Study of pseudoscalar mesons (π^+ , K^+) with a symmetric vertex model in the light front.

XIII International Workshop on Hadron Physics-2015

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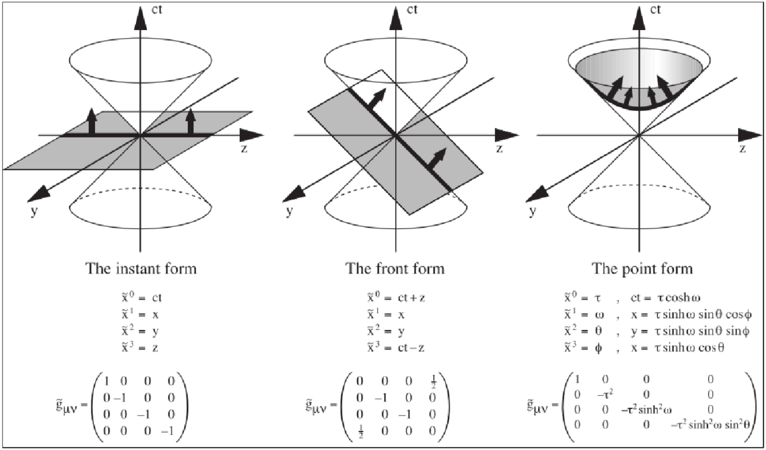
Light-Front Motivations

- **Ligh-Front is the Ideal Framework to Describe Hadronic Bound States**
- **Constituent Picture and Unambiguous Partons Content of the Hadronic System**
- **Light-Front Wavefunctions: Representation of Composite Systems in QFT**
- **Invariant Under Boosts**
- **Light-Front Vacuum is Trivial**
- **After Integrate in k^-: Bethe-Salpeter Amplitude (Wave Function)**
- **LF Lorentz Invariant Hamiltonian:** $P^2 = P^+ P^- - P_\perp^2$

- Obviously, one has many possibilities to parameterize space-time by introducing some generalized coordinates $\tilde{x}(x)$.
- But one should exclude all those which are accesible by a "Lorentz transformation". This limits considerably the freedom and excludes, for example, almost all rotation angles.
- **Dirac**, in 1949, proposed there are no more than three basically diffeent parameterizations.



Dirac's three forms of Hamiltonian dynamics.

Light-Front Coordinates

Four-Vector $\Longrightarrow$ $x^\mu = (x^0, x^1, x^2, x^3) \Longrightarrow (x^+, x^-, x_\perp)$

$x^+ = t + z$ $\quad$ $x^+ = x^0 + x^3$ $\Longrightarrow$ **Time**

$x^- = t - z$ $\quad$ $x^- = x^0 - x^3$ $\Longrightarrow$ **Position**

Metric Tensor and Scalar product

$$x \cdot y = x^\mu y_\mu = x^+ y_+ + x^- y_- + x^1 y_1 + x^2 y_2 = \frac{x^+ y^- + x^- y^+}{2} - \vec{x}_\perp \vec{y}_\perp$$

$p^+ = p^0 + p^3\ ,\ p^- = p^0 - p^3\ ,\ p^\perp = (p^1, p^2)$

Dirac Matrix and Electromagnetic Current

$\gamma^+ = \gamma^0 + \gamma^3 \implies$ **Electr. Current** $J^+ = J^0 + J^3$
$\gamma^- = \gamma^0 - \gamma^3 \implies$ **Electr. Current** $J^- = J^0 - J^3$
$\gamma^\perp = (\gamma^1, \gamma^2) \implies$ **Electr. Current** $J^\perp = (J^1, J^2)$

$$p^\mu x_\mu = \frac{p^+x^- + p^-x^+}{2} - \vec{p}_\perp \vec{x}_\perp$$

$$x^+, x^-, \vec{x_\perp} \implies p^+, p^-, \vec{p}_\perp$$

$p^- \implies$ **Light-Front Energy**

$$p^2 = p^+ p^- - (\vec{p}_\perp)^2 \implies \quad p^- = \frac{(\vec{p}_\perp)^2 + m^2}{p^+}$$

