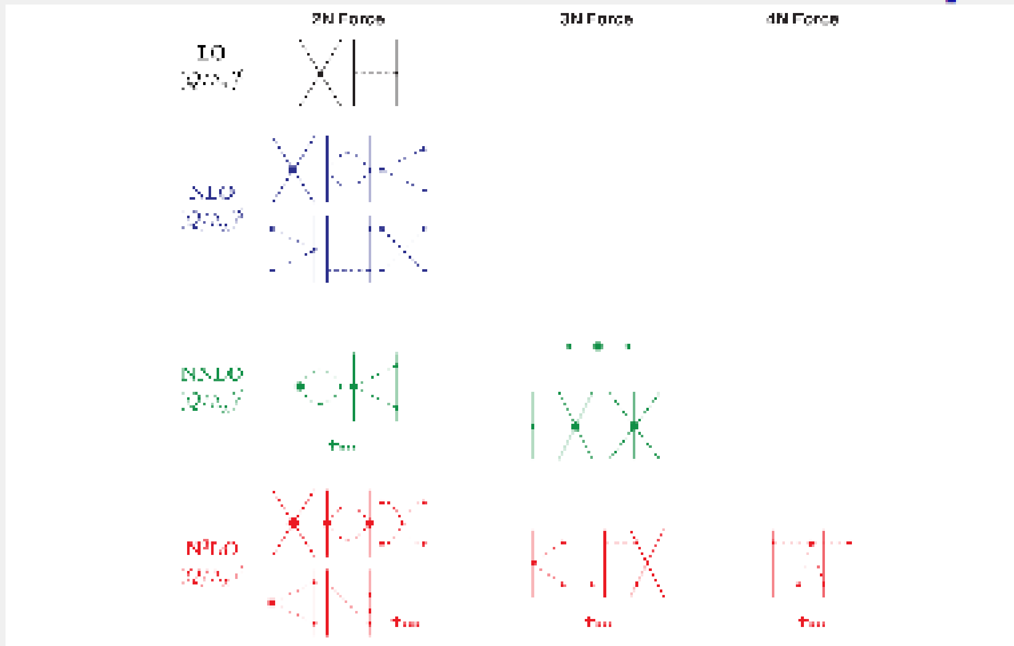
$$|\Psi_0\rangle = \lim_{\tau\to\infty} \left[\exp\left(\mathcal{H}\tau\right) \Psi_{\text{trial}}\right]$$

Gandolfi et al. (2015)

- Restricted to nearly local potentials
Gezerlis et al. making local versions of chiral interactions for QMC

- Fail to properly describe light nuclei and nuclear saturation
e.g. Akmal et al. get -12.61 MeV instead of -16 MeV for saturation

Chiral effective theory



- QCD (except for mass terms) has a SU(3)×SU(3) chiral symmetry
- This symmetry is spontaneously broken, and pions are nearly massless Goldstone bosons
- At low momenta, nucleon-nucleon interaction dominated by pions
- This works up to a scale, $\Lambda_\chi = 500 - 1000$ MeV
- Use an effective interaction with undetermined coupling constants: fixed from light nuclei
- Neutron matter is "perturbative" at low densities

Machleidt and Entem (2011); Epelbaum et al. (2009)

Energy density functionals

- There is a energy density functional for a many-body system and the ground state density is the minimum of this functional
  Hohenberg and Kohn
- Gradient expansion:

$$\varepsilon(n_n, n_p) = \tau_n + \tau_p + \varepsilon_{\rm int}(n_n, n_p) + \varepsilon_{\rm grad}(n_n, n_p)\nabla(n_n + n_p)^2$$

$$+\varepsilon_{\rm grad,isovec}(n_n, n_p)\nabla(n_n - n_p)^2 + \text{higher gradient terms}$$

