Gluon Correlations and the Deconfinement Phase Transition

Tereza Mendes

in collaboration with A. Cucchieri

Instituto de Física de São Carlos

University of São Paulo

Debye Screening and Gluon Propagator

At high T, reorganization of moving charges in a plasma determines screening except for nearby region, delimited by the Debye length λ_D, inverse of the Debye mass (classical)

Debye Screening and Gluon Propagator

At high T, reorganization of moving charges in a plasma determines screening except for nearby region, delimited by the Debye length λ_D, inverse of the Debye mass (classical)

Scales: $g(T) \ll 1, \;\; m_E \approx g(T)\, T, \;\; m_M \approx g^2(T)\, T$

$$2\pi T \;\gg\; g(T)T \;\gg\; g^2(T)T$$

$\Rightarrow$ dimensional reduction: temporal modes decouple except for zero mode; 3d theory + scalar field; real m_E (screening) given by longitudinal propagator (found in lattice simulations)

Debye Screening and Gluon Propagator

At high T, reorganization of moving charges in a plasma determines screening except for nearby region, delimited by the Debye length λ_D, inverse of the Debye mass (classical)

Scales: $g(T) \ll 1, \;\; m_E \approx g(T)\, T, \;\; m_M \approx g^2(T)\, T$

$$2\pi T \;\gg\; g(T)T \;\gg\; g^2(T)T$$

$\Rightarrow$ dimensional reduction: temporal modes decouple except for zero mode; 3d theory + scalar field; real m_E (screening) given by longitudinal propagator (found in lattice simulations)

Around deconfinement: screening not directly observable (but would relate e.g. to quarkonia melting...)
Recent study using holograhy: Finazzo & Noronha PRD 2014

Plan

Gluonic correlations of Yang-Mills theory at nonzero temperature are encoded in the chromoelectric sector of the gluon propagator, which is a gauge-dependent quantity

Plan

Gluonic correlations of Yang-Mills theory at nonzero temperature are encoded in the chromoelectric sector of the gluon propagator, which is a gauge-dependent quantity

It would be interesting to establish how strongly this propagator feels the deconfinement transition and to learn more about the propagator's analytic structure, which may then be related to (screening) mass scales in the theory

Plan

Gluonic correlations of Yang-Mills theory at nonzero temperature are encoded in the chromoelectric sector of the gluon propagator, which is a gauge-dependent quantity

It would be interesting to establish how strongly this propagator feels the deconfinement transition and to learn more about the propagator's analytic structure, which may then be related to (screening) mass scales in the theory

We address these issues analyzing data from finite-temperature simulations of the gluon propagator in SU(2) Landau gauge on large lattices

Gluon at Nonzero Temperature

Gluon propagator is the most fundamental quantity of QCD and is believed to be closely related to the phenomenon of color confinement in the theory

Gluon at Nonzero Temperature

Gluon propagator is the most fundamental quantity of QCD and is believed to be closely related to the phenomenon of color confinement in the theory

As temperature T is turned on, propagator may:

- change qualitatively its behavior (?)

Gluon at Nonzero Temperature

Gluon propagator is the most fundamental quantity of QCD and is believed to be closely related to the phenomenon of color confinement in the theory

As temperature T is turned on, propagator may:

- change qualitatively its behavior (?)
- signal deconfinement around the critical temperature T_c (?)

Gluon at Nonzero Temperature

Gluon propagator is the most fundamental quantity of QCD and is believed to be closely related to the phenomenon of color confinement in the theory

As temperature T is turned on, propagator may:

- change qualitatively its behavior (?)
- signal deconfinement around the critical temperature T_c (?)
- display Debye screening of the color charge (at high T), i.e. show exponential fall-off, defining a screening mass

Gluon at Nonzero Temperature

Gluon propagator is the most fundamental quantity of QCD and is believed to be closely related to the phenomenon of color confinement in the theory

As temperature T is turned on, propagator may:

- change qualitatively its behavior (?)
- signal deconfinement around the critical temperature T_c (?)
- display Debye screening of the color charge (at high T), i.e. show exponential fall-off, defining a screening mass

Note: chromoelectric (respec. chromomagnetic) screening related to longitudinal (respec. transverse) gluon propagator with momentum component $p_0 = 0$;

Gluon at Nonzero Temperature

Gluon propagator is the most fundamental quantity of QCD and is believed to be closely related to the phenomenon of color confinement in the theory

As temperature T is turned on, propagator may:

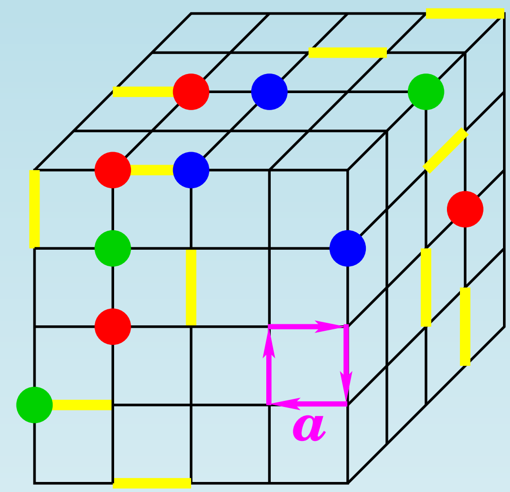
- change qualitatively its behavior (?)
- signal deconfinement around the critical temperature T_c (?)
- display Debye screening of the color charge (at high T), i.e. show exponential fall-off, defining a screening mass

Note: chromoelectric (respec. chromomagnetic) screening related to longitudinal (respec. transverse) gluon propagator with momentum component $p_0 = 0$; propagator is gauge-dependent, but poles are believed to be gauge-independent

Lattice Gluon Propagator

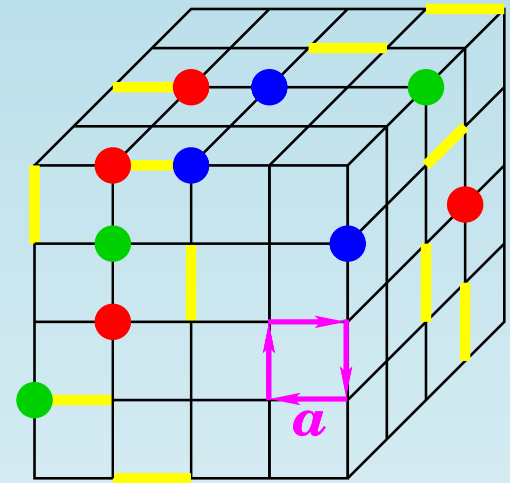
Wilson action

- written in terms of the gauge links
  $U_{x,\mu} \equiv e^{i g_0 a A^b_\mu(x) T_b}$
- reduces to continuum action for $a \to 0$
- gauge-invariant



Lattice Gluon Propagator

Wilson action



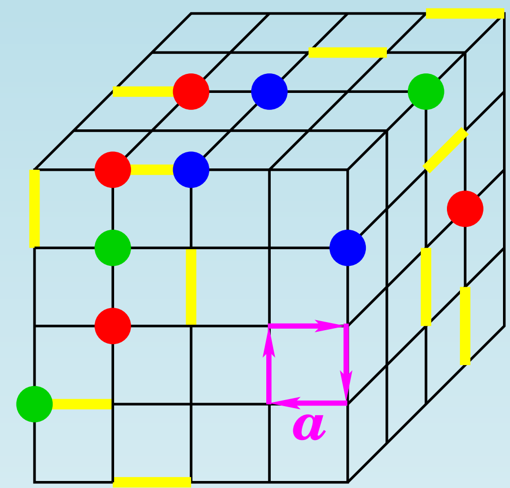
- written in terms of the gauge links
  $U_{x,\mu} \equiv e^{i g_0 a A^b_\mu(x) T_b}$
- reduces to continuum action for $a \to 0$
- gauge-invariant

Also: finite-size lattices $\Rightarrow$ IR cut for small momenta $p \approx 1/L$

$\Rightarrow$ IR limit corresponds to large lattice sizes...

Lattice Gluon Propagator

Wilson action



- written in terms of the gauge links
  $U_{x,\mu} \equiv e^{i g_0 a A_\mu^b(x) T_b}$
- reduces to continuum action for $a \to 0$
- gauge-invariant

Also: finite-size lattices $\Rightarrow$ IR cut for small momenta $p \approx 1/L$

$\Rightarrow$ IR limit corresponds to large lattice sizes...

