

Resummation of non-global logarithms at finite N_c

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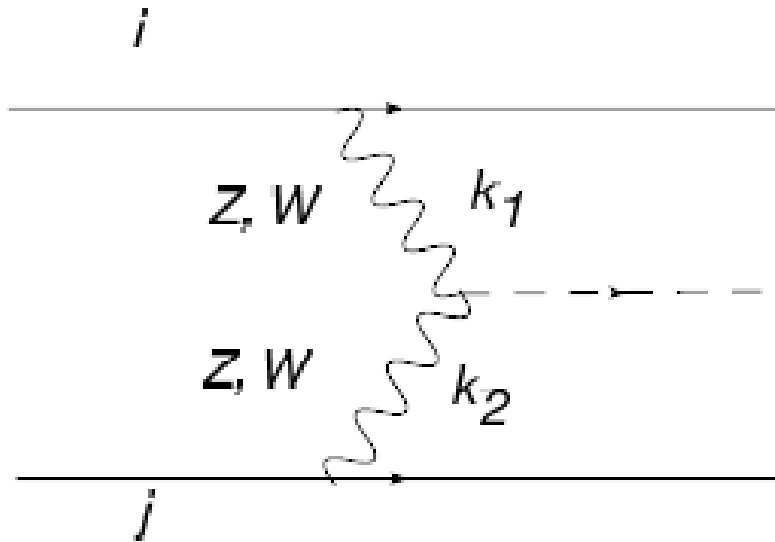
(Yukawa institute, Kyoto U.)

Based on: **Nucl. Phys. B874 (2013) 808** with
Takahiro Ueda (Karlsruhe)

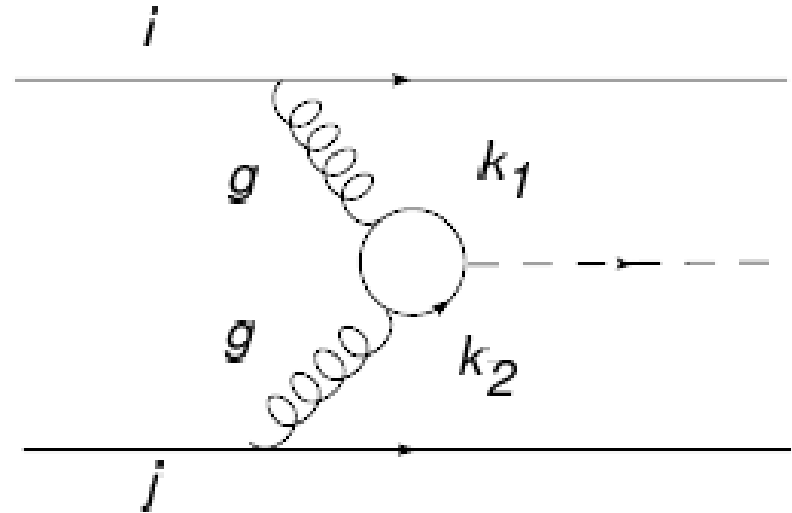
Outline

- Motivation : Jet vetoing at the LHC
- Non-global logarithms
- Relation to saturation physics
- Weigert's approach and its refinement
- Numerical result

Higgs plus di-jets at the LHC



Vector boson fusion



Gluon fusion

Different patterns of soft gluon radiation

Could be used to extract Higgs couplings, suppress large backgrounds

Cox, Forshaw, Pilkington; Englert, Spannowsky, Takeuchi,...