- τ_i is the non-relativistic non-interacting Fermi gas energy
- Generalize to finite temperature by $\tau_i \rightarrow \tau_i(T)$
- Compute systems in the mean-field approximation, but implicitly include correlations
- More complicated functionals include pairing and spin-orbit densities: state of the art for heavy nuclei

Thomas-Fermi or Hartree-Fock

- If you have an energy density functional, then the ground state is formed from the variation of

$$\delta \int d^3r \; \left[\mathcal{H} - \mu_n n_n(r) - \mu_p n_p(r)\right]$$

with respect to the neutron and proton densities

Hartree-Fock

- Represent the many-body wave function as a Slater determinant:

$$\phi(x_1, x_2, \ldots, x_A) = \frac{1}{\sqrt{A!}} \det |\phi_i(x_j)|$$

- Then,

$$\rho_q(r) = \sum_{i,\sigma} |\phi_i(r,\sigma,q)|^2$$

$$\tau_q(r) = \sum_{i,\sigma} \left|\vec{\nabla}\phi_i(r,\sigma,q)\right|^2$$

- and again, we minimize the energy.

Vautherin and Brink (1972)

Covariant mean-field theory

- Walecka model:

$$\mathcal{L} = \bar{\psi}_i \left(i\partial\!\!\!/ - m_i + g_\sigma \sigma - g_\omega \omega\!\!\!/ - \frac{g_\rho}{2} \vec{\rho\!\!\!/} \cdot \vec{\tau} \right) \psi_i$$

$$+ \frac{1}{2} (\partial^\mu \sigma) \left(\partial_\mu \sigma \right) - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{bM}{3} (g_\sigma \sigma)^3 - \frac{c}{4} (g_\sigma \sigma)^4$$

$$+ \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu + \frac{1}{2} m_\rho^2 \rho^\mu \rho_\mu - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4} \vec{B}_{\mu\nu} \vec{B}^{\mu\nu}$$

- Mean-field approximation; Dirac equation for nucleons and meson field equations
- Relativistic, so in principle can go to higher densities
- Spin-orbit force appears more naturally
- No problem with $c_s^2 > c$
- In this form, limited control of symmetry energy
- Natural generalization to include hyperons

Density-dependent couplings

- Promote couplings g_σ, g_ω and g_ρ to functions of density
  Typel et al. (1999)