Gluon propagator in Landau gauge

$$D^{ab}_{\mu\nu}(p) = \sum_x e^{-2i\pi k\cdot x} \langle A^a_\mu(x)\, A^b_\nu(0)\rangle$$

$$= \delta^{ab} \left(g_{\mu\nu} - \frac{p_\mu\, p_\nu}{p^2} \right) D(p^2)$$

Expected Behavior

At high T expect real electric mass $D_L(z) \approx e^{-m_E z}$

Expected Behavior

At high T expect real electric mass $D_L(z) \approx e^{-m_E z}$

At the same time, dimensional-reduction picture (based ont the 3D-adjoint-Higgs model) suggests a confined magnetic gluon, associated to a nontrivial magnetic mass

Expected Behavior

At high T expect real electric mass $D_L(z) \approx e^{-m_E z}$

At the same time, dimensional-reduction picture (based ont the 3D-adjoint-Higgs model) suggests a confined magnetic gluon, associated to a nontrivial magnetic mass

These predictions are confirmed at high T: Heller, Karsch & Rank (1995); Cucchieri, Karsch & Petreczky (2001)

Expected Behavior

At high T expect real electric mass $D_L(z) \approx e^{-m_E z}$

At the same time, dimensional-reduction picture (based ont the 3D-adjoint-Higgs model) suggests a confined magnetic gluon, associated to a nontrivial magnetic mass

These predictions are confirmed at high T: Heller, Karsch & Rank (1995); Cucchieri, Karsch & Petreczky (2001)

It is not clear if/how a screening mass would show up around T_c

Expected Behavior

At high T expect real electric mass $D_L(z) \approx e^{-m_E z}$

At the same time, dimensional-reduction picture (based ont the 3D-adjoint-Higgs model) suggests a confined magnetic gluon, associated to a nontrivial magnetic mass

These predictions are confirmed at high T: Heller, Karsch & Rank (1995); Cucchieri, Karsch & Petreczky (2001)

It is not clear if/how a screening mass would show up around T_c

On the other hand, studies of the gluon propagator at $T = 0$ have shown a (dynamical) mass, so we can try to use this knowledge to define temperature-dependent masses for the region $T \approx T_c$

Gluon at Criticality

First (small lattice) studies of SU(2) theory around T_c found:

- $D_T(p^2)$ is IR-suppressed and decreases as T increases
- $D_L(p^2)$ shows IR plateau, which has a sharp peak around T_c

Gluon at Criticality

First (small lattice) studies of SU(2) theory around T_c found:

- $D_T(p^2)$ is IR-suppressed and decreases as T increases
- $D_L(p^2)$ shows IR plateau, which has a sharp peak around T_c

Questions: is peak consistent with a divergence at infinite volume? is behavior different for $SU(3)$ and unquenched cases?

Gluon at Criticality

First (small lattice) studies of SU(2) theory around T_c found:

- $D_T(p^2)$ is IR-suppressed and decreases as T increases
- $D_L(p^2)$ shows IR plateau, which has a sharp peak around T_c

Questions: is peak consistent with a divergence at infinite volume? is behavior different for $SU(3)$ and unquenched cases?

More recently: peak and/or sensitivity of $D_L(p^2)$ to T_c seen by Fischer et al. (2010), Bornyakov & Mitrjushkin (2010, 2011), Aouane et al. (2012), Maas et al. (2012) and Silva et al. (2014)

Gluon at Criticality

First (small lattice) studies of SU(2) theory around T_c found:

- $D_T(p^2)$ is IR-suppressed and decreases as T increases
- $D_L(p^2)$ shows IR plateau, which has a sharp peak around T_c

Questions: is peak consistent with a divergence at infinite volume? is behavior different for $SU(3)$ and unquenched cases?

More recently: peak and/or sensitivity of $D_L(p^2)$ to T_c seen by Fischer et al. (2010), Bornyakov & Mitrjushkin (2010, 2011), Aouane et al. (2012), Maas et al. (2012) and Silva et al. (2014)

Strong response of D_L to the transition implies that it contains information about the location of T_c.

Gluon at Criticality

First (small lattice) studies of SU(2) theory around T_c found:

- $D_T(p^2)$ is IR-suppressed and decreases as T increases
- $D_L(p^2)$ shows IR plateau, which has a sharp peak around T_c

Questions: is peak consistent with a divergence at infinite volume? is behavior different for $SU(3)$ and unquenched cases?

More recently: peak and/or sensitivity of $D_L(p^2)$ to T_c seen by Fischer et al. (2010), Bornyakov & Mitrjushkin (2010, 2011), Aouane et al. (2012), Maas et al. (2012) and Silva et al. (2014)

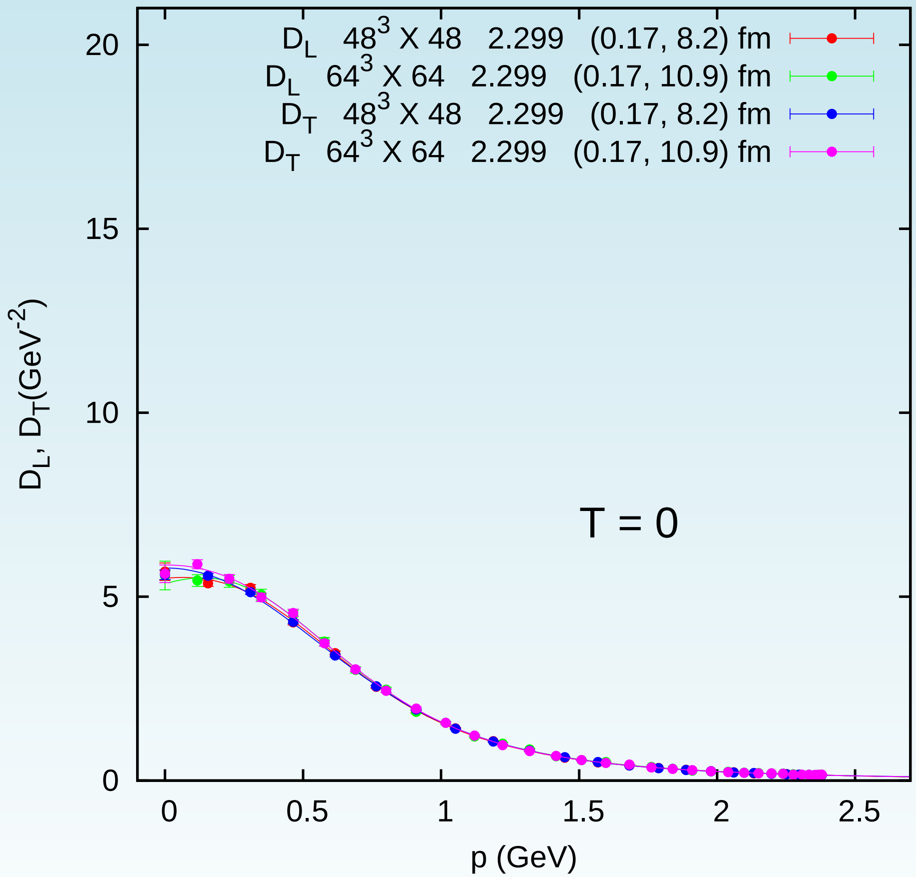
Strong response of D_L to the transition implies that it contains information about the location of T_c. If this info is unrelated to the center symmetry restoration, one could define an alternative order parameter for the deconfinement transition.

