Opening angle as an evolution variable for parton distributions

Emmanuel Gräve de Oliveira
Universidade Federal de Santa Catarina
Florianópolis, SC – Brazil

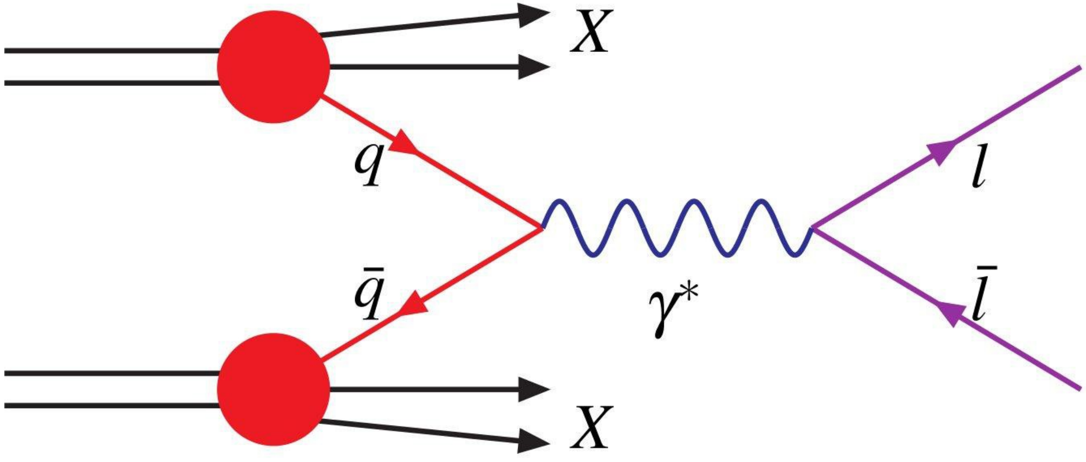
Cross section and parton distribution functions

- LHC: pp collisions -- seen as parton collisions

$$d\sigma/d^3p \;=\; \int dx_1 dx_2 \,\mathrm{PDF}(x_1,\mu_F)\,|\mathcal{M}(p;\mu_F,\mu_R)|^2\,\mathrm{PDF}(x_2,\mu_F)\;,$$

- Longitudinal momentum fraction *x*

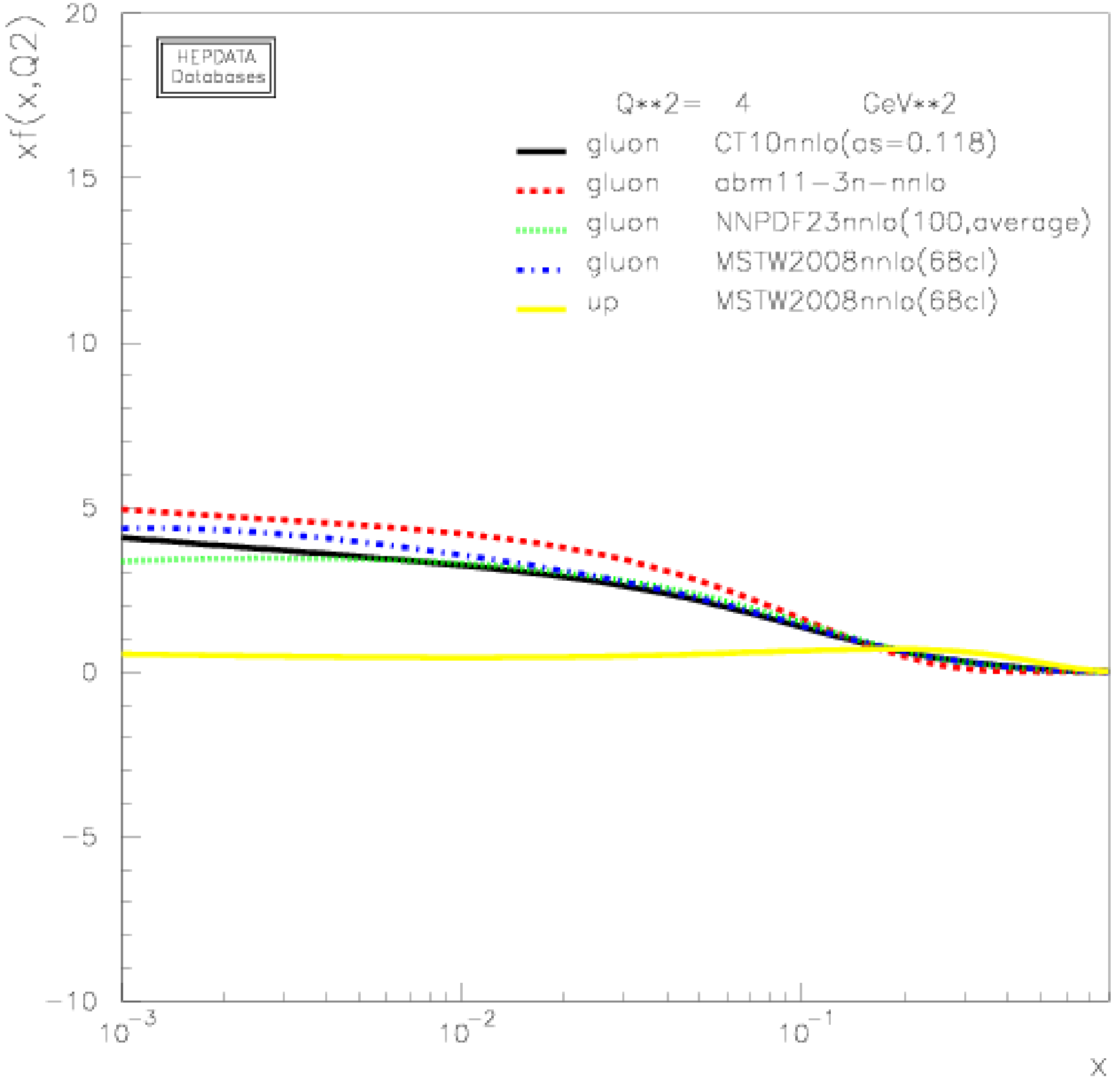
$$x_{1,2} = \frac{m_{\rm hard}}{\sqrt{s}}\exp(\pm y)$$



