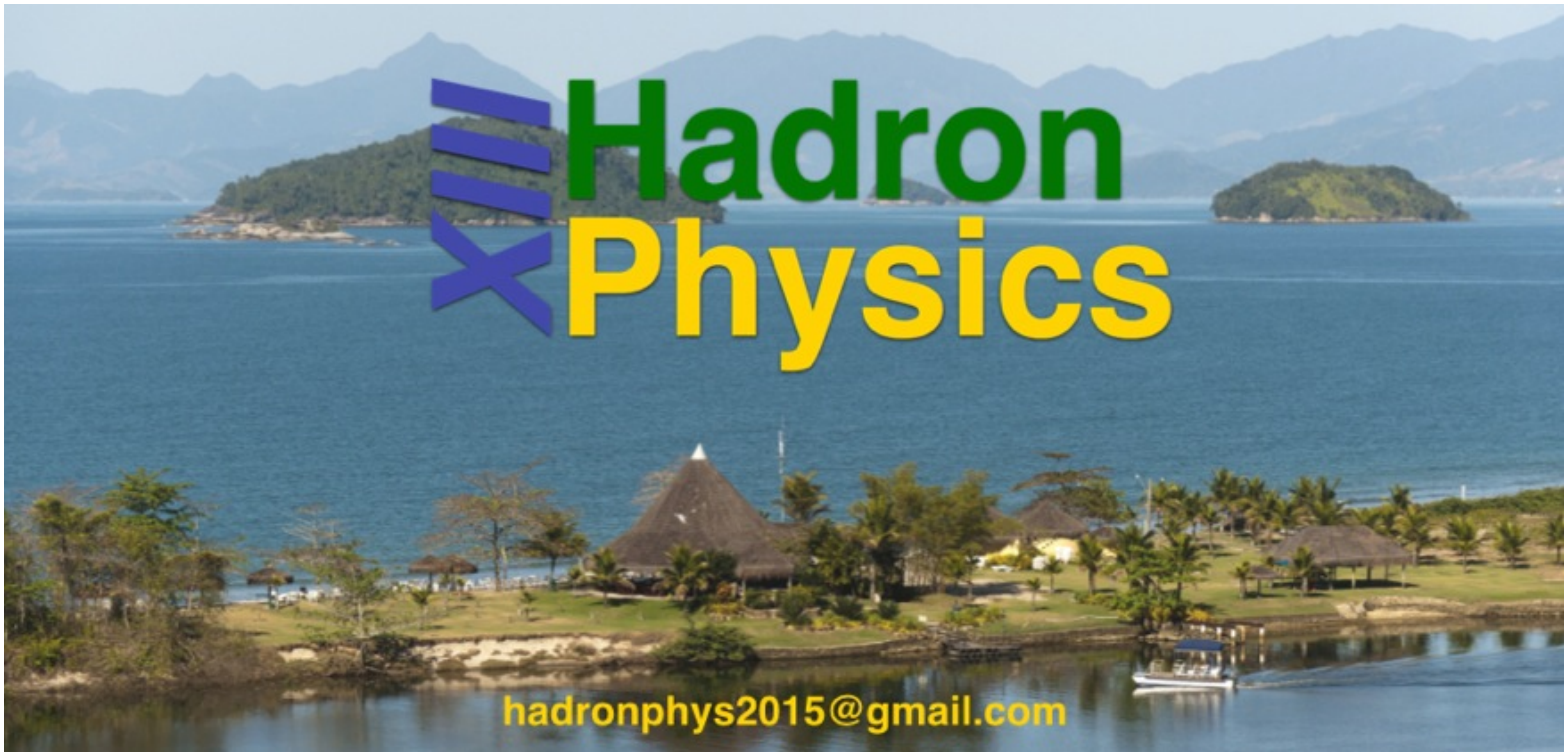
Modelling Hadronic Matter



Colloquium - D.P. Menezes - 24/03/2015

Particle Physics

Hadron Physics

Nuclear Physics

Lattice QCD

QCD - solutions?

Effective models

A bit of history
Liquid drop model (1929)

- The liquid is **incompressible -** the nucleus has low compressibility due to its almost cte internal density

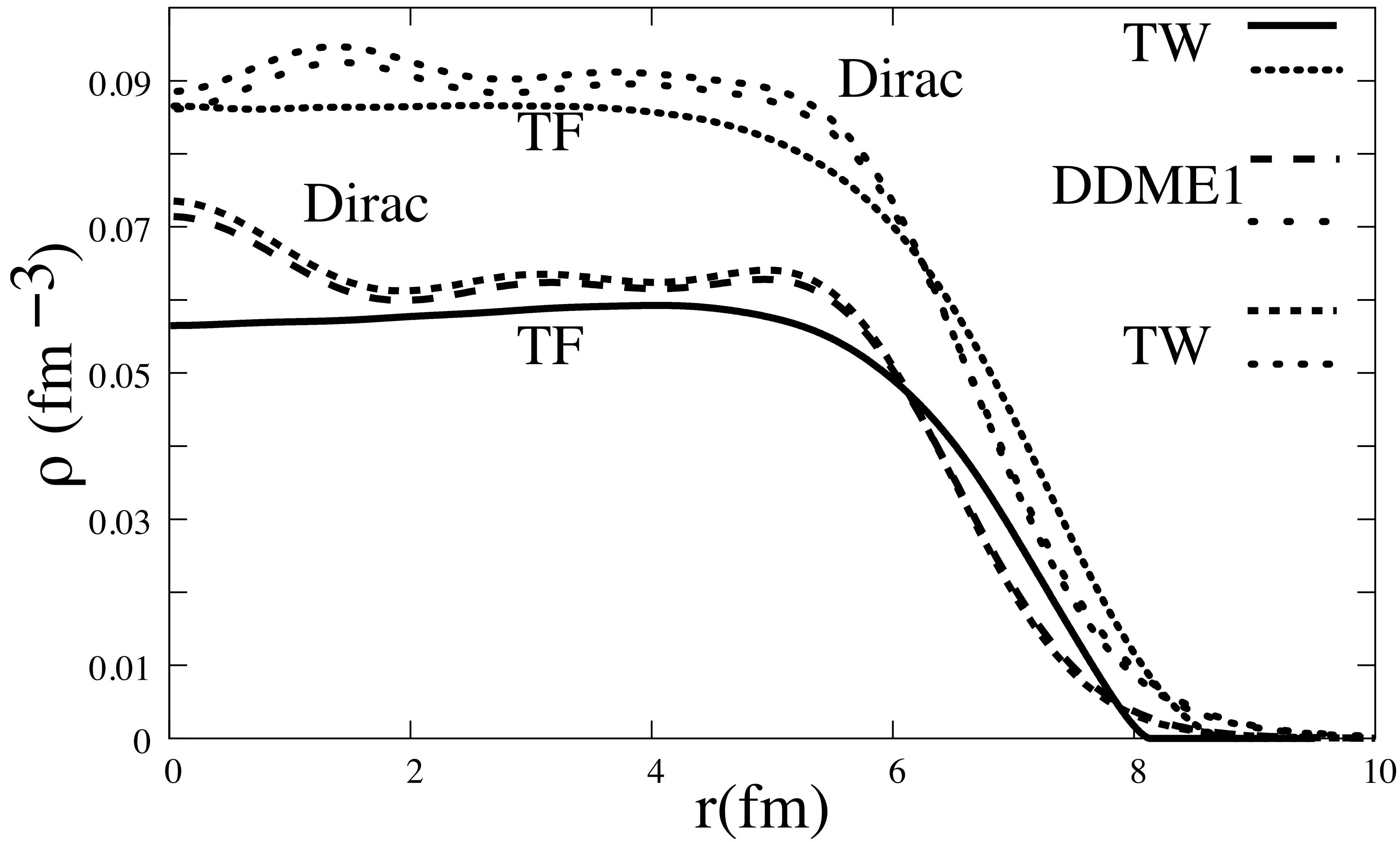
- Nuclei present **well defined surface**

- Radius vary with the number of nucleons such as

$$R = R_0 A^{1/3}$$

- The nuclear force **saturates** and it is **isospin independent**

nuclear density profiles



A bit of history

Semi-empirical mass formula - Bethe and Weizsäcker - 1935

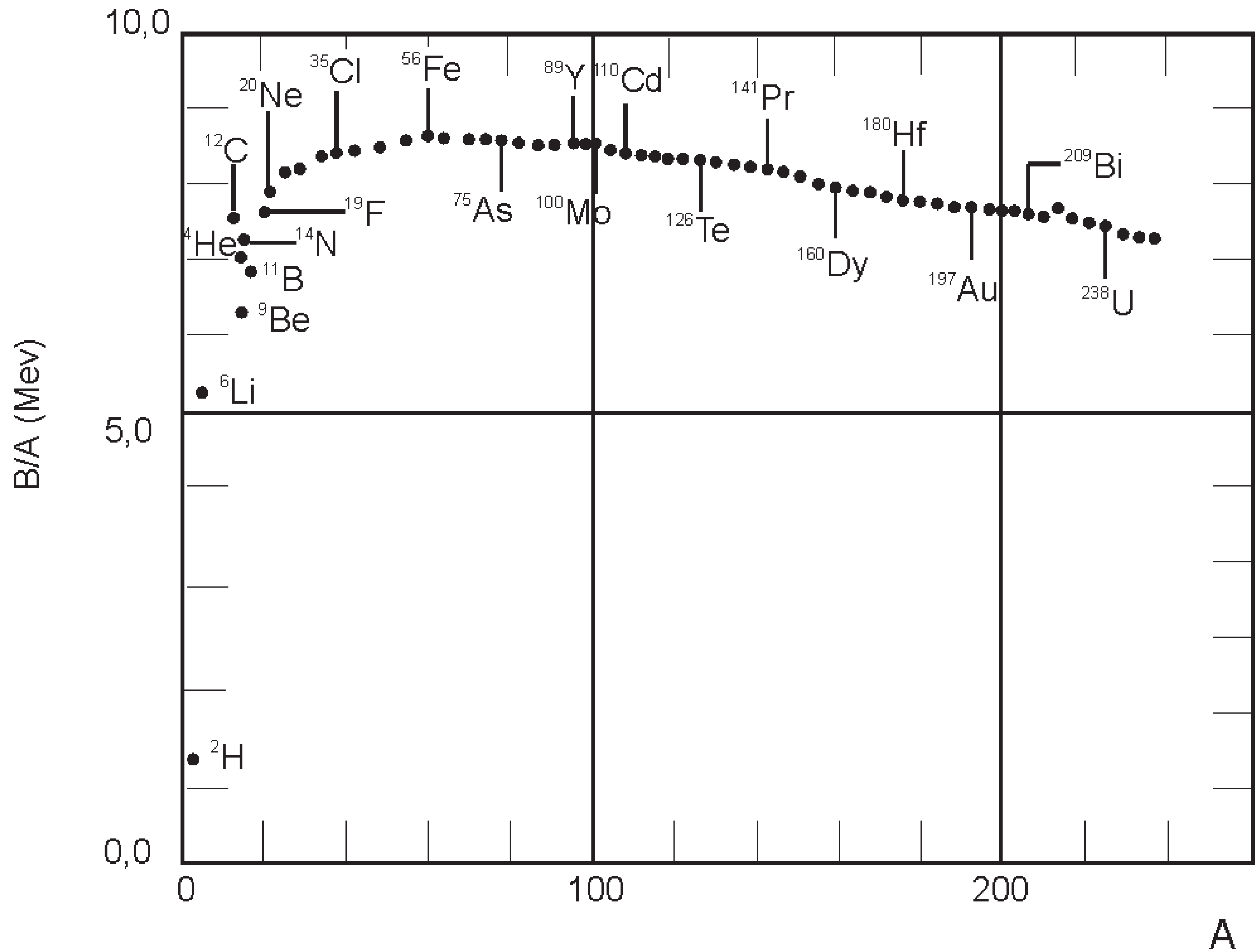
$$m(Z,A) = Zm(^1H) + Nm_n - B(Z,A)/c^2$$

$$B(Z,A) = a_v A - a_s A^{\frac{2}{3}} - a_c e^2 \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_i \frac{(N-Z)^2}{A} + \delta(A)$$

$$a_v = 15,68 \text{ MeV}, a_s = 18,56 \text{ MeV}, a_c \times e^2 = 0,72 \text{ MeV}, a_i = 18,1 \text{ MeV}$$

$$\delta = \pm 34 \quad A^{-3/4}, 0 \quad \text{MeV} - \text{even} - \text{even} \ , \ \text{odd} - \text{odd} \ , \ \text{even} - \text{odd} \ \ \text{nucle}$$

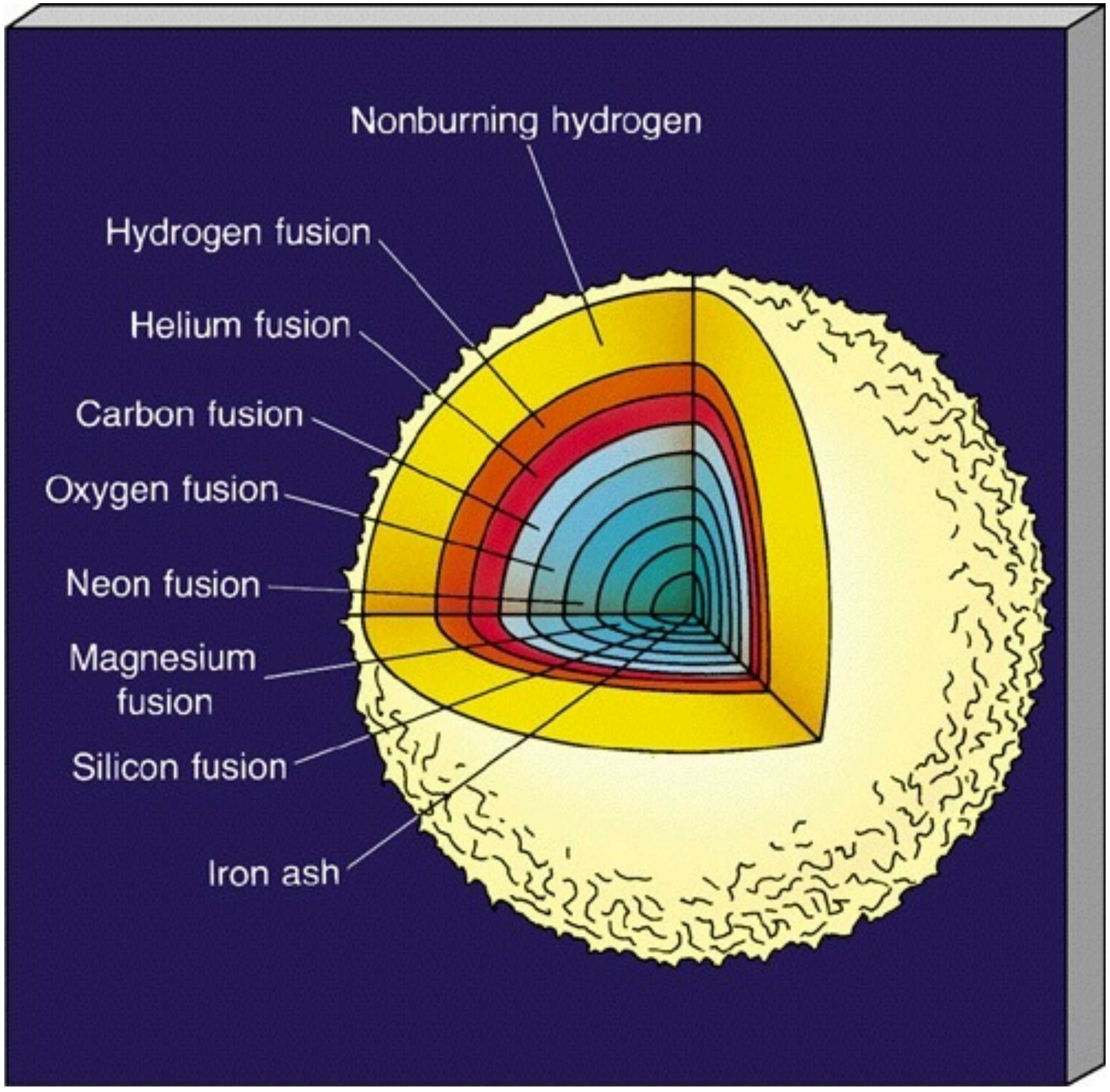
Many other possible parameter sets are possible