- More parameters and more control

- Not as much work on nuclear structure side

- Not clear if this is as successful as modern energy density functionals

Hyperons

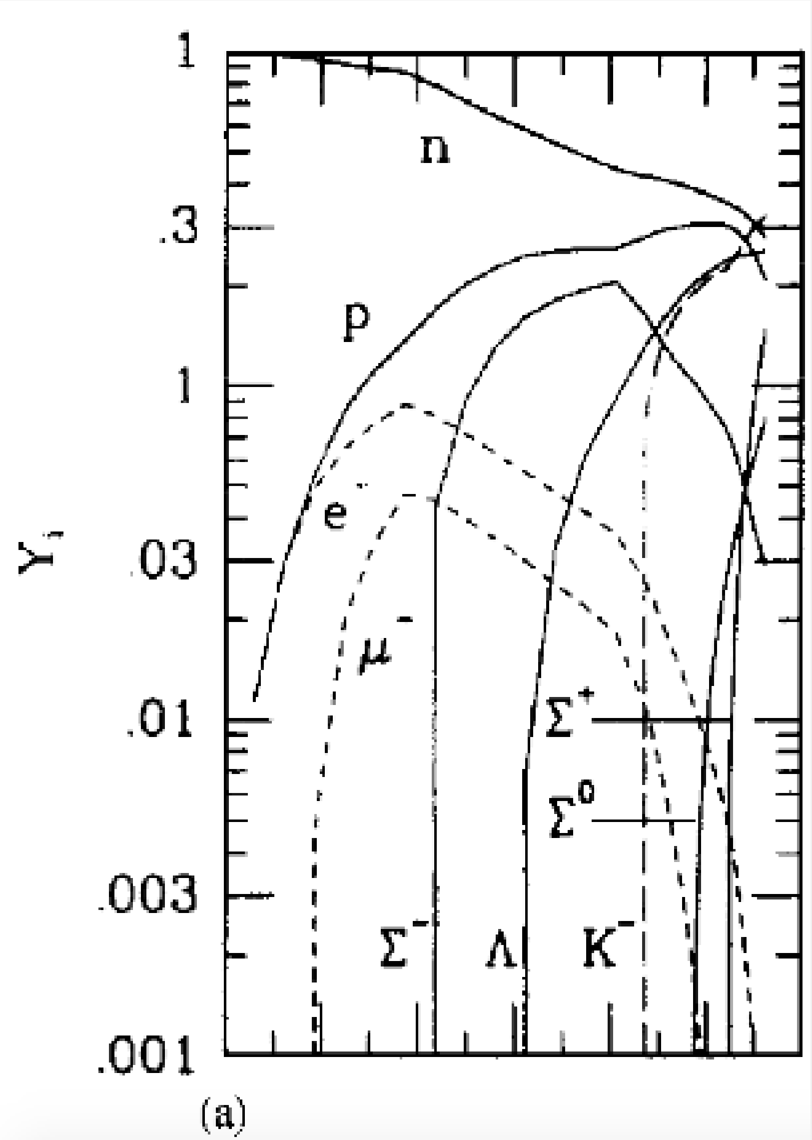
- Appear when, e.g. $\mu_\Lambda(n_B, n_{\Lambda=0}) > \mu_n(n_B)$ or $\mu_{\Sigma^-}(n_B, n_{\Sigma^-=0}) > \mu_n(n_B) + \mu_e(n_B)$
- Hyperons decrease the pressure (less degeneracy)
- Binding energy of Λ in nuclear matter $\sim$ 30 MeV
- Nucleons only models imply $\mu_n \sim$ 1600 MeV in the core
- Density-functional, Brueckner, or Walecka-type models
- "Hyperon problem" - 2 $M_\odot$ neutron star

TABLE III. Λ separation energies (in MeV) obtained using the two-body plus three-body hyperon-nucleon interaction with the set of parameters (II). The results already include the CSB contribution. In the last column are the expected B_Λ values. No experimental data for $A = 17, 18, 41, 49, 91$ exist. For ${}^{17}_{\Lambda}$O the reference separation energy is a semiempirical value. For $A = 41, 49, 91$ the experimental hyperon binding energies are those of the nearest hypernuclei ${}^{40}_{\Lambda}$Ca, ${}^{51}_{\Lambda}$V, and ${}^{89}_{\Lambda}$Y respectively.

System	AFDMC B_Λ	Expt. B_Λ
${}^{3}_{\Lambda}$H	−1.22(15)	0.13(5) [12]
${}^{4}_{\Lambda}$H	0.95(9)	2.04(4) [12]
${}^{4}_{\Lambda}$He	1.22(9)	2.39(3) [12]
${}^{5}_{\Lambda}$He	3.22(14)	3.12(2) [12]
${}^{6}_{\Lambda}$He	4.76(20)	4.25(10) [12]
${}^{7}_{\Lambda}$He	5.95(25)	5.68(28) [22]
${}^{13}_{\Lambda}$C	11.2(4)	11.69(12) [13]
${}^{16}_{\Lambda}$O	12.6(7)	12.42(41) [65]
${}^{17}_{\Lambda}$O	12.4(6)	13.0(4) [31]
${}^{18}_{\Lambda}$O	12.7(9)	
${}^{41}_{\Lambda}$Ca	19(4)	18.7(1.1) [14]
${}^{49}_{\Lambda}$Ca	20(5)	19.97(13) [66]
${}^{91}_{\Lambda}$Zr	21(9)	23.11(10) [66]