- Factorization scale
-

Gluon distributions

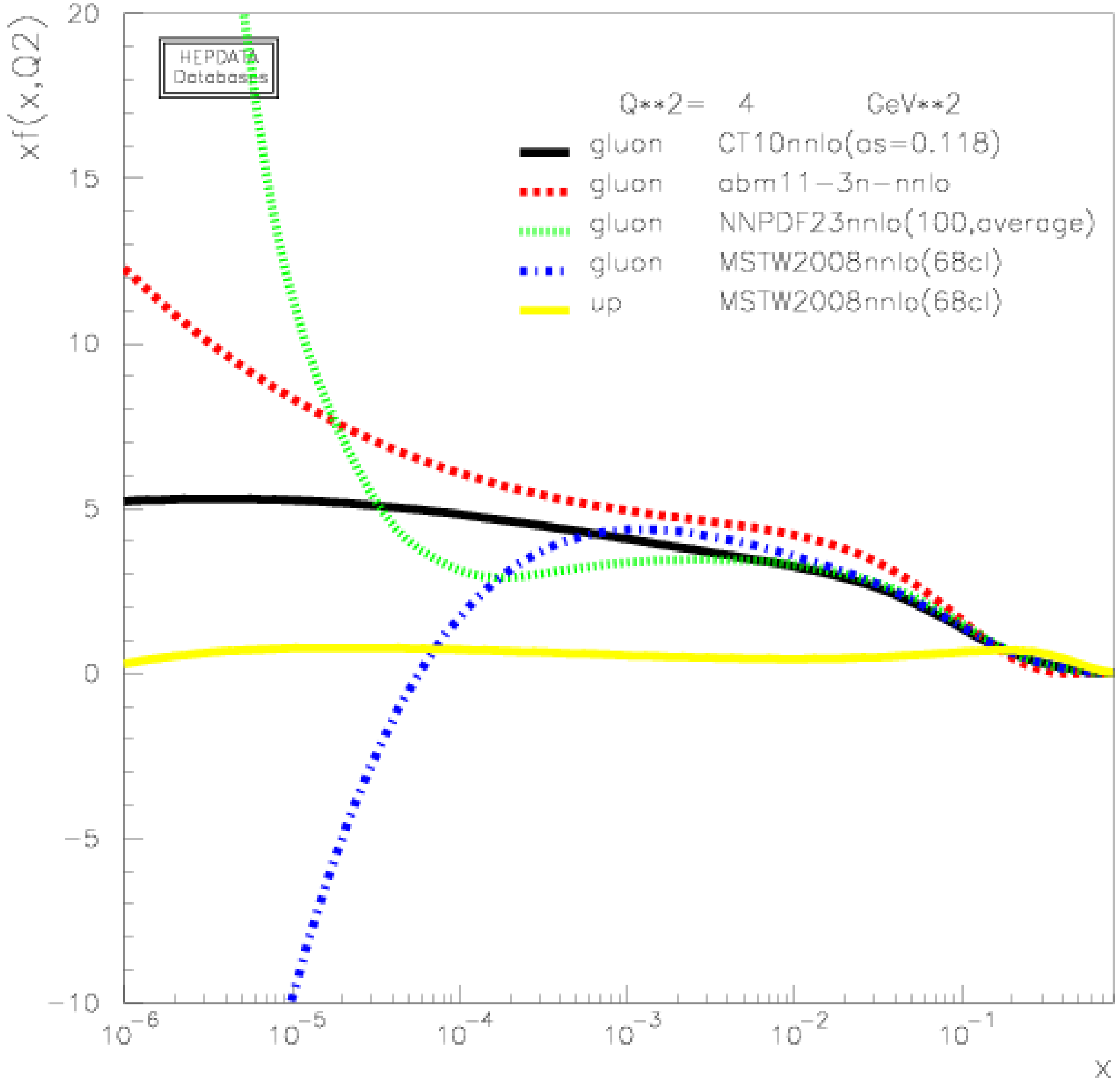
- Global analysis using the same theory.
- Gluon dominates at small *x*.



