The inclusive gluon production in the BFKL-Bartels approach with a running coupling

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I. BFKL equation

BFKL equation:

$$\Big(j-1-\omega(q_1)-\omega(q_2)\Big)\psi(q_1,q_2) = \lambda_R \int \frac{d^2q_1'}{(2\pi)^2} K(q_1,q_2|q_1',q_2')\psi(q_1',q_2')$$

where $q_1 + q_2 = q_1' + q_2' = q$, $R = v, g$, $\lambda_v = 1$, $\lambda_g = 1/2$

$$\omega(q) = -\frac{1}{2} N_c g^2 \int \frac{d^2 q_1}{(2\pi)^3} \frac{q^2}{q_1^2 q_2^2}$$

$$V(q_1,q_2|q_1',q_2') = \frac{N_c g^2}{2\pi}\Big[\Big(\frac{q_1^2)}{{q_1'}^2} + \frac{q_2^2}{q_2')^2}\Big)\frac{1}{(q_1-q_1')^2} - \frac{(q_1+q_2)^2}{{q_1'}^2{q_2'}^2}\Big].$$

The bootstrap relation is the unitarity condition for the gluon t-channel amplitude

$$\frac{1}{2}\int \frac{d^2 q_1'}{(2\pi)^2} V(q_1, q_2|q_1', q_2') = \omega(q) - \omega(q_1) - \omega(q_2).$$

Remarkably the bootstrap relation is preserved if one substitutes

$$\frac{2\pi}{g^2} q^2 = \eta(q)$$

where $\eta(q)$ is an arbitrary function.
The generalized trajectory and interaction take the forms

$$\omega(q) = -\frac{1}{2} N_c \int \frac{d^2 q_1}{(2\pi)^2} \frac{\eta(q)}{\eta(q_1)\eta(q_2)},$$

$$V(q_1, q_2|q_1', q_2') = N_c \Big[\Big(\frac{\eta(q_1)}{\eta(q_1')} + \frac{\eta(q_2)}{\eta(q_2')}\Big) \frac{1}{\eta(q_1 - q_1')} - \frac{\eta(q_1 + q_2)}{\eta(q_1')\eta(q_2')}\Big].$$

One can use this freedom to introduce the running of the coupling [Braun, 1994]. From the asymptotic of the unintegrated gluon distribution at large q obtained via the generalized BFKL equation one finds

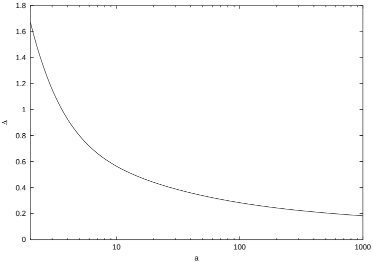
$$\eta(q) = \frac{1}{2\pi} b q^2 \ln \frac{q^2}{\Lambda^2}, \quad q^2 >> \Lambda^2,$$

where Λ is the standard QCD parameter and $b = \frac{1}{12}(33N_c - 2N_f)$. As to the behaviour of $\eta(q)$ at small momenta, we shall assume $\eta(0) = 0$, which guarantees that the gluon trajectory $\omega(q)$ passes through zero at $q = 0$ in accordance with the gluon properties. The asymptotic and condition at $q = 0$ are the only properties of $\eta(q)$ which follow from the theoretical reasoning. A concrete form of $\eta(q)$ interpolating between $q = 0$ and $q >> \Lambda$ may be chosen differently. One hopes that the following physical results will not strongly depend on the choice.

The bootstrap method correctly reproduces the running of the coupling in the reggeon interaction both in the forward and non-forward directions [Braun-Vacca, 1999] explicitly calculated in [Fadin,Lipatov,Camici,Ciafaloni,1998].
As a possible choice

