

# Forward jets using new tools for high energy factorization (and beyond)

Piotr Kotko  
ピョートル コトコ

Institute of Nuclear Physics (Cracow)

supported by  
LIDER/02/35/L-2/10/NCBiR/2011

in collaboration with  
A. van Hameren, K. Kutak,  
C. Marquet, S. Sapeta



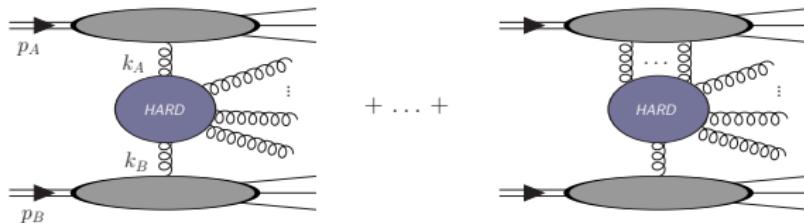
# Outline

- ① new results for forward jets within high energy factorization were presented  
⇒ see Krzysztof's and Cyrille's talks
- ② this talk ⇒ the Monte Carlo tools we have used
- ③ further developments in off-shell matrix elements evaluation ⇒ a new tool for analytic, automatized calculation of tree-level matrix elements of Wilson lines

# High Energy Factorization

"Hybrid" high energy factorization (HEF) formula relevant for forward jets

[e.g. M. Deak, F. Hautmann, H. Jung, K. Kutak, JHEP 0909 (2009) 121]



$$k_A = x_A p_A + k_{TA}, \quad k_A^2 = k_{TA}^2, \quad k_B = x_B p_B, \quad k_B^2 = 0, \quad x_A \ll x_B$$

$$d\sigma_{AB \rightarrow X} = \int \frac{d^2 k_{TA}}{\pi} \int \frac{dx_A}{x_A} \int dx_B \mathcal{F}(x_A, k_{TA}, \mu) f_{b/B}(x_B, \mu) d\sigma_{g^* b \rightarrow X}(x_A, x_B, k_{TA}, \mu)$$

- collinear PDFs  $f_{b/B}(x_B, \mu)$
  - unintegrated gluon density (UGD)  $\mathcal{F}(x_A, k_{TA}, \mu)$   
the hard scale dependence is important for some observables
  - off-shell gauge invariant tree-level matrix elements reside in  $d\sigma_{g^* b \rightarrow X}$   
subleading effects are accounted for in the UGD
- ★ In general  $k_T$ -factorization does not hold for hadron-hadron collisions  
⇒ see the talks of Cyrille and Anna

# Recent results using HEF and various UGDs

- forward-central dijets in a configuration measured by CMS (**Krzysztof's talk**)  
[A. van Hameren, P.K., K. Kutak, S. Sapeta, arXiv:1404.6204]
  - **KS** UGD (unified BFKL+DGLAP with nonlinear BK term; fitted to HERA data by K. Kutak and S. Sapeta)  
[K. Kutak, A. Stasto, Eur.Phys.J. C41, 343 (2005)]  
[K. Kutak, S. Sapeta, Phys.Rev. D86, 094043 (2012)]
  - **KS** UGD supplemented with the Sudakov resummation effects
  - **KMR** UGD  
[M. Kimber, A. D. Martin, and M. Ryskin, Phys.Rev. D63, 114027 (2001)]
- forward-forward dijets (**Cyrille's talk**)  
[A. van Hameren, P.K., K. Kutak, C. Marquet, S. Sapeta, Phys.Rev. D89, 094014 (2014)]  
study of saturation effects in p+p and p+Pb with **KS** UGD and **rcBK** (BK equation in the momentum space with running coupling)  
[I. Balitsky, G.A. Chirilli, Phys.Rev. D77, 014019 (2008)]  
[J. L. Albacete, N. Armesto, J.G. Milhano, P. Quiroga-Arias, C.A. Salgado, Eur. Phys. J. C 71 (2011) 1705]
- trijets in forward and forward-central configurations  
[A. van Hameren, P.K., K. Kutak, Phys.Rev. D88, 094001 (2013)]
- very forward dijets with possible extension of CASTOR (not published; prepared for a yellow report)