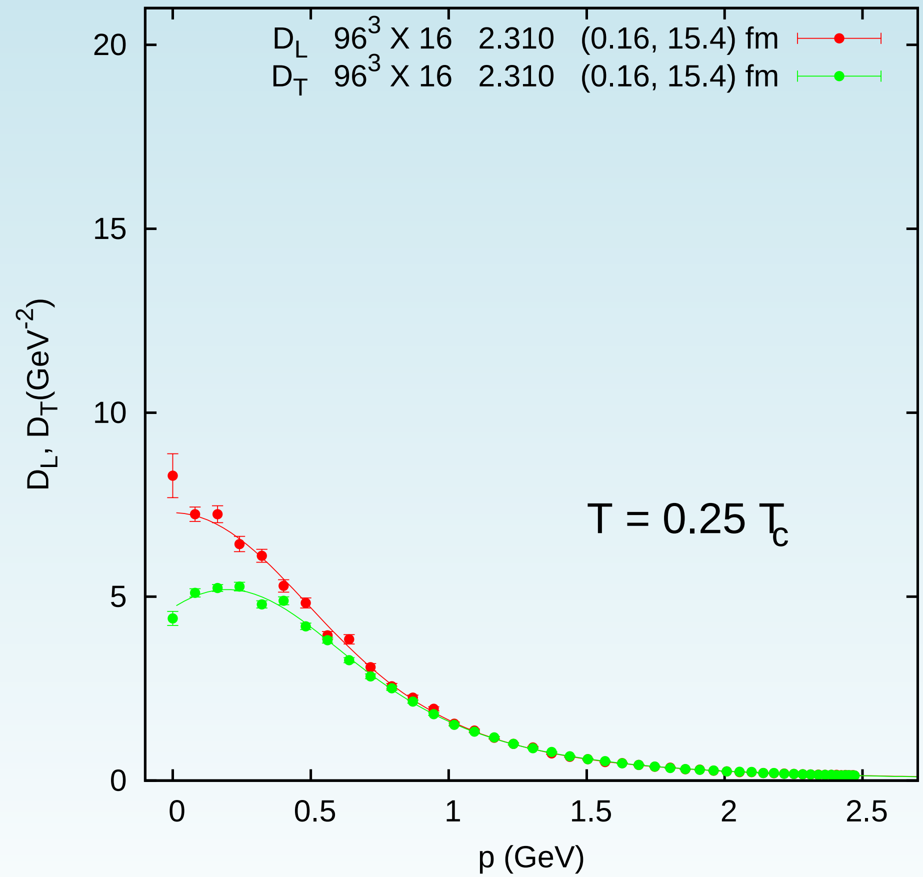
This Work (Finite T): Parameters

- pure SU(2) case, with a standard Wilson action
- cold start, projection on positive Polyakov loop configurations
- Landau-gauge fixing using stochastic overrelaxation
- lattice sizes ranging from $48^3 \times 4$ to $192^3 \times 16$
- several β values, allowing several values of the temperature $T = 1/N_t\, a$ around T_c
- gluon dressing functions normalized to 1 at 2 GeV
- masses extracted from Gribov-Stingl behavior (fits shown in plots below)

Results: Low Temperatures

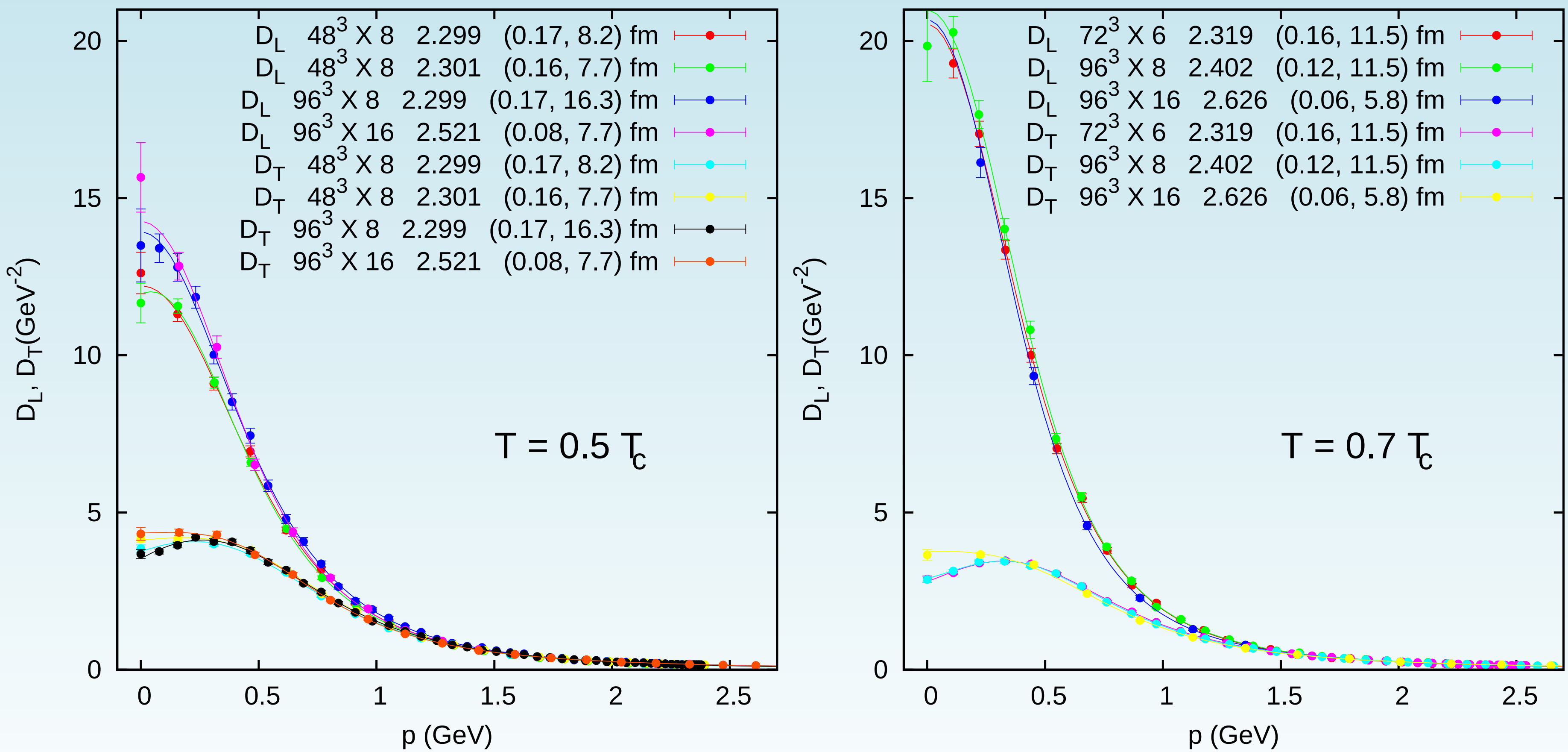
As T is turned on, magnetic propagator gets more strongly suppressed (3d-like), electric one increases





Results: Low Temperatures

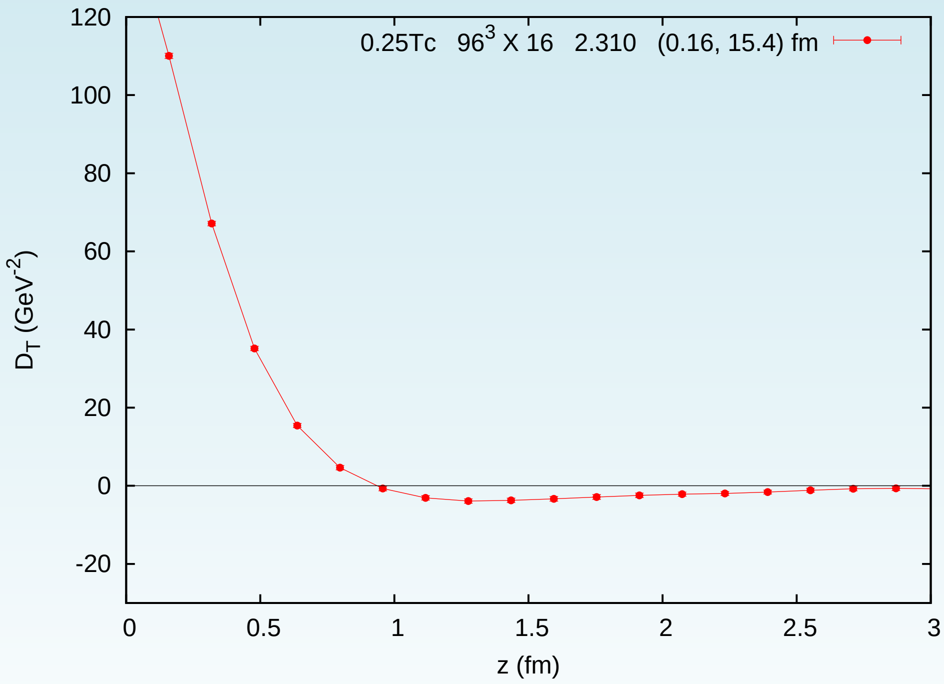
At larger T, magnetic propagator slightly more suppressed, electric one increases (showing IR plateau?)