Boson condensates

- Equilibrium: $\mu_{\pi^-} = \mu_e$
- π^- and K^- condensates most common
- Decrease electron pressure and maximum mass
- Models based on chiral symmetry
  Kaplan and Nelson (1986)
- There is also π^0 condensation implicitly inside Akmal et al. (1998)



Prakash et al. (1997)

Phase transitions in dense quark matter

- Chiral symmetry breaking: order parameter is $\langle \bar{q}q \rangle$
- Confinement
- Quark superconductivity
  - Color-flavor-locked phase:

$$E_0 = \sqrt{p^2 + 2\Delta^2} \quad E_{i=1\ldots8} = \sqrt{p^2 + \Delta^2}$$

  - Two flavor superconducting phase:

$$E_{dr-ug} = \sqrt{p^2 + \Delta^2} \quad E = \sqrt{p^2 + m^2}$$

- Nearly free quarks, strange quark mass, CFL gap

$$P = -B + \frac{3(1-c)}{4\pi^2}\mu^4 - \frac{3}{4\pi^2}\left(m_s^2 - 4\Delta^2\right)\mu^2$$

Nambu Jona-Lasinio model

$$\mathcal{L} = \bar{q}_{i\alpha}\left(i\partial\delta_{\alpha\beta} - m_{ij}\delta_{\alpha\beta} - \mu_{ij,\alpha\beta}\gamma^0\right) q_{j\beta} + G\sum_{a=0}^{8}\left[\left(\bar{q}\lambda^a q\right)^2 + \left(\bar{q}i\gamma_5\lambda^a q\right)^2\right]$$

$$+G_\Delta \sum_k \sum_\gamma \left(\bar{q}_{i\alpha}\epsilon_{ijk}\epsilon_{\alpha\beta\gamma}q^C_{j\beta}\right)\left(\bar{q}^C_{i'\alpha'}\epsilon_{i'j'k}\epsilon_{\alpha'\beta'\gamma}q_{j'\beta'}\right)$$

$$+G_\Delta \sum_k \sum_\gamma \left(\bar{q}_{i\alpha}i\gamma_5\epsilon_{ijk}\epsilon_{\alpha\beta\gamma}q^C_{j\beta}\right)\left(\bar{q}^C_{i'\alpha'}i\gamma_5\epsilon_{i'j'k}\epsilon_{\alpha'\beta'\gamma}q_{j'\beta'}\right)$$

Hatsuda and Kunihiro (1987); Klevansky (1992); Buballa (2005); from Steiner et al. (2002)

- Exhibits a chiral phase transition
- Model for color superconductivity
- Same symmetries as QCD
- Often used in the mean-field approximation

$$\bar{q}_1 q_2 \bar{q}_3 q_4 \to \langle\bar{q}_1 q_2\rangle \bar{q}_3 q_4 + \bar{q}_1 q_2 \langle\bar{q}_3 q_4\rangle - \langle\bar{q}_1 q_2\rangle\langle\bar{q}_3 q_4\rangle$$

- Maximize pressure w.r.t. $\langle\bar{q}q\rangle$, $\left\langle\bar{q}q^C\right\rangle$, and $\mu_{ij,\alpha\beta}$
- Non-renormalizable, UV cutoff at $p = \Lambda$

't Hooft interaction

- Six-fermion interaction for $U_A(1)$ breaking

$$\mathcal{L}_{'t\,Hooft} = -K\left[\mathrm{det}_f\,\bar{q}\,(1-i\gamma_5)\,q + \mathrm{det}_f\,\bar{q}\,(1+i\gamma_5)\,q\right]$$

't Hooft (1986)