On-shell

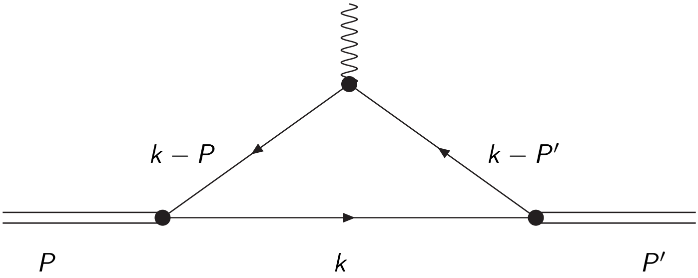
Bosons $\implies \quad S_F(p) = \frac{1}{p^2 - m^2 + \imath\epsilon}$

Fermions $\implies \quad S_F(p) = \frac{\not{p} + m}{p^2 - m^2 + \imath\epsilon} + \frac{\gamma^+}{2p^+}$

Review Papers:

- **Phys. Rept. 301, (1998) 299-486, Brodsky, Pauli and Pinsky**
- **A. Harindranath, Pramana, Journal of Indian Academy of Sciences Physics Vol.55, Nos 1 & 2, (2000) 241.**
- **An Introduction to Light-Front Dynamics for Pedestrians Avaroth Harindranath**

Light-Front book organizers: James Vary and Frank Wolz,(1997)



- Symmetry Vertex function

$$\Lambda(k,P) = \frac{C}{(k^2 - m_R^2 + i\epsilon)} + \frac{C}{((P-k)^2 - m_R^2 + i\epsilon)}$$

The covariant electromagnetic form factor is defined as

$$(p_\mu + p'_\mu)F_{M_0^-}(q^2) = \langle M_0^-(p')|J_\mu|M_0^-(p)\rangle,$$

where $M_0^- = \pi^+, K^+$ denote the light pseudoscalar mesons, $q = p - p'$ and $J_\mu = e_q\bar{\psi}_q\gamma_\mu\psi_q$ is the electromagnetic current.
In the impulse approximation, the form factor is given by a triangle diagram which represents an amplitude via a single integral:

$$\begin{aligned}(p_\mu + p'_\mu)F_{M_0^-}(q^2) &= -i2e\frac{m^2}{f_\pi^2}N_c\int\frac{d^4k}{(2\pi)^4}Tr[S_{\bar{q}}(k)\gamma^5 S_q(k-P')\gamma^\mu \\ &\times S_q(k-P)\gamma^5]\Lambda(k,P')\Lambda(k,P)\end{aligned}$$

where

$$S_q(p) = (\not{p} - m_q + i\epsilon)^{-1}$$

, $q = u$, $\bar{q} = \bar{d}, \bar{s}$ denotes the quark propagator with quark masses m_q, e_q is the quark's charge, and $N_c = 3$ is the number of colors.

In our analysis we consider **Breit frame**, where the momentum transfer q^μ has the spatial component parallel to the $z - x$ plane.
By using Front-Form variables, i.e.

$$k^+ = k^0 + k^3, \quad k^- = k^0 - k^3, \vec{k}_\perp \equiv (k^1, k^2)$$

one has

$$\begin{aligned} q^+ = -q^- &= \sqrt{-q^2}\sin\alpha, \quad q_x = \sqrt{-q^2}\cos\alpha, \quad q_y = 0 \\ q^2 &= q^+ q^- - (\vec{q}_\perp)^2 \end{aligned}$$