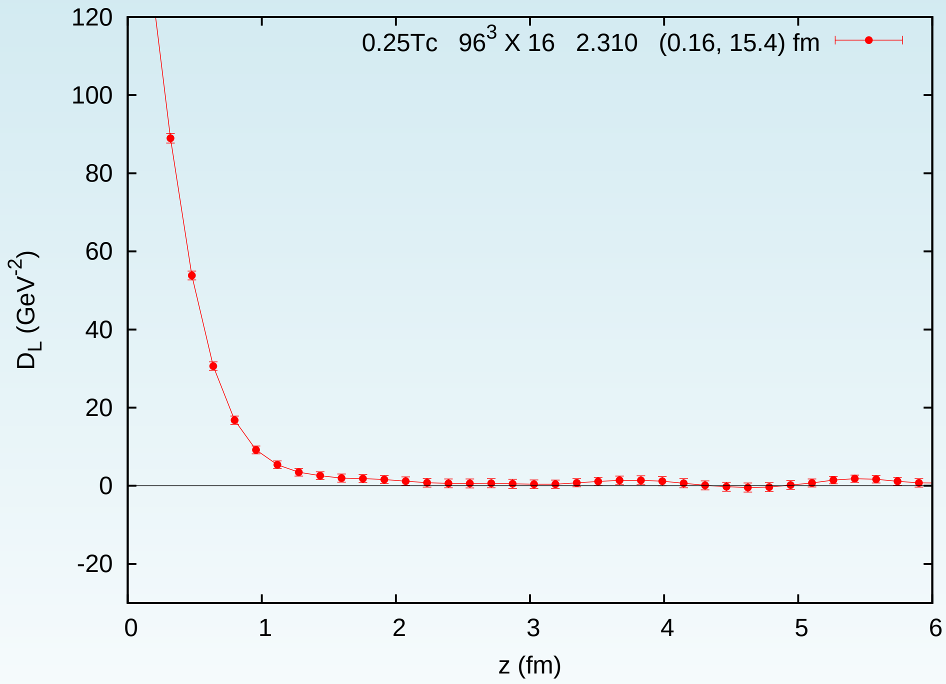


Real-Space Propagator at $T \neq 0$

Another qualitative response of the propagator to temperature: D_L ceases to show violation of reflection positivity as T is turned on, while such violation is still observed in the magnetic sector.

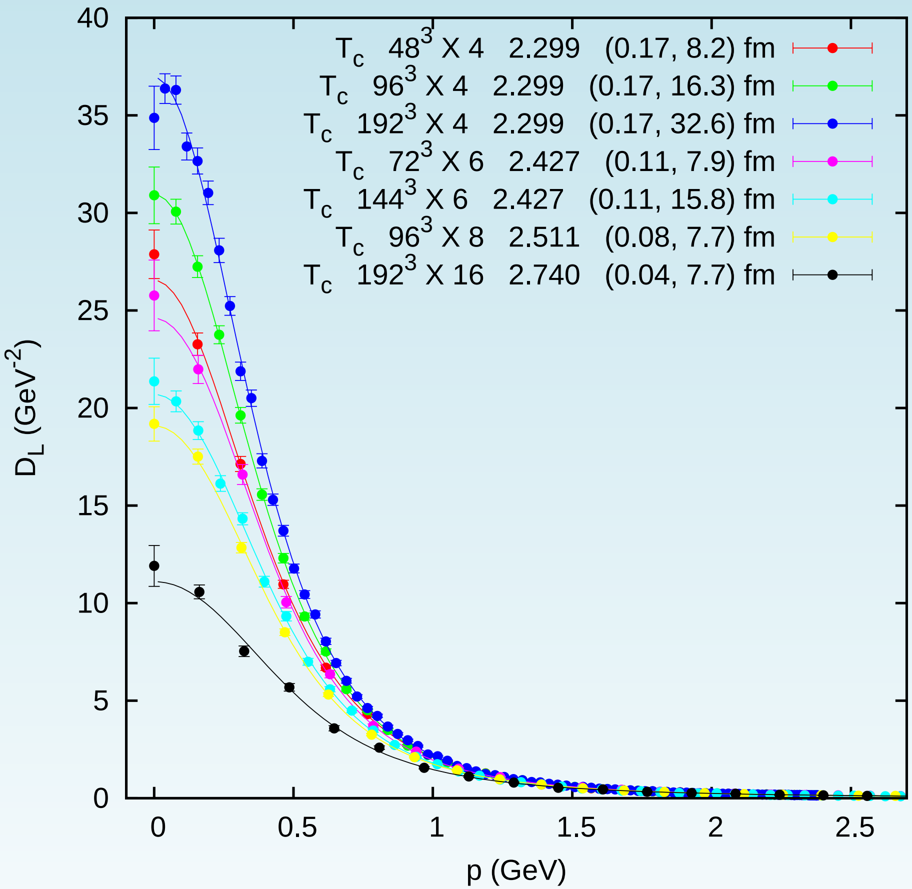
Plots of transverse and longitudinal real-space propagator at $T = 0.25\, T_c$:

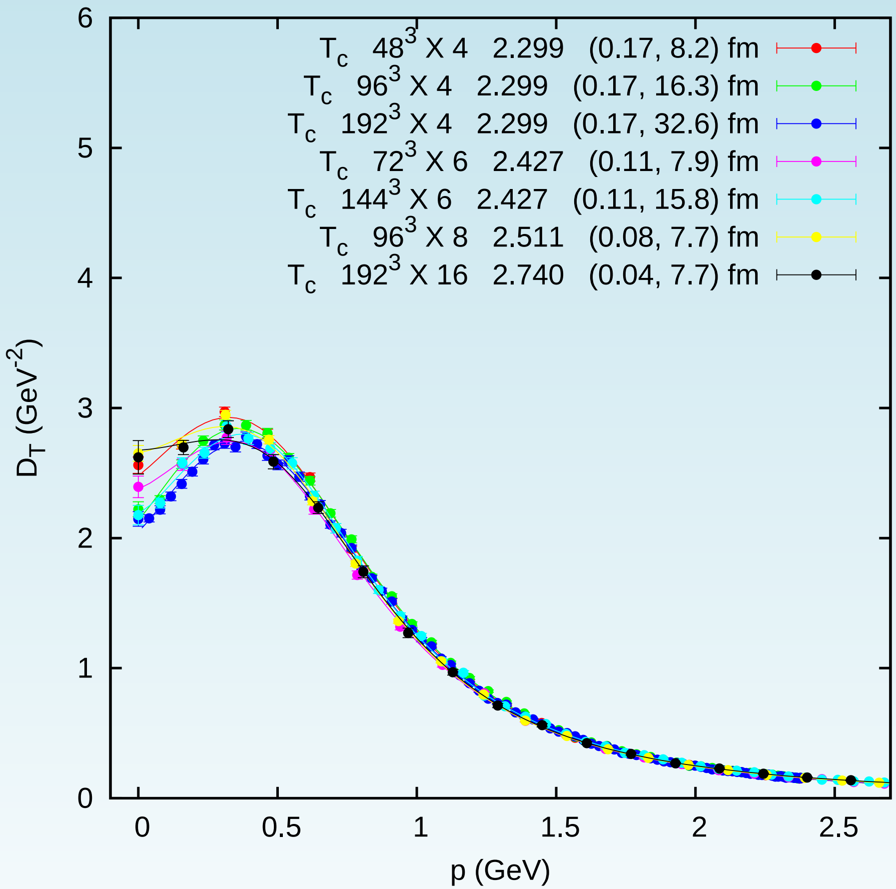




Longitudinal and transverse gluon at T_c

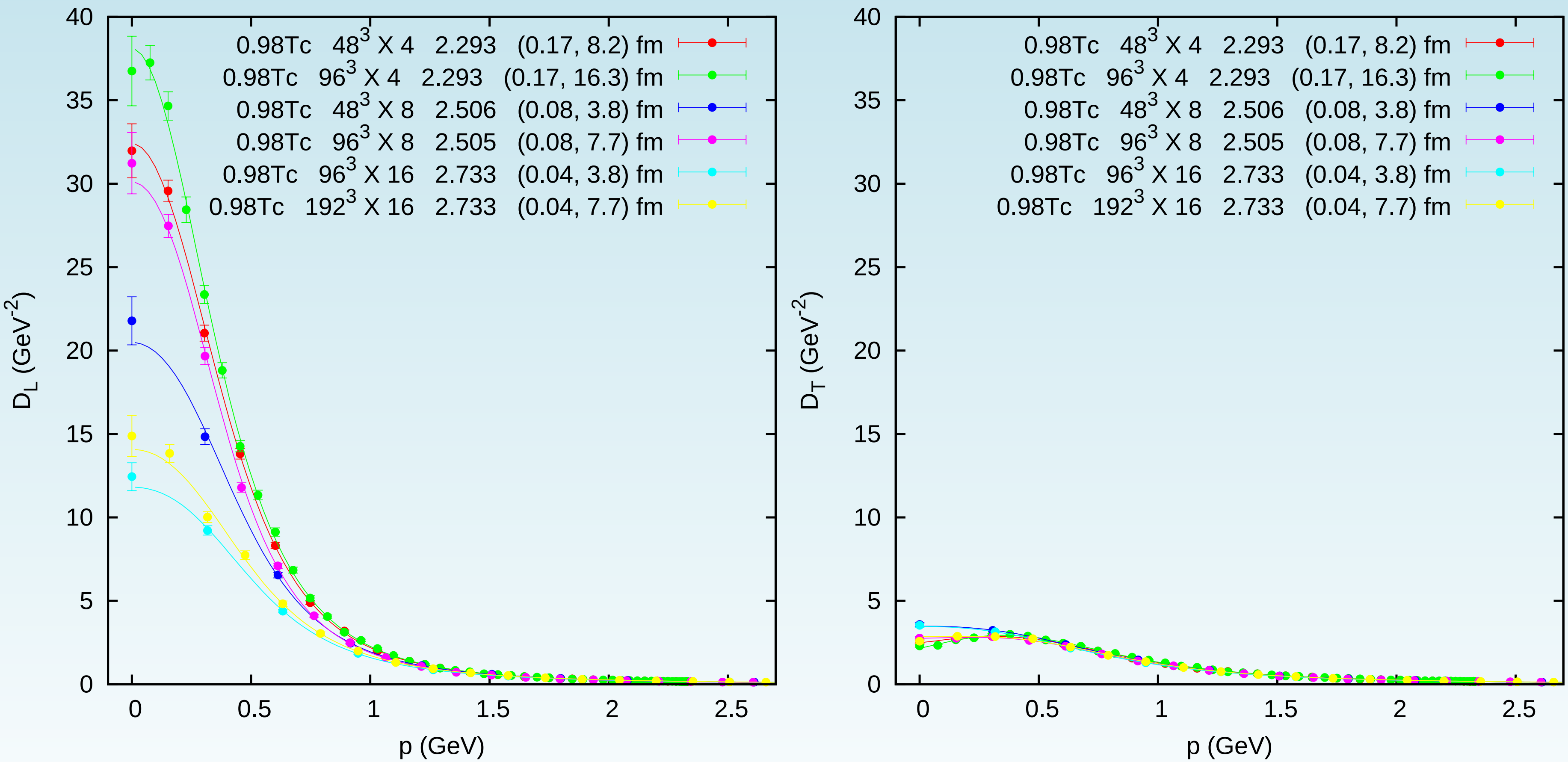
Electric (left) and magnetic (right) propagators at T_c