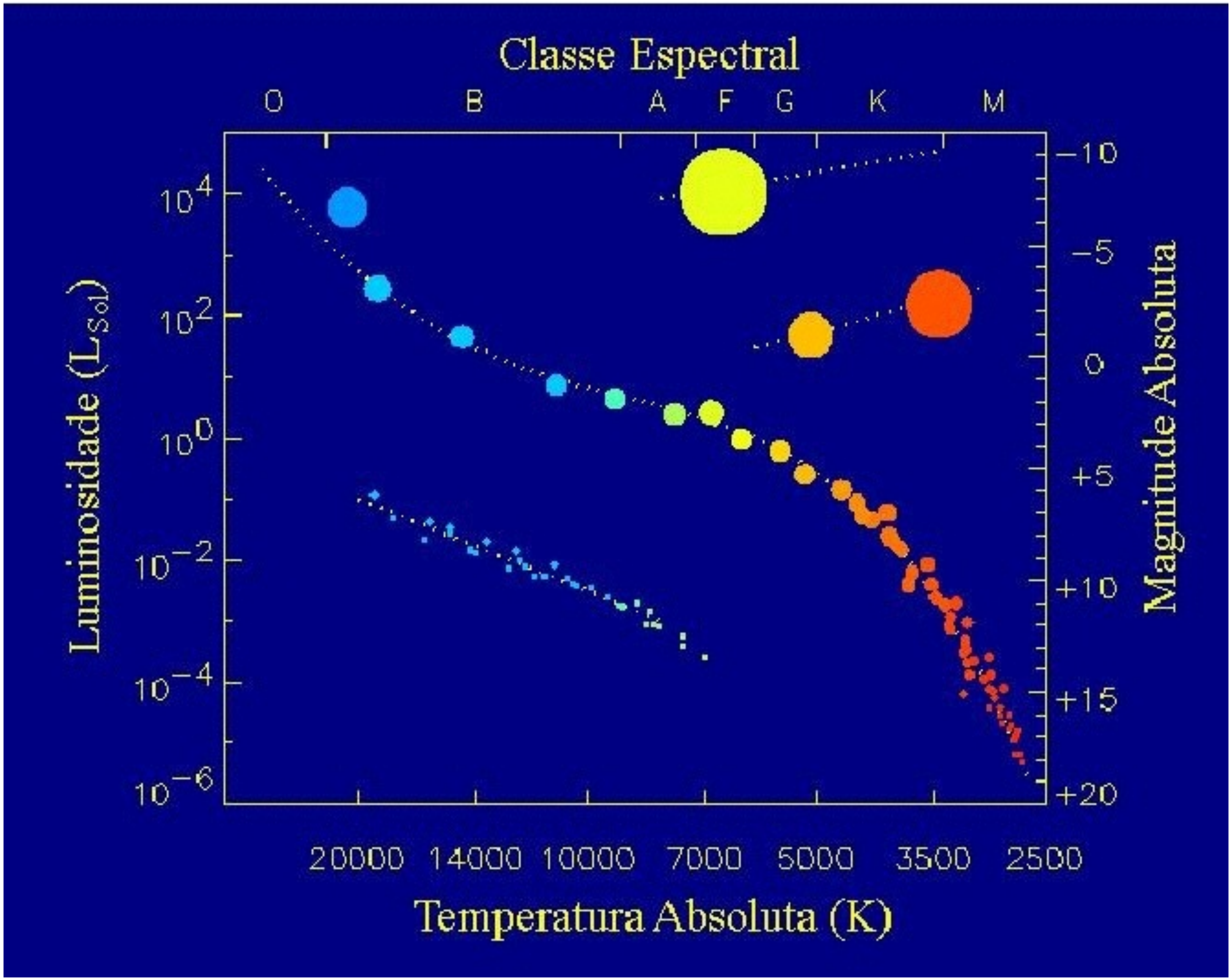
Although very naive, it works well even to explain fission - Coulomb X surface terms

Chemical elements in the Universe

H - 71%; He 27%; C - Ne - 1.8%; Ne - Ti - 0.2 %
Fe - 0.02 %; A > 60 - 0.0001 %



Life and death of a star - Hertzspring & Russel Diagram

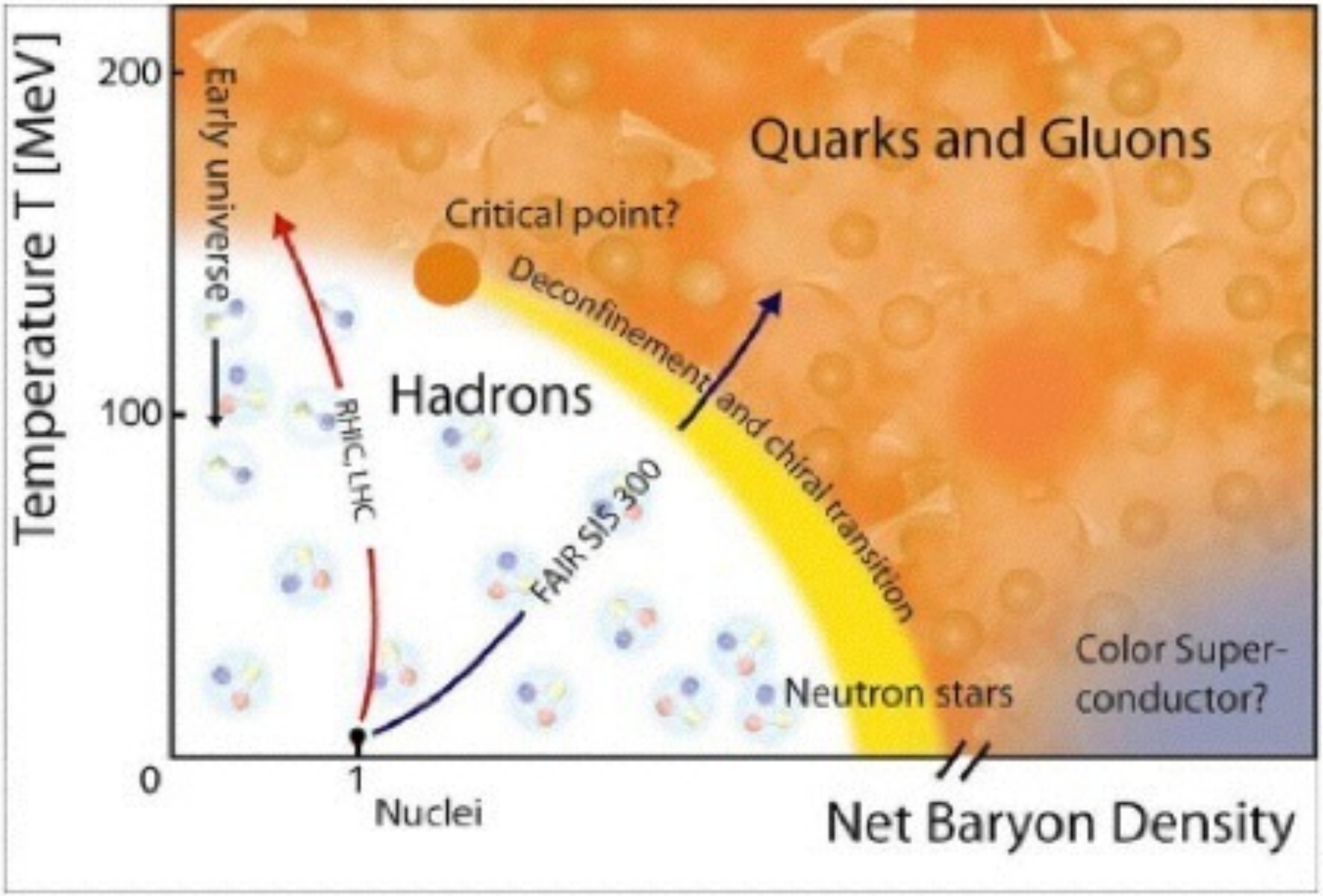


When a star burns all its fuel, it may become a **black hole** or a **neutron star** ($M > 8\ M_{sun}$) or a **white dwarf**

Nuclear physics (hadronic) models can explain:

- the life of the stars (**fusion** reactions)
- some of the compact objects (NS)

QCD Phase Diagram - what parts can we explain?



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Existing models

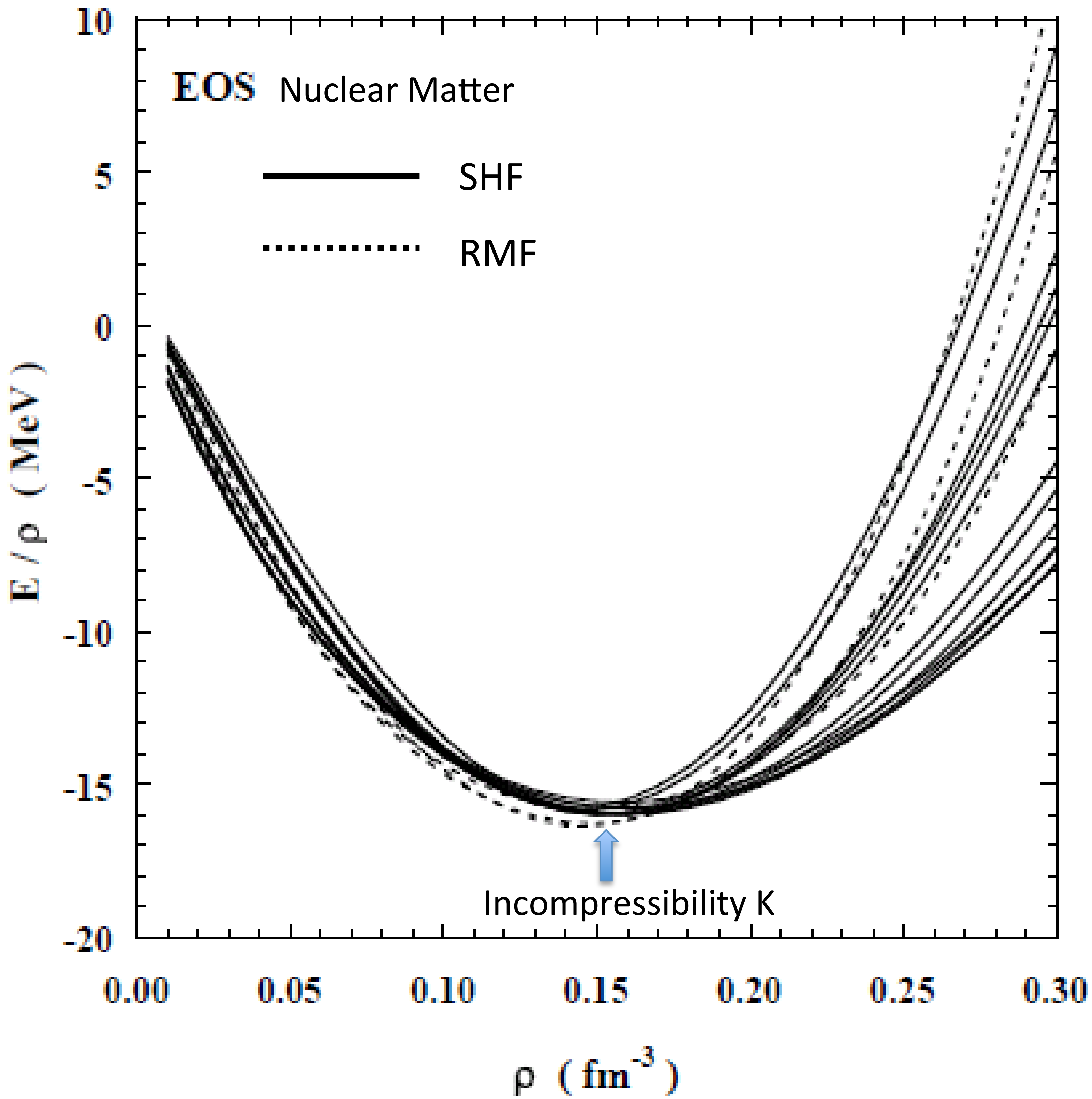
- Parameter dependent nuclear models should satisfy some **nuclear bulk properties:**

- Saturation density, binding energy , symmetry energy (and its derivatives), (in)compressibility

- Many parameters lead to many different models that satisfy them, both **non-relativistic** (Skyrme-type) and **relativistic** ones.

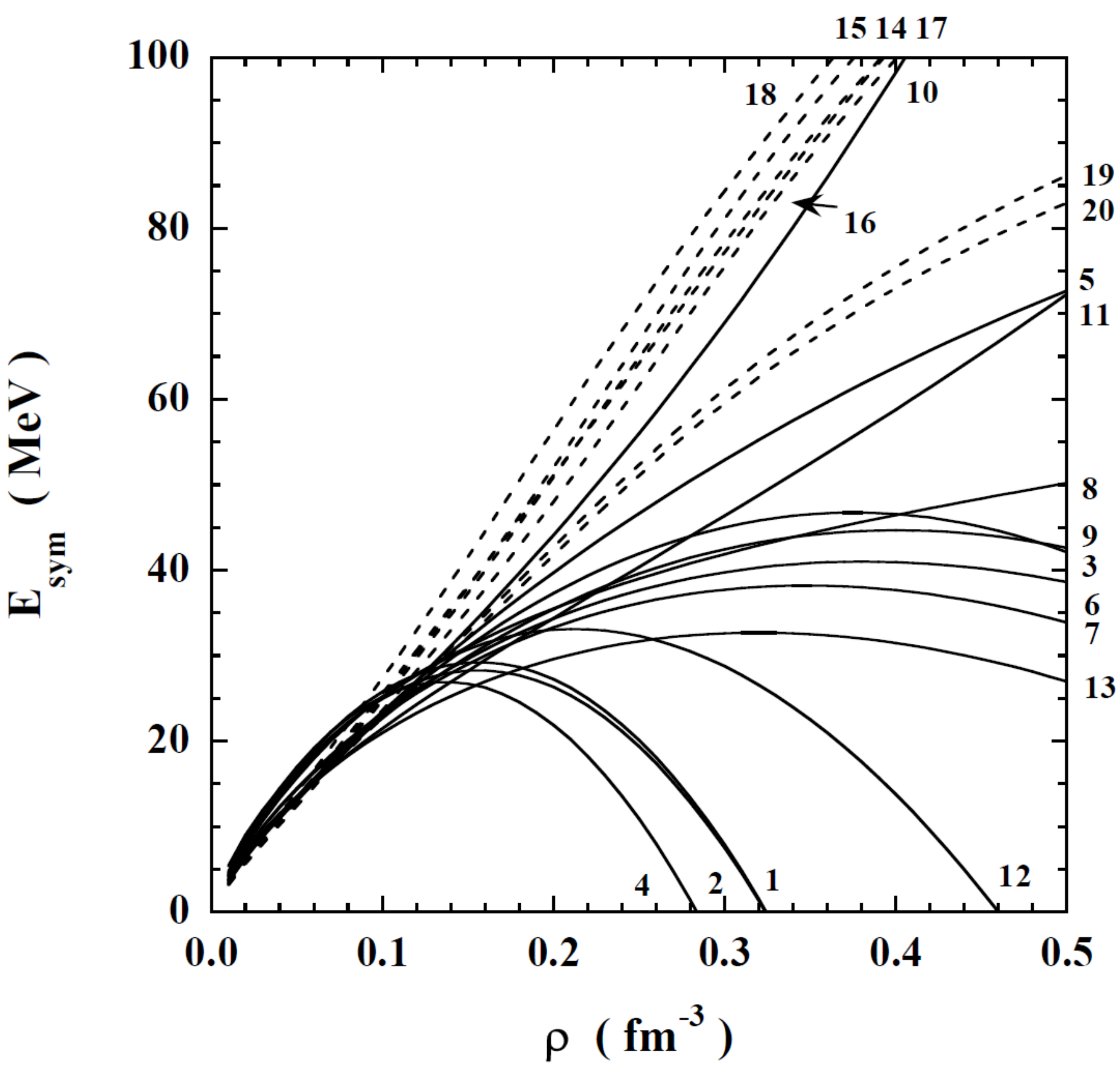
- How to choose?

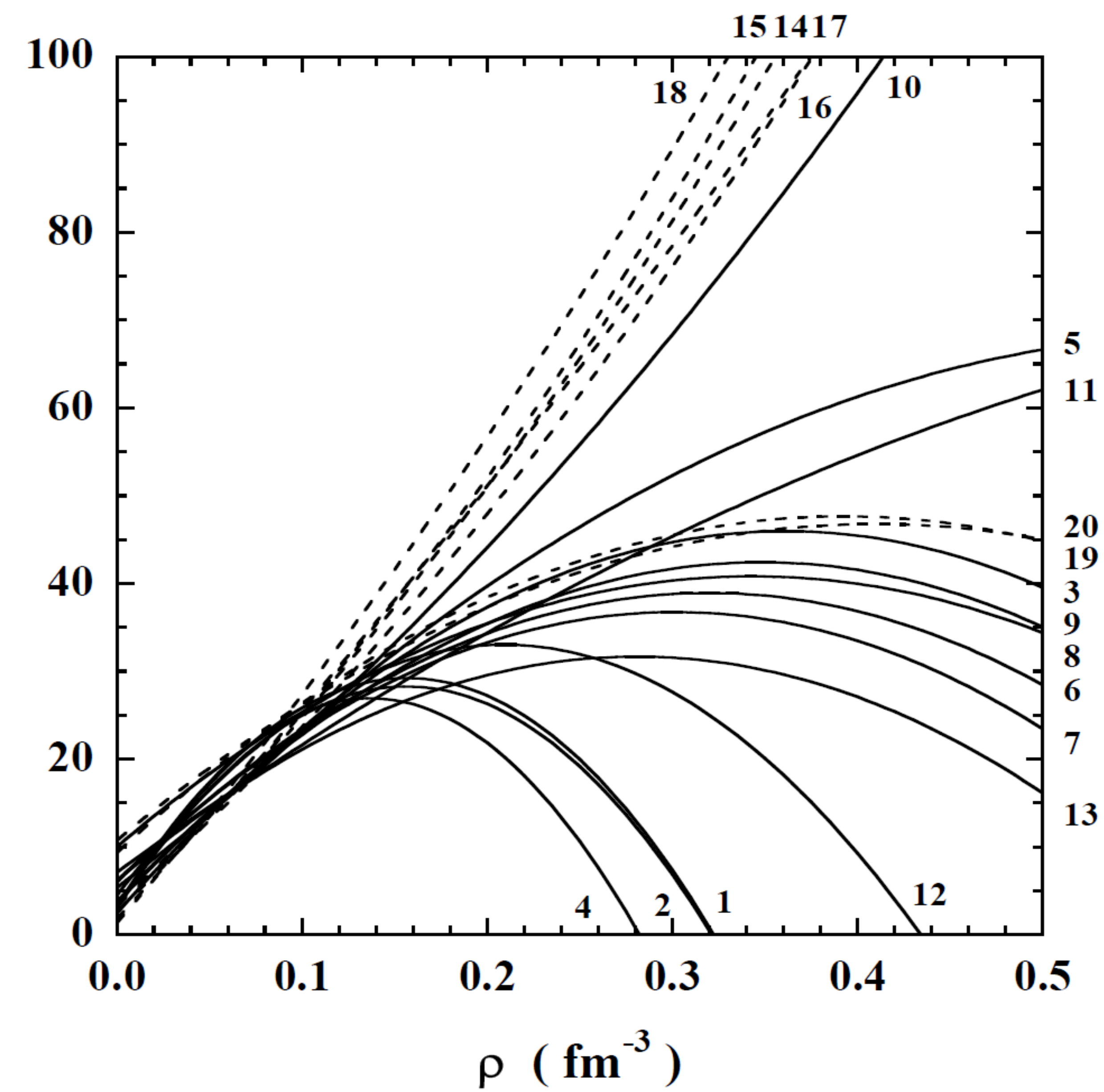
What happens when we move to higher densities?
What about finite temperature?



$$\varepsilon_\delta(\rho) = \frac{1}{2} \lim_{I\to 0} \frac{\partial^2}{\partial I^2} \left(\frac{H_{nm}}{\rho}\right)$$

$$\varepsilon_\delta(\rho) = J + \frac{L}{3}\frac{\rho-\rho_{nm}}{\rho_{nm}} + \frac{K_{sym}}{18}\left(\frac{\rho-\rho_{nm}}{\rho_{nm}}\right)^2.$$





Constraints can be obtained from **experimental data**:

1) isoscalar giant monopole resonances (GMR), isovector giant dipole resonances (GDR) : **incompressibility**

2) heavy ion collisions, pygmy dipole resonances, isobaric analog states, GMR, GDR : **symmetry energy and its slope**

3) neutron skin thickness, isospin diffusion calculations, GMR :

volume part of the isospin incompressibility

240 non-relativistic Skyrme models were assessed, in describing nuclear matter up to about 3 times nuclear saturation density. **16 were approved - Phys. Rev. C 85, 035201 (2012)**

263 Relativistic mean-field (**RMF**) models were also tested in a comparable approach. **35 were approved - Phys. Rev. C 90, 055203 (2014)**

Common problems of Skyrme models:

- Many EoS derived with non relativistic formalisms are only suited at low densities; the EoS becomes acausal (the speed of sound exceeds the speed of light at high densities);
- Non-relativistic models lead to symmetry energies that decrease too much beyond $3\rho_0$; this is a serious deficiency for neutron stars (highly asymmetric systems). These deficiencies can be cured with the inclusion of a three-body force (too complicated).