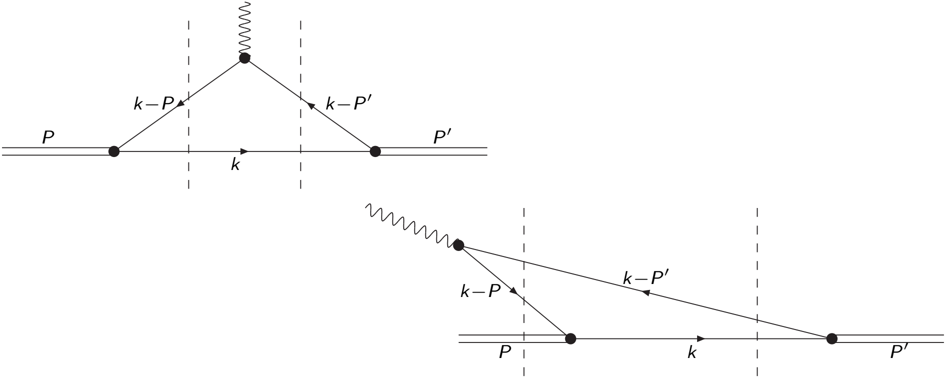
The Drell-Yan condition $q^+ = 0$ is recovered with $\alpha = 0$.
In the light-front variables the initial and final momenta of the composite spin-0 bound state are:

$$\vec{P}'_\perp = -\vec{P}_\perp = \frac{\vec{q}_\perp}{2} \text{ and } P'_z = -P_z = \frac{q^+}{2}.$$

To calculate the electromagnetic form factors, we use the plus component of the current

$$J^{+} = -i2e\frac{m^2}{f_\pi^2}N_c \int \frac{d^4k}{(2\pi)^4} Tr[S(k)\gamma^5 S(k-P')\gamma^+ S(k-P)\gamma^5] \times \Lambda(k,P')\Lambda(k,P)$$

It is worth full to mention here that sometimes we need the restoration covariance term $J^+ = J^- +$ restoration covariance term



- After transformation to light cone variable, the component J^+, has two nonvanishing contributions to the form factors

$$F_\pi(q^2) = F_\pi^{(I)}(q^2, \alpha^2) + F_\pi^{(II)}(q^2, \alpha^2)$$

- $F_\pi^{(I)}(q^2, \alpha^2)$ has the loop integration on k^+ constrained by $0 \leq k^+ < P^+$ (called the valance contribution).

- $F_\pi^{(II)}(q^2, \alpha^2)$ has the loop integration on k^+ in the interval $P^+ \leq k^+ < P^{+\prime}$ (called the nonvalance contribution)

The two contributions to the from factor obtained from J^+ are given by the following expressions

$$F_\pi^{(I)}(q^2,\alpha^2) = -i\frac{m^2}{(P^+ + P'^+)f_\pi^2}\frac{N_c}{(2\pi)^4} \times \int \frac{d^2k_\perp dk^+ dk^- \theta(k^+)\theta(P^+ - k^+)}{k^+(P^+ - k^+)(P'^+ - k^+)}\Pi(k,P,P')$$

and

$$F_\pi^{(II)}(q^2,\alpha^2) = -i\frac{m^2}{P^+ + P'^+ f_\pi^2}\frac{N_c}{(2\pi)^4} \times \int \frac{d^2k_\perp dk^+ dk^- \theta(k^+ - P^+)\theta(P'^+ - k^+)}{k^+(P^+ - k^+)(P'^+ - k^+)}\Pi(k,P,P')$$

where

$$\Pi(k,P,P') = \frac{Tr[O^+]\lambda(k,P)\lambda(k,P')}{(k^- - k^-_{on} + i\epsilon)(P^- - k^- - (P-k)^-_{on} + \frac{i\epsilon}{P^+ - k^+})} \times \frac{1}{(P'^- - k^- - (P'-k)^-_{on} + i\epsilon)},$$

with the on-energy-shell values of the individual momenta are given by

$$k^-_{on} = \frac{k_\perp^2 + m^2}{k^+}, \qquad (P-k)^-_{on} = \frac{(P-k)_\perp^2 + m^2}{P^+ - k^+}$$