Cross section with a jet veto

Require that no jets with transverse momentum greater than p_t^{veto} is produced in $gg \rightarrow H$

Double logarithms $(\alpha_s L^2)^n$ $L = \ln \frac{m_H}{p_t^{veto}}$

→ exponentiate

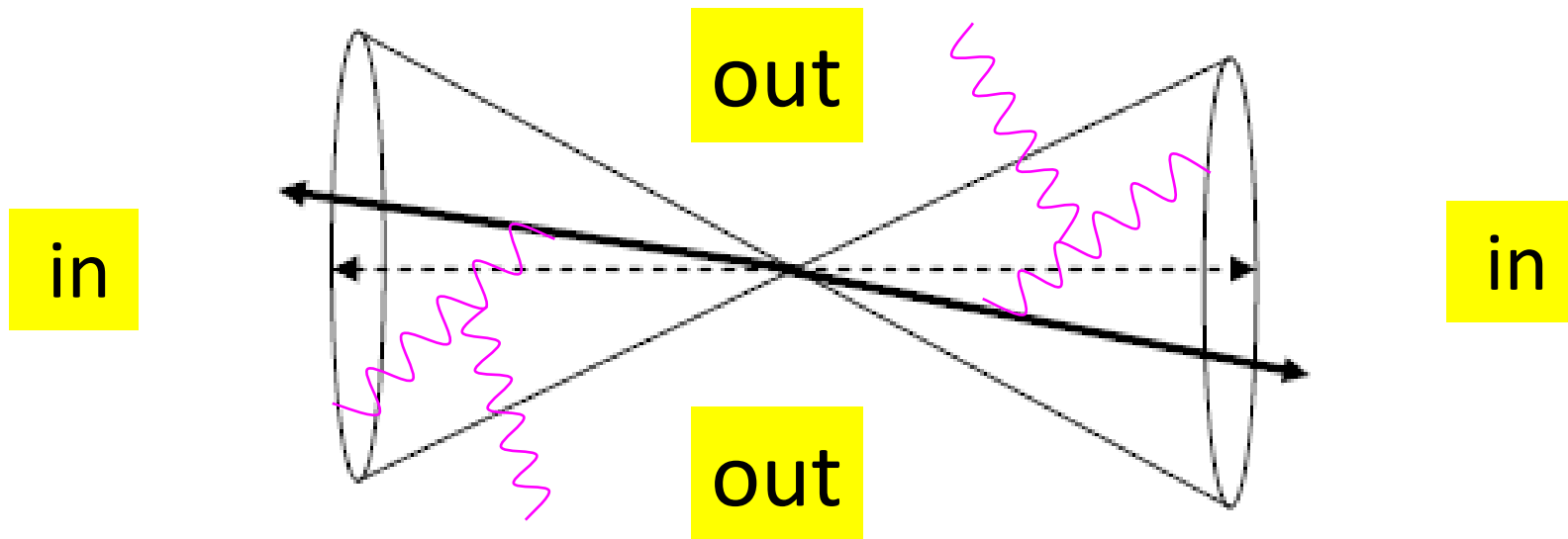
$$\exp \left(\underbrace{Lg_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} + \cdots \right)$$

Berger, Macantonini, Stewart, Tackmann, Waalewijn;
Banfi, Salam, Zanderighi; Becher, Neubert

Note: the observable is **global**,
i.e. **all** particles in the final state are measured.

Non-global observables

Measurement is done only in a **part** of the phase space **excluding** jets



Gluons are emitted at large angle, resum only the **soft** logarithms

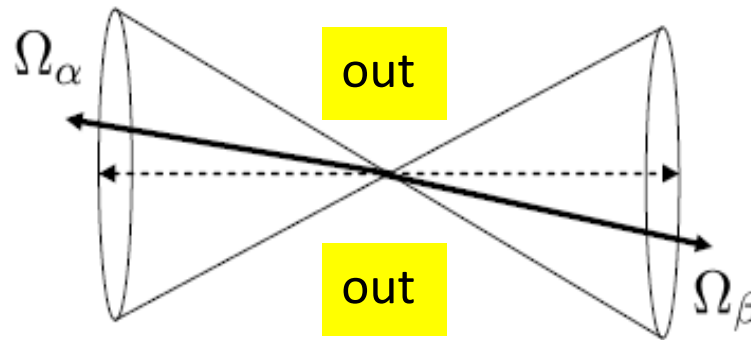
Do **not** exponentiate,

Resummation done only at **large- N_c**

Dasgupta, Salam (2001)