- Color-superconducting six-fermion interaction

$$\mathcal{L}_{CSC\,'t\,Hooft} = -K_\Delta \epsilon^{ijm}\epsilon^{k\ell n}\epsilon^{\alpha\beta\varepsilon}\epsilon^{\gamma\delta\eta}\left(\bar{q}_{i\alpha} i\gamma_5 q^C_{j\beta}\right)\left(\bar{q}^C_{k\gamma} i\gamma_6 q_{\ell\delta}\right)\left(\bar{q}_{m\varepsilon} q_{n\eta}\right)$$

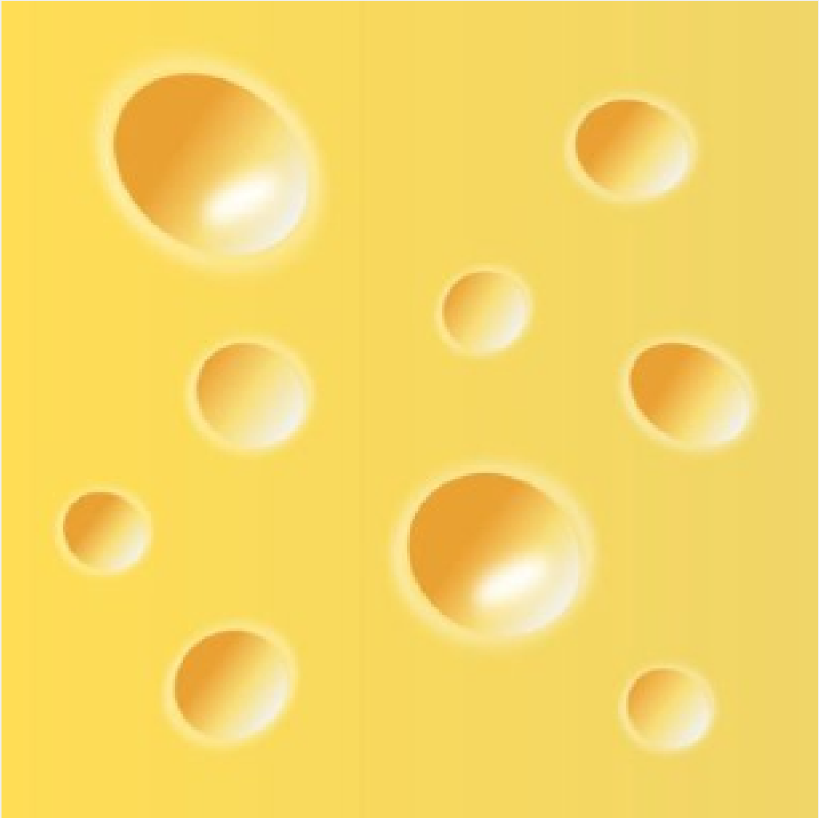
First analyzed in Steiner (2005)

Alternatives

- High-density effective theory: similar results to NJL models
  D.K. Hong and T. Schäfer

- Polyakov-quark-meson model
  e.g. Mintz et al. (2013)

Gibbs Phase Construction



- Transition at a surface: mechanical $P_1 = P_2$, chemical $\mu_{i,1} = \mu_{i,2}$, and thermal equilibrium, $T_1 = T_2$
- A mixture of the two phases: volume fraction of the low-density phase, $\chi \to$
- Local charge neutrality

  Glendenning

$$n_Q = 0 = \chi n_{Q,\mathrm{low}} + (1-\chi) n_{Q,\mathrm{high}}$$

$$n_B = \chi n_{B,\mathrm{low}} + (1-\chi) n_{B,\mathrm{high}}$$

- Minimize "internal" energy density, $\varepsilon(s, n)$
- Minimize free energy density, $f(n, T)$
- Maximize pressure, $P(\mu, T)$

Surface energy

- Surface tension is surface energy per unit area, $\sigma = S/(4\pi r^2)$
- Quark spheres embedded in hadronic matter are analogous to nuclear matter embedded in vacuum
- Sharp transition is limit of large surface tension
- Gibbs phase transition is often computed with zero surface tension
- We can compute these energies with Thomas-Fermi or with Hartree-Fock