and $$(P'-k)^-_{on} = \frac{(P'-k)_\perp^2 + m^2}{P'^+ - k^+}$$

The trace $Tr[O^+]$ of the opertaor

$$O^+ = (\not{k} + m)\gamma^5(\not{k} - \not{P}' + m)\gamma^+(\not{k} - \not{P} + m)\gamma^5,$$

is given by:

$$\begin{aligned}\frac{1}{4}Tr[O^+] &= -k^-(P'^+ - k^+)(P^+ - k^+) + (k_\perp^2 + m^2)(k^+ - P^+ - P'^+) \\ &- \frac{1}{2}\vec{k}_\perp \cdot (\vec{P}'_\perp - \vec{P}_\perp)(P'^{+-P^+}) + \frac{1}{4}k^+ q_\perp^2.\end{aligned}$$

The explicit form of the symmetric regulator function in Front-form momentum coordinates is given by

$$\begin{aligned}\lambda(k,P) = \quad & C[k^+(k^- - \frac{k_\perp^2 + m_R^2 - i\epsilon}{k^+})]^{-1} \\ & + C[(P^+ - k^+)(P^- - k^- - \frac{(P-k)_\perp^2 + m_R^2 - i\epsilon}{P^+ - k^+})]^{-1}\end{aligned}$$

where the position of the poles for k^- clearly appears.

Since the integration range of k^+ is $0 \leq k^+ < P^+$ and $P^+ \leq k^+ < P'^+$, then, the sign of the imaginary part of some of the poles in the k^--complex plane changes.
The poles that have their imaginary part modified are

$$k^-_{(1)} = P^- - (P-k)^-_{on} + \frac{i\epsilon}{P^+ - k^+} = P^- - \frac{(P-k)^2_\perp + m^2}{P^+ - k^+} + \frac{i\epsilon}{P^+ - k^+},$$

and from the vertex function

$$k^-_{(2)} = P^- - \frac{(P-k)^2_\perp + m^2_R}{P^+ - k^+} + \frac{i\epsilon}{P^+ - k^+}$$

Note: The difference in the sign of the imaginary parts of $k^-_{(1)}$ and $k^-_{(2)}$ for the intervals $0 \leq k^+ < P^+$ and $P^+ \leq k^+ < P'^+$ is the mathematical signature of the pair production mechanism, which appears just in the second interval.

The other two poles are:

$$k^-_{(3)} = P'^- - \frac{(P'-k)^2_\perp + m^2_R}{P'^+ - k^+} + \frac{i\epsilon}{P'^+ - k^+},$$
$$k^-_{(4)} = P'^- - \frac{(P'-k)^2_\perp + m^2_R}{P'^+ - k^+} + \frac{i\epsilon}{P'^+ - k^+}$$

- The sum of the contributions $F_\pi^{(I)}(q^2,\alpha)$ and $F_\pi^{(II)}(q^2,\alpha)$ yields the covariant result, dependent upon q^2 only i.e. $F_\pi(q^2)$.
- Then the different directions of $\vec{q}$ in the Breit frame can only change the values of $F_\pi^{(I)}(q^2,\alpha)$ and $F_\pi^{(II)}(q^2,\alpha)$, but not their sum.
- For instance, by choosing $q^+ = 0$ (i.e., $\alpha = 0$) $F_\pi^{(II)}(q^2,\alpha)$ vanishes and therefore $F_\pi^{(I)}(q^2,\alpha)$ alone gives the whole, covariant result.