Banfi-Marchesini-Smye (BMS) equation

Probability that energy flow into the “out” region is **less** than p_t^{veto}



$$\partial_\tau P_{\alpha\beta} = N_c \int \frac{d\Omega_\gamma}{4\pi} \mathcal{M}_{\alpha\beta}(\gamma) \left(\Theta_{in}(\gamma) P_{\alpha\gamma} P_{\gamma\beta} - P_{\alpha\beta} \right)$$

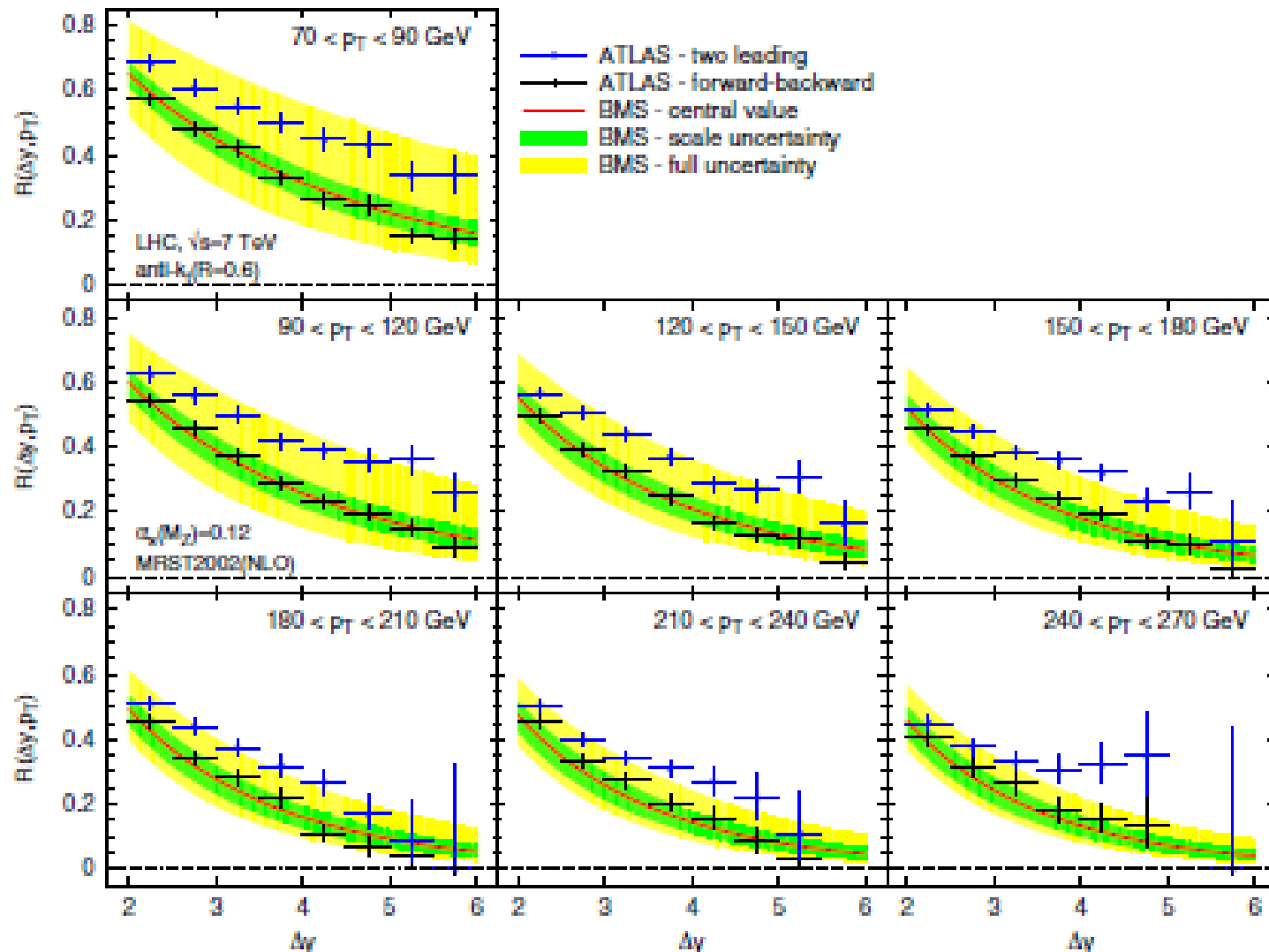
$$\mathcal{M}_{\alpha\beta}(\gamma) \equiv \frac{1 - \cos \theta_{\alpha\beta}}{(1 - \cos \theta_{\alpha\gamma})(1 - \cos \theta_{\gamma\beta})}$$