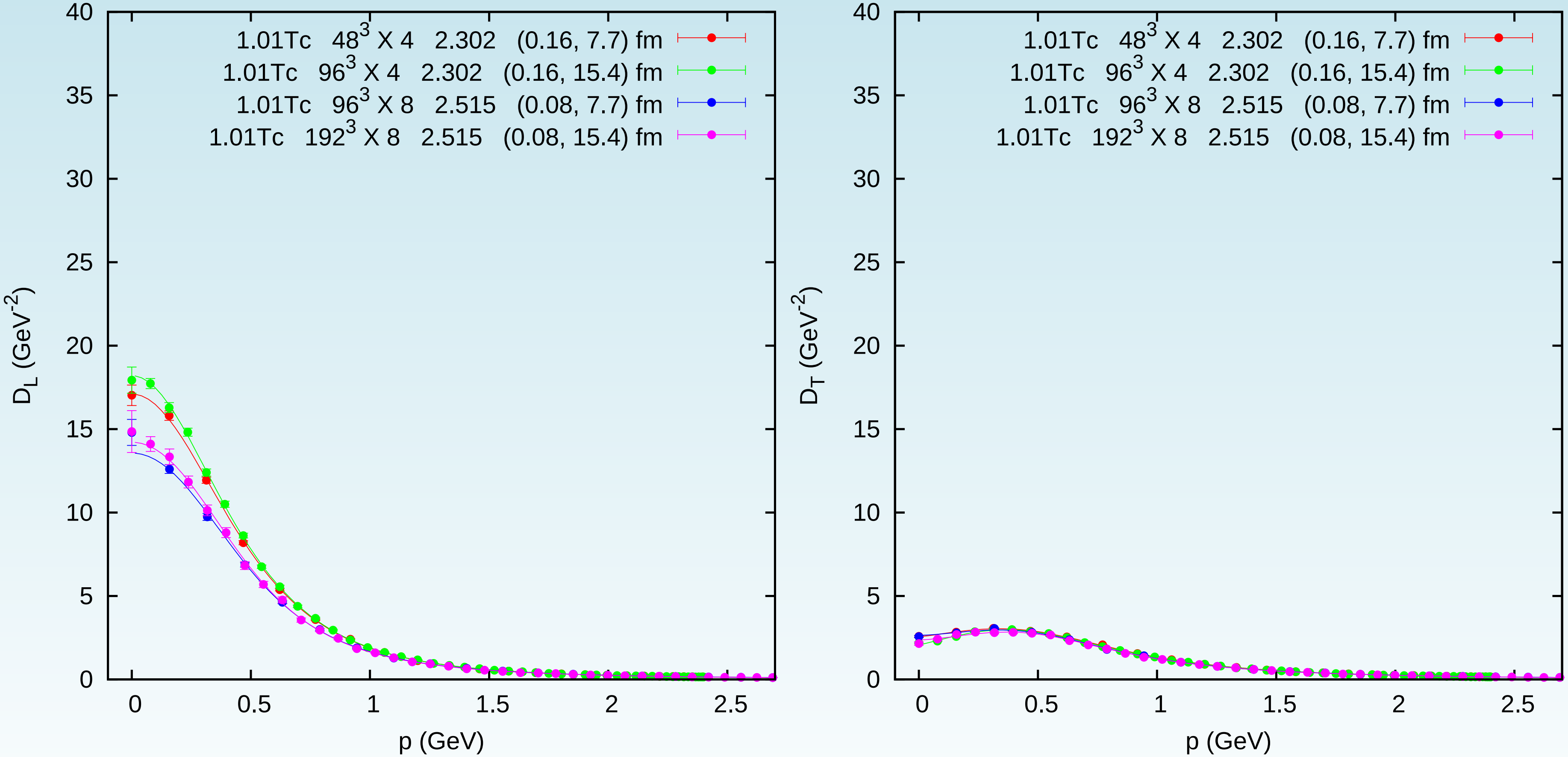
Results: Propagators at 0.98 T_c

Just below T_c, systematic errors for $D_L(p)$ are already present



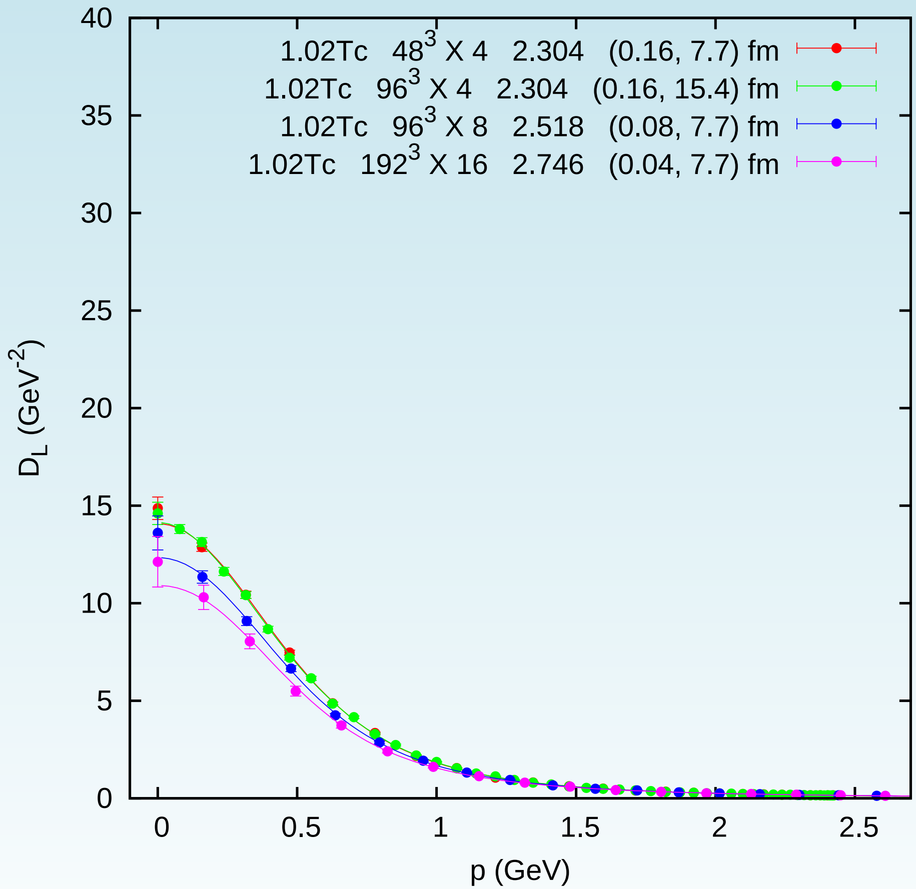
Results: Propagators at 1.01 T_c

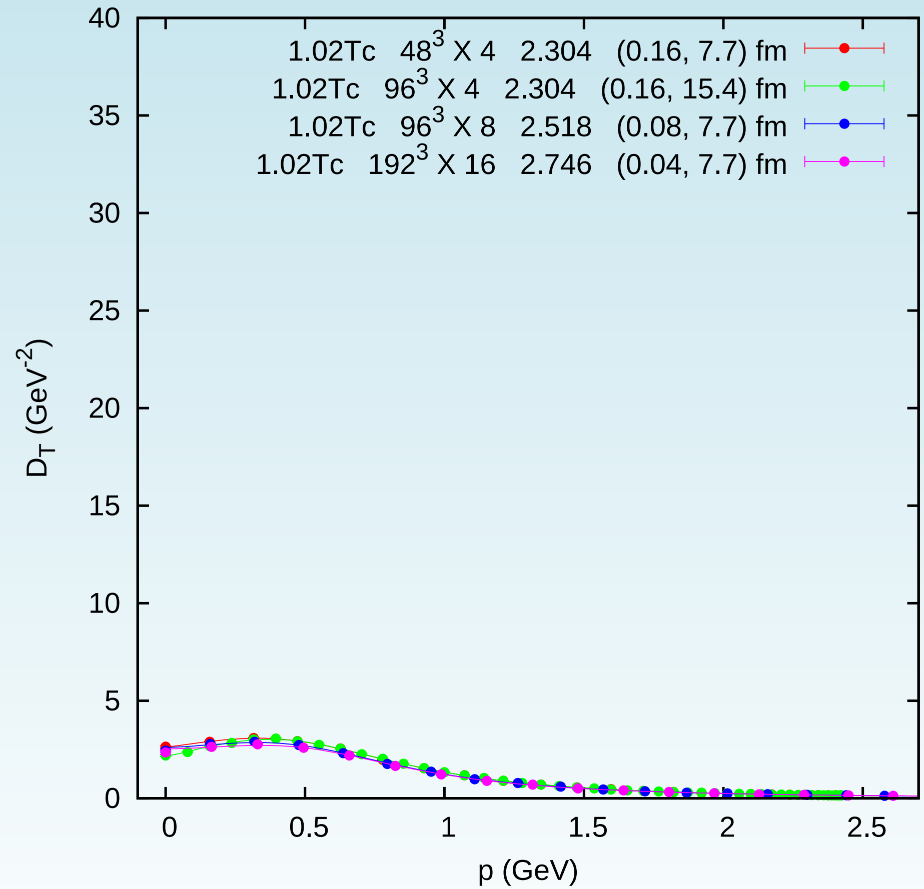
Just above T_c, systematic errors for $D_L(p)$ seem much less severe, IR plateau for $D_L(p)$ drops significantly for $N_t \leq 8$



Results: Propagators at 1.02 T_c

Just above T_c, systematic errors for $D_L(p)$ seem much less severe, IR plateau for $D_L(p)$ drops somewhat for $N_t \leq 8$





Discussion

Clearly, the thing that stands out more about T_c is the presence of very large finite-size corrections, but the (large-volume) behavior of D_L itself seems to be smooth around the critical region

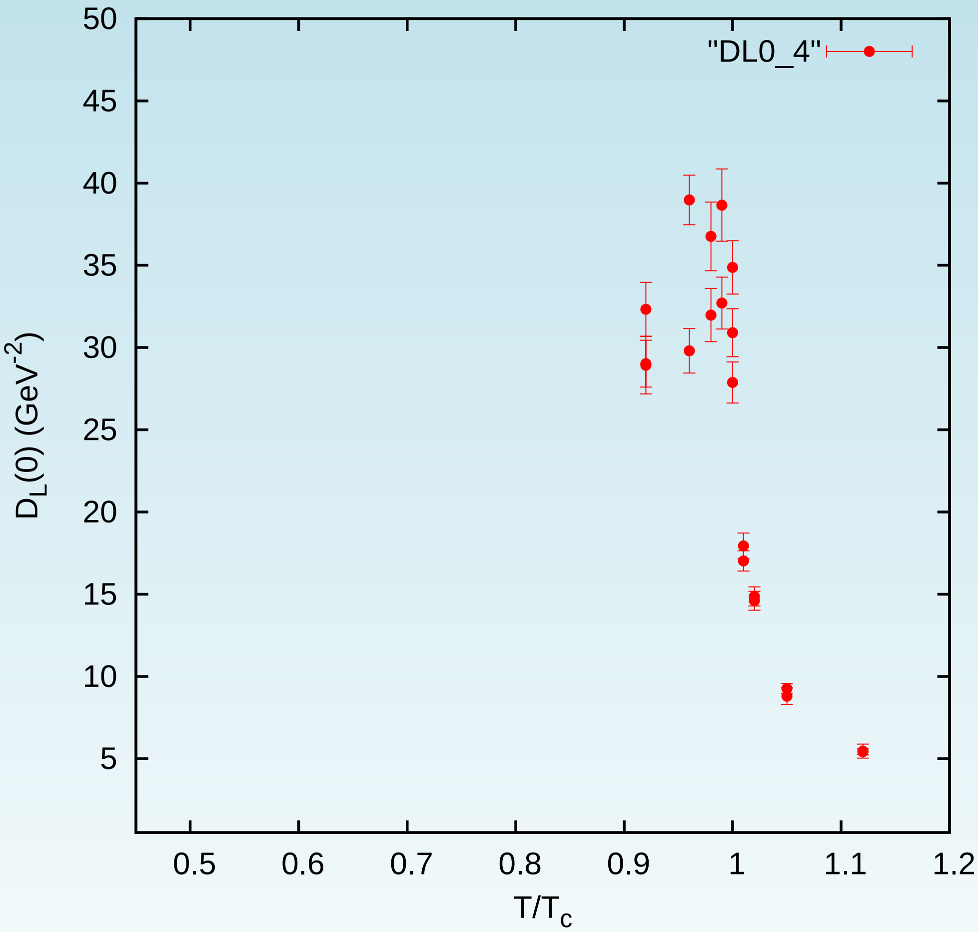
Discussion

Clearly, the thing that stands out more about T_c is the presence of very large finite-size corrections, but the (large-volume) behavior of D_L itself seems to be smooth around the critical region

$\Rightarrow$ To get an idea let us consider $D_L(0)$ as a function of the temperature

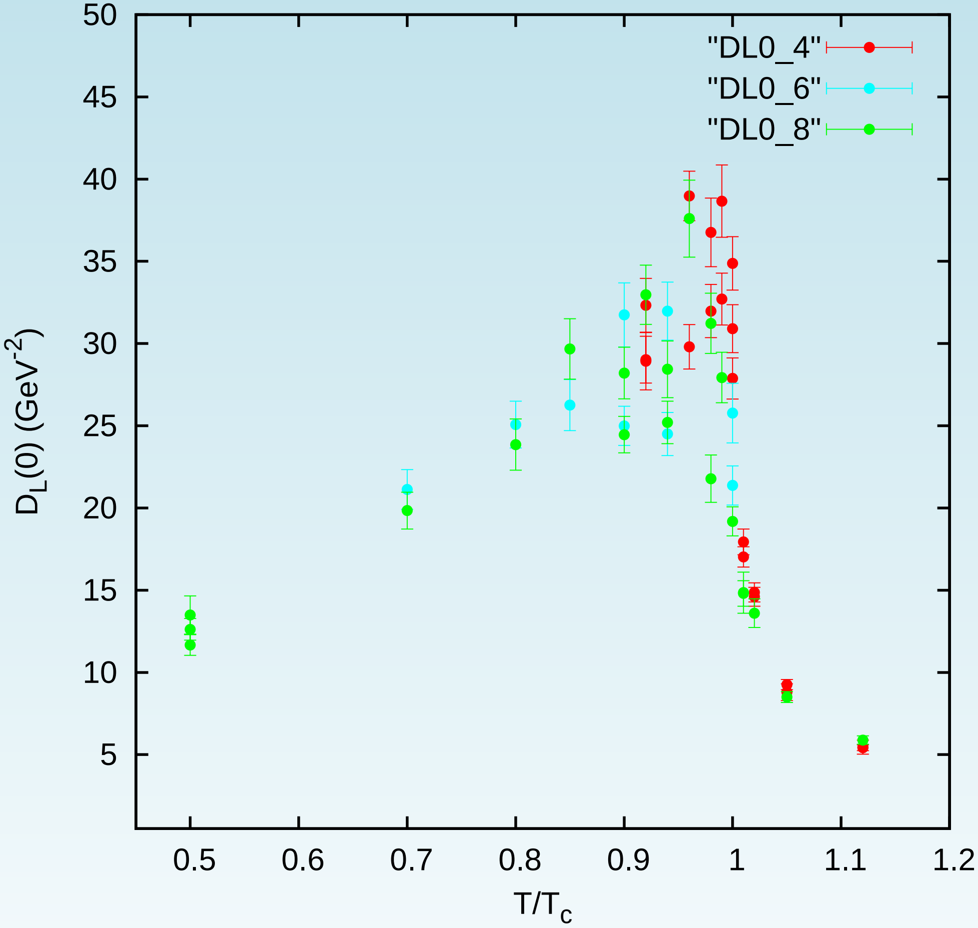
Infrared Plateau for $D_L(p)$ vs. T

IR plateau [from $D_L(0)$]:



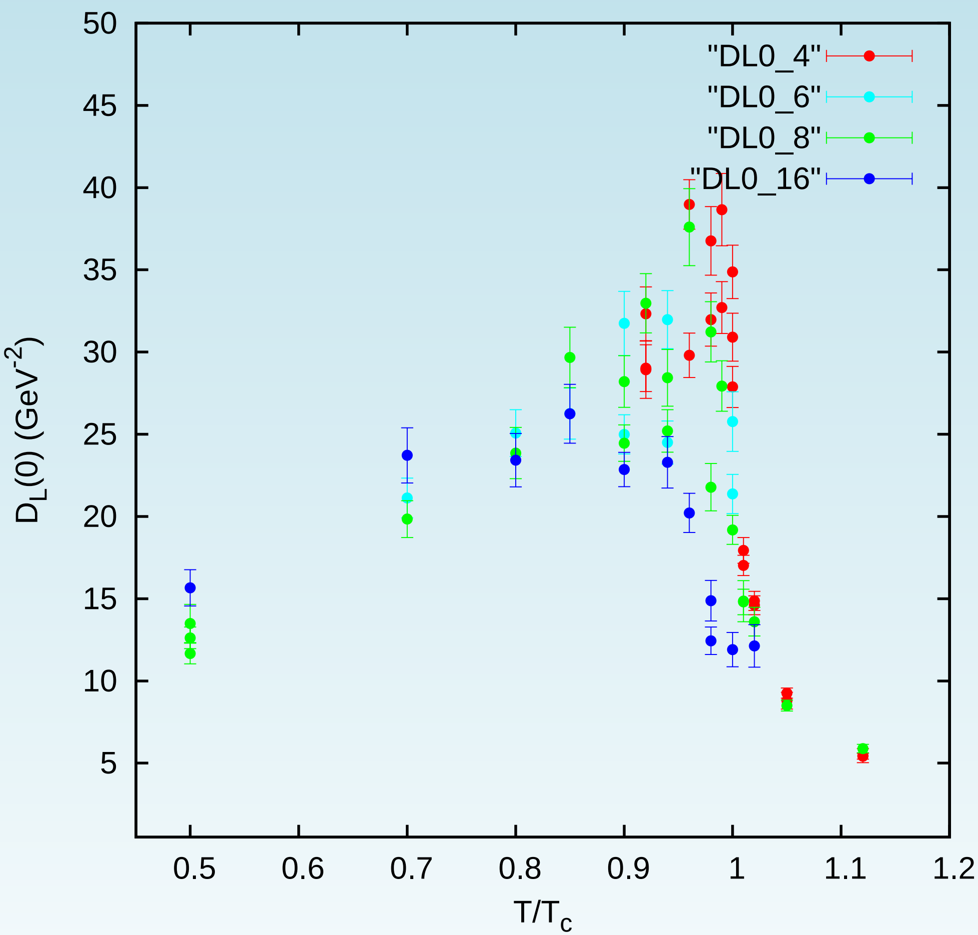
Infrared Plateau for $D_L(p)$ vs. T

IR plateau [from $D_L(0)$]:



Infrared Plateau for $D_L(p)$ vs. T

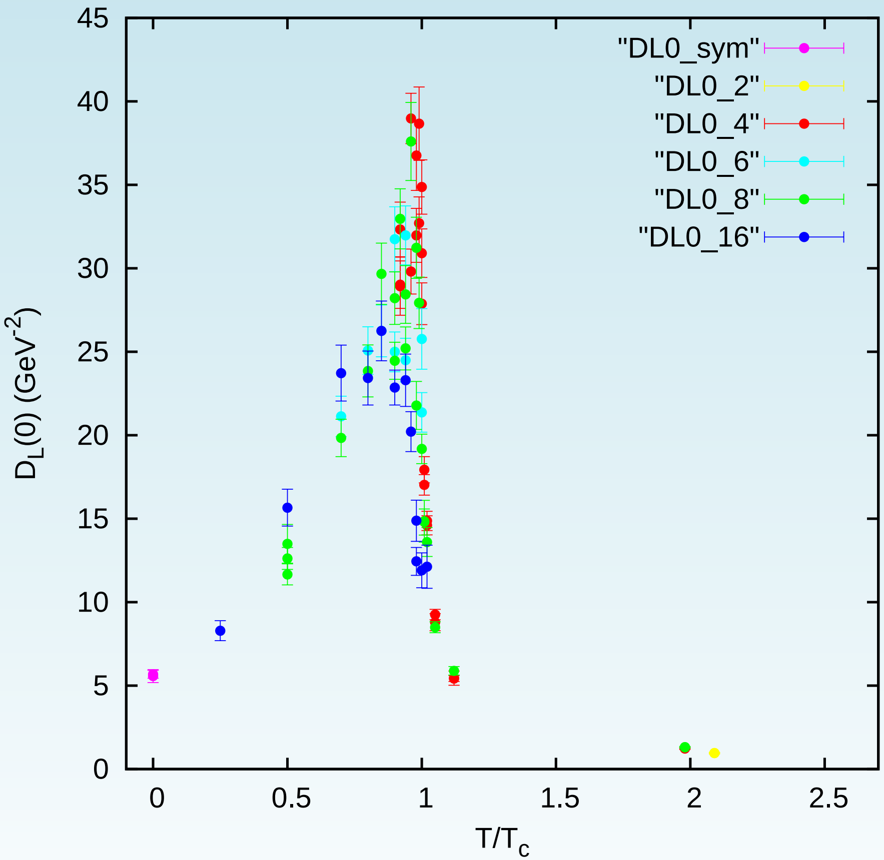
IR plateau [from $D_L(0)$]:

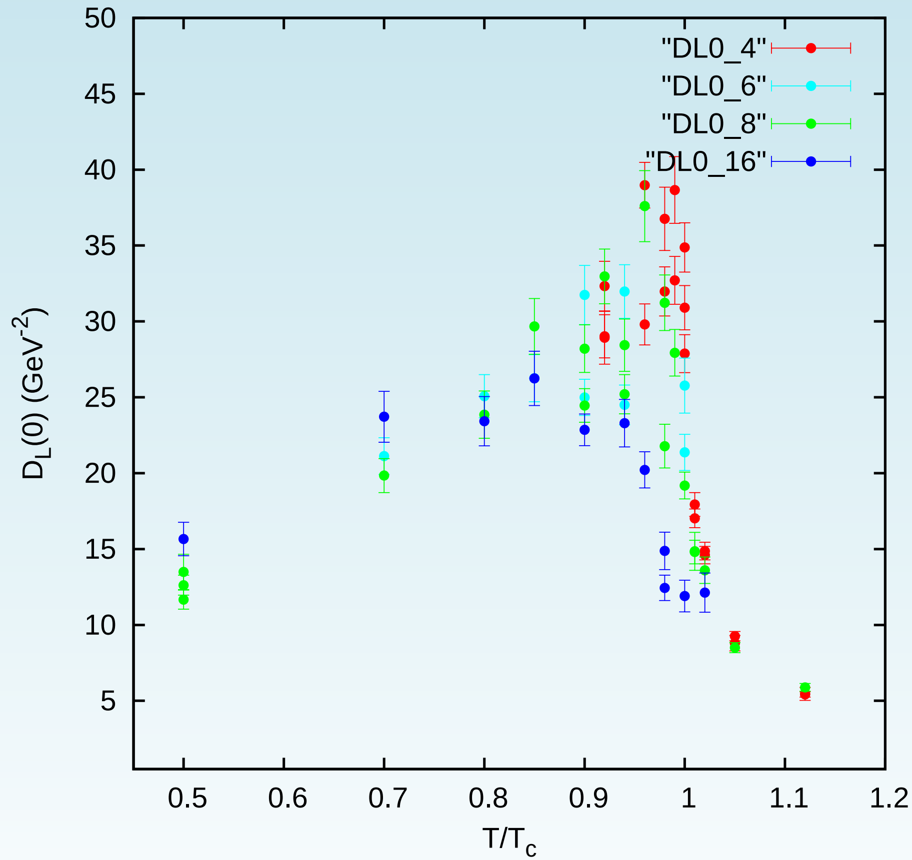


Peak at T_c for $N_t = 4 \Rightarrow$ finite maximum at $\lesssim$ 0.9 T_c for $N_t = 16$

Infrared Plateau for $D_L(p)$ vs. T

IR plateau value [estimated as $D_L(0)$] for all T values (left) and smaller range (right).