Gluon distributions at small *x*

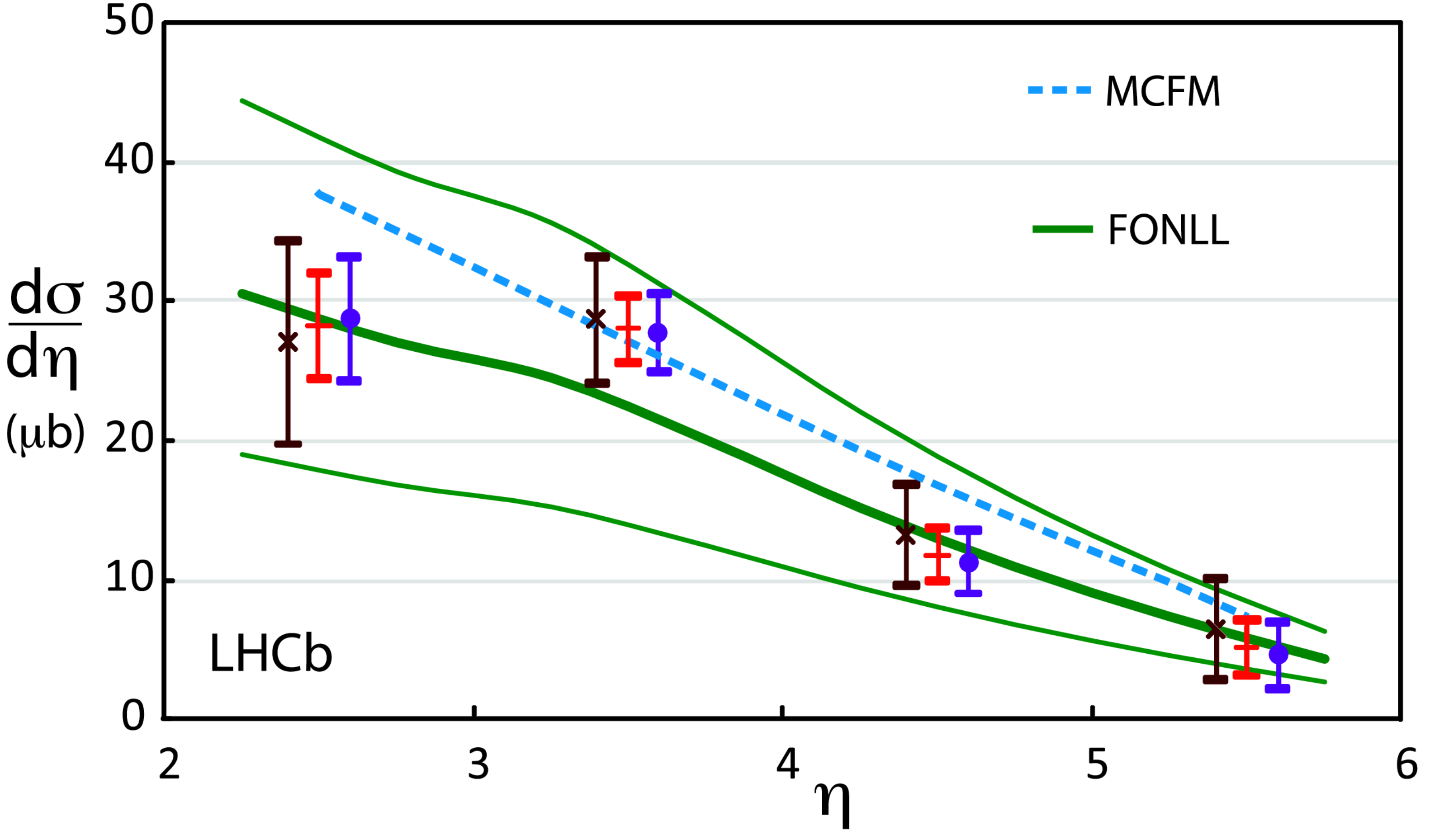
- Global analysis using the same theory.
- Gluon dominates at small *x*.
- What happens at small *x*?

  → Very important at the LHC.