$$\tau = \frac{\alpha_s}{\pi} \ln \frac{p_t}{p_t^{veto}}$$

Equation derived at **large- N_c**

Describing ATLAS jet veto data

YH, Marquet, Royon, Soyez, Ueda, Werder (2013)



Similarity to the BK equation

BK

$$\partial_\tau \langle S_{xy} \rangle_\tau = N_c \int \frac{d^2 z}{2\pi} \mathcal{M}_{xy}(z) \left(\langle S_{xz} \rangle_\tau \langle S_{zy} \rangle_\tau - \langle S_{xy} \rangle_\tau \right)$$

$$\mathcal{M}_{xy}(z) = \frac{(x-y)^2}{(x-z)^2(z-y)^2}$$

BMS

$$\partial_\tau P_{\alpha\beta} = N_c \int \frac{d\Omega_\gamma}{4\pi} \mathcal{M}_{\alpha\beta}(\gamma) \left(\Theta_{in}(\gamma) P_{\alpha\gamma} P_{\gamma\beta} - P_{\alpha\beta} \right)$$

$$\mathcal{M}_{\alpha\beta}(\gamma) \equiv \frac{1 - \cos \theta_{\alpha\beta}}{(1 - \cos \theta_{\alpha\gamma})(1 - \cos \theta_{\gamma\beta})}$$

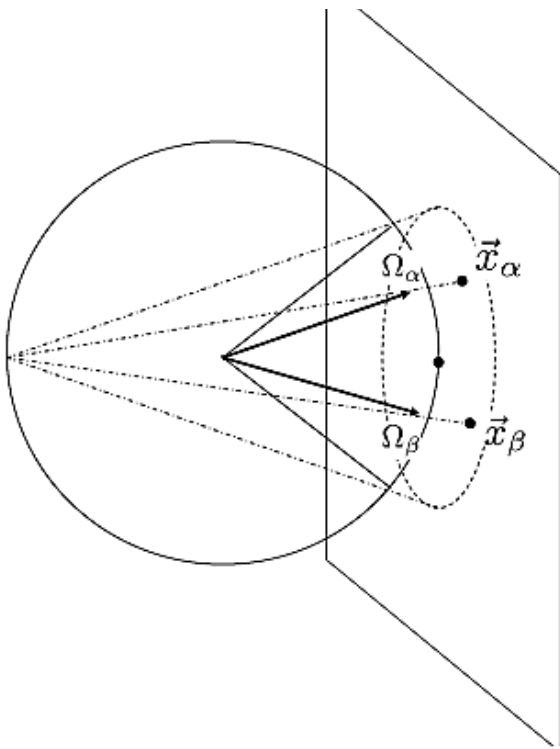
For BK, finite- N_c generalization (**JIMWLK**) is known.

Exact correspondence

YH (2008)

Stereographic projection exactly maps the two equations

$$\frac{d^2 z}{2\pi} \frac{(x - y)^2}{(x - z)^2 (z - y)^2} = \frac{d\Omega_\gamma}{4\pi} \frac{1 - \cos \theta_{\alpha\beta}}{(1 - \cos \theta_{\alpha\gamma})(1 - \cos \theta_{\gamma\beta})}$$