- Finally, by using the vertex functions $\lambda(k, P)$ and $\lambda(k, P')$, the two contributions $F_\pi^{(I)}(q^2, \alpha)$ and $F_\pi^{(II)}(q^2, \alpha)$ has four terms,

$$F_\pi^{(I)}(q^2, \alpha) = F_\pi^{(I)a} + F_\pi^{(I)b} + F_\pi^{(I)c} + F_\pi^{(I)d}$$
$$F_\pi^{(II)}(q^2, \alpha) = F_\pi^{(II)a} + F_\pi^{(II)b} + F_\pi^{(II)c} + F_\pi^{(II)d}$$

- To understand the analytic integration on k^- i.e. how to perform the contour integration on k^-, consider the first term of $F_\pi^{(I)a}$

$$F_\pi^{(I)a} = -\frac{i}{2\pi} N \int \frac{d^2k_\perp dk^+ dk^- \theta(k^+)\theta(P^+ - k^+)}{(k^+)^3 (P^+ - k^+)(P^{+\prime} - k^+)} Tr[O^+(k^-)]$$
$$\times \frac{1}{(k^- - k^-_{on} + i\epsilon)(P^- - k^- - (P - k)^-_{(on)} + i\epsilon)}$$
$$\times \frac{1}{(P'^- - k^- - (P' - k)^-_{on} + i\epsilon)(k^- - k^-_R + i\epsilon)^2},$$

where

$$k_R^- = \frac{k_\perp^2 + m_R^2}{k^+}, \qquad N = \frac{m^2 C^2}{(P^+ + P'^+) f_\pi^2} \frac{N_c}{(2\pi)^3}.$$

- For the sake of algebraic simplicity the contour of integration in $F_\pi^{(I)a}$ can be close in the upper complex semi- plane of k^- where only the poles

$$k_{(1)}^- = P^- - \frac{(P-k)_\perp^2 + m^2}{P^+ - k^+} + \frac{i\epsilon}{P^+ - k^+},$$
$$k_{(3)}^- = P'^- - \frac{(P'-k)_\perp^2 + m_R^2}{P'^+ - k^+} + \frac{i\epsilon}{P'^+ - k^+},$$

are present.

By evaluating the residues of the integrand one has

$$
\begin{aligned}
F_{\pi}^{(I)a} &= -\frac{i}{2\pi}N\int\frac{d^2k_{\perp}dk^{+}dk^{-}\theta(k^{+})\theta(P^{+}-k^{+})}{(k^{+})^3(P^{+}-k^{+})(P^{+\prime}-k^{+})} \\
&\times \frac{1}{(P'-P^{-}+(P-k)^{-}_{on}-(P'-k)^{-}_{on})} \\
&\times \left[\frac{Tr[O^{+}(P^{-}-(P-k)^{-}_{on})]}{(P^{-}-(P-k)^{-}_{on}-k^{-}_{on})(P^{-}-(P-k)^{-}_{on}-k^{-}_{R})^2}\right. \\
&- \left.\frac{Tr[O^{+}(P'^{-}-(P'-k)^{-}_{on})]}{(P'^{-}-(P'-k)^{-}_{on}-k^{-}_{on})(P'^{-}-(P'-k)^{-}_{on}-k^{-}_{R})^2}\right]
\end{aligned}
$$

similar procedure apply for other terms.

Charge radii

- To find out the charge radius of pion $\langle r_\pi \rangle$ and kaon $\langle r_K \rangle$, we use the following relation between form factors and the radius

$$F_{\pi(K)}(q^2) \simeq 1 - \frac{1}{6}\langle r^2_{\pi(K)} \rangle q^2$$

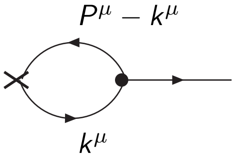
differentiating with respect to q^2, we get

$$\langle r^2_{\pi(K)} \rangle = -6\left[\frac{dF_{\pi(K)}(q^2)}{dq^2}\right]_{q^2=0}$$