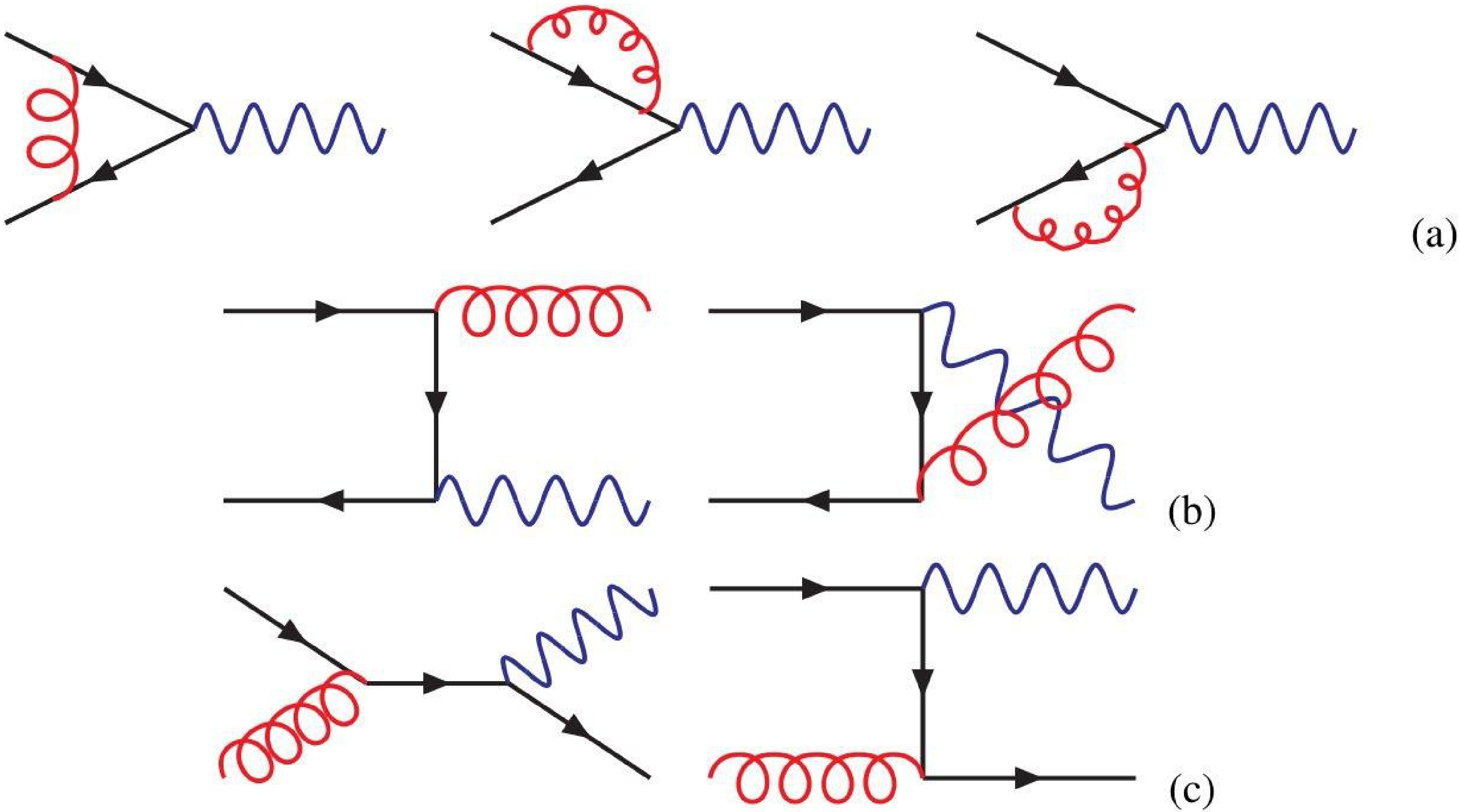


Factorization scale dependence

- LHCb – Phys.Lett.B694:209-216,2010

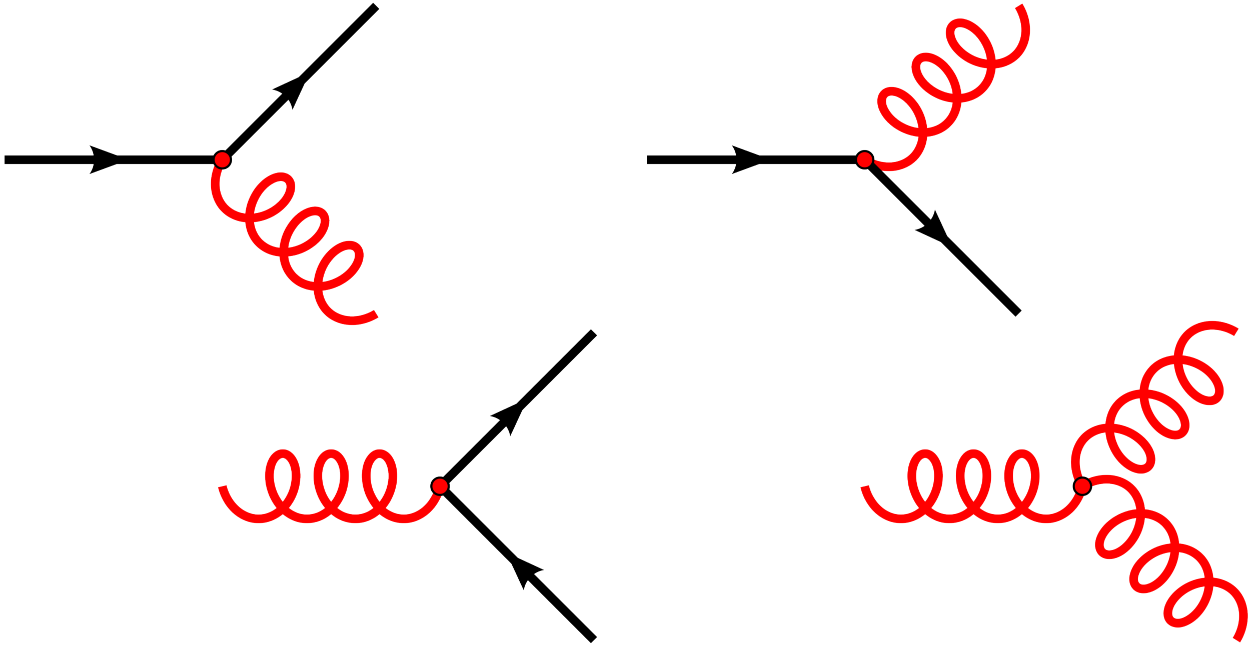


Next-to-leading order diagrams (Drell—Yan example)



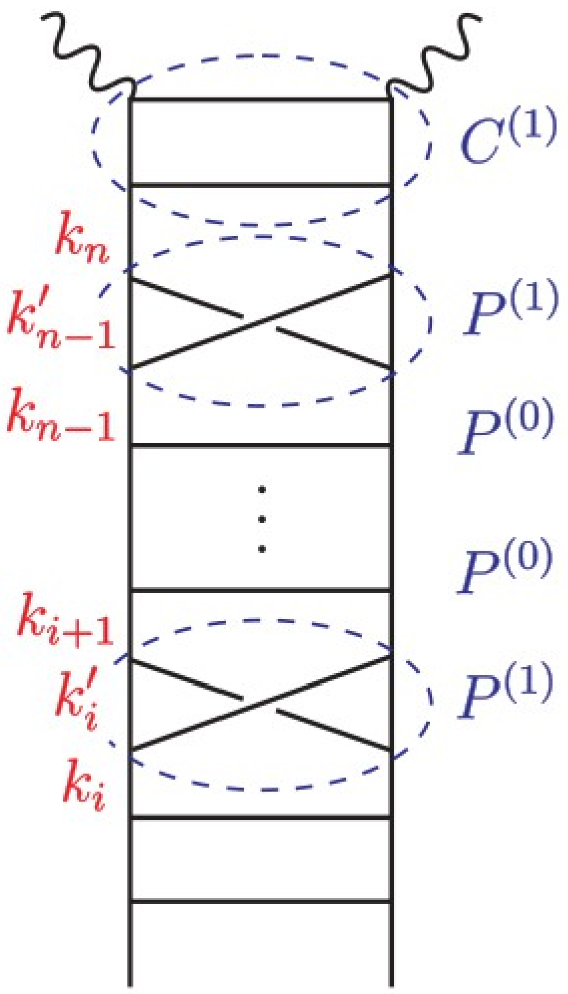
DGLAP evolution

- Partons in the distributions can emit more partons



- Part of this splitting is taken into account in the parton distribution.