# The tools for HEF

It is convenient to implement HEF in a Monte Carlo program. The calculations have been carried and cross checked using three independent programs:

- **oscars** (Off-Shell Currents And Related Stuff)  
FORTRAN code by A. van Hameren (not public yet), similar to HELAC; off-shell amplitudes are calculated efficiently using recently developed BCFW recursion for off-shell amplitudes [A. van Hameren, arXiv:1404.7818]
- **LxJet**  
C++ program by P.K., see next slide
- **forward**  
C++ program of S. Sapeta (available upon request), currently for dijets; off-shell MEs taken from [M. Deak, F. Hautmann, H. Jung, K. Kutak, JHEP 0909 (2009) 121]

Another program using  $k_T$  factorization  $\Rightarrow$  LMZ for photoproduction [A.V. Lipatov, M.A. Malyshev, N.P. Zotov] (see the talk of A. Iudin)

# LxJet Monte Carlo program

- uses FOAM – cellular Monte Carlo generator  
[S. Jadach, Comput.Phys.Commun. 152 (2003)]
- uses ROOT (easy histograming, allows e.g. to save events and reuse them without recalculating, etc.)
- operates on helicity amplitudes level (implemented spinor algebra)
- weight and unweighted events

## Gauge invariant off-shell matrix elements

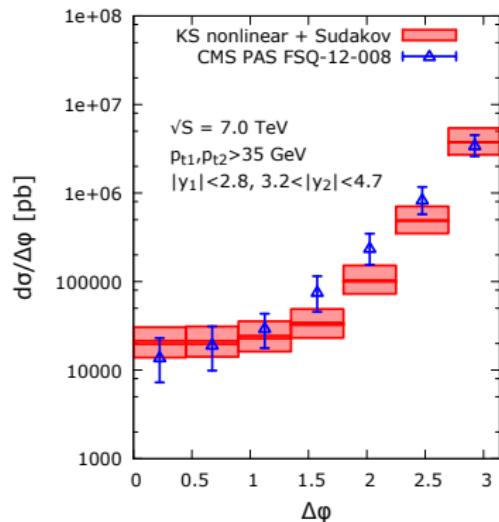
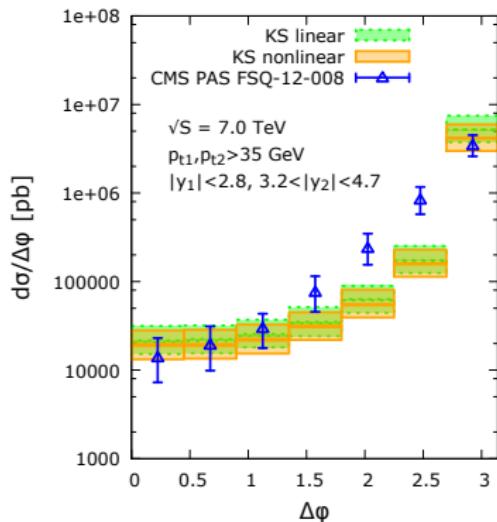
- helicity amplitudes for  $g^*g \rightarrow g \dots g$  calculated from gauge invariant extension of the Berends-Giele recursion  
[A. van Hameren, P.K., K. Kutak, JHEP 1212 (2012) 029]
- implementation of analytic formulae for  $g^*a \rightarrow X$ ,  $a = g, q$ ,  $\# \{X\} = 2, 3$  calculated using OGIME (Off-shell Gauge Invariant Matrix Elements)