Advantages of relativistic models

- **Same** relativistic models can be applied to describe the physics involved in:

  nuclear matter; stellar matter and heavy-ion reactions

- They are Lorentz invariant and strictly causal;
- Anti-particles appear naturally;
- Mesonic degrees of freedom are explicitly treated.

Structure and evolution of compact stars

- Essential ingredients for astrophysical model calculations can be obtained from appropriate **Equations of State** (EoS) of dense matter:

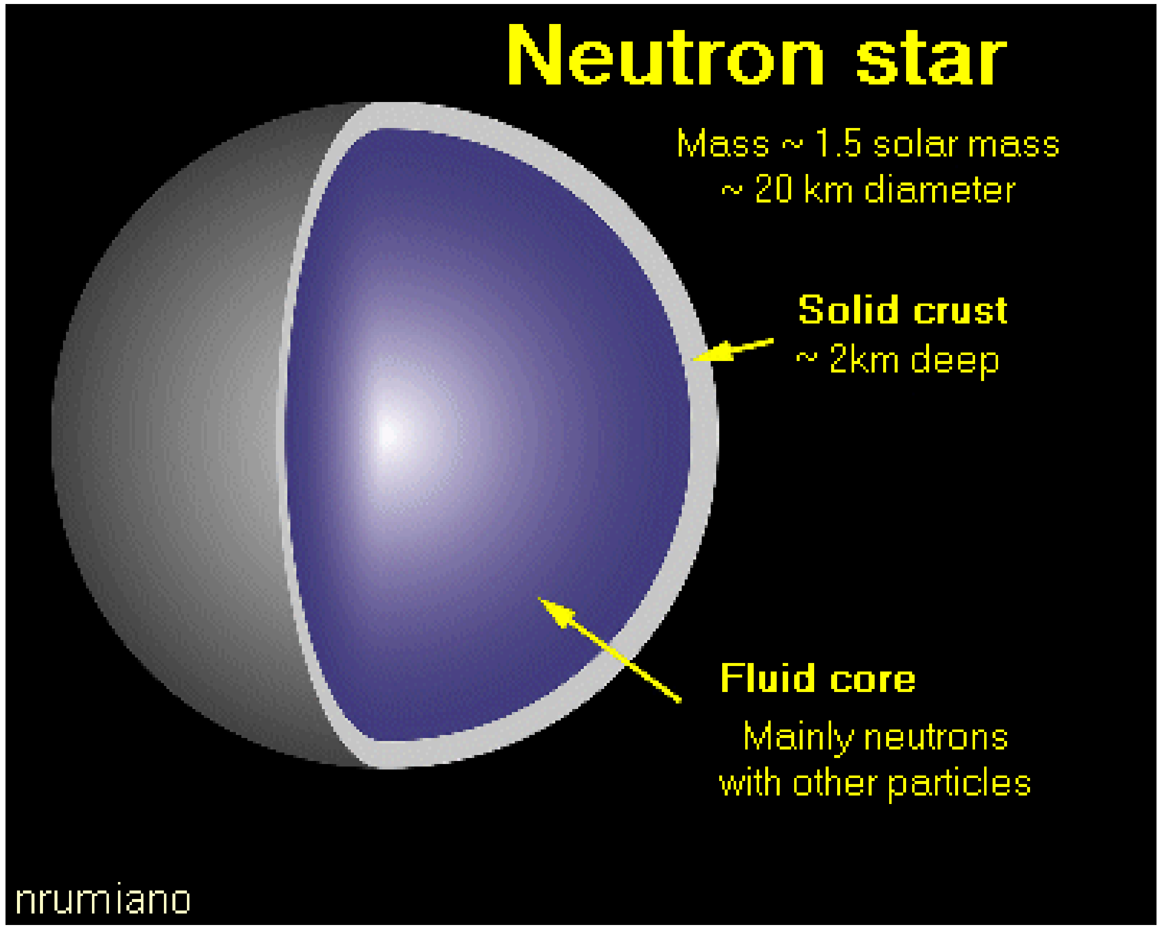
- static properties of (proto)neutron stars (radius, mass, moment of inertial, etc);

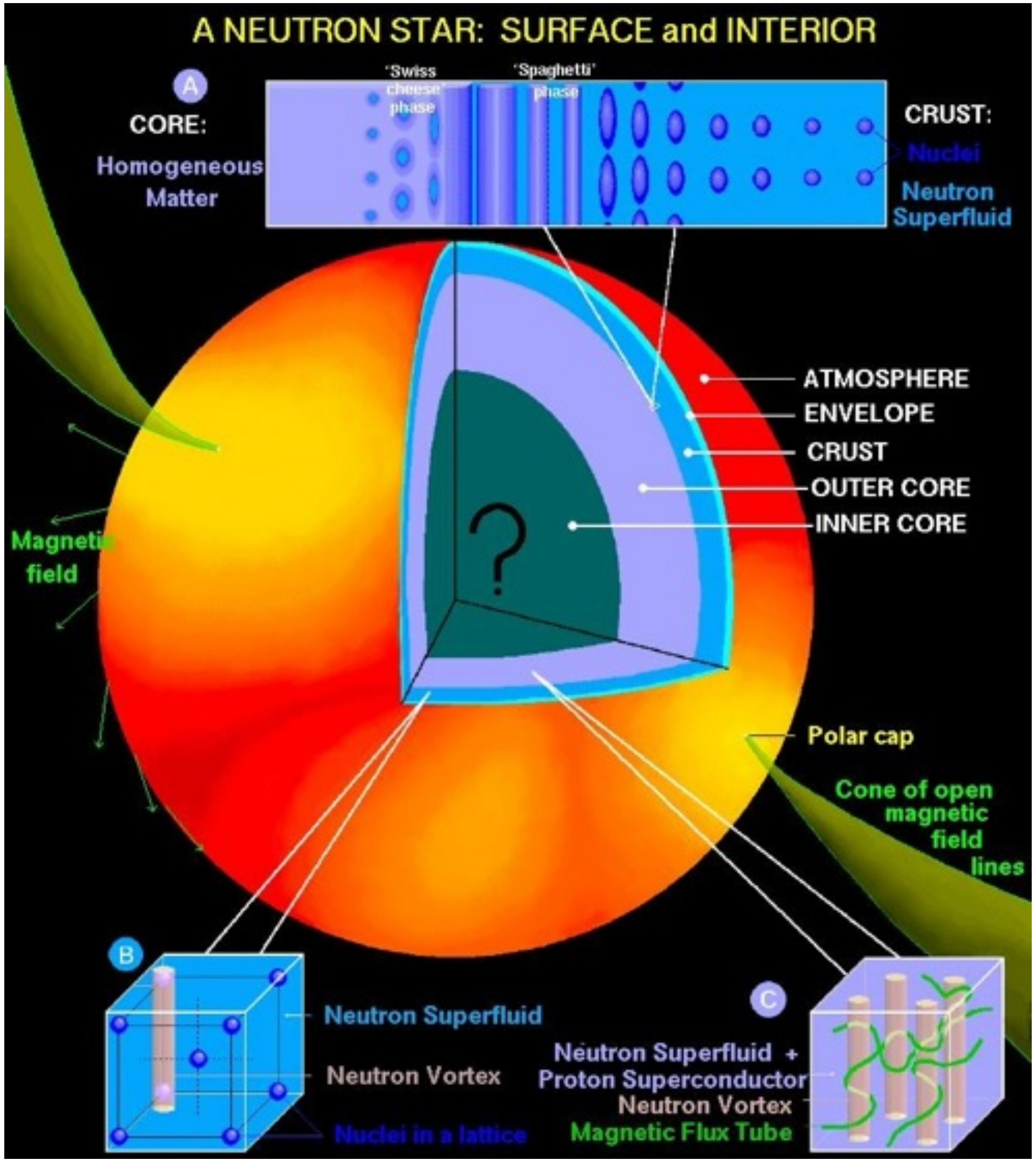
- conditions for nucleosynthesis;

- dynamical evolution of supernova;

- energetics, chemical composition, transport properties,etc

Structure of a NS





Non-linear models

$$
\begin{aligned}
\mathcal{L}_{\mathrm{nm}} &= \overline{\psi}(i\gamma^\mu\partial_\mu - M)\psi + g_\sigma\sigma\overline{\psi}\psi - g_\omega\overline{\psi}\gamma^\mu\omega_\mu\psi - \frac{g_\rho}{2}\overline{\psi}\gamma^\mu\vec{\rho}_\mu\vec{\tau}\psi + g_\delta\overline{\psi}\vec{\delta}\vec{\tau}\psi, \\
\mathcal{L}_\sigma &= \frac{1}{2}(\partial^\mu\sigma\partial_\mu\sigma - m_\sigma^2\sigma^2) - \frac{A}{3}\sigma^3 - \frac{B}{4}\sigma^4, \\
\mathcal{L}_\omega &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{C}{4}(g_\omega^2\omega_\mu\omega^\mu)^2, \\
\mathcal{L}_\rho &= -\frac{1}{4}\vec{B}^{\mu\nu}\vec{B}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu, \\
\mathcal{L}_\delta &= \frac{1}{2}(\partial^\mu\vec{\delta}\partial_\mu\vec{\delta} - m_\delta^2\vec{\delta}^2), \\
\mathcal{L}_{\sigma\omega\rho} &= g_\sigma g_\omega^2\sigma\omega_\mu\omega^\mu\left(\alpha_1 + \frac{1}{2}{\alpha_1}'g_\sigma\sigma\right) + g_\sigma g_\rho^2\sigma\vec{\rho}_\mu\vec{\rho}^\mu\left(\alpha_2 + \frac{1}{2}{\alpha_2}'g_\sigma\sigma\right) \\
&\quad + \frac{1}{2}{\alpha_3}'g_\omega^2 g_\rho^2\omega_\mu\omega^\mu\vec{\rho}_\mu\vec{\rho}^\mu
\end{aligned}
$$

Baryon	M (MeV)	q content	J	$\vec{\tau}$	τ_3	S	electric charge
p	938.28	uud	1/2	1/2	+1/2	0	1
n	939.57	udd	1/2	1/2	-1/2	0	0
Λ	1115.6	uds	1/2	0	0	-1	0
Σ^+	1189.4	uus	1/2	1	+1	-1	+1
Σ^0	1192.5	uds	1/2	1	0	-1	0
Σ^-	1197.3	dds	1/2	1	-1	-1	+1
Ξ^0	1314.9	uss	1/2	1/2	+1/2	-2	0
Ξ^-	1321.3	dss	1/2	1/2	-1/2	-2	-1

Stellar matter is subjet to **chemical equilibrium** and **charge neutrality** conditions:

$$\mu_{\Sigma^+} = \mu_p = \mu_n - \mu_e, \quad \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_\Lambda = \mu_n, \quad \mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e$$

$$\sum_B q_B \rho_B + \sum_l q_l \rho_l = 0$$

RMF - usual steps:

Mean-field approximation (the meson fields are treated as classical fields)

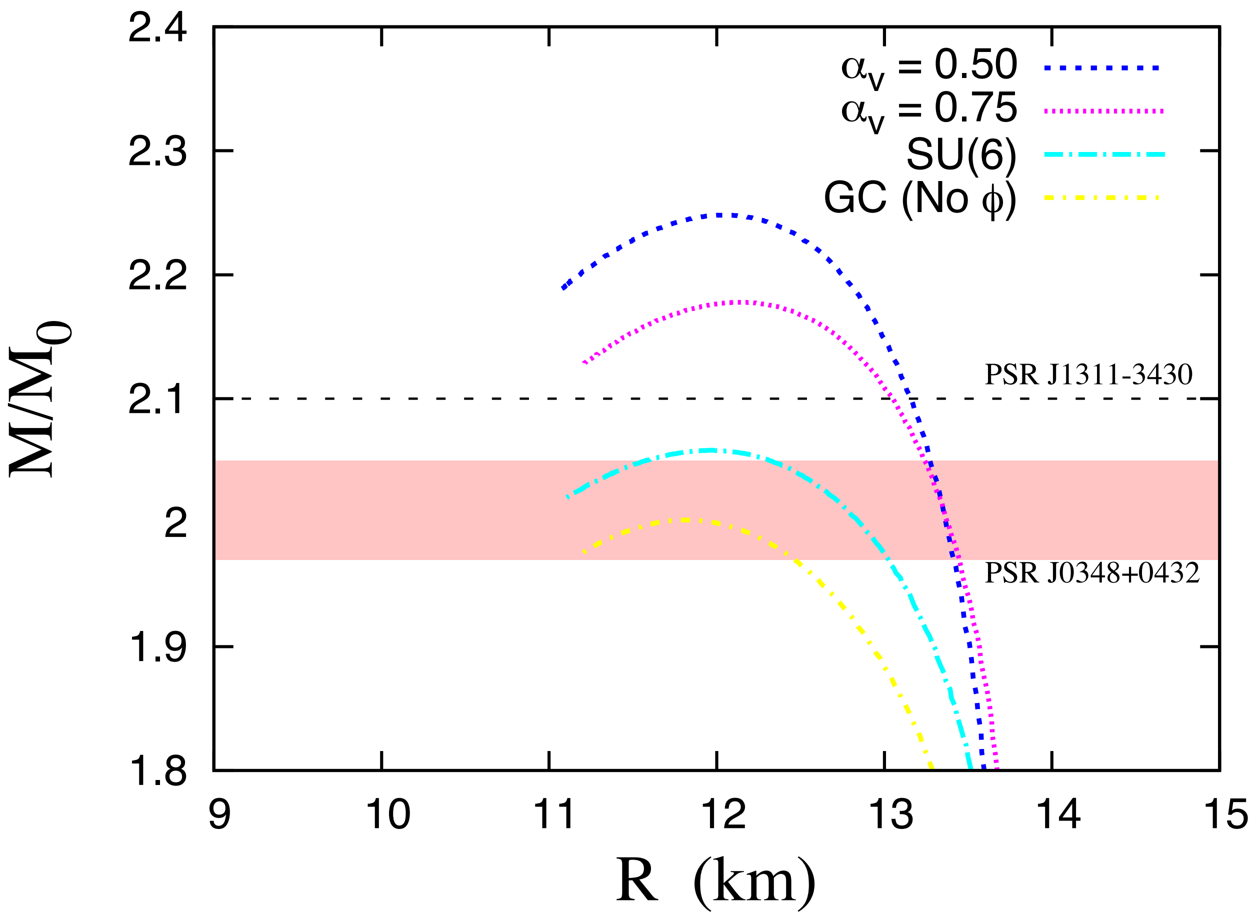
Euler-Lagrange equations → equations of motion (translational and rotational invariance) → energy-momentum tensor → EoS