Tolman-Oppenheimer-Volkov equations

- Einstein's field equations for a static, spherically symmetric, non-rotating compact object
- Units where $c = 1$

$$\frac{dP}{dr} = -\frac{G\varepsilon m}{r^2}\left(1+\frac{P}{\varepsilon}\right)\left(1+\frac{4\pi P r^3}{m}\right)\left(1-\frac{2Gm}{r}\right)^{-1}$$

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon$$

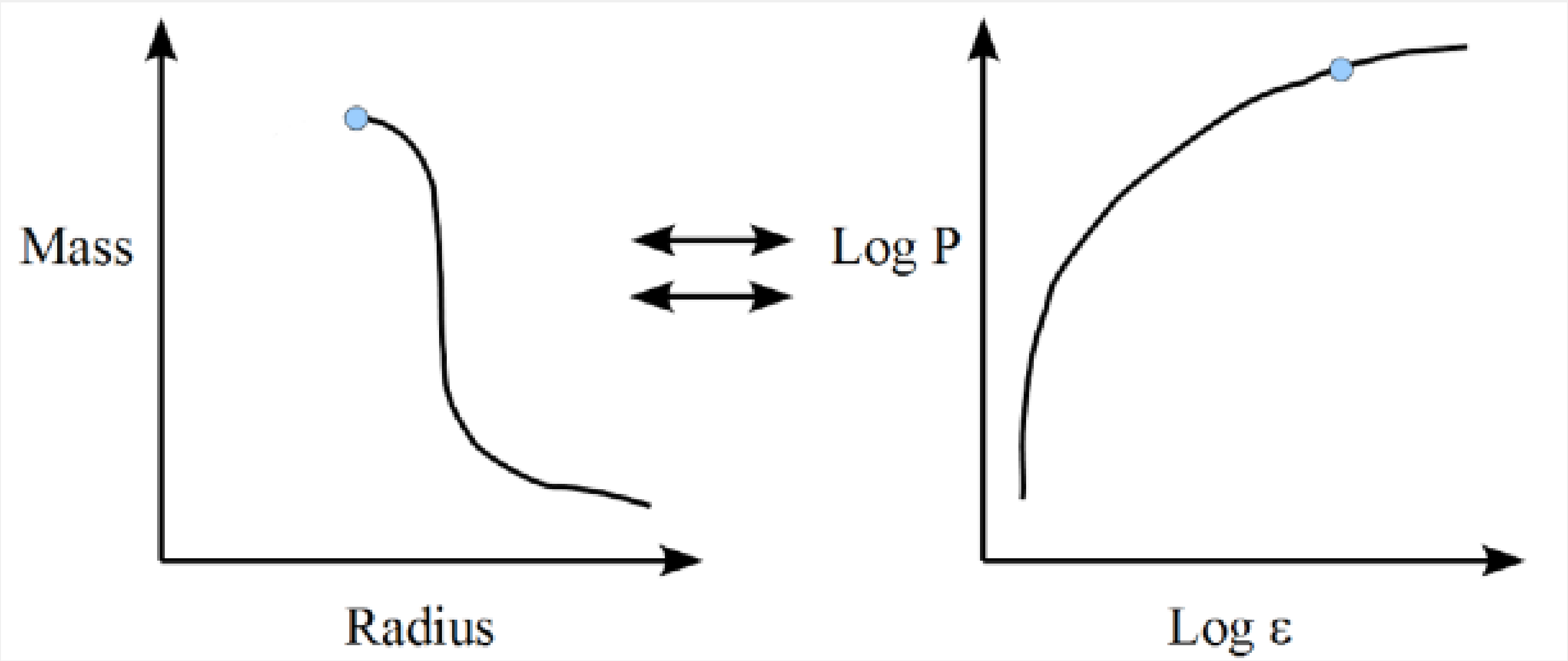
$$\frac{d\Phi}{dr} = -\frac{1}{\varepsilon}\frac{dP}{dr}\left(1+\frac{P}{\varepsilon}\right)^{-1}$$

- Boundary conditions, $P(r=R)=0, m(r=R)=M$
  $\Phi(r=R) = \frac{1}{2}\ln\left(1-\frac{2GM}{R}\right)$

Several solvers online: LORENE, O2scl, etc.