DGLAP ladder
Multiple splitting



- DGLAP ressums large factorization scale logarithms

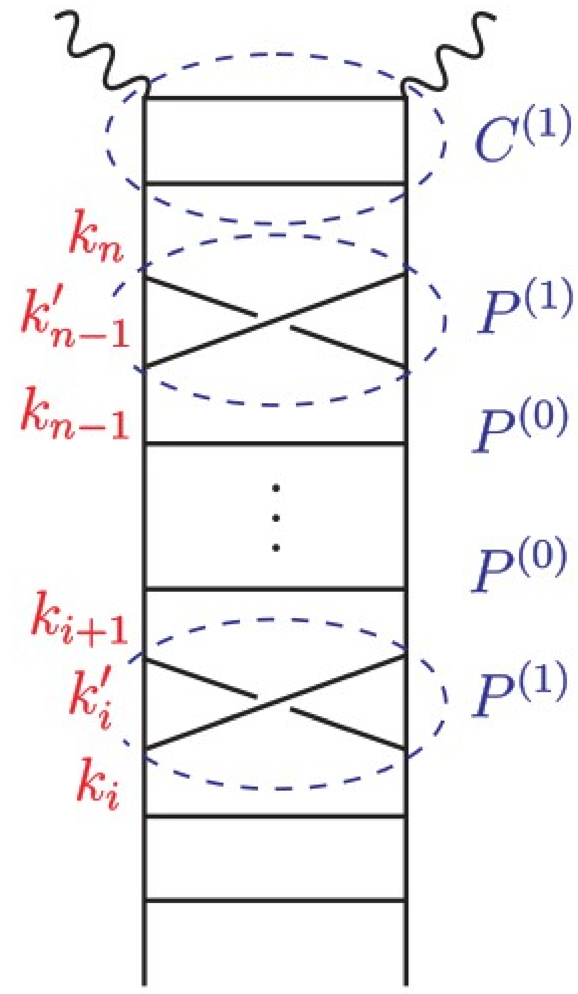
$$x_i > x_{i+1}$$

$$k_{i,t} \ll k_{i+1,t}$$

- For small longitudinal momentum *x*, there is a big probability of multiple splittings

Nuclear unintegrated pdfs depend on transverse momentum

- In collinear factorization, there is no transverse momentum dependence
- All dependence is the coefficient function
- KMR approach: do all steps like collinear except by the last one
- Our work: start with nuclear integrated pdfs



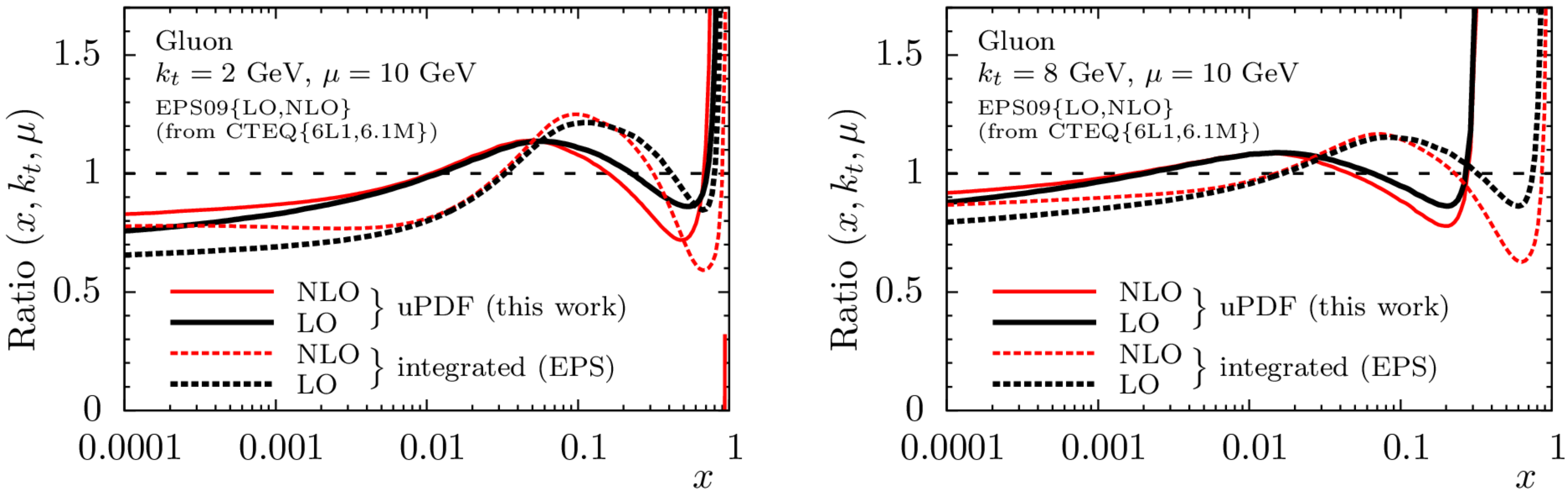
Derivation of last step in evolution

$$f_a(x, k_t^2, \mu^2) \;=\; \int_x^1 \mathrm{d}z \; T_a(k^2, \mu^2) \; \frac{\alpha_S(k^2)}{2\pi}$$
$$\times \sum_{b=q,g} \tilde{P}_{ab}\,(z, \Delta)\; \frac{x}{z} b\left(\frac{x}{z}, k^2\right) \Theta(1 - z - k_t^2/\mu^2),$$