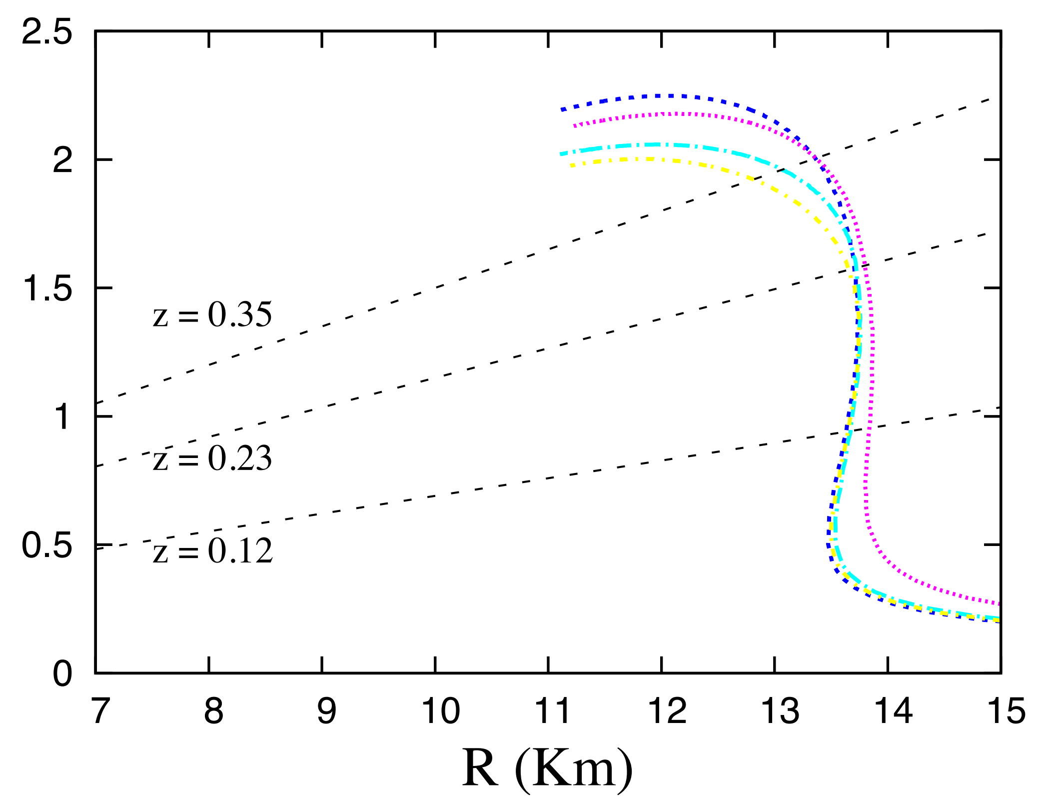
EoS is input to the Tolman-Oppenheimer-Volkoff equations:

$$\frac{dP}{dr} = -\frac{G}{r}\frac{[\varepsilon + P]\left[M + 4\pi r^3 P\right]}{(r - 2GM)}$$

$$\frac{dM}{dr} = 4\pi r^2 \varepsilon$$

Tuning the EoS to describe massive stars

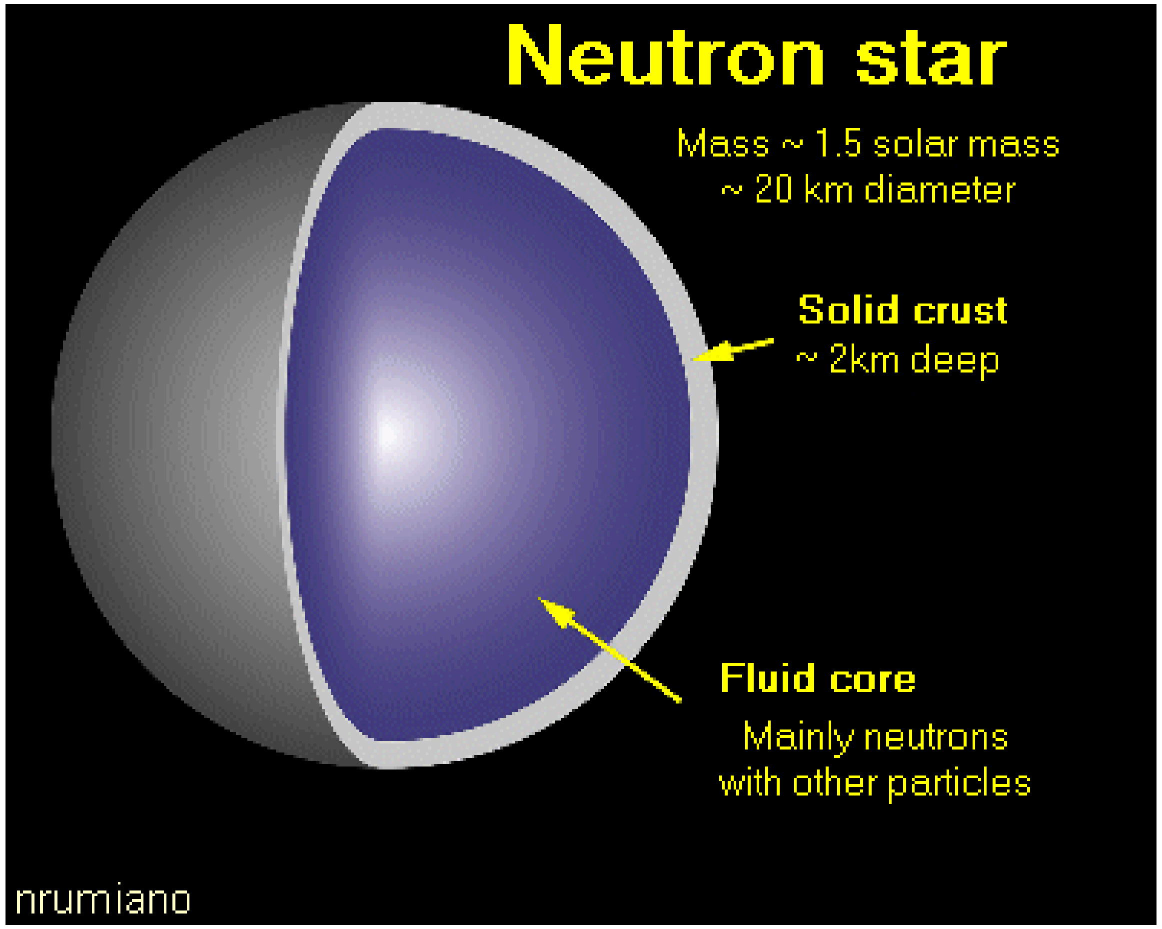


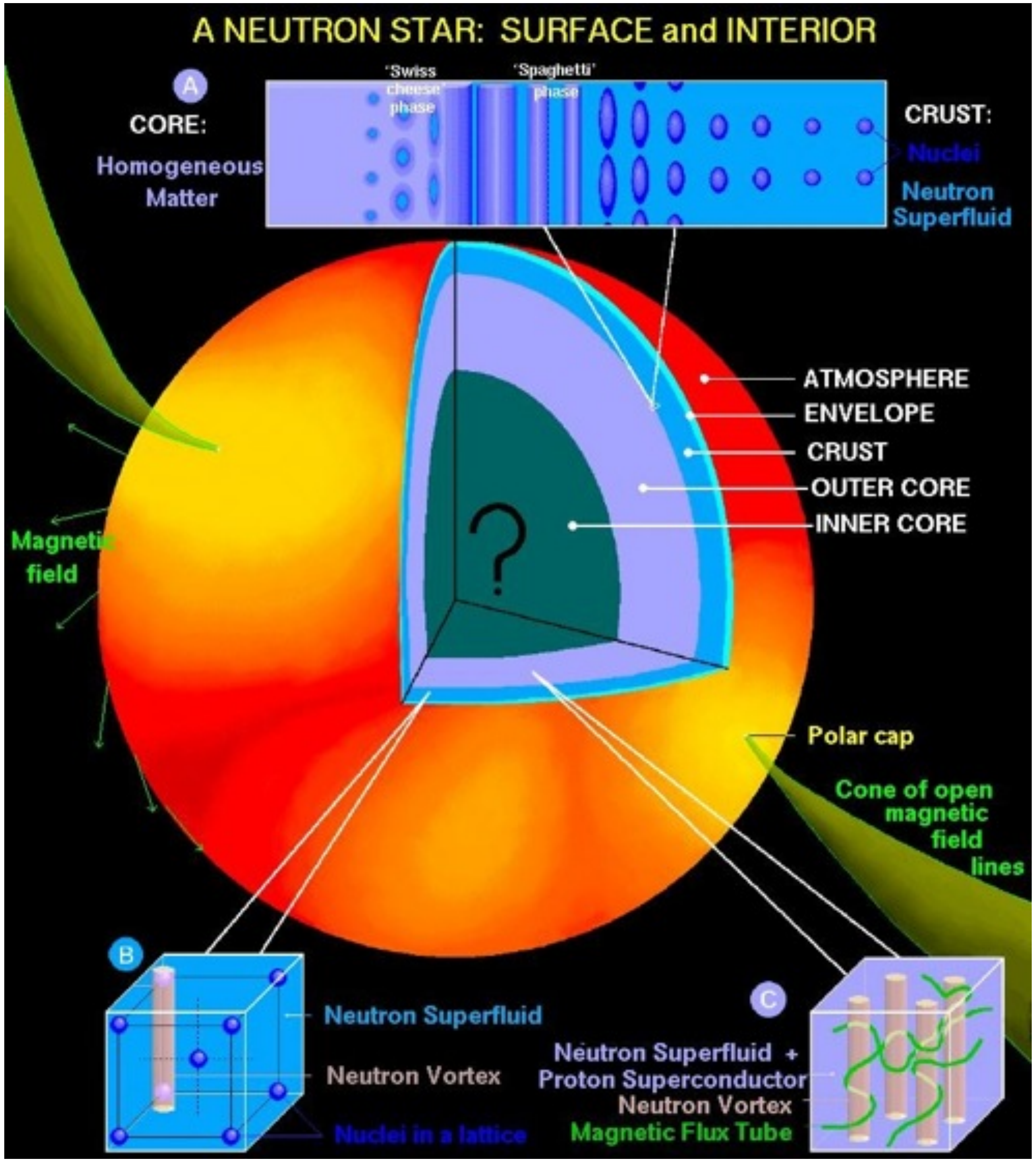


EoS for astrophysical applications

- **Detailed aspects are well-known:**
- matter at zero temperature;
- symmetric nuclear matter, pure neutron matter;
- low density matter; high density matter, matter in beta-eq
- inhomogeneous matter (pasta phase)
- **Still to be improved:**
- One EoS that covers the complete parameter space (T, μ_b)
- in a single model by combining different approaches

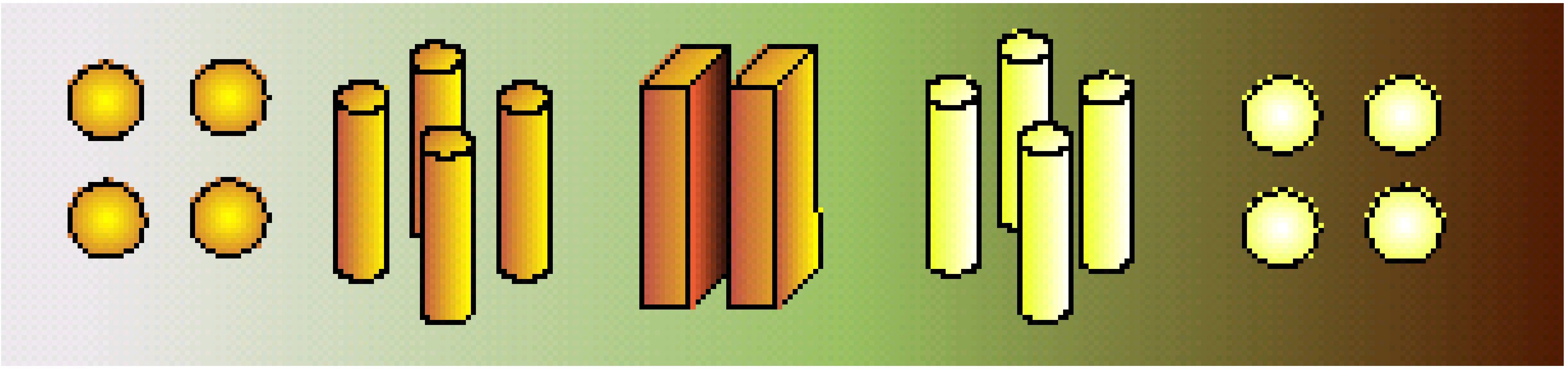
Structure of a NS



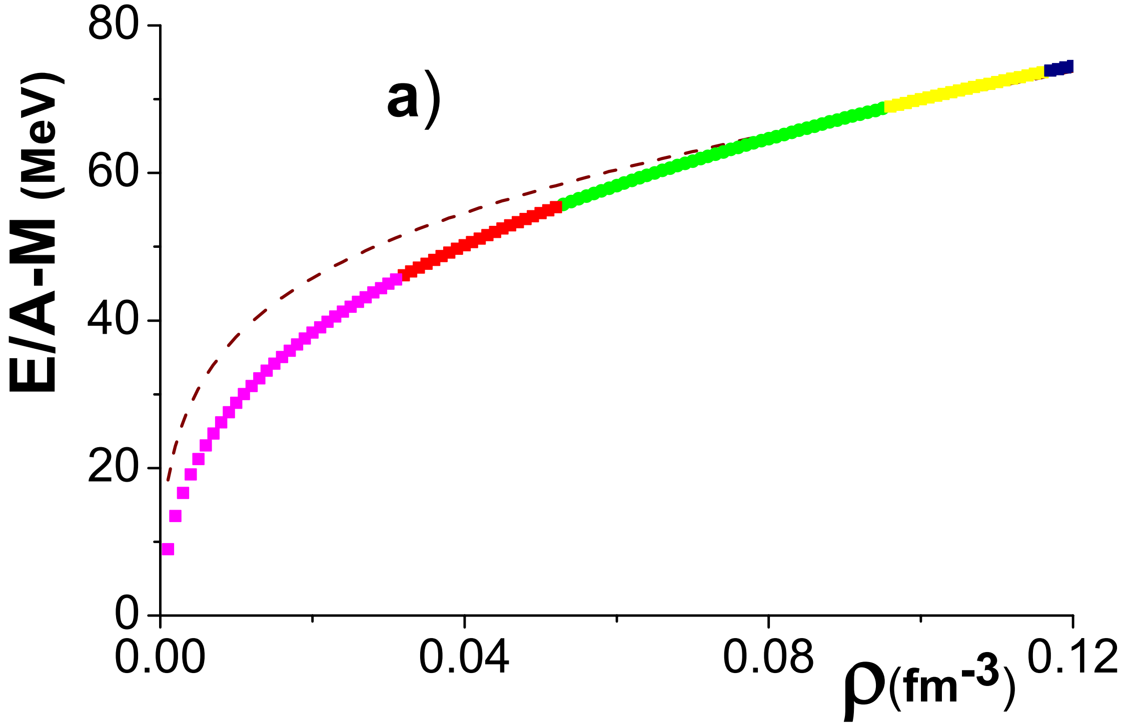


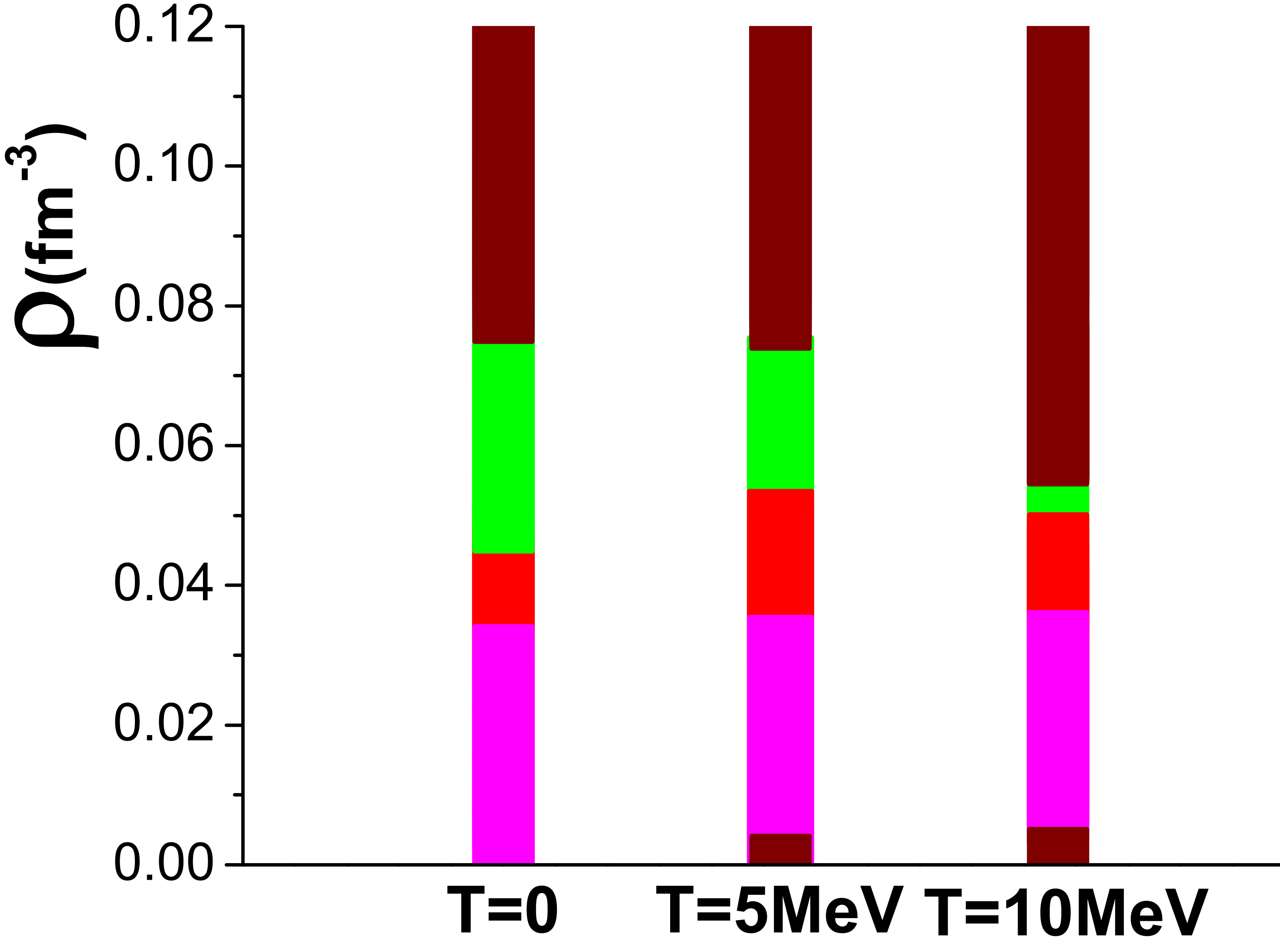
Pasta phase

It is the result of a **frustrated** system. Normally the short and large distance scales related to the nuclear and Coulomb interactions are well separated so that nucleons bind into nuclei but at very low densities, these length scales are comparable and a variety of complex structures exist: **droplet** (meatball, 3D), **rod** (spaghetti, 2D), lazagna (**slab,** 1D), penne (**tube**, 2D), Swiss cheese (**bubble**, 3D).



The pasta phase is the ground state configuration if its free energy is lower than the corresponding homogeneous phase.