## Versions

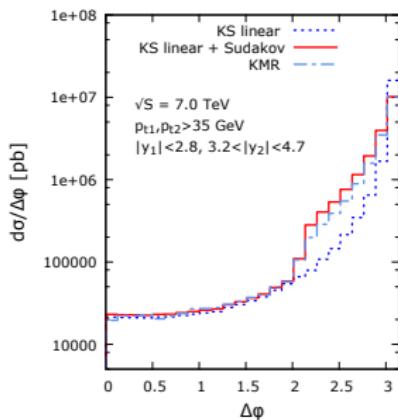
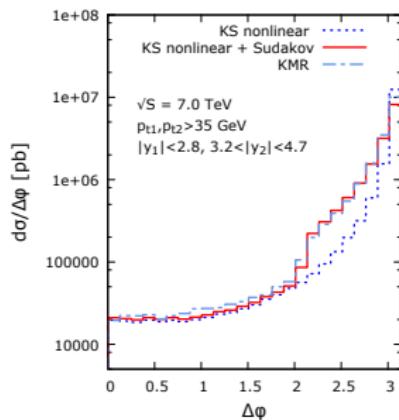
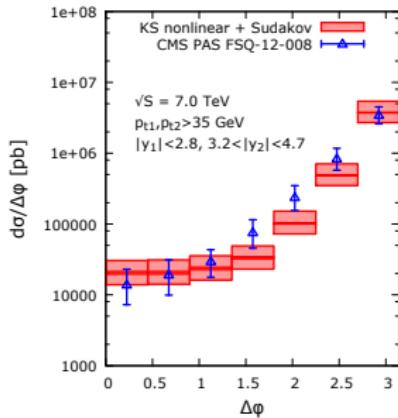
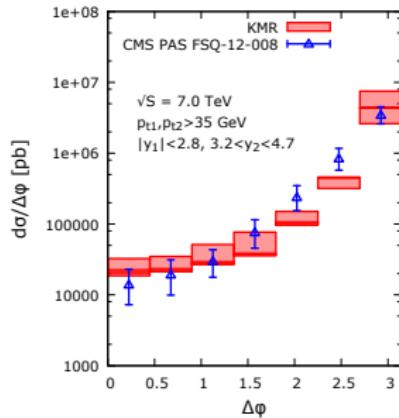
- v1.1, ready to use for dijets and trijets with KS linear UGD  
[<http://annapurna.ifj.edu.pl/~pkotko/LxJet.html>]
- v1.2 (to appear), accounts for hard scale dependence in UGD; distributed with a plugin to make “the Sudakov resummation”

# Example LxJet application

- new measurement of CMS: decorrelations of forward-central high  $p_T$  jets [CMS-PAS-FSQ-12-008]
- HEF factorization underestimates the number of events with unbalanced  $p_T$  of the order of the hard scale  
⇒ this is cured by introducing hard scale dependence via “the Sudakov resummation” [A. van Hameren, P.K., K. Kutak, S. Sapeta, arXiv:1404.6204]



# Sudakov resummation model vs KMR



# OGIME – Off-shell Gauge Invariant Matrix Elements

Original idea: design a tool for **analytic, automatic** calculation of tree-level off-shell amplitudes with one off-shell leg (to be used in HEF, e.g. in LxJet)

- it expanded to a more general tool: several off-shell legs with arbitrary “polarizations” of the off-shell gluons  
⇒ it enables to use them outside small x physics
- written in **FORM** – an open source symbolic manipulation system by J. Vermaseren (very fast, can deal with plenty of terms, but “low level”)
- method: matrix elements of Wilson lines (see next slides)
- automatic Wick contractions, momentum conservation, simplification, etc.
- limitations: analytic results already for 6 gauge fields are huge

## Versions

- v1.2, only gluons [<http://annapurna.ifj.edu.pl/~pkotko/LxJet.html>]
- v1.3, quarks added (not public yet)
- v2.0, electroweak interactions added, under development

# Off-shell amplitudes and Wilson lines

Off-shell gauge invariant amplitude  $\tilde{M}_{e_1 \dots e_n}(k_1, \dots, k_n; X)$  for

$$g^*(k_1, e_1) \dots g^*(k_n, e_n) \rightarrow X$$

where  $k_i, e_i$  are momentum and “polarization” vector of an off-shell gluon can be defined as [P.K. arXiv:1403.4824, accepted to JHEP]

$$\langle 0 | \mathfrak{R}_{e_1}^{c_1}(k_1) \dots \mathfrak{R}_{e_n}^{c_n}(k_n) | X \rangle = \delta(k_1 \cdot e_1) \dots \delta(k_n \cdot e_n) \delta^4(k_1 + \dots + k_n - X) \tilde{M}_{e_1 \dots e_n}(k_1, \dots, k_n; X)$$

where (almost-)infinite (almost-)straight Wilson lines are defined as

$$\mathfrak{R}_{e_i}^{c_i}(k_i) = \int d^4y e^{iy \cdot k_i} \text{Tr} \left\{ \frac{1}{\pi g} t^{c_i} \mathcal{P} \exp \left[ ig \int_{-\infty}^{\infty} ds \frac{dz_{i\mu}(s)}{ds} A_b^\mu(z) t^b \right] \right\}$$

where  $t^a$  are color generators and the path is parametrized as

$$z_i^\mu(s) = y^\mu + \frac{2}{\epsilon} \tanh\left(\frac{\epsilon s}{2}\right) e_i^\mu, \quad s \in (-\infty, \infty)$$