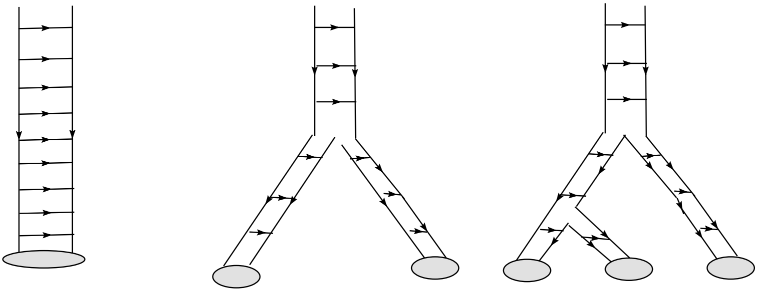
$$\eta(q) = \frac{1}{2\pi} b q^2 \ln\left(\frac{q^2}{\Lambda^2} + a^2\right), \quad a > 1$$

with the only free parameter a.
This running coupling freezes at $q = 0$ at value $\alpha_s(0) = \pi/(2b \ln a)$
However the asymptotical behaviour of $\psi(y, q) \sim s^{\Delta}$ strongly depends on a



II. Balitski-Kovchegov equation

In the reggeized gluon technique the BK equation is obtained as a sum of pomeron fan diagrams with the pomeron splitting in two via the triple pomeron vertex Γ



a *b* *c*

Vertex Γ can be expressed via the kernel $K_{2\to 3}$ for transition of two reggeons into three and the reggeon trajectory [Bartels,Wuesthoff,1995].

$$\Gamma(1,2,3,4|1',2') = 2g^2 G(1,23,4|1',2')$$

where the final pomerons momenta are $1 \equiv q_1$, 2 and 3,4, $23 \equiv 2+3$, $1234 = 1'2'$ and

$$G(1,2,3|1',2') = -K_{2\to 3}(1,2,3|1',2')$$

$$-\delta^2(1'-1)\Big(\omega(2)-\omega(23)\Big) - \delta^2(2'-3)\Big(\omega(2)-\omega(12)\Big)$$

It is important that the splitting kernel $K_{2\to 3}$ is expressed via the BFKL interaction

$$K_{2\to 3}(1,2,3|1',3') = V(2,3|1'-1,3') - V(12,3|1'3').$$

One can use this relation to introduce the running coupling. Define the generalized splitting kernel by the same relation with the new intergluon interaction expressed via function $\eta(q)$. As a result one gets the triple pomeron vertex with the running coupling constant. In the forward direction

$$\Gamma(1,-1,4,-4|1',-4') = 2N_c \int \frac{d^2q_2}{(2\pi)^2}$$

$$\Big\{-\delta^2(2-1)\Big(\frac{\eta(14)}{\eta(1-1')\eta(4-4')}-\frac{\eta(1)}{\eta(1-1')\eta(4')}-\frac{\eta(4)}{\eta(1')\eta(4-4')}\Big)$$
$$-\frac{1}{2}\delta^2(1'-1)\Big(\frac{\eta(14)}{\eta(2)\eta(14-2)}-\frac{\eta(1)}{\eta(2)\eta(1-2)}\Big)$$
$$-\frac{1}{2}\delta^2 4'-4\Big(\frac{\eta(14)}{\eta(2)\eta(14-2)}-\frac{\eta(4)}{\eta(2)\eta(4-2)}\Big)\Big\}$$

Expression for the triple pomeron vertex is simplified in the coordinate space. One finds

$$G(r_1, r_2, r_3|r_1', r_3') = \frac{1}{2} N_c \delta^2(r_{13} - r_{13}') F(r_{12}, r_{32}),$$

where $r_{12} = r_1 - r_2$ etc.

$$F(r_1, r_2) = \int d^2\rho \tilde{\eta}(\rho) \Big(\xi(r_1 - \rho) - \xi(r_2 - \rho) \Big)^2$$

and $\tilde{\eta}(r)$ and $\xi(r)$ are Fourier trasforms of $\eta(q)$ and $1/\eta(q)$.
The sum of pomeron fan diagrams leads to the evolution equation

$$\frac{\partial}{\partial y} \Phi(y, r) = -\frac{1}{2} N_c \int d^2 r_1 F(r_1 - r, r_1)$$

$$\Big(\Phi(y, r_1, b) + \Phi(y, r_1 - r, b) - \Phi(y, r, b) - \Phi(y, r_1, b) \Phi(y, r_1 - r, b) \Big).$$