$$k^2 = k_t^2/(1-z)$$

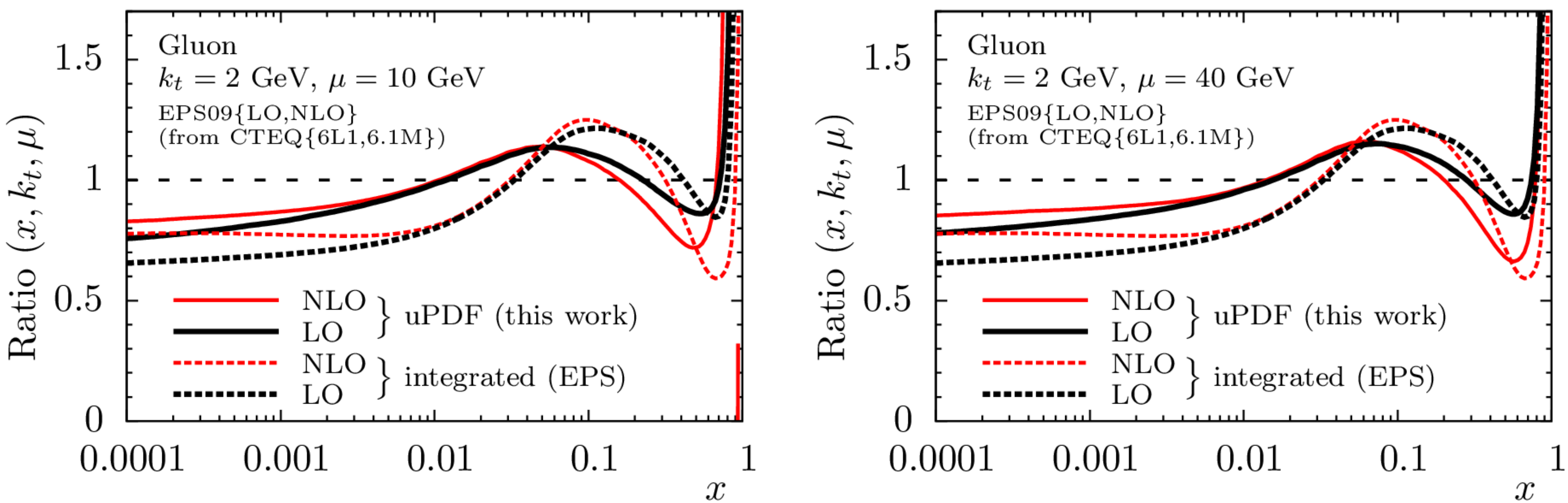
- Last step splitting
- Energy conservation
- Sudakov factor

Nuclear unintegrated pdfs at a given scale



- Nuclear effects (ratio)
- Shift in *x* and smaller effects for higher transverse momentum

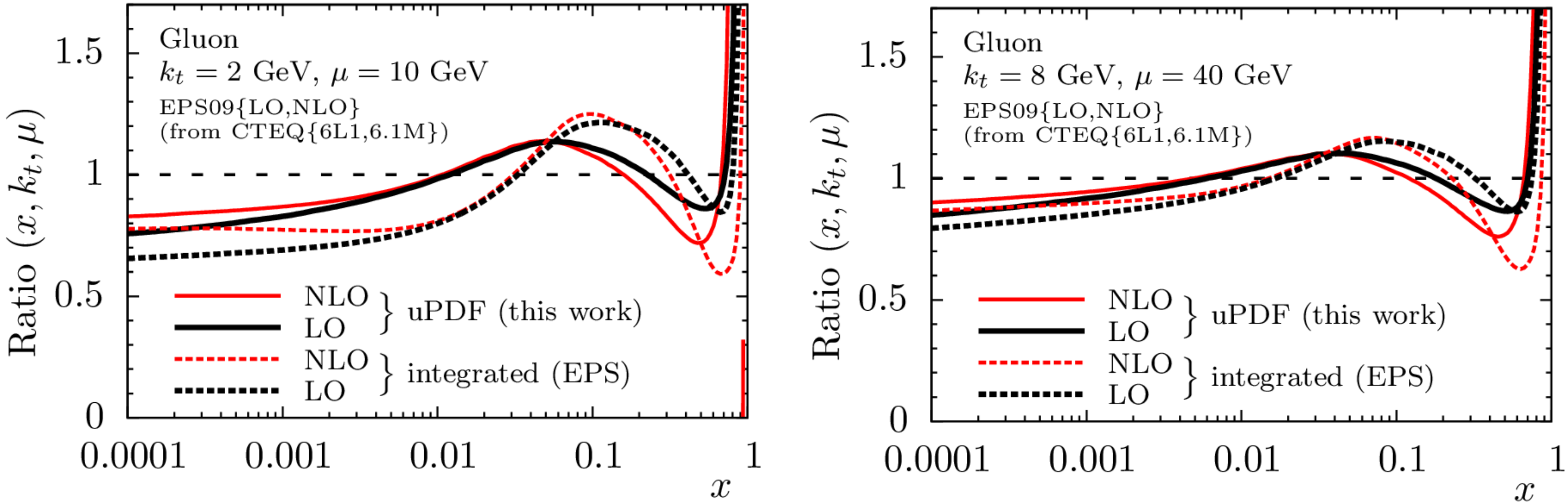
Nuclear unintegrated pdfs at a given transverse momentum



- Very small change in unintegrated pdfs

→ Smaller factorization scale dependence

Nuclear unintegrated PDFs at a given ratio



- Smaller nuclear effects for higher scales

Summary (1)

- Nuclear unintegrated pdfs calculated with correct $\log(Q^2)$ terms for the first time.
- Valid for all *x* and also for quarks.
- Comparison with log(x) approaches is needed.
- Reduces the factorization scale dependence

BFKL evolution: strong *x* ordering

- The BFKL for the unintegrated gluon distribution is given by:

$$f(x,k_t) = f_0(x,k_t) + \frac{\alpha_s}{2\pi}\int_0^\infty d^2k_t' \int_x^1 \frac{dx'}{x'} \mathcal{K}(k_t,k_t') f(x',k_t')$$

with kernel

$$\mathcal{K}(k_t,k_t') f(x',k_t') = 2N_c \frac{k_t^2}{k_t'^2}\left[\frac{f(x',k_t') - f(x',k_t)}{|k_t'^2 - k_t^2|} + \frac{f(x',k_t)}{\sqrt{4k_t'^4 + k_t^4}}\right]$$

- Transverse momentum is not ordered

Adding DGLAP + BFKL

- When adding both, the double log part must be subtracted.

$$f(x,k_t) = f_0(x,k_t) + \frac{\alpha_s}{2\pi}\left(\int_0^\infty d^2k_t' \int_x^1 \frac{dx'}{x'}\mathcal{K}(k_t,k_t')f(x',k_t')\right.$$
$$\left. + \int_{Q_0^2}^{k_t^2} \frac{dk_t'^2}{k_t'^2}\int_x^1 dz P(z) f\left(\frac{x}{z},k_t'\right) - DL\right),$$