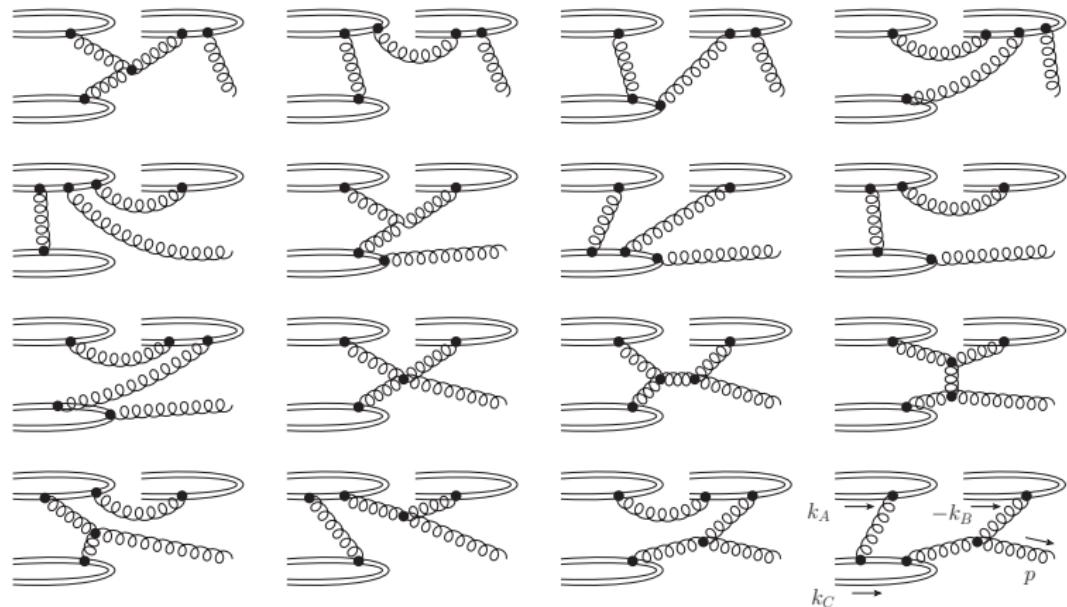
In the matrix element definition the limit  $\epsilon \rightarrow 0$  is assumed.

- regularization with hyperbolic paths  $\Rightarrow$  formal derivation of generalized functions
- “polarization” vectors are arbitrary  $\Rightarrow$  application outside small  $x$  physics

## Example

Consider example:  $g^*(k_A, e_A) g^*(k_C, e_C) \rightarrow g^*(k_B, e_B) g(p)$

There are 16 color-ordered (for simplicity) diagrams:



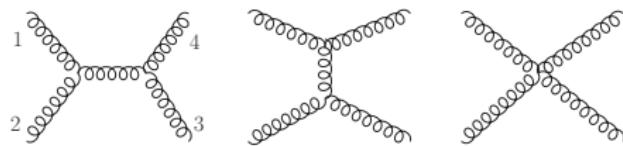
For  $e_A, e_B, e_C \in \{n_+, n_-\}$  some of the diagrams vanish and the result is consistent with **RRRg** Lipatov's vertex

[M.A. Braun, M.Yu. Salykin, S.S. Pozdnyakov, M.I. Vyazovsky, Eur.Phys.J. C72 (2012) 2223]

# Gauge invariant decompositions

Our off-shell amplitudes are defined for any “polarization vectors”. This allows to use them outside high-energy physics.