Liquid-gas phase transitions

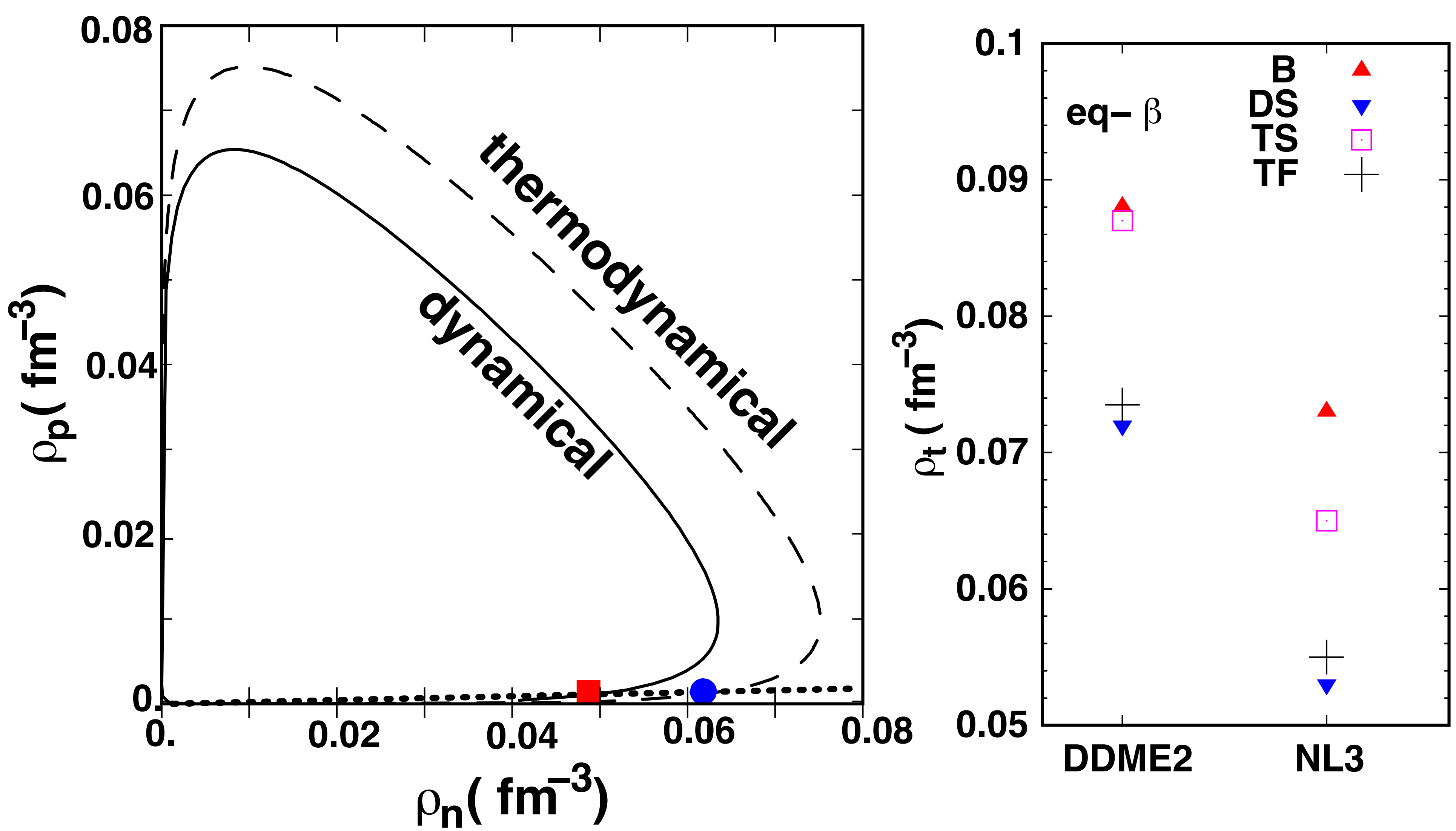
Spinodal instabilities:

thermodynamical - obtained from the determinant of the symmetric matrix:

$$\mathcal{F}_{ij} = \left(\frac{\partial^2 \mathcal{F}}{\partial \rho_i \partial \rho_j} \right)_T ,$$

dynamical - obtained from the Vlasov equation:

$$\frac{\partial f_{i\pm}}{\partial t} + \{f_{i\pm}, h_{i\pm}\} = 0, \qquad i = p,\ n,\ e,$$



0.08
0.06
0.04
0.02
0.
ρ_p(fm^{-3})
thermodynamical
dynamical
0.
0.02
0.04
0.06
0.08
ρ_n(fm^{-3})
0.1
0.09
0.08
0.07
0.06
0.05
ρ_t (fm^{-3})
eq- β
B
DS
TS
TF
DDME2
NL3

H.Müller and B. Serot, Phys. Rev. C 52, 2072 (1995)

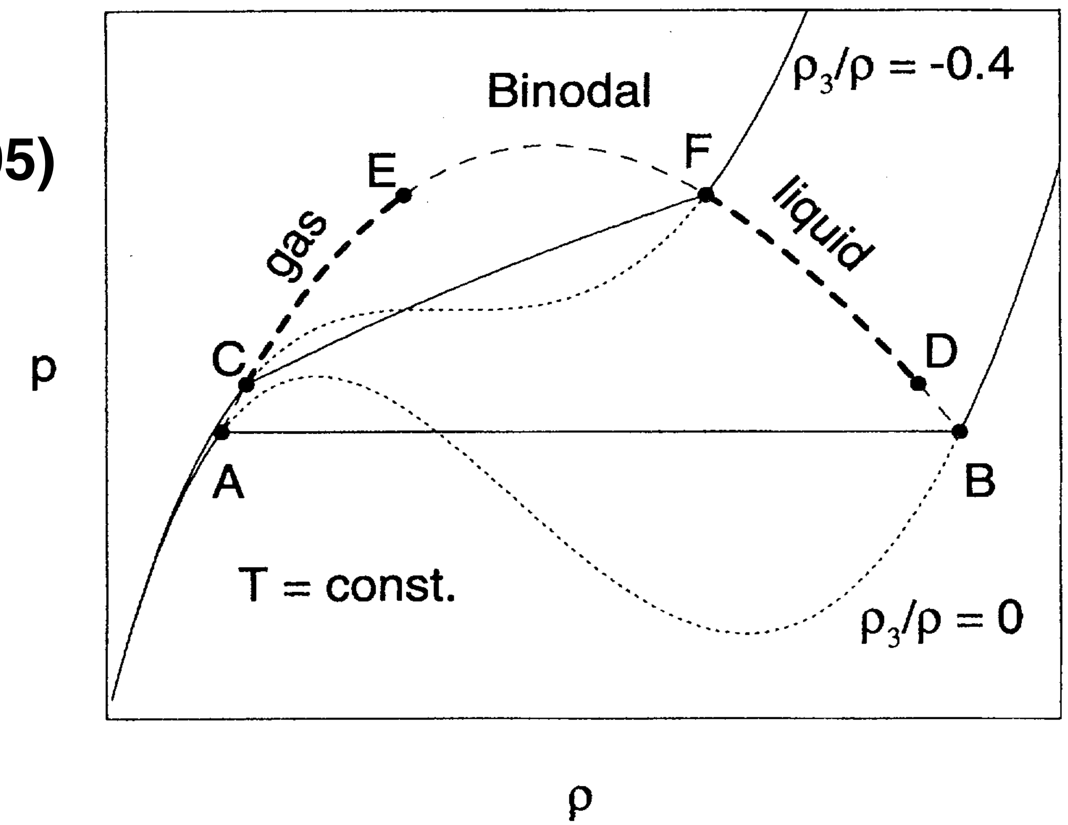
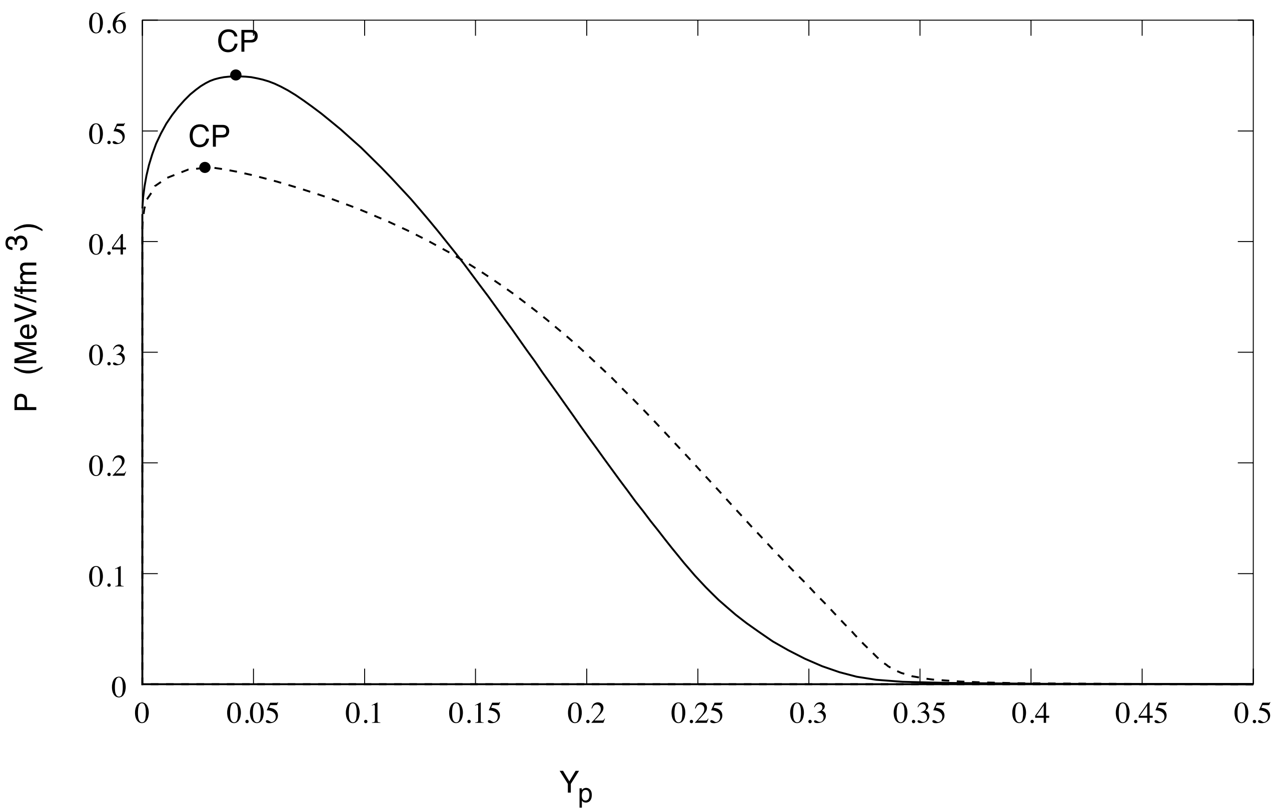


FIG. 3. The Maxwell construction for symmetric and asymmetric systems. The construction for symmetric matter ($\rho_3/\rho=0$) is indicated by the segment AB and is the same as for a one-component system. The construction for asymmetric matter with $\rho_3/\rho=-0.4$ is indicated by the segment CF and shows the qualitatively new behavior allowed in a two-component system. The asymmetry is held constant throughout the phase separation. The (dashed) binodal line is obtained from similar isotherms at other values of the asymmetry.

Phase Transitions in multicomponent systems:

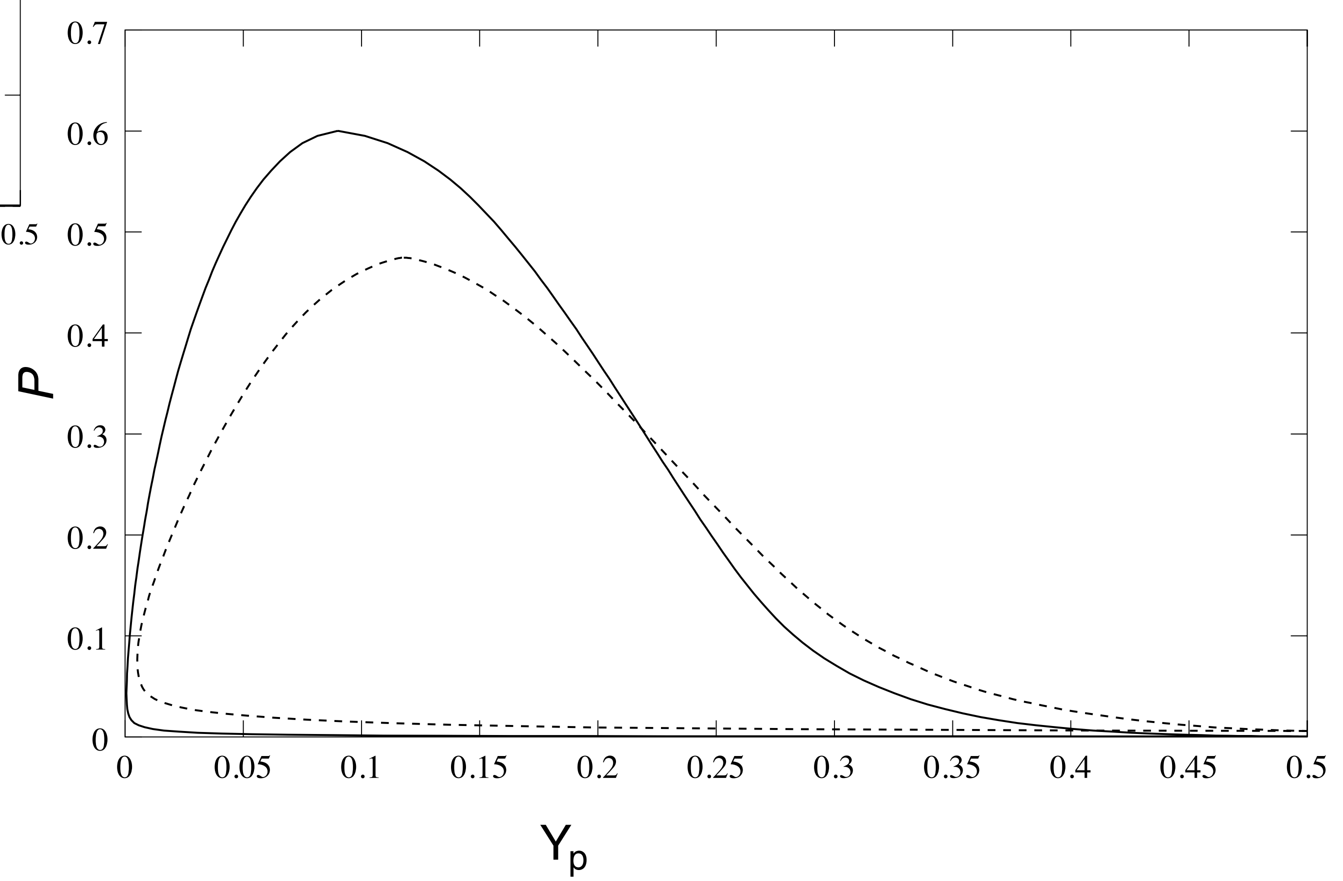
Kmax=n+2 phases can coexist in a system with n conserved charges (more than 2 phases can coexist if and only if each pair of phases form a binodal and if all the binomials have a common region of intersection.

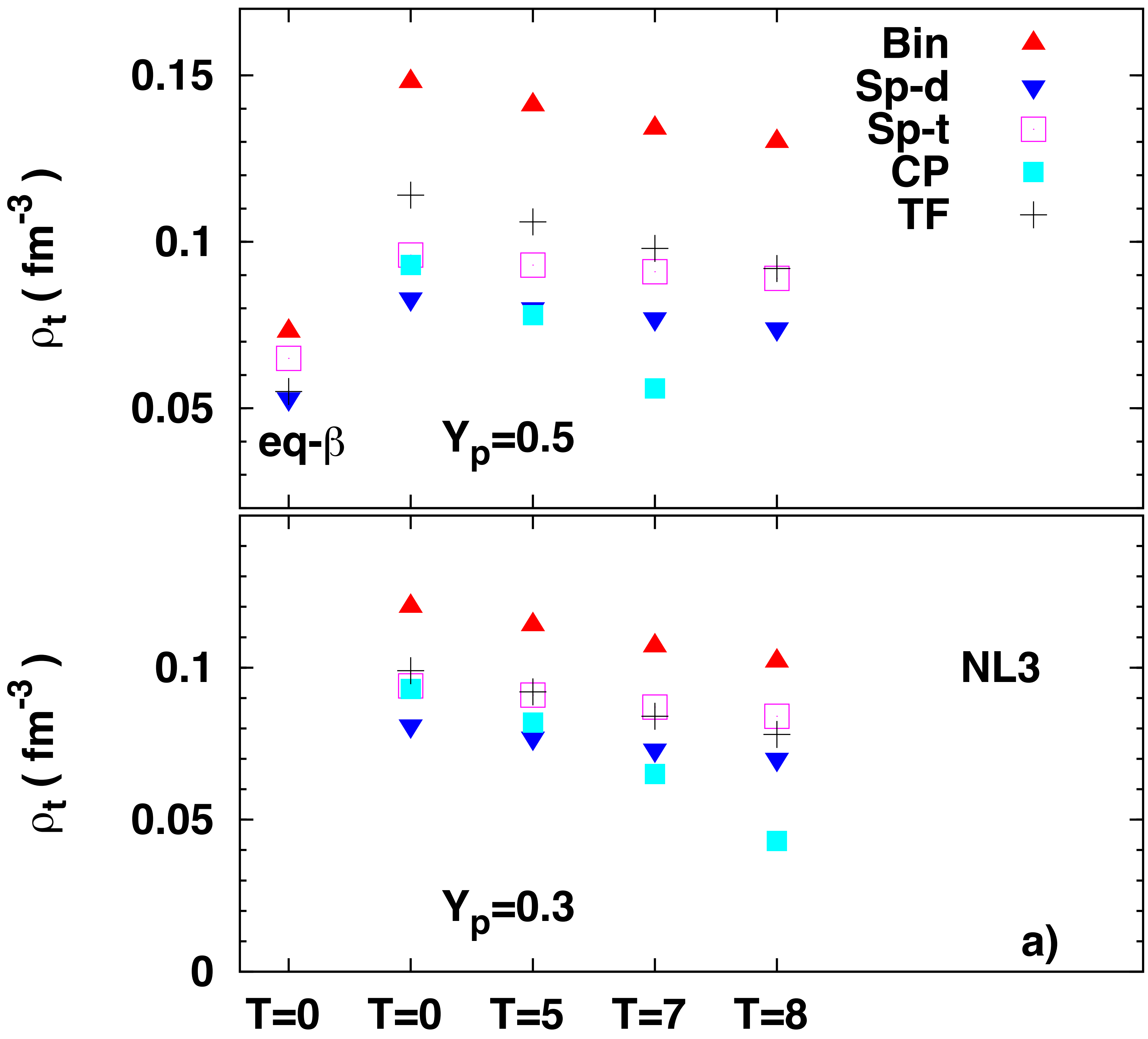
- **Binodal sections** - obtained from the conditions of phase coexistence:



T=0
solid - QMC
dashed - NL1

solid- T=5 MeV
dashed - T=7.5 MeV

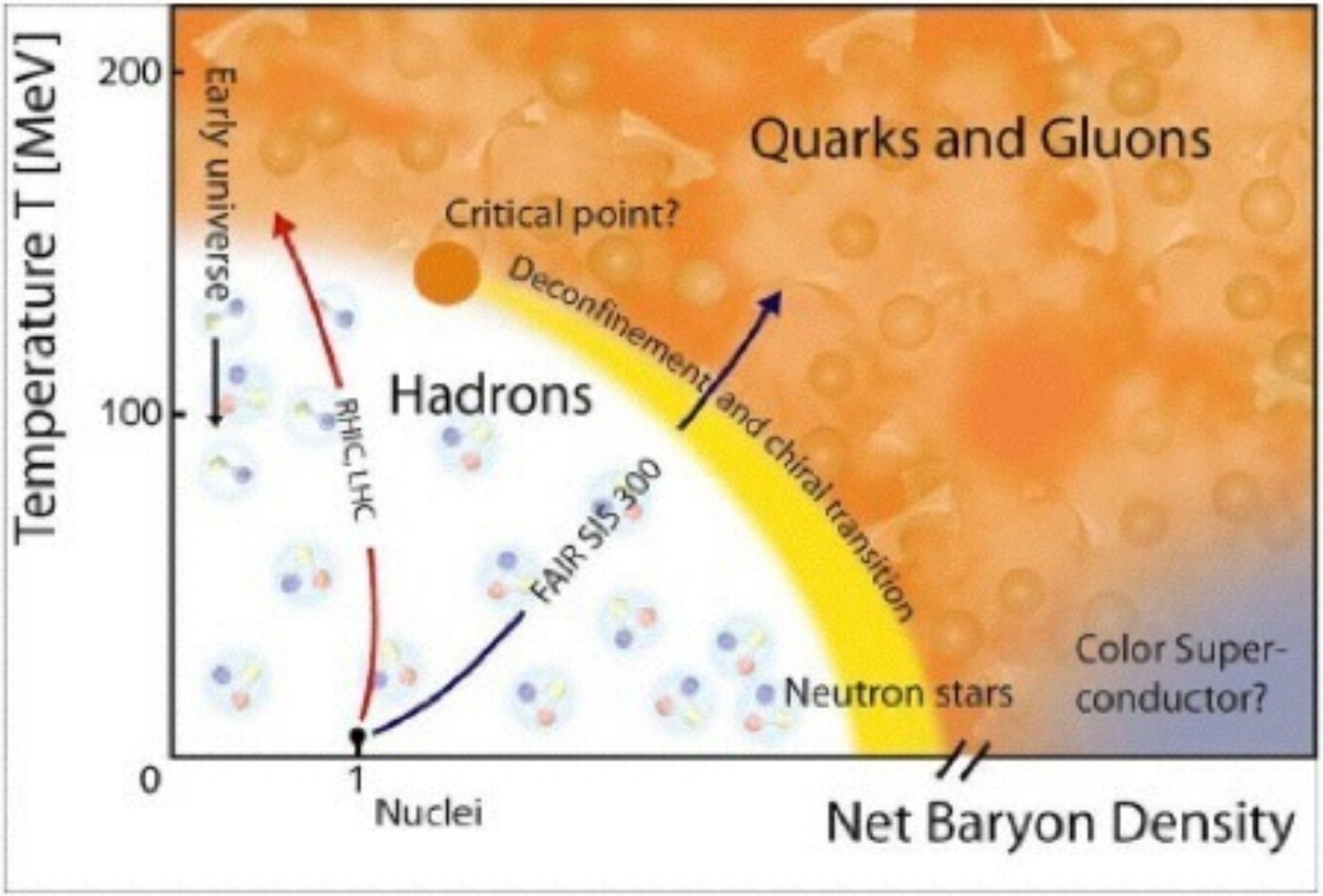




Bin
Sp-d
Sp-t
CP
TF
0.15
0.1
0.05
ρt (fm-3)
eq-β
Yp=0.5
0.1
0.05
0
NL3
Yp=0.3
a)
T=0
T=0
T=5
T=7
T=8

QCD Phase Diagram

•



What would happen if matter were subject to strong magnetic fields ?

Heavy ion collisions

Facility	Location	Ions	Energy
AGS (1986-2000)	BNL	Au + Au	2.6 - 4.3 GeV
SPS (1986-2003)	CERN	Pb + Pb	8.6 - 17.2 GeV
RHIC (2000-?)	BNL	Au + Au	200 GeV
LHC (2009-?)	CERN	Pb + Pb	5.5 TeV

LHC collisions:

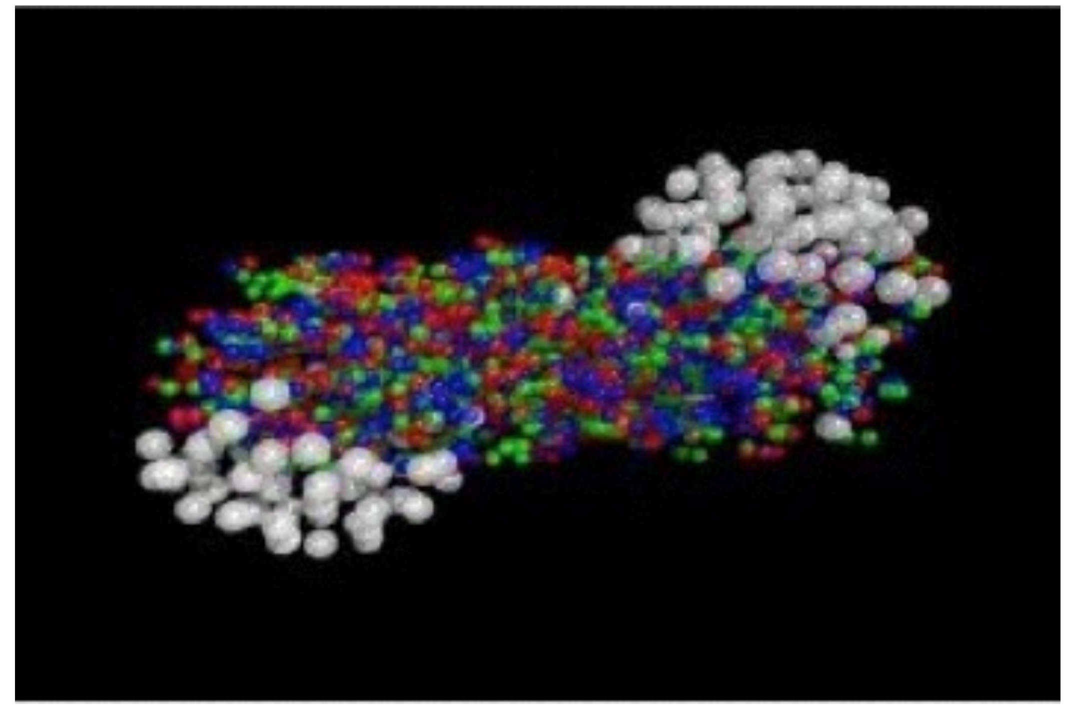
p-p : as much energy as possible in the smallest possible volume - aim is to produce elementary particles with the possible highest masses (Higgs-like particles)

Pb - Pb : The idea is not to produce new particles, but to understand how the ones we know interact with each other by investigating the properties of the fluid (very low viscosity) produced in the collision

p - Pb : The aim is to obtain benchmarking information and to study the partonic distribution inside the incoming ion

Comparing both reactions, one can also identify **density effects**

Motivation: why magnetic fields?



Magnetars - $eB \simeq 0.5 m^2_\pi$

$m^2_\pi \simeq 3.5 \times 10^{18}$ G

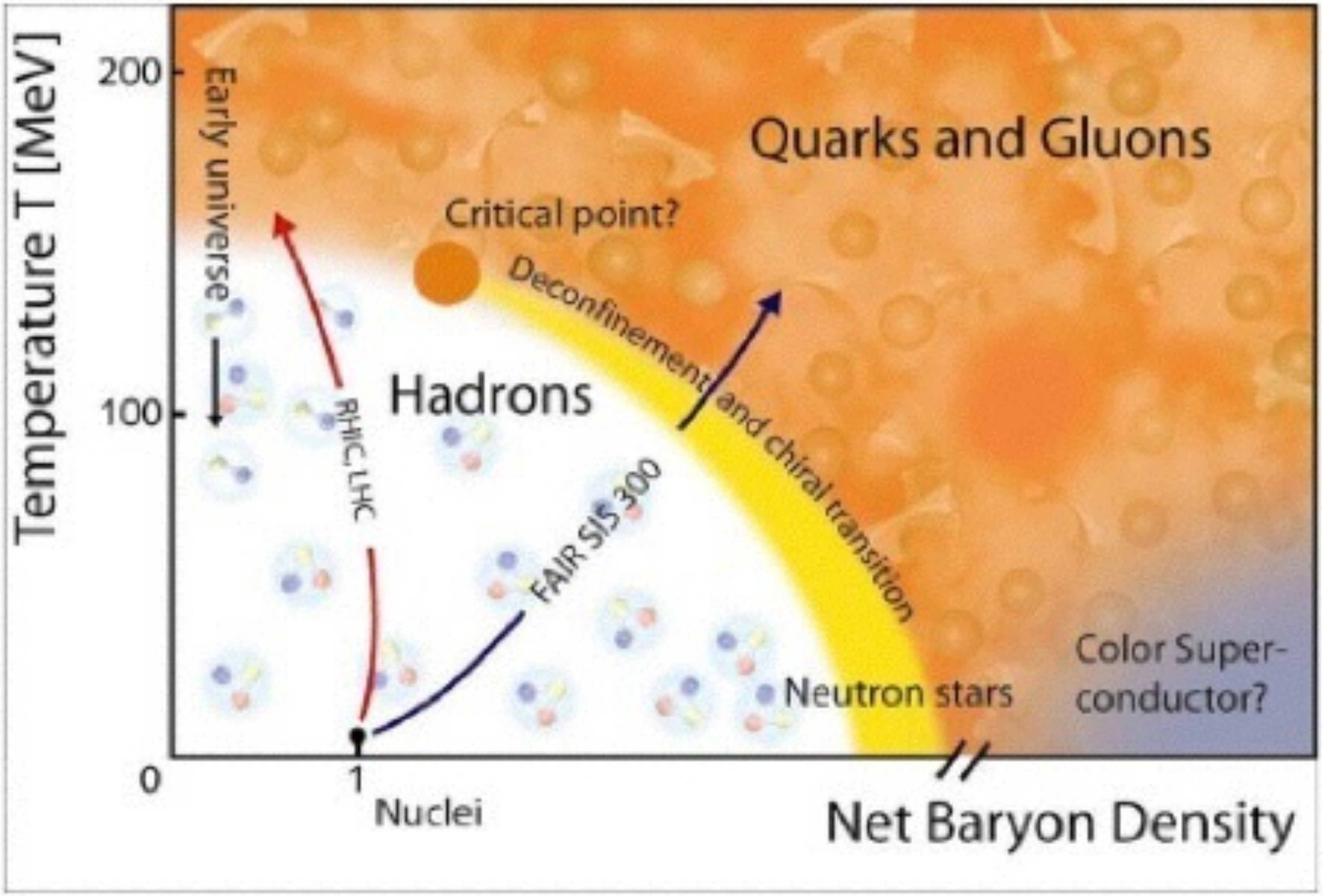
Non-central HIC - $eB \simeq 5 - 15 m^2_\pi$

Early Universe - $eB \simeq 30 m^2_\pi$

in natural units: $eB = 1\ \mathrm{GeV}^2 \quad B = 1.69 \times 10^{20}\mathrm{G}$

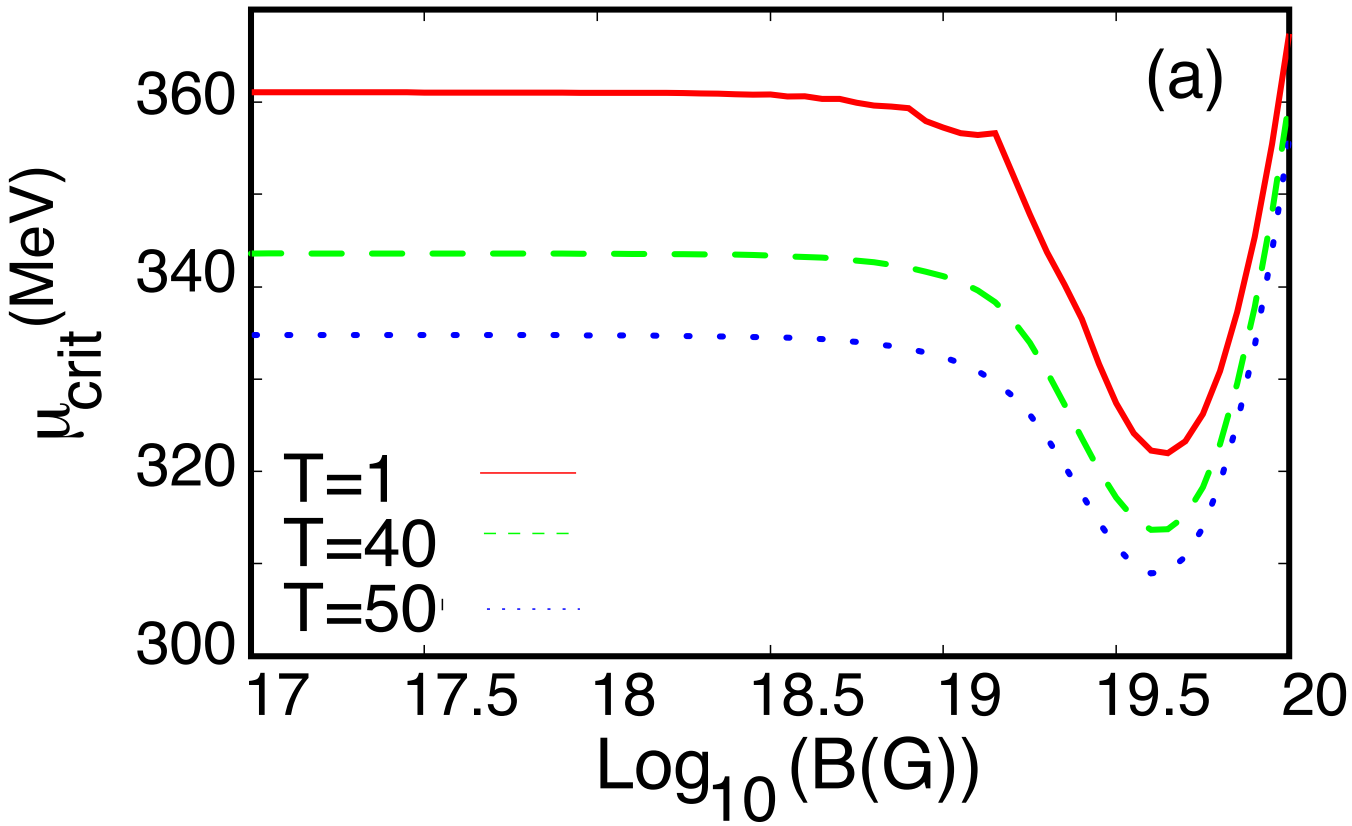
Heaviside-Lorentz, Gaussian and natural units lead to different conversions!