Decay Constants

- Another relevant quantity to be used for constraining the parameters of our model are the decay constants, f_π and f_K.
- It is defined through the matrix element of the partially conserved axial-vector current

$$P_\mu \langle 0 | A_i^\mu(0) | \pi_j(K_j) \rangle = i m^2_{\pi(K)} f_{\pi(K)} \delta_{ij}.$$



where

$$A_i^{\mu} = \bar{q}\gamma^{\mu}\gamma^{5}\frac{\tau_i}{2}q$$

- To find out the decay constant we need the light-front wave function which can be obtained from the Bethe-Salpetr amplitude eliminating the relative light-front time.
- In the present model the Bathe-Salpeter amplitude is

$$\psi(k,P) = \frac{m}{f_{\pi}}\frac{\not{k}+m}{k^2-m^2+i\epsilon}\gamma^5\Lambda(k,P)\frac{\not{k}-\not{P}+m}{(k-P)^2-m^2+i\epsilon}.$$

The momentum part of the valence component of the light-front wave function, $\phi(k^+, \vec{k}_\perp; P^+, \vec{P}_\perp)$, can be obtained by eliminating out from the above equation:

- The instantaneous terms,
- The factors containing gamma matrices in the numertaor, and
- The k^+ and $(P^+ - k^+)$ factors appearing in the denominator.

Then, after introducing the explicit expression for Λ, one has to integrate over k^-,

$$\begin{aligned}\phi(k^+, \vec{k}_\perp; P^+, \vec{P}_\perp) &= i\mathcal{N} \int \frac{dk^-}{2\pi} \\ &\times \frac{1}{(k^- - k^-_{on} + \frac{i\epsilon}{k^+})(P^- - k^- - (P-k)^-_{on} + \frac{i\epsilon}{P^+ - k^+})} \\ &\times \left(\frac{1}{k^2 - m_R^2 + i\epsilon} + \frac{1}{(P-k)^2 - m_R^2 + i\epsilon} \right),\end{aligned}$$

where $\mathcal{N} = \sqrt{N_c} C \frac{m}{f_\pi}$.

Performing the k^- integration in $\phi(k^+, \vec{k}_\perp; P^+, \vec{P}_\perp)$, we get

$$\begin{aligned}\phi(k^+, \vec{k}_\perp; P^+, \vec{P}_\perp) &= \frac{P^+}{m^2_{\pi(K)} - M_0^2}\Bigg[\frac{\mathcal{N}}{(1-x)(m^2_{\pi(K)} - \mathcal{M}^2(m^2, m_R^2))} \\ &+ \frac{\mathcal{N}}{x(m^2_{\pi(K)} - \mathcal{M}^2(m_R^2, m^2))}\end{aligned}$$

where

$$x = \frac{k^+}{P^+}, \text{ with } 0 \le x \le 1;$$

$$\mathcal{M}^2(m_a^2, m_b^2) = \frac{k_\perp^2 + m_a^2}{x} + \frac{(P-k)_\perp^2 + m_b^2}{1-x} - P_\perp^2;$$

and the squae of the free mass is $M_0^2 = \mathcal{M}^2(m^2, m^2)$.

- Therefor, by using the same ansatz for the pion-$q\bar{q}$ vertex function, (i.e. symmetric by the exchange of the momenta of the two constituents) we obtain

$$iP^2 f_{\pi(K)} = \frac{\hat{m}}{f_{\pi(K)}} N_c \int \frac{d^4k}{(2\pi)^4} Tr[\not{P}\gamma^5 S(k)\gamma^5 S(k-P)]\Lambda(k,P),$$

and integrating on k^-, one gets $f_{\pi(K)}$ in terms of the valence component of our model:

$$f_{\pi(K)} = \frac{\hat{m}\sqrt{N_c}}{4\pi^3} \int \frac{d_\perp^k dk^+}{k^+(m_{\pi(K)-k^+})} \phi(k^+, \vec{k}_\perp; m_{\pi(K)}, \vec{0})).$$

where $\hat{m} = \frac{m_q + m_{\bar{q}}}{2}$.