Defining the running coupling constant in the coordinate space by relation

$$f(r_1, r_2) = -\frac{1}{\pi^2} \frac{\alpha_s(r_1)\alpha_s(r_2)}{\alpha_s(r_1, r_2)} \frac{\mathbf{r}_1\mathbf{r}_2}{r_1^2 r_2^2},$$

one can rewrite the evolution equation in the same form as in the dipole approach [Kovchegov, Weigert,2007]

$$\frac{\partial}{\partial y}\Phi(y, r, b) =$$

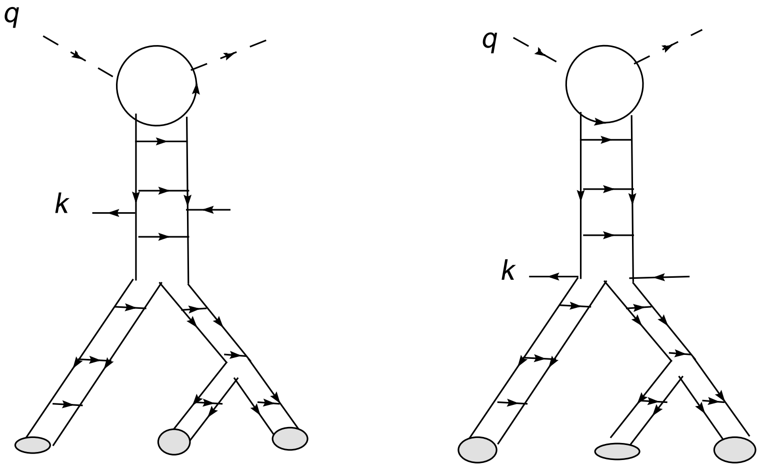
$$\frac{1}{2\pi^2} N_c \int d^2 r_2 d^2 r_3 \delta(r - r_1 + r_2) \Big(\frac{\alpha_s(r_1)}{r_1^2} + \frac{\alpha_s(r_2)}{r_2^2} - 2 \frac{\alpha_s(r_1)\alpha_s(r_2)}{\alpha_s(r_1, r_2)} \frac{\mathbf{r}_1\mathbf{r}_2}{r_1^2 r_2^2} \Big)$$

$$\Big(\Phi(y, r_1, b) + \Phi(y, r_2, b) - \Phi(y, r, b) - \Phi(y, r_1, b)\Phi(y, r_2, b) \Big).$$

However the meaning of the running coupling constants is somewhat different: they all are determined by $\eta(q)$.

III. Inclusive cross-sections

Diagrammatically contributions to the forward amplitude for the collision of the projectile with the nucleus are



Here the new form of the reggeized gluon trajectory, interaction between the reggeons and the splitting vertex are implied. The inclusive cross-sections are obtained either by cutting the interaction in the uppermost pomeron (before all splittings) or by cutting the first splitting vertex. All the rest contributions will be cancelled by the AGK cancellations.
This inclusive cross-sections do not refer to precisely gluon production. The intermediate s-channel states include not only the single real gluon state but also states which contribute to the one-loop β-function, namely the quark-antiquark states and two gluon states. So the inclusive cross-section obtained by fixing the intermediate s-state momentum p actually refers to all possible states having this momentum. It does not disriminate between contributions from gluons (single or in pairs) and (anti)quarks. So it is rather the inclusive cross-section for emission of a jet with a total transverse momentum p. This quantity has a well-defined physical meaning and is accessible for experimental observation.

Jet production from the pomeron

If only a single pomeron is exchanged between the projectile and target

$$\frac{(2\pi)^3 d\sigma}{dy d^2 p} = \frac{4N_c}{\eta(p)} \int \frac{d^2 k}{(2\pi)^2} \eta(k)\eta(p-k) P(Y-y, p-k) P(y,k)$$

Here it is assumed that the "semi-amputated" forward pomeron wave function $\phi(y,p) = \eta(p)P(y,p)$ in the momentum space satisfies the equation

$$\frac{\partial \phi(y,p)}{\partial y} = 2\omega(p)\phi(y,p) + 2N_c \int \frac{d^2 k}{(2\pi)^2} \frac{\phi(y,k)}{\eta(p-k)}$$

For a nucleus target the pomeron coupled to the target has to be substituted by the solution of the Balitski-Kovchegov equation. at fixed impact parameter b

$$\frac{(2\pi)^3 d\sigma_1}{dy d^2 p d^2 b} = \frac{4N_c}{\eta(p)} \int \frac{d^2 k}{(2\pi)^2} \eta(k)\eta(p-k) P(Y-y, p-k) \Phi(y,k,b)$$

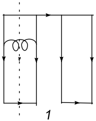
Jet production from the vertex

The three-pomeron vertex has a fixed rapidity and does not include evolution. This makes it possible to study the contribution to jet emission from the vertex in the lowest order of perturbation theory, that is for the target consisting of only two centers. It also allows to simplify treatment choosing for the projectile and target quarks modeling the pomeron exchanges by colorless double reggeon exchanges.