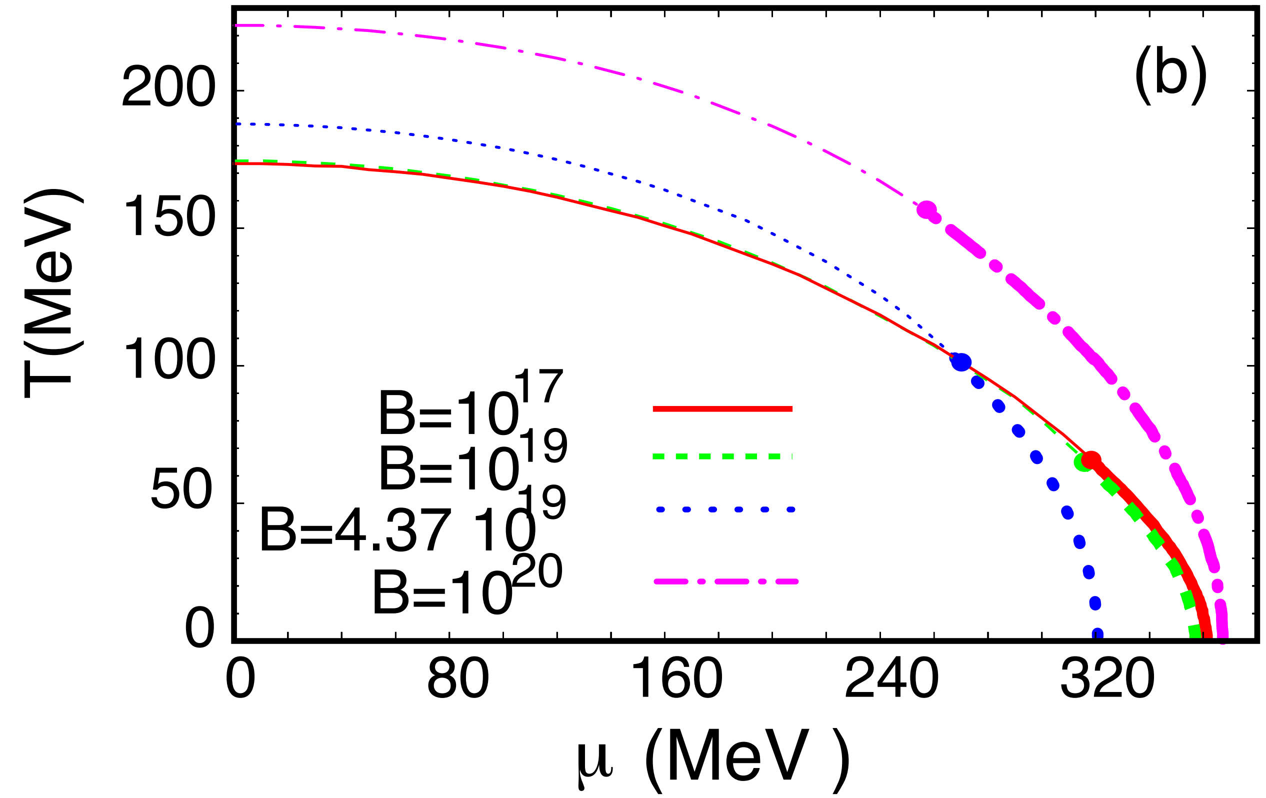
QCD Phase Diagram

- 

What would happen if matter were subject to strong magnetic fields ?

NJL model



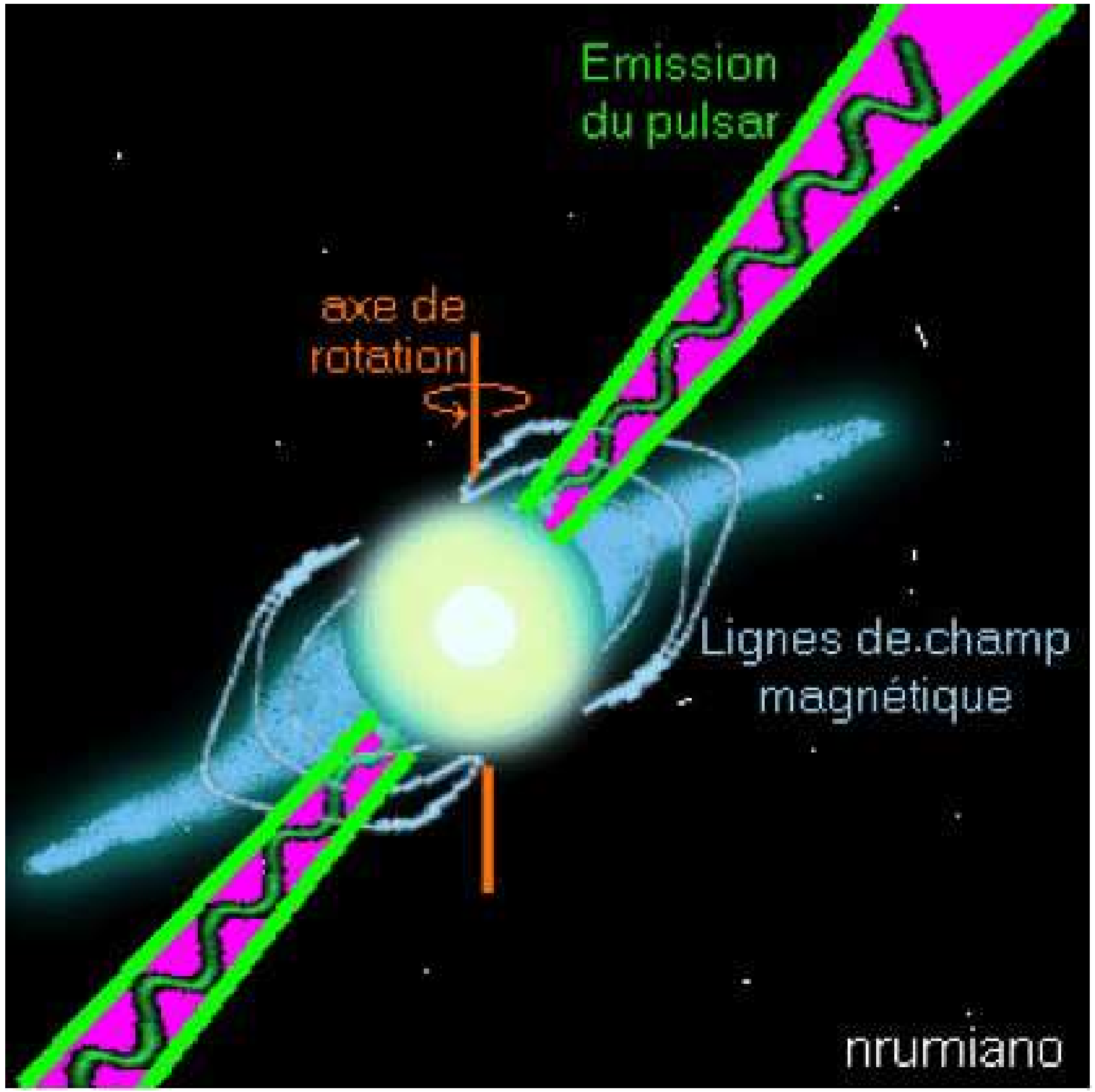


S.S. Avancini, D.P. Menezes, M.B. Pinto and C. Providência - Phys. Rev. D 85, 091901(R) (2012)

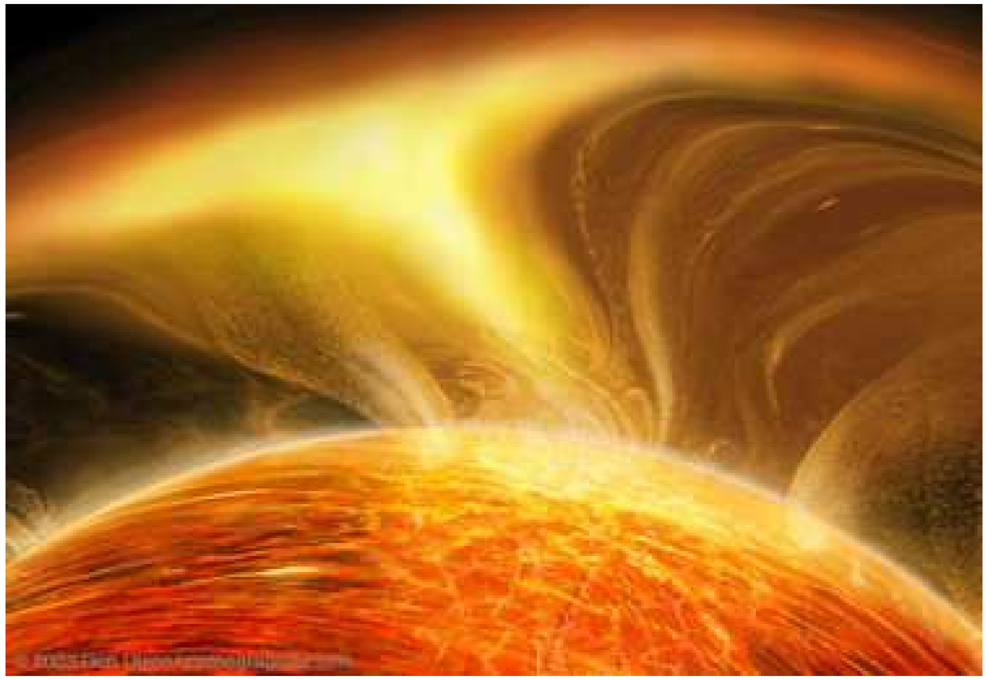
We would like to understand magnetic field effects:

- at **high** densities and **low** temperatures (NS):
- at **low** densities and **high** temp. (heavy ion collisions);
- at **low** densities and **low** temperatures (pasta phase):
- if the **CEP** exists, how its **location** would change

Pulsares (NS) X Magnetares



$B = 10^{12}$ G na superfície



$B = 10^{15}$ G na superfície

Main NS manifestations:

• Pulsars - powered by rotation energy (1900 observed in radio-frequency)

• Accreting X-Ray Binaries - powered by gravitational energy (typical rotation periods 0.0015 - 1000 s)

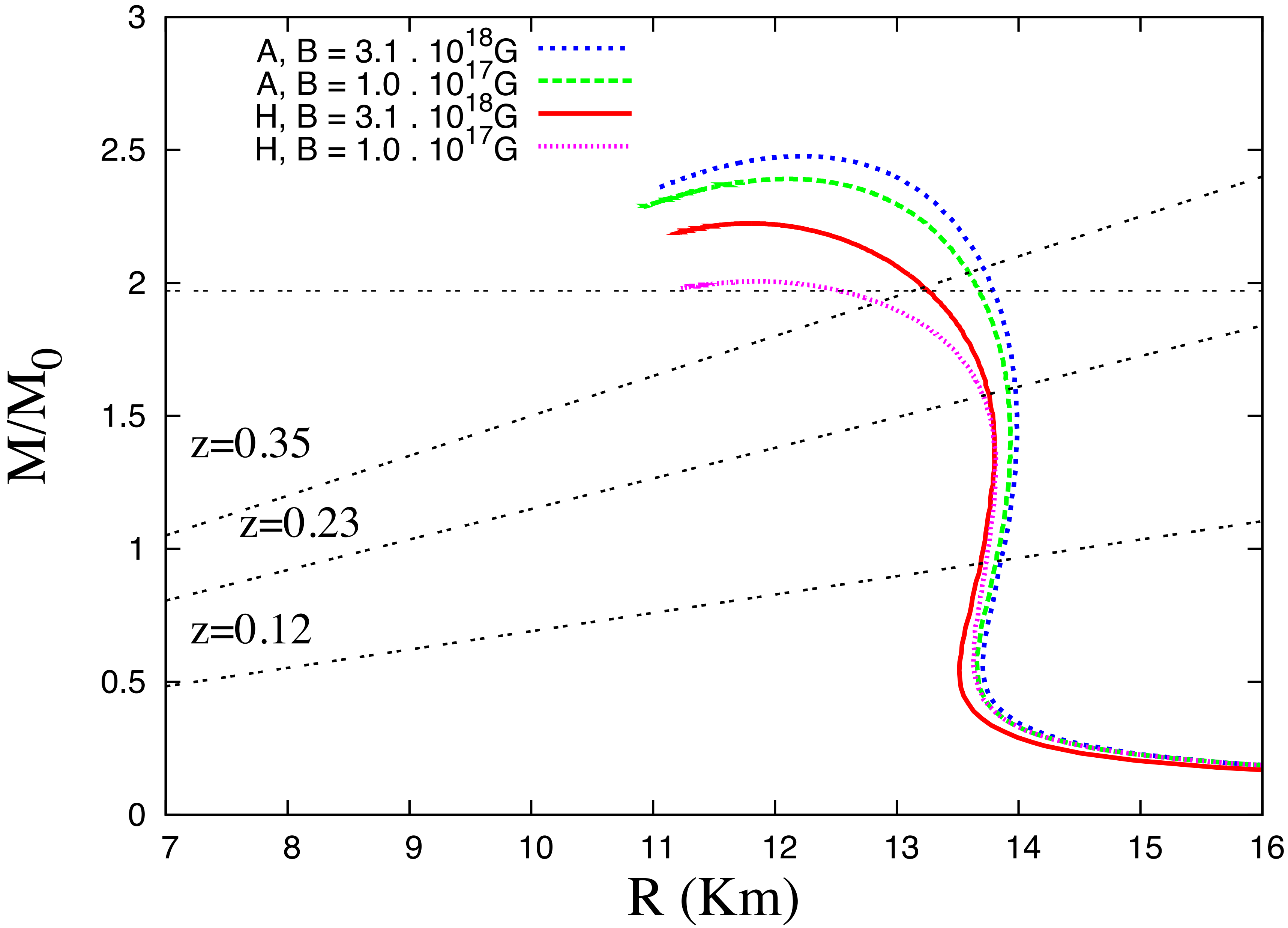
Magnetars don’t fit into these categories! They are normally isolated NS whose main power source is the magnetic field.

There are 2 classes of magnetars (25 confirmed):

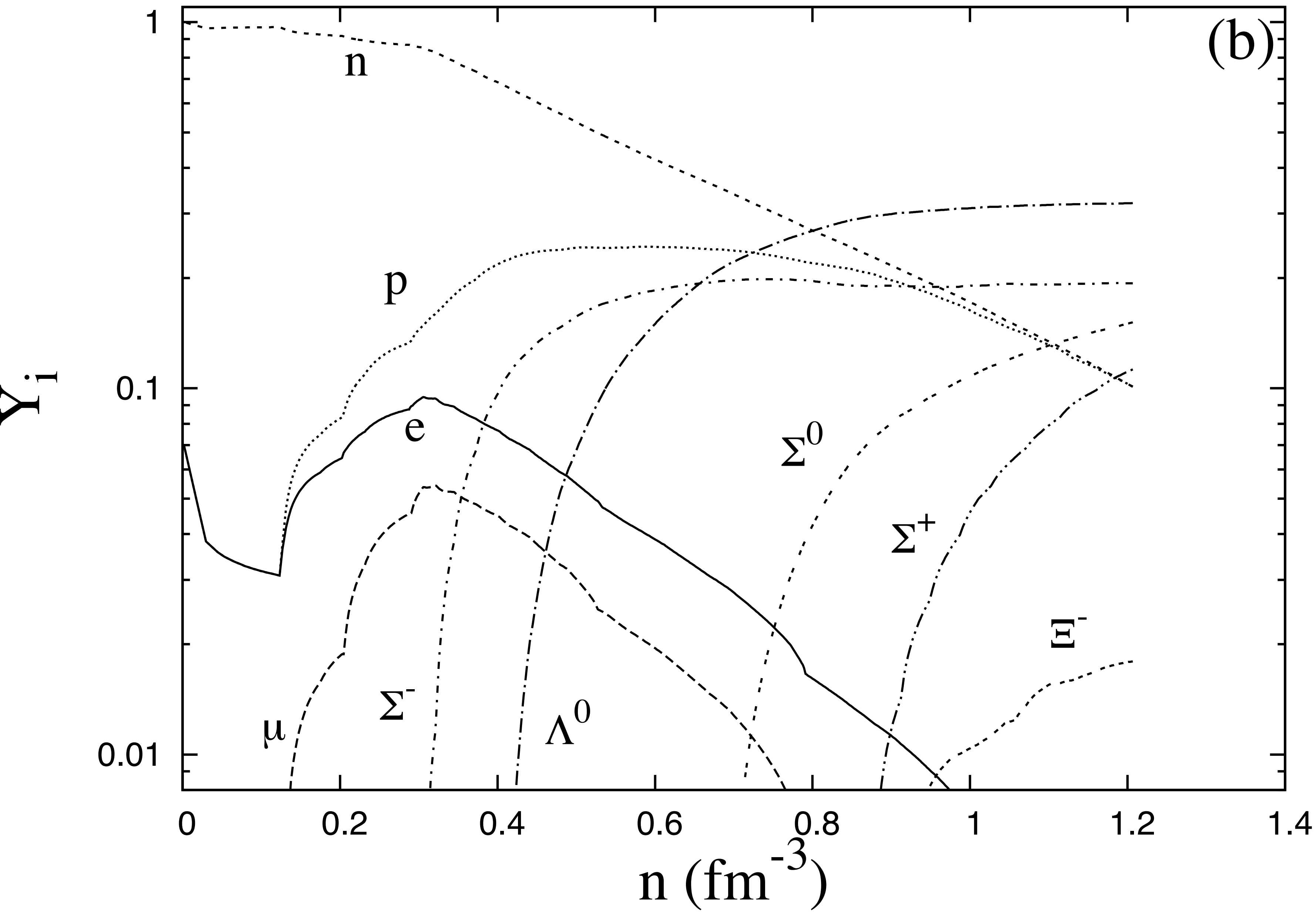
- Soft gamma-ray repeaters (discovered in 1979 as transient X-ray sources and giant flares);

- Anomalous X-ray pulsars (identified in 1990 as a class of persistent
- X-ray with no sign of a binary companion);

Magnetars - NLWM



B makes the EoS **harder**, which results in **higher** maximum masses



The kinks are related to the filling of the LL

Particle yields in HIC

We model matter as a free gas of baryons and mesons consisting of **54** particles (18 baryons, 18 antibaryons and 18 mesons) under the influence of a constant magnetic field.

Heaviside-Lorentz units:

$$\hbar = c = 1, \epsilon_0 = \mu_0 = 1, e = \sqrt{4\pi\alpha}, \alpha = \frac{1}{137}$$

Gauge:

$$A^\mu = \delta_{\mu 2} x_1 B \quad \rightarrow \quad A^0 = 0 \quad \text{and} \quad \vec{A} = (0, x_1 B, 0)$$

$$\vec{\nabla} \cdot \vec{A} = 0, \quad \vec{\nabla} \times \vec{A} = B\hat{e}_3, \quad D_\mu = \partial^\mu - i\epsilon_q |q| B x_1 \hat{e}_2$$

Au+Au (0-5%) collision at $\sqrt{s_{NN}} = 200$ A GeV.