Exact correspondence also in the strong coupling limit of N=4 super Yang-Mills

Comment on conformal symmetry

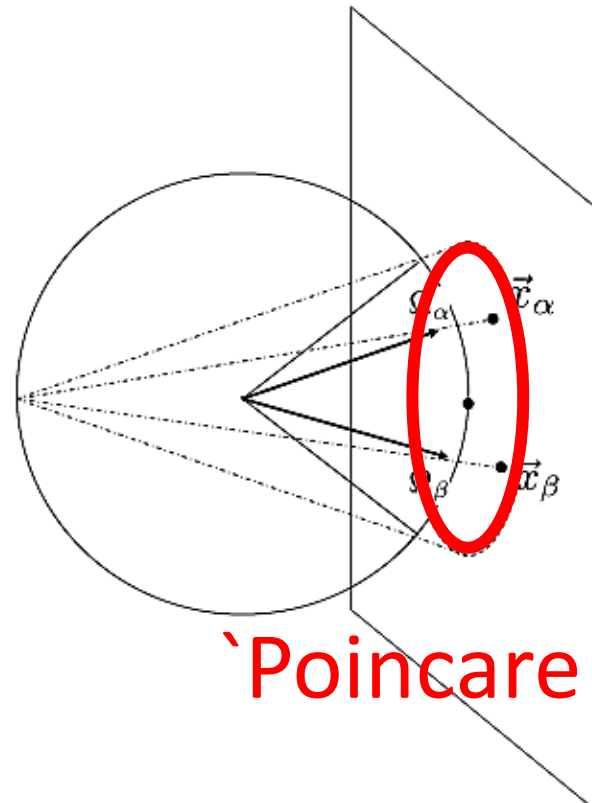
BK: Equation is conformal ($SL(2,C)$ symmetry),
but the solution is not (broken by the initial condition).

BMS: Jet cone breaks conformal symmetry down to $SL(2,R)=SU(1,1)$
unbroken by the initial condition

YH, Ueda (2009)

Non-global logs calculated to 5-loops
using $SU(1,1)$ symmetry

Schwartz, Zhu (2014)



Solving the JIMWLK equation

Operator form : $\partial_\tau \langle S_{xy} \rangle_\tau = -\langle \hat{H} S_{xy} \rangle_\tau$

$$\hat{H} = \int d^2x d^2y \frac{d^2z}{2\pi} \mathcal{K}_{xy}(z) \nabla_x^a \left(1 + \tilde{U}_x^\dagger \tilde{U}_y - \tilde{U}_x^\dagger \tilde{U}_z - \tilde{U}_z^\dagger \tilde{U}_y \right)^{ab} \nabla_y^b$$

Can be viewed as a **Fokker-Planck equation**

Solve the associated **Langevin** equation

Blaizot-Iancu-Weigert: Rummukainen, Weigert

For this purpose, it is crucial that the kernel

$$\mathcal{K}_{xy}(z) = \frac{(x-z) \cdot (z-y)}{(x-z)^2 (z-y)^2} \quad \text{is factorized}$$

Weigert's approach

Finite-Nc generalization of BMS in operator form

$$\partial_\tau \langle P_{\alpha\beta} \rangle_\tau = -\langle \hat{H} P_{\alpha\beta} \rangle$$

$$\hat{H} = \frac{1}{2} \int d\Omega_\alpha d\Omega_\beta \frac{d\Omega_\gamma}{4\pi} \mathcal{M}_{\alpha\beta}(\gamma) \nabla_\alpha^a \left(1 + \tilde{U}_\alpha^\dagger \tilde{U}_\beta - \Theta_{in}(\gamma) (\tilde{U}_\alpha^\dagger \tilde{U}_\gamma + \tilde{U}_\gamma^\dagger \tilde{U}_\beta) \right)^{ab} \nabla_\beta^b$$

The kernel is factorized

$$\mathcal{M}_{\alpha\beta}(\gamma) \equiv \frac{1 - \cos \theta_{\alpha\beta}}{(1 - \cos \theta_{\alpha\gamma})(1 - \cos \theta_{\gamma\beta})} = \frac{p_\alpha \cdot p_\beta}{(p_\alpha \cdot k_\gamma)(k_\gamma \cdot p_\beta)}$$

→ Langevin equation

A sign problem

$$\mathcal{M}_{\alpha\beta}(\gamma) = \frac{p_\alpha \cdot p_\beta}{(p_\alpha \cdot k_\gamma)(k_\gamma \cdot p_\beta)}$$

The kernel is indeed factorizedbut in **four-momentum** space

“Gaussian” noise

$$\langle \xi_a^{(I)\mu} \xi_b^{(J)\nu} \rangle \sim \delta_{ab} \delta^{IJ} \underline{g^{\mu\nu}}$$

Not positive definite!