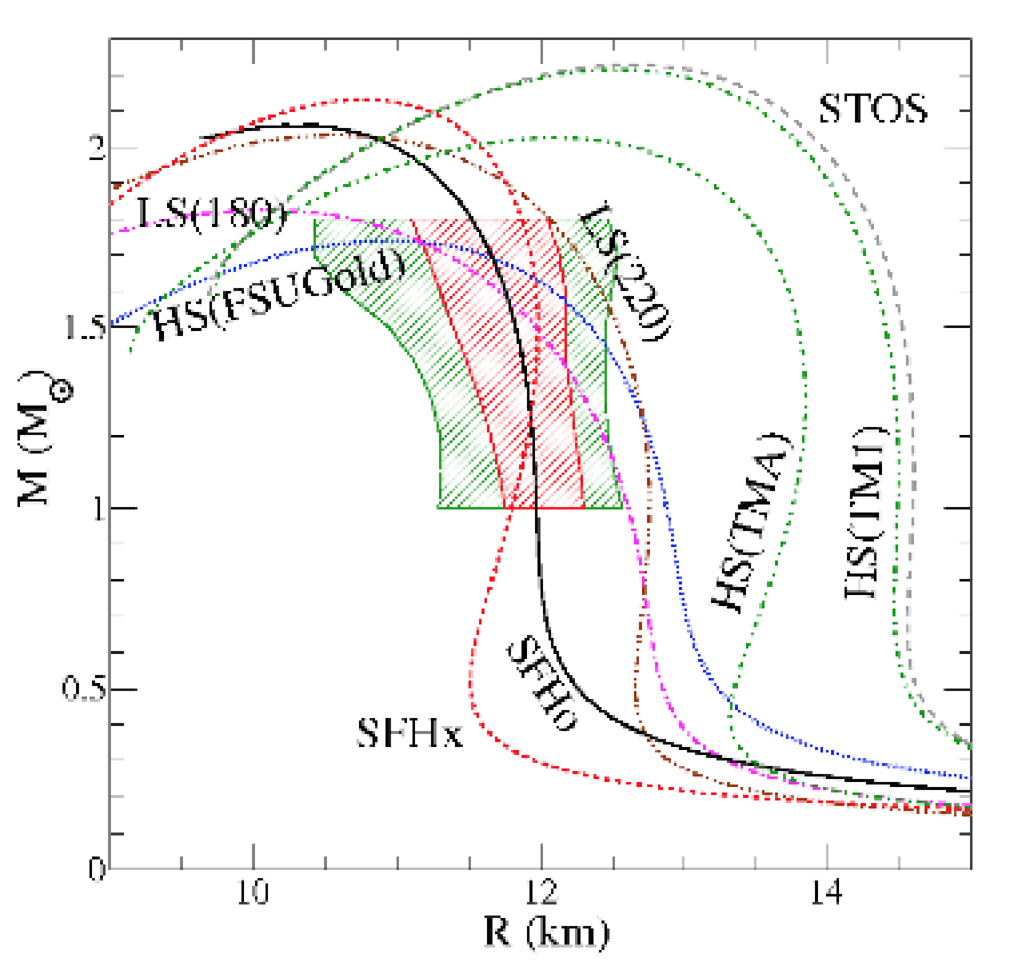
Neutron Star Masses and Radii and the EOS