So?

We can see that the suggested sharp peak at T_c observed for $N_t = 4$ turns into a finite maximum around $T \lesssim 0.9 T_c$ for $N_t = 16$

So?

We can see that the suggested sharp peak at T_c observed for $N_t = 4$ turns into a finite maximum around $T \lesssim 0.9T_c$ for $N_t = 16$

Thus no critical information from D_L 

So?

We can see that the suggested sharp peak at T_c observed for $N_t = 4$ turns into a finite maximum around $T \lesssim 0.9T_c$ for $N_t = 16$

Thus no critical information from D_L 

In that sense we can say that the information contained in $D_L(p^2)$ is independent of the one coming from the Polyakov Loop...

So?

We can see that the suggested sharp peak at T_c observed for $N_t = 4$ turns into a finite maximum around $T \lesssim 0.9T_c$ for $N_t = 16$

Thus no critical information from D_L 

In that sense we can say that the information contained in $D_L(p^2)$ is independent of the one coming from the Polyakov Loop...

It is still interesting to characterize the behavior of the gluon propagator at these temperatures in terms of its analytic structure, performing fits to extract mass scales; can make a comparison with $T = 0$ case

Fitting forms

Usual estimates for screening masses, taken as $D_L(0)^{-1/2}$, can only be based here on small ranges (for rather small momenta)

Fitting forms

Usual estimates for screening masses, taken as $D_L(0)^{-1/2}$, can only be based here on small ranges (for rather small momenta)

Might try interpolation (inspired by dimensional reduction in transverse case) of more elaborated fits used for the $T = 0$ 4d and 3d cases:

$$D_{4d}(p^2) \; = \; C \, \frac{p^2 \, + \, d}{p^4 \, + \, u^2 \, p^2 \, + \, t^2}$$

$$D_{3d}(p^2) = C \, \frac{(p^2 \, + \, d) \, (p^2 \, + \, 1)}{(p^4 \, + \, u^2 \, p^2 \, + \, t^2) \, (p^2 \, + \, v)}$$

Fitting forms

Usual estimates for screening masses, taken as $D_L(0)^{-1/2}$, can only be based here on small ranges (for rather small momenta)

Might try interpolation (inspired by dimensional reduction in transverse case) of more elaborated fits used for the $T = 0$ 4d and 3d cases:

$$D_{4d}(p^2) \;=\; C\,\frac{p^2 \,+\, d}{p^4 \,+\, u^2\,p^2 \,+\, t^2}$$

$$D_{3d}(p^2) = C\,\frac{(p^2 \,+\, d)\,(p^2 \,+\, 1)}{(p^4 \,+\, u^2\,p^2 \,+\, t^2)\,(p^2 \,+\, v)}$$

These (polynomial) Gribov-Stingl forms allow for complex-conjugate poles. At nonzero T they do not work well...

Our Proposal

Consider generalized versions of Gribov-Stingl forms above, e.g.

$$D_{L,T}(p^2) \;=\; C\,\frac{1\,+\,d\,p^{2\eta}}{(p^2+a)^2\,+\,b^2} \quad \text{or} \quad C\left[\frac{p^2\,+\,d}{(p^2+a)^2\,+\,b^2}\right]^{\eta}$$

Our Proposal

Consider generalized versions of Gribov-Stingl forms above, e.g.

$$D_{L,T}(p^2) \;=\; C\,\frac{1\;+\;d\,p^{2\eta}}{(p^2+a)^2\;+\;b^2} \quad \text{or} \quad C\left[\frac{p^2\;+\;d}{(p^2+a)^2\;+\;b^2}\right]^{\eta}$$

Both fits correspond to poles at masses

$$m^2 = a \pm ib \Rightarrow m = m_R + i m_I$$

(Expect $m_I \to 0$ at high T)

Our Proposal

Consider generalized versions of Gribov-Stingl forms above, e.g.

$$D_{L,T}(p^2) \;=\; C\,\frac{1\,+\,d\,p^{2\eta}}{(p^2+a)^2\,+\,b^2} \quad \text{or} \quad C\left[\frac{p^2\,+\,d}{(p^2+a)^2\,+\,b^2}\right]^{\eta}$$

Both fits correspond to poles at masses

$$m^2 = a \pm ib \Rightarrow m = m_R + i m_I$$

(Expect $m_I \to 0$ at high T)

These fits (shown in above plots) work quite well. The masses obtained have comparable real and imaginary parts and are smooth around the transition. At higher T: imaginary part gets smaller in longitudinal case.

Electric and Magnetic Masses vs. T

T/T_c	$N_s^3 \times N_t$	$m_R^{(E)}$	$m_I^{(E)}$	$m_R^{(M)}$	$m_I^{(M)}$
0	$64^3 \times 64$	0.83 GeV	0.43 GeV	0.86 GeV	0.51 GeV
0.25	$96^3 \times 16$	0.61 GeV	0.28 GeV	0.57 GeV	0.28 GeV
0.5	$48^3 \times 8$	0.51 GeV	0.13 GeV	0.59 GeV	0.36 GeV
0.7	$96^3 \times 8$	0.31 GeV	0.13 GeV	0.37 GeV	0.24 GeV
0.9	$96^3 \times 16$	0.10 GeV	0.06 GeV	0.15 GeV	0.10 GeV
0.98	$96^3 \times 8$	0.19 GeV	0.10 GeV	0.28 GeV	0.20 GeV
1.0	$96^3 \times 8$	0.23 GeV	0.09 GeV	0.25 GeV	0.19 GeV
1.05	$96^3 \times 8$	0.29 GeV	0.09 GeV	0.24 GeV	0.18 GeV
2.0	$96^3 \times 8$	0.27 GeV	0.07 GeV	0.19 GeV	0.14 GeV

Conclusions

- $D_L(p)$ around the transition: large-lattice results indicate no divergence, only a finite maximum around 0.9 T_c
  $\Rightarrow$ Might explain why the same qualitative behavior is seen for $SU(2)$ and $SU(3)$ cases

Conclusions

- $D_L(p)$ around the transition: large-lattice results indicate no divergence, only a finite maximum around 0.9 T_c
  $\Rightarrow$ Might explain why the same qualitative behavior is seen for $SU(2)$ and $SU(3)$ cases
- Freakishly large systematic effects for $D_L(p)$ (volume dependence at fixed and small N_t) strongest at and just below the critical point

Conclusions

- $D_L(p)$ around the transition: large-lattice results indicate no divergence, only a finite maximum around 0.9 T_c
  $\Rightarrow$ Might explain why the same qualitative behavior is seen for $SU(2)$ and $SU(3)$ cases
- Freakishly large systematic effects for $D_L(p)$ (volume dependence at fixed and small N_t) strongest at and just below the critical point
- Good fits (for transverse and longitudinal cases) to several generalized Gribov-Stingl forms, including an exponentiaded form, suggesting the presence of branch cuts in addition to simple poles

Conclusions

- $D_L(p)$ around the transition: large-lattice results indicate no divergence, only a finite maximum around 0.9 T_c
  $\Rightarrow$ Might explain why the same qualitative behavior is seen for $SU(2)$ and $SU(3)$ cases
- Freakishly large systematic effects for $D_L(p)$ (volume dependence at fixed and small N_t) strongest at and just below the critical point
- Good fits (for transverse and longitudinal cases) to several generalized Gribov-Stingl forms, including an exponentiaded form, suggesting the presence of branch cuts in addition to simple poles
- Main qualitative feature of gluonic correlations in the deconfined phase seems to be lack of violation of reflection positivity for $D_L(x)$ (observed however for all $T \neq 0$)