Alternative JIMWLK Hamiltonian

YH, Iancu, Itakura, McLerran (2004)

$$\hat{H} = \int d^2x d^2y \frac{d^2z}{2\pi} \mathcal{K}_{xy}(z) \nabla_x^a \left(1 + \tilde{U}_x^\dagger \tilde{U}_y - \tilde{U}_x^\dagger \tilde{U}_z - \tilde{U}_z^\dagger \tilde{U}_y \right)^{ab} \nabla_y^b$$



equivalent

$$\hat{H} = \frac{1}{2} \int d^2x d^2y \frac{d^2z}{2\pi} \mathcal{M}_{xy}(z) \nabla_x^a \left(1 + \tilde{U}_x^\dagger \tilde{U}_y - \tilde{U}_x^\dagger \tilde{U}_z - \tilde{U}_z^\dagger \tilde{U}_y \right)^{ab} \nabla_y^b$$

$$\mathcal{K}_{xy}(z) = \frac{(x - z) \cdot (z - y)}{(x - z)^2 (z - y)^2}$$

$$\mathcal{M}_{xy}(z) = \frac{(x - y)^2}{(x - z)^2 (z - y)^2}$$

Effective kernel in jet physics

BK/JIMWLK

$$\mathcal{M}_{xy}(z) = \frac{(x-y)^2}{(x-z)^2(z-y)^2}$$



$$\mathcal{K}_{xy}(z) = \frac{(x-z) \cdot (z-y)}{(x-z)^2(z-y)^2}$$

BMS

$$\mathcal{M}_{\alpha\beta}(\gamma) \equiv \frac{1 - \cos \theta_{\alpha\beta}}{(1 - \cos \theta_{\alpha\gamma})(1 - \cos \theta_{\gamma\beta})}$$



$$\mathcal{K}_{\alpha\beta}(\gamma) = \frac{(n_\alpha - n_\gamma) \cdot (n_\gamma - n_\beta)}{2(1 - n_\alpha \cdot n_\gamma)(1 - n_\gamma \cdot n_\beta)}$$

New !

factorized
in 3D Euclidean space

The Langevin equation

$$U_{\alpha}(\tau + \varepsilon) = e^{iA_{\alpha}^L} U_{\alpha}(\tau) e^{iA_{\alpha}^R}$$

$$A_{\alpha}^L = \sqrt{\frac{\varepsilon}{4\pi}} \int d\Omega_{\gamma} \frac{(n_{\alpha} - n_{\gamma})^k}{1 - n_{\alpha} \cdot n_{\gamma}} \left(-\Theta_{in}(\gamma) U_{\gamma} t^a U_{\gamma}^{\dagger} \xi_{\gamma a}^{(1)k} + \Theta_{out}(\gamma) t^a \xi_{\gamma a}^{(2)k} \right)$$

$$A_{\alpha}^R = \sqrt{\frac{\varepsilon}{4\pi}} \int d\Omega_{\gamma} \frac{(n_{\alpha} - n_{\gamma})^k}{1 - n_{\alpha} \cdot n_{\gamma}} t^a \xi_{\gamma a}^{(1)k}$$