Best fit of the model's parameters

We have also found the best fit values of the model's parameters $m_u = m_d$, m_s and m_R by matching the closest experimental values of the decay constant $f_{\pi(K)}$ and charge radius $\langle r_{\pi(K)} \rangle$.

m_R=0.6 GeV;

m_u=m_d=0.22 GeV; m_s=0.50 GeV.

f_π=93.12 MeV (f_π^{exp}=92.42 MeV)(0.76%),

$\langle r_\pi \rangle$=0.736 fm (r_π^{exp}=0.672 fm)(8.7%),

f_K=115.1 MeV (f_K^{exp}=110.4 MeV)(4.1%),

$\langle r_K \rangle$=0.714 fm (r_K^{exp}=0.560 fm)(21.5%),

m_R=0.6 GeV;

m_u=m_d=0.25 GeV; m_s=0.50 GeV.

f_π=101.85 MeV (f_π^{exp}=92.42 MeV)(9.3%),

$\langle r_\pi \rangle$=0.670 fm (r_π^{exp}=0.672 fm)(0.35%),

f_K=118.4 MeV (f_K^{exp}=110.4 MeV)(6.8%),

$\langle r_K \rangle$=0.655 fm (r_K^{exp}=0.560 fm)(14.45%),

All observables are less than 15%.

- The experimental value of the ratio of the kaon decay constant f_K and the pion decay constant f_π:

$$\frac{f_{K^-}^{exp}}{f_{\pi^-}^{exp}} = 1.197 \pm 0.002 \pm 0.006.$$

- In our calculation, we have found the ratio:

m_R=0.6 GeV;

m_u=m_d=0.22 GeV; m_s=0.50 GeV.

$$\frac{f_{K^+}}{f_{\pi^+}} = 1.163$$

approximately 2.85% less than the experimental value.

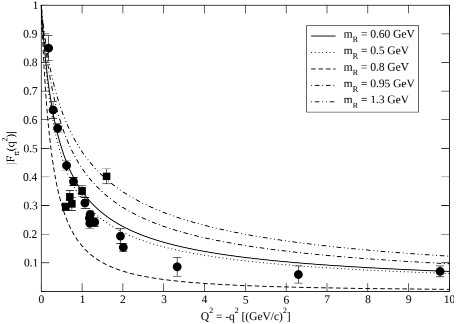
m_R=0.6 GeV;

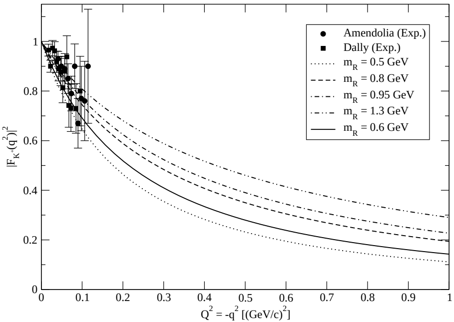
m_u=m_d=0.25 GeV; m_s=0.50 GeV.

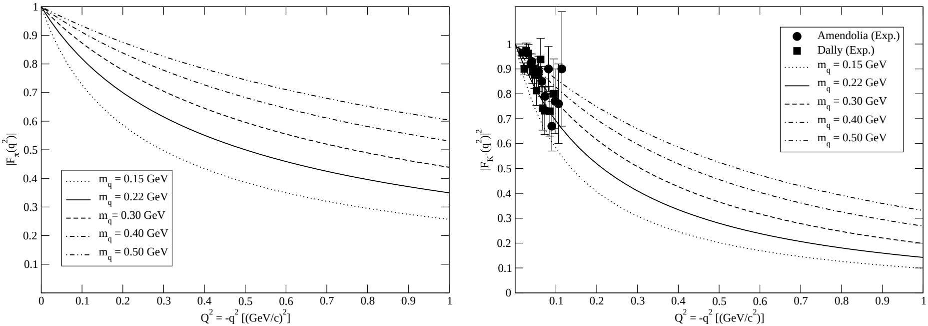
$$\frac{f_{K^+}}{f_{\pi^+}} = 1.236$$

approximately 3.31% larger than the experimental value.