- Easier to subtract it from BFKL.

$$\overline{\mathcal{K}}(k_t,k_t')f(x',k_t') = 2N_c\frac{k_t^2}{k_t'^2}\left[\frac{f(x',k_t') - f(x',k_t)}{|k_t'^2 - k_t^2|} + \frac{f(x',k_t)}{\sqrt{4k_t'^4 + k_t^4}} - \frac{f(x',k_t')}{k_t^2}\right].$$

Kinematical constraint

- For real emission, virtuality has to increase:

$$k_t'^2 < \frac{k_t^2}{z} , \qquad \text{where} \quad z = x/x'$$

since for BFKL virtuality is approximated by transverse momentum. The kernel becomes:

$$\overline{\mathcal{K}}(k_t, k_t') f(x', k_t') = 2N_c \frac{k_t^2}{k_t'^2} \left[\frac{\Theta(k_t^2/z - k_t'^2) f(x', k_t') - f(x', k_t)}{|k_t'^2 - k_t^2|} + \right.$$
$$\left. + \frac{f(x', k_t)}{\sqrt{4k_t'^4 + k_t^4}} - \frac{\Theta(k_t^2 - k_t'^2) f(x', k_t')}{k_t^2} \right] .$$

Angle variable

- We define the emission angle to be:

$$\theta = k_t/xp \text{ and } \theta' = k'_t/x'p.$$

- The integrated distribution

$$xg(x,k_t^2) = \int^{k_t^2} \frac{dk_t'^2}{k_t'^2} f(x,k_t'^2).$$

- Becomes

$$xg(x,\theta) = \int^{\theta^2} f(x,\theta') \frac{d\theta'^2}{\theta'^2}.$$

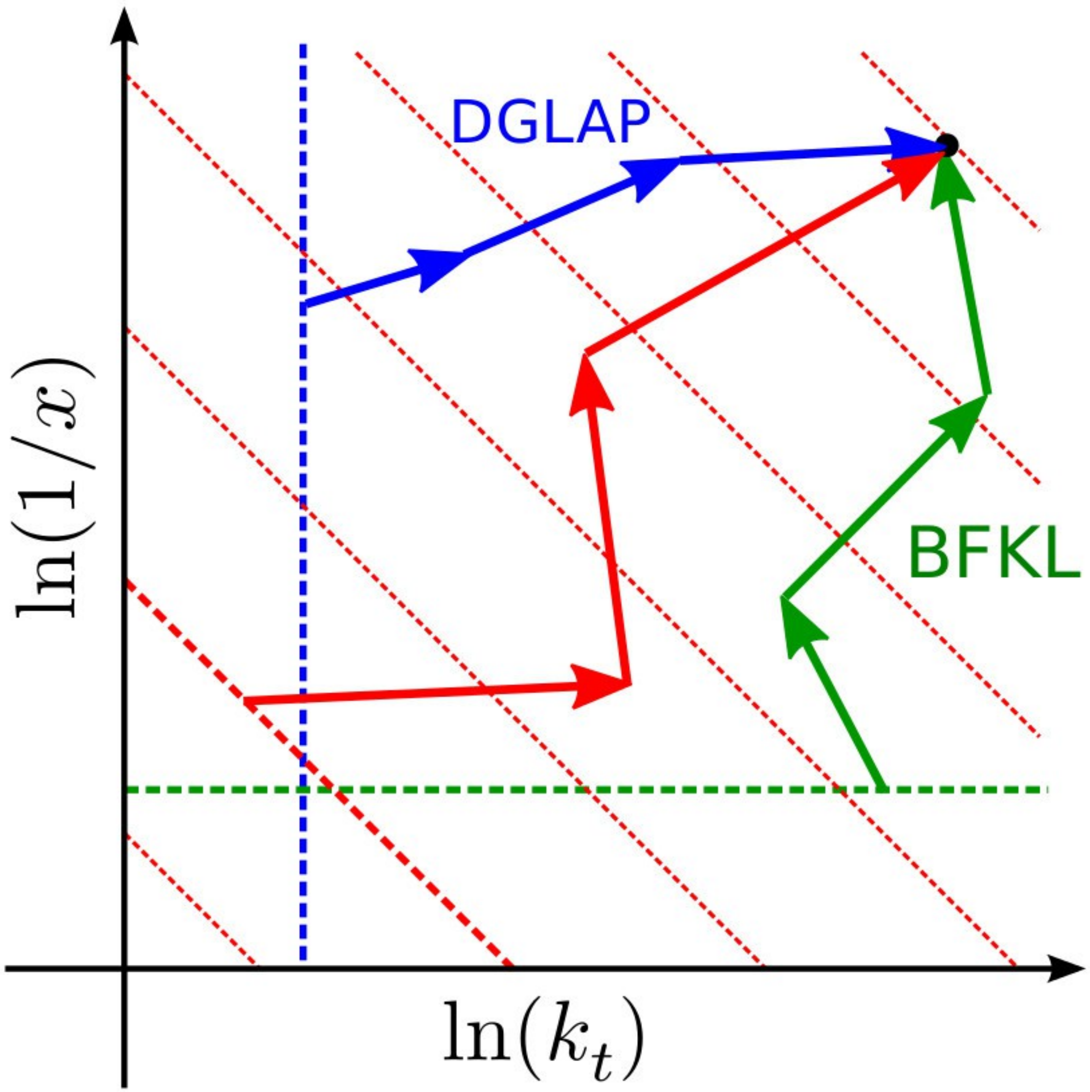
Evolution in angle

$$f(x,k_t) = f_0(x,k_t) + \frac{\alpha_s}{2\pi}\left(\int_0^\infty d^2k'_t \int_x^1 \frac{dx'}{x'}\mathcal{K}(k_t,k'_t)f(x',k'_t) + \int_{Q_0^2}^{k_t^2}\frac{dk'^2_t}{k'^2_t}\int_x^1 dz P(z) f\left(\frac{x}{z},k'_t\right) - DL\right),$$