- Neutron stars (to better than 10%) all lie on one universal mass-radius curve
  (Largest correction is rotation - work in progress)
- Recent measurement of two $2\ M_\odot$ neutron stars
  Demorest et al. (2010), Antoniadis et al. (2013)
- As of 2007 neutron star radii constrained to 8-15 km, now 10-13 km
  Lattimer and Prakash (2007); Steiner, Lattimer and Brown (2013)



- Einstein's field equations provide a 1-1 correspondence

Maximum mass and causality



Several mass-radius curves based on equations of state for core-collapse supernova simulations from Steiner et al. (2012)

- Stability analysis shows that stars to left of maximum collapse
- Two $2\ M_\odot$ neutron stars
  Demorest et al. (2010), Antoniadis et al. (2013)
- Hydrodynamic stability: $dP/d\varepsilon > 0$
- Causality: $dP/d\varepsilon = c_s^2 < 1$
  Effectively prohibits very compact, (R/M small), neutron stars

Rotation and magnetic fields

- Slowly-rotating approximation: $\omega(r)$ is the rotation rate of the inertial frame, Ω is the angular velocity in the fluid frame, and $\bar{\omega}(r) \equiv \Omega - \omega(r)$ is the angular velocity of a fluid element at infinity. The function $\bar{\omega}(r)$ is the solution of

$$\frac{d}{dr}\left(r^4 j \frac{d\bar{\omega}}{dr}\right) + 4r^3 \frac{dj}{dr}\bar{\omega} = 0$$

where the function $j(r)$ is defined by

$$j = e^{-\Lambda-\Phi} = \left(1 - \frac{2Gm}{r}\right)^{1/2} e^{-\Phi}\,.$$

Approximate I, but $R_{\rm eq} = R_{\rm pol}$.

- Full rotation

LORENE, RNS, etc.

- Magnetic fields
  Cardall et al. (2000); Broderick et al. (2015)
  - Evolution difficult to describe
  - Hard to find stable configurations

Data analysis

- What makes nuclear astrophysics exciting is that we can connect data and observations
- Fit N data points to M parameters
- Minimize

$$\chi^2 = \frac{1}{N-M} \sum_{i=0}^{N} \frac{[T_i(\{p_j\}) - D_i]^2}{U_i^2} \qquad \tilde{\chi}^2 \equiv \chi^2/(N-M)$$

- $\tilde{\chi}^2$ follows a probability distribution; $\tilde{\chi}^2 \sim 1$
- Gaussian likelihood function, $\mathcal{L} = \exp(-\chi^2/2)$
- Uncertainty (statistical) by varying parameters so that $\chi^2 \sim N - M + 1$
- Not quite right for counting statistics
- Must do something else for $M > N$
- Model with $\tilde{\chi}^2 = 0.8$ and model with $\tilde{\chi}^2 = 1.2$?
- Not invariant under all variable transformations

Bayesian analysis

- Can handle $M > N$

- Bayes theorem:

$$\mathcal{P}[M|D] \propto \mathcal{P}[D|M]\mathcal{P}[M]$$

- Prior distributions

- Marginal estimation:

$$P[\hat{p}_i] = \int \delta(p_i - \hat{p}_i)\mathcal{P}[D|M]\mathcal{P}[M]d\{p\}$$

- Much like computing quantum expectation values

- MCMC often used; Computing availability important

Systematic uncertainties

- In many cases, we are dominated by systematic uncertainties

- Sometimes we can just go to the next order in perturbation theory

- In some cases, one can match to a more accurate model, e.g. Lattice instead of NJL

- If not, one must try other models!

- If a model fits, it is not necessarily the correct model. A successful fit only means the model is not yet ruled out

Taking nuclear theoretical astrophysics to the next century

- There are (at least) thousands of models of dense matter
- Also (at least) hundreds of ways of fitting those models to data
- Each model has several parameters, and a countably infinite set of parameterizations
- Should we write a paper for each one? No.
- New models should have a clear purpose, or one should characterize a large class of models