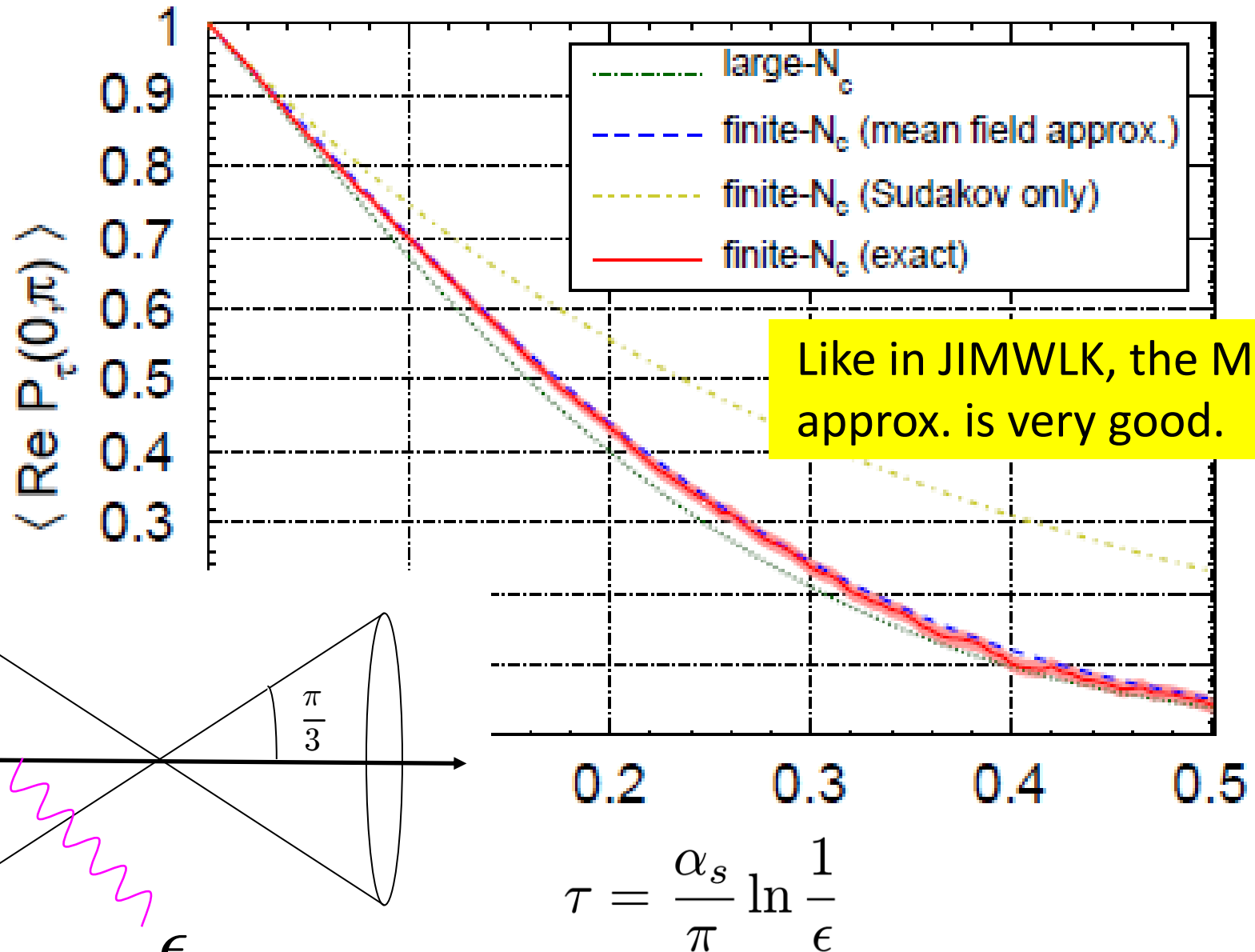
noise

Calculate the average

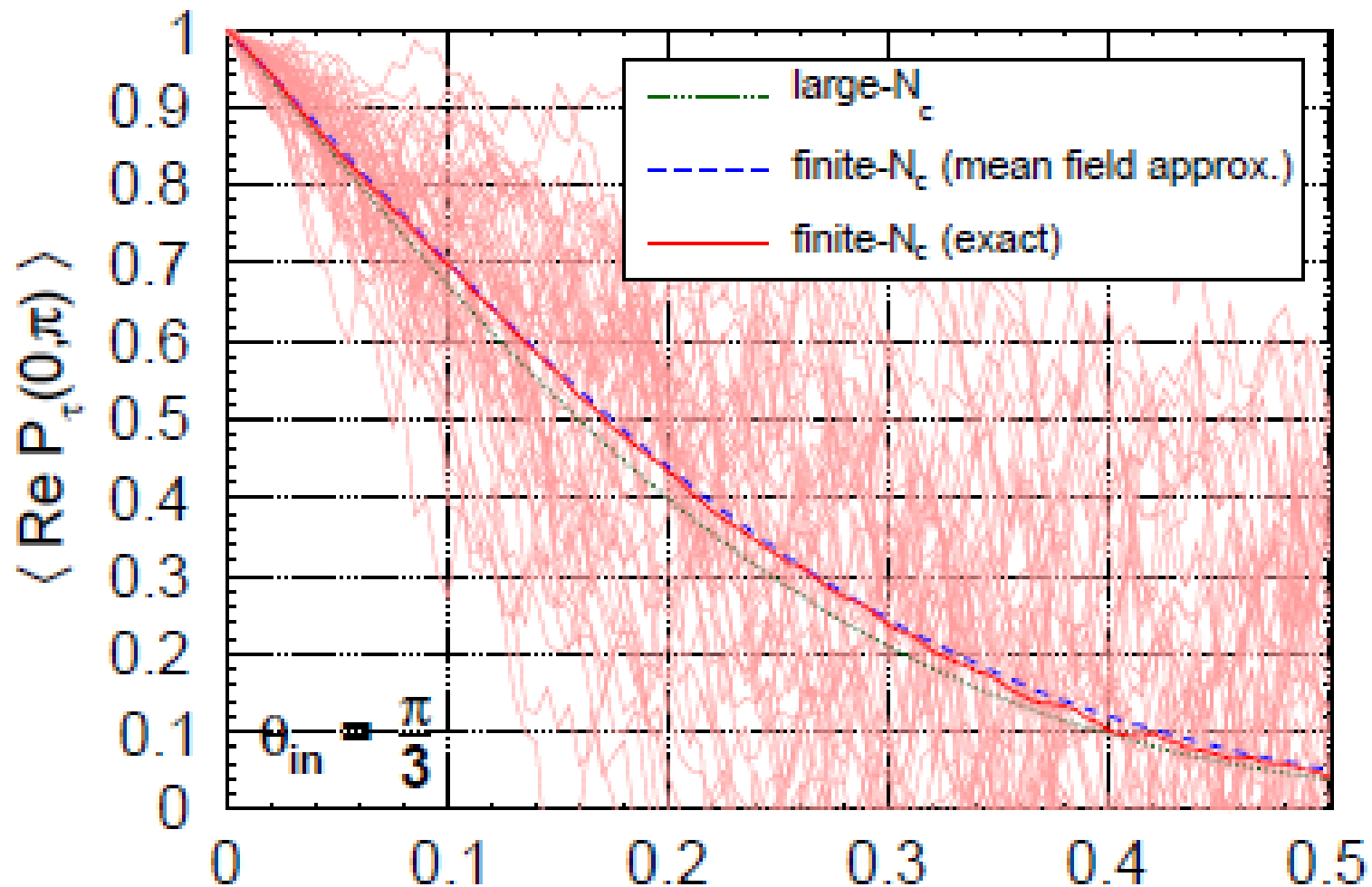
$$\frac{1}{N_c} \text{tr}(U_{\alpha}(\tau) U_{\beta}^{\dagger}(\tau))$$

over many random walk trajectories

Result for e+e- annihilation



Fluctuation of random walks



Unlike in JIMWLK, the fluctuation is enormous.

Conclusions

- First quantitative result of the resummation of non-global logs at finite N_c .
Sign problem overcome.
- Fluctuations very big, will significantly affect jet-veto cross section in hadron collisions.