- DGLAP $\quad \theta' = z\theta < \theta.$

- BFKL $\quad \theta' = \frac{zk'_t}{xp} < \sqrt{z}\frac{k_t}{xp} < \sqrt{z}\theta.$

Evolution space



Energy-momentum conservation

- To keep energy-momentum conservation, the kernel should be subtracted:

$$\frac{\alpha_s}{2\pi}\left(\int_x^1 \frac{dz}{z}\int_0^\infty d^2k_t'\,\overline{\mathcal{K}}(k_t,k_t')f(\frac{x}{z},k_t') - \int_0^1 dz\int_0^\infty d^2k_t'\,\overline{\mathcal{K}}(k_t,k_t')f(x,k_t')\right).$$

- Problem: not an evolution equation anymore
- Solution: solve it iteratively, since it is a small contribution (NLO).

Final result

$$
\begin{aligned}
\frac{\partial[xg(x,\theta)]}{\partial \mathrm{ln}\theta^2} &= f_0(x,\theta_0) \\
&+ \frac{\alpha_s}{2\pi}\Bigg[\int_{\theta_0}^{\theta}\int_0^{\infty} d^2k_t'\,\overline{\mathcal{K}}(k_t,k_t')\frac{\partial[x'g(x',\theta')]}{\partial\ln\theta'^2}\frac{d\theta'}{\theta'} \\
&- \int_0^1 dz\int_0^{\infty} d^2k_t'\,\overline{\mathcal{K}}(k_t,k_t')\frac{\partial[xg(x,\theta')]}{\partial\ln\theta'^2} \\
&+ \int_x^1 dzP(z)\frac{x}{z}g\left(\frac{x}{z},z\theta\right)\Bigg]\ ,
\end{aligned}
$$

Summary (2)

- Angle is a good variable too bring uniformity over DGLAP and BFKL
- Ressuming both large logs should make the evolution more precise
- Already implemented by David Toton: Phys. Rev. D 91, 054003 (2015)

BFKL for an integrated distribution

- Integrated distribution:

$$F(x,q) = \int^{q^2} \frac{dk_t^2}{k_t^2} f(x,k_t) = \int^{q^2} d\ln k_t^2 \, f(x,k_t).$$

- BFKL equation

$$f(x,k_t) = f_0(x,k_t) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \int_{k_0}^{\infty} \frac{d^2 k_t'}{\pi} \, \mathcal{K}(k_t, k_t', z) \; f(x/z, k_t'),$$

- Integration by parts: *u dv = d (u v) – v du*

Result

- After some calculations ...

$$
\begin{aligned}
F(x,q) &= F(x,k_0) + F_0(x,q) - F_0(x,k_0) \\
&+ \frac{N_c\alpha_s(q^2)}{\pi}\int_x^1 \frac{dz}{z}\left\{ \int_{k_0^2}^{q^2} dk_t^2 \frac{F(x',k_t)-F(x',k_0)}{|k_t^2-k_0^2|} + \int_{k_0^2}^{q^2/z} dk_t^2 \frac{q^2}{k_t^2}\frac{F(x',k_t)-F(x',q)}{|k_t^2-q^2|} \right. \\
&- \int_{zk_0^2}^{k_0^2} \frac{dk_t^2}{k_t^2}\frac{F(x',k_t/\sqrt{z})-F(x',k_t)}{|1/z-1|} + \ln(1-z)[F(x',q)-F(x',k_0)] \\
&+ \left. \ln\left(\frac{\sqrt{4k_0^4+q^4}+q^2}{2k_0^2}\right)F(x',q) - \ln\left(\frac{\sqrt{5}+1}{2}\right)F(x',k_0) - \int_{k_0^2}^{q^2} dk_t^2\left[\frac{F(x',k_t)}{\sqrt{4k_0^4+k_t^4}}\right]\right\} \\
&- \text{energy–momentum conservation term.}
\end{aligned}
$$

Extrapolation to small scales

- Can we take the lower limit to be zero?

$$
\begin{aligned}
F(x,q) &= F_0(x,q) + \frac{N_c\alpha_s}{\pi}\int_x^1 \frac{dz}{z}\left\{\int_{k_0^2}^{q^2/z}\frac{dk_t^2}{k_t^2}\left[q^2\frac{F(x',k_t)-F(x',q)}{|{k_t}^2-q^2|}\right]\right. \\
&+ \left.\ln(1-z)F(x',q) + \ln\left(\frac{q^2}{k_0^2}\right)F(x',q)\right\} \\
&- \text{energy–momentum conservation term.}
\end{aligned}
$$

$$
\begin{aligned}
F(x,q) &= F_0(x,q) + \frac{N_c\alpha_s}{\pi}\int_x^1 \frac{dz}{z}\left\{\int_0^{q^2/z}\frac{dk_t^2}{k_t^2}\frac{q^2F(x',k_t)-(q^2-|q^2-{k_t}^2|)F(x',q)}{|{k_t}^2-q^2|}\right. \\
&+ \left.\ln[z(1-z)]F(x',q)\right\} - \text{energy–momentum conservation term}
\end{aligned}
$$

Summary (3)

- Integrated distributions are easier to handle.
- More groups work with them
- There are more tools to work with them
- Most of available cross sections are calculated for integrated distributions.
- To do:
  - merge integrated BFKL with DGLAP.
  - Global analysis.

Thank you

- Thank the organizers for this amazing event.
- Thank very much for my collaborators
  - *E.G.O., A.D.Martin, F.S.Navarra; M.G.Ryskin; JHEP 2013, 158.*
  - *E.G.O., A.D.Martin, M.G.Ryskin; Eur. Phys. J. C 74 (2014) 3118.*
  - *E.G.O., A.D.Martin, M.G.Ryskin; Eur. Phys. J. C 74 (2014) 3030.*

emmanuel.de.oliveira@ufsc.br

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