B ($\times 10^{19}$G)	0	0.1	0.5	1	**3**	5	STAR/RHIC
eB (m_π^2)	0	0.3	1.5	3	**9**	15	
T (MeV)	140	141	152	165	**198**	215	
μ_B (MeV)	19	20	21	23	**26**	29	
χ^2/ndf	14.4	17.7	14.5	11.6	**9.4**	10.6	
μ_{I3} (MeV)	-1.42	-1.50	-1.70	-2.01	**-2.85**	-3.89	
μ_S (MeV)	2.24	2.42	3.29	4.54	**7.16**	8.64	
R (fm)	38.3	37.0	29.7	23.4	**14.9**	12.09	
$\rho \times 10^{-3}$ (fm^{-3})	1.67	1.86	3.60	7.30	**29.0**	53.0	
π^-/π^+	1.000	1.000	1.000	1.000	**1.000**	1.000	1.015±0.051
K^-/K^+	0.978	0.976	0.968	0.957	**0.941**	0.936	0.965±0.048
$\bar{p}/p$	0.770	0.761	0.767	0.766	**0.781**	0.778	0.769±0.055
K^-/π^-	0.218	0.218	0.215	0.210	**0.208**	0.212	0.151±0.018
$\bar{p}/\pi^-$	0.034	0.035	0.041	0.047	**0.055**	0.053	0.082±0.010
K^+/π^+	0.223	0.223	0.222	0.219	**0.221**	0.226	0.159±0.019
p/π^+	0.044	0.045	0.054	0.061	**0.070**	0.068	0.108±0.013

CEP - NJL / PNJL

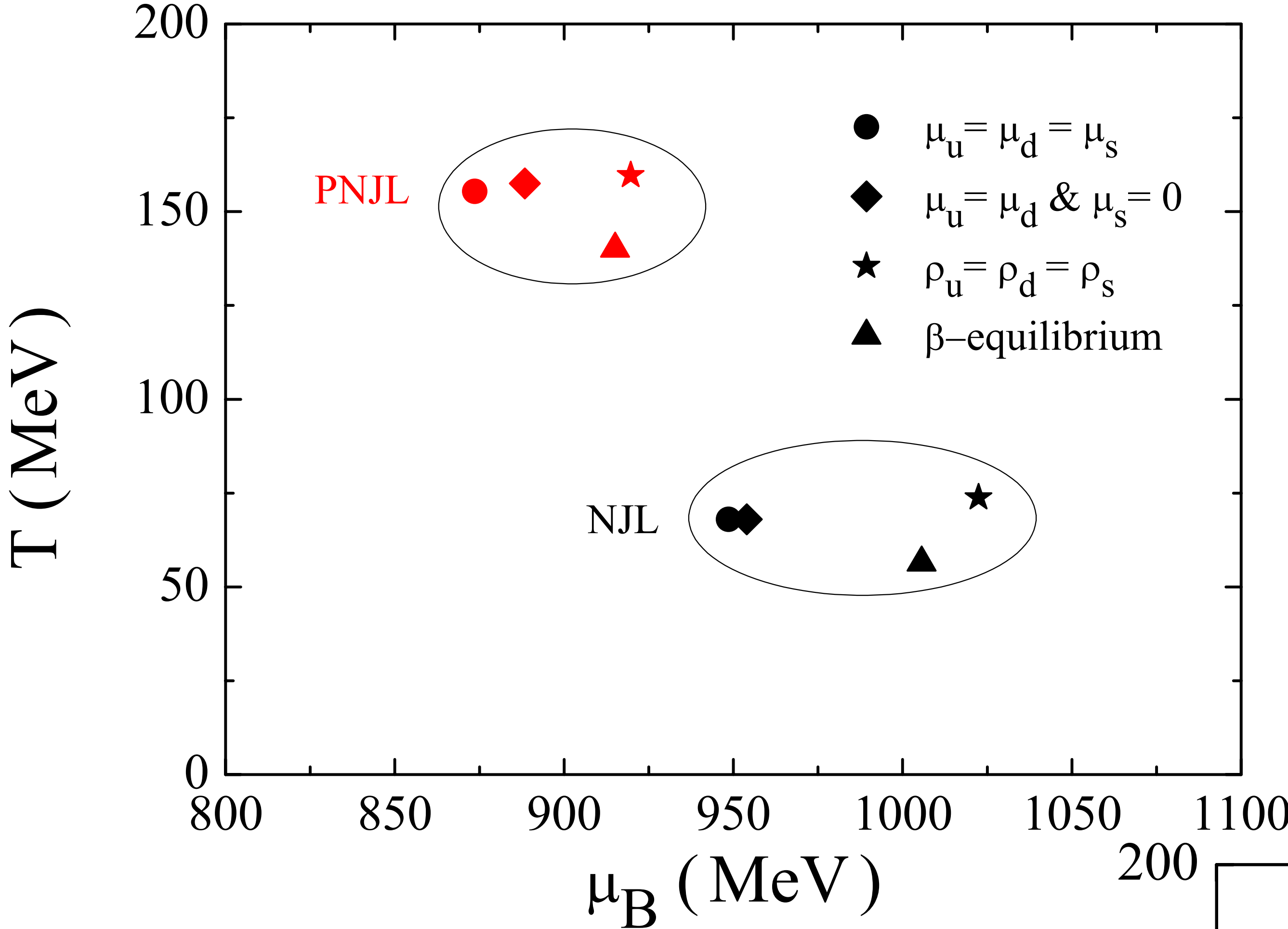
- **RKH parametrization / B=0 and strong B**
- **Different scenarios :**

$$\mu_u = \mu_d = \mu_s$$

$$\mu_u = \mu_d, \quad \mu_s = 0$$

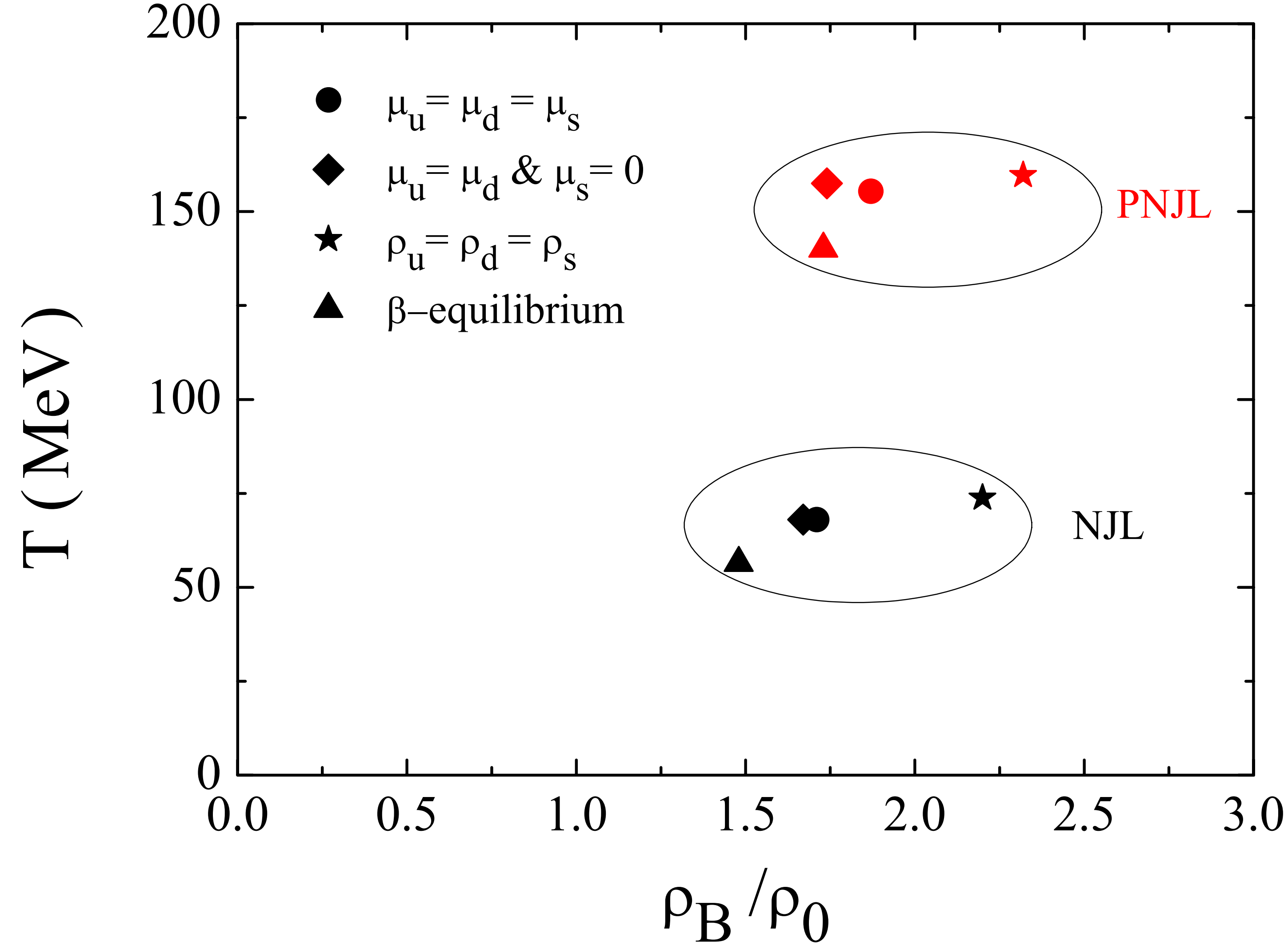
$$\rho_u = \rho_d = \rho_s$$

$$\beta - \text{equilibrium}$$

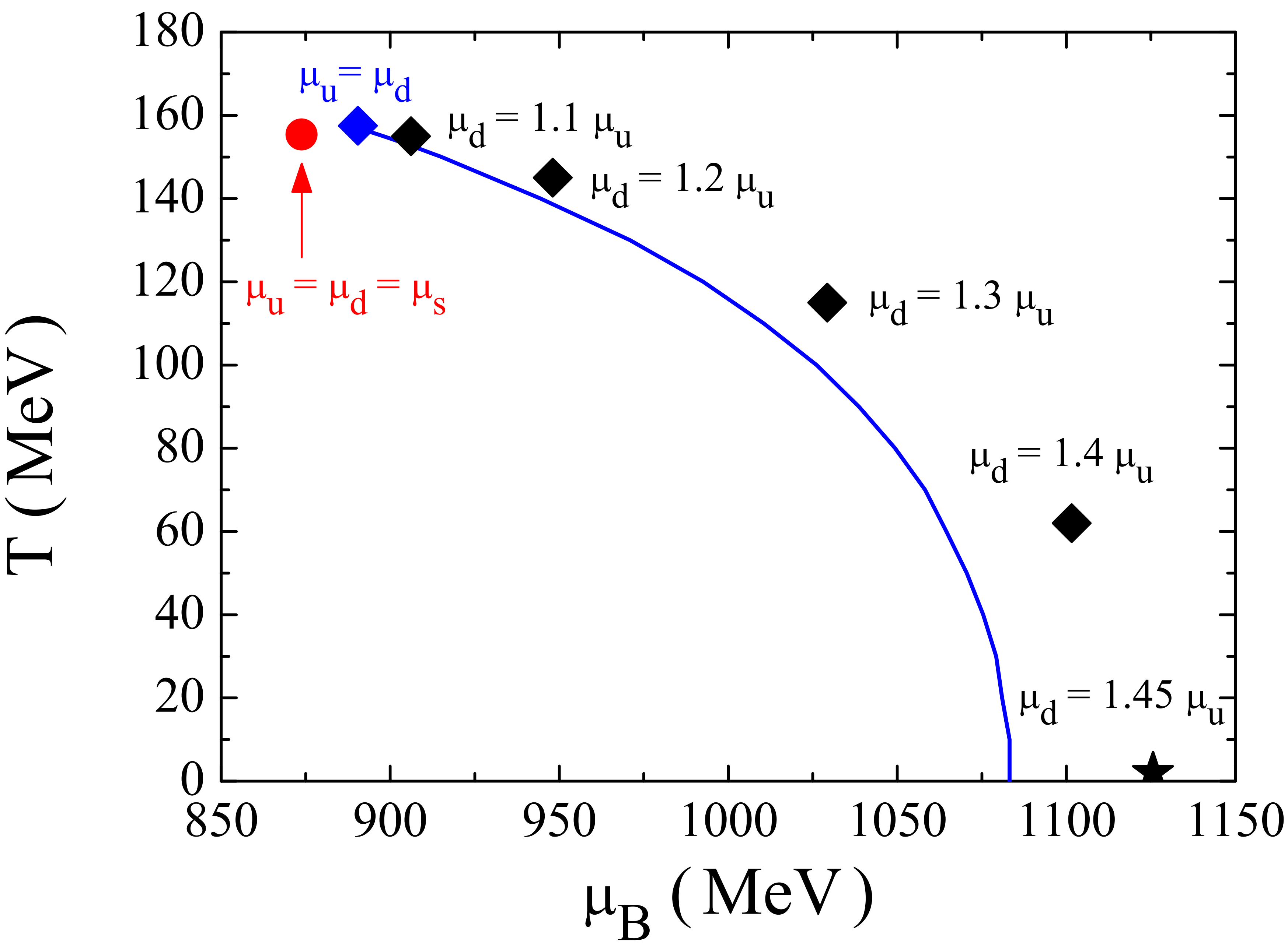


T (MeV)
μ_B (MeV)
PNJL
NJL
$\mu_u = \mu_d = \mu_s$
$\mu_u = \mu_d$ & $\mu_s = 0$
$\rho_u = \rho_d = \rho_s$
β–equilibrium

B=0



T (MeV)
ρ_B / ρ_0
PNJL
NJL
$\mu_u = \mu_d = \mu_s$
$\mu_u = \mu_d$ & $\mu_s = 0$
$\rho_u = \rho_d = \rho_s$
β–equilibrium

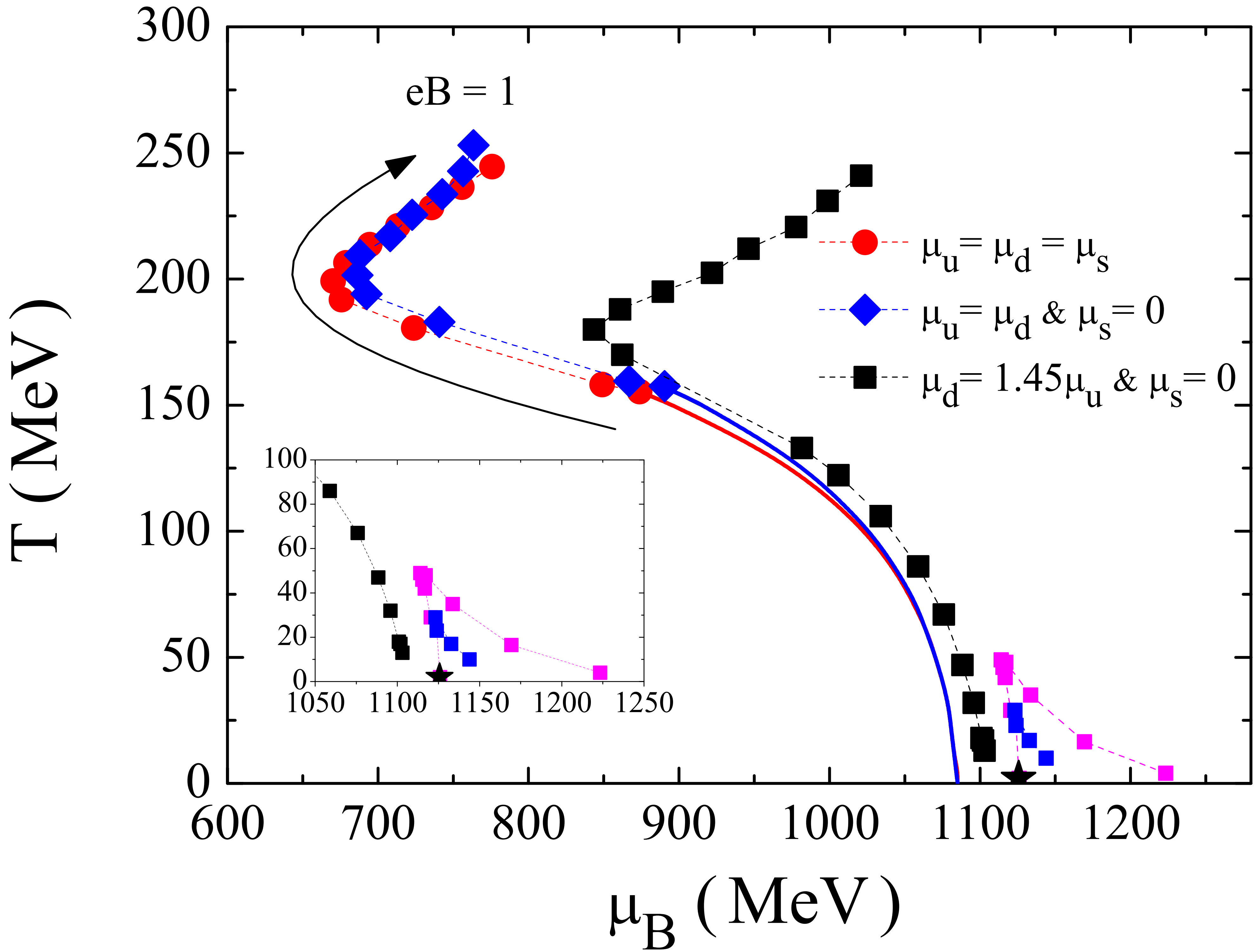


B=0

Effect of the isospin on the location of the CEP (PNJL)

the line corresponds to zero isospin

$$\mu_u = \mu_d, \quad \mu_s = 0$$



Full lines= 1st order transitions at eB=0; two CEPs at low T and strong B (pink and blue)

- **For matter in beta-equilibrium, the CEP occurs at smaller Ts and densities (no B)**
- **For very asymmetric matter, no 1st order phase transition to a deconfined phase occurs (no B)**
- **CEP occurs at very small Ts if eB < 0.1 GeV^2 and a complicate structure appears, i.e., more than one CEP**
- **Strong Bs can drive the system without a CEP to a 1st order phase transition**

Final Remarks

Hadron physics is a very rich field, with many aspects still to be investigated

Different (T, chemical potential) of the QCD phase diagram can be described

In general, it involves multidisciplinary areas (thermodynamics, statistical mechanics, astrophysics,…)

Modelling hadronic matter properly remains a challenge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Thank you !

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