We have plotted the $|F_\pi(q^2)|$ and $|F_K(q^2)|^2$ as a function of q^2. These plotes show that in both cases the electromagnetic form factors are the decreasing function of q^2. In addition to see the dependence on the mass parameters m_R we choose the difference valuses of m_R where one can notice that the value of the form-factors is sensitive to the m_R.

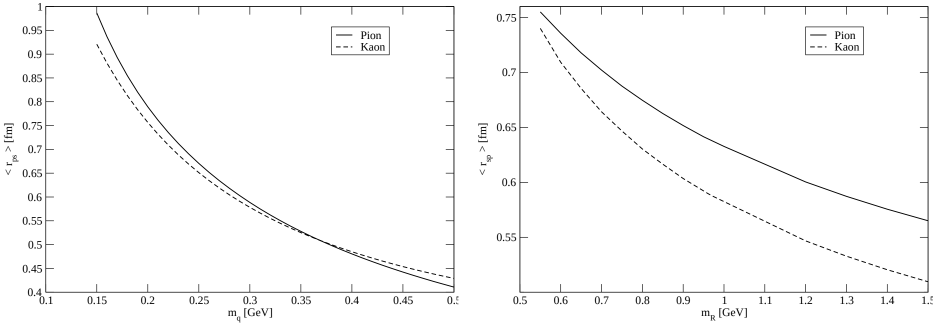






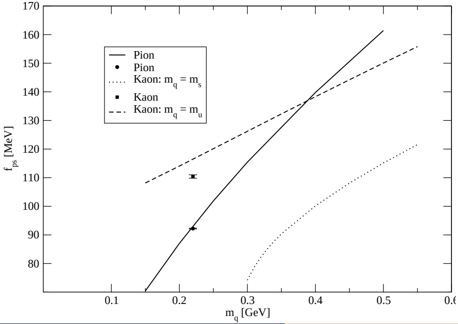
Similarly, to see the dependence on the m_q, we have plotted the $|F_\pi(q^2)|$ and $|F_K(q^2)|^2$ as a function of q^2. These plotes show that in both cases the electromagnetic form factors are are also sensitive to the m_q.

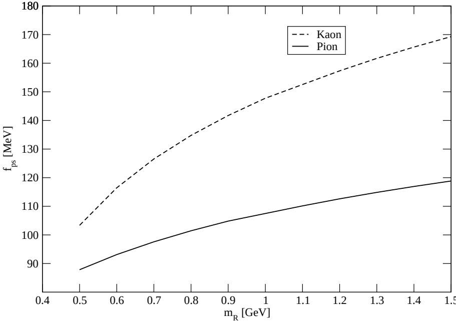
The dependence is in both cases nonlinear and somewhat more pronounced for variations of the quark mass m_q than of the regulaor mass m_R.
We remark that a larger regulator or quark mass results in a smaller charge radius, as expected.



We note that the decay constants are also sensitive to both the regulator and quark masses.

We remark that for the case of the pion and kaon decay constants, $m_R = 0.6$ GeV yields the best adjustment to the experimental value of the decay constants.





Conclusions.

- We reassessed the light-front model in view of available data on $|F_\pi|$ and $|F_K|^2$ and due to the need of a better determination and restriction of the model parameters.
- The regulator mass, $m_R = 0.6$ GeV, using a symmetric vertex model for the bound state, is found to simultaneously satisfy the experimental data on the spacelike form factors, the decay constans, and the charge radii of the pion and the kaon within reasonable theoretical uncertainties.
- These parameters values are useful for, eg., heavy-meson and vector decay constants or heavy-to-light transition form factors.

The next task: To calculate the heavy meson transition form factors, such as, $D \to \pi$, $B \to \pi$,etc.

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