For example, consider a standard color-ordered four gluon amplitude



$$\begin{aligned} \mathcal{M}^{(1234)} = & J_\mu^{(12)} \frac{ig^{\mu\nu}}{k_{12}^2} J_\nu^{(34)} \\ & + J_\mu^{(41)} \frac{ig^{\mu\nu}}{k_{14}^2} J_\nu^{(23)} + iV_4^{(1234)} \end{aligned}$$

It is possible to write this amplitude in a manifestly gauge invariant way

$$\mathcal{M}^{(1,2,3,4)} = i \left( k_{12}^2 \tilde{J}^{(1,2)} \cdot \tilde{J}^{(3,4)} + k_{14}^2 \tilde{J}^{(4,1)} \cdot \tilde{J}^{(2,3)} + \tilde{V}_4^{(1,2,3,4)} \right)$$

where

$$\tilde{J}^{(ab)} \cdot \tilde{J}^{(cd)} = \sum_{i=0}^2 \tilde{J}_i^{(ab)} \tilde{J}_i^{(cd)} d_i, \quad \tilde{J}_i(\varepsilon_1, \varepsilon_2; k_{12}) \stackrel{*}{=} \langle k_1, \varepsilon_1; k_2, \varepsilon_2 | \mathcal{R}_{\varepsilon_i}(k_{12}) | 0 \rangle$$

and  $k \cdot \epsilon_i(k) = 0$ ,  $\epsilon_i(k) \cdot \epsilon_j(k) = d_i(k) \delta_{ij}$ ,  $d_0(k) = \pm 1$ ,  $d_1(k) = d_2(k) = -1$ ,  $\sum_{i=0}^2 \epsilon_i^\nu(k) \epsilon_i^\mu(k) d_i(k) = g^{\mu\nu} - k^\mu k^\nu / k^2$ .

# Summary

- new results for forward jets at the LHC within High Energy factorization are available, e.g.:
  - forward-central jets  $\Rightarrow$  reasonable agreement with the data is encouraging (see Krzysztof's talk)
  - forward-forward dijets as saturation probes (see Cyrille's talk)
- convenient calculations require Monte Carlo tools  $\Rightarrow$  here `LxJet` has been presented
- off-shell gauge invariant amplitudes can be calculated using matrix elements of Wilson lines  $\Rightarrow$  a practical analytic realization is `OGIME` program

# Backup

# Off-shell Multigluon Amplitude

Color ordered result for  $g^* g \rightarrow g \dots g$

$$\begin{aligned}\widetilde{\mathcal{A}}(\varepsilon_1, \dots, \varepsilon_N) = & -\left| \vec{k}_{TA} \right| \left[ k_{TA} \cdot J(\varepsilon_1, \dots, \varepsilon_N) \right. \\ & \left. + \left( \frac{-g}{\sqrt{2}} \right)^N \frac{\varepsilon_1 \cdot p_A \dots \varepsilon_N \cdot p_A}{k_1 \cdot p_A (k_1 - k_2) \cdot p_A \dots (k_1 - \dots - k_{N-1}) \cdot p_A} \right]\end{aligned}$$

where

$$\begin{aligned}J^\mu(\varepsilon_1, \dots, \varepsilon_N) = & \frac{-i}{k_{1N}^2} \left( g_\nu^\mu - \frac{k_{1N}^\mu p_{A,\nu} + k_{1N,\nu} p_A^\mu}{k_{1N} \cdot p_A} \right) \\ & \left\{ \sum_{i=1}^{N-1} V_3^{\nu\alpha\beta}(k_{1i}, k_{(i+1)N}) J_\alpha(\varepsilon_1, \dots, \varepsilon_i) J_\beta(\varepsilon_{i+1}, \dots, \varepsilon_N) \right. \\ & \left. + \sum_{i=1}^{N-2} \sum_{j=i+1}^{N-1} V_4^{\nu\alpha\beta\gamma} J_\alpha(\varepsilon_1, \dots, \varepsilon_i) J_\beta(\varepsilon_{i+1}, \dots, \varepsilon_j) J_\gamma(\varepsilon_{j+1}, \dots, \varepsilon_N) \right\}\end{aligned}$$

where  $k_{ij} = k_i + k_{i+1} + \dots + k_j$ ,  $V_3$  and  $V_4$  are three and four-gluon vertices.

The red piece was obtained using the Slavnov-Taylor identities and correspond to bremsstrahlung from the straight infinite Wilson line along  $p_A$  (in axial gauge).