The inclusive cross-section will be obtained from the diagrams for the forward scattering off two centers, in which reggeon interactions and splittings are described by functions V and W with the running coupling. The contribution from the vertex is obtained after subtraction of the one from the so-called reggeized term or alternatively from the Glauber initial condition (in the dipole formalism)

All the diagrams may be divided into two configurations depending on the way the four final reggeons are combined into pomerons.

The diffractive: diagrams with two consecutive colourless exchanges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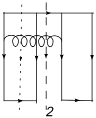
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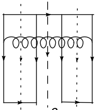
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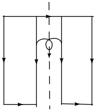
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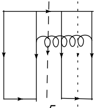
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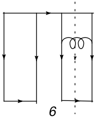
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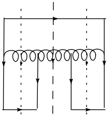

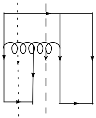
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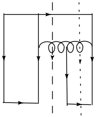
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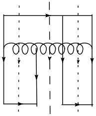
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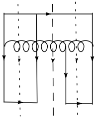
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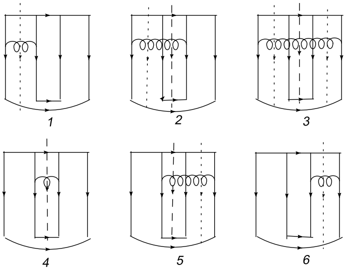
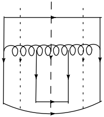
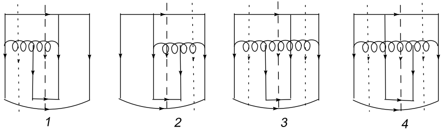
The non-diffractive: with parallel colourless exchanges with one of the colourless pair of reggeons enclosed in the other.







To obtain the inclusive cross-section for the production of a jet with momentum p one has to fix function $\eta(p)$ in intermediate states in the s channel. These intermediate states are obtained by cutting the diagrams in the s-channel. Different cuts may pass through one of the targets (single cuts, S), through both targets (double cuts, DC) or do not pass through targets at all (diffractive cuts, D). In the diffractive configuration only diffractive and single cuts are possible. In the non-diffractive configuration only single and double cuts are possible. According to the AGK rules the relative weights of the contributions from diffractive, single and double cuts are 1:-1:2.

The diffractive contribution

We denote the reggeon momenta of the first final pomeron $\pm q_1$ and of the second as $\pm q_4$. Suppressing the final pomerons and integrations over their momenta as well as the overall colour coefficient N_c^4 we find

$$Diff =$$

$$-W(q_1, -q_1 - q_4, q_4 | p, -p)$$

$$+W(q_1, -q_1 - q_4, q_4 | q_1 + p, -q_1 - p) + W(q_1, -q_1 - q_4, q_4 | -q_4 + p, q_4 - p)$$

$$-\frac{1}{2} W(q_1, -q_1, -q_4 | p, -q_4 - p) - \frac{1}{2} W(-q_1, -q_4, q_4 | -q_1 + p, -p).$$

The non-diffractive contribution

$$Nondiff =$$

$$\frac{1}{4}\Big(V(-q_4,-q_1|-q_4+p,-q_1p)+V(q_1,q_4|q_1p,q_4-p)$$

$$+(V(-q_4,q_1|-q_4+p,q_1-p)+V(-q_1,q_4|-q_1+p,q_4-p)\Big)$$

$$-2W(-q_4,0,q_4|q_1-q_4+p,-q_1+q_4-p)$$

$$-W(-q_4,q_1,-q_1|-q_4+q_1p,-q_1p)-W(q_1,-q_1,q_4|q_1p,q_4-q_1p)$$

$$+\frac{1}{4}W(-q_4,q_1,q_4|q_1-q_4+p,q_4-p)+\frac{1}{4}W(-q_4,-q_1,q_4|-q_4+p,q_4-q_1p)$$

$$+\frac{1}{2}W(-q_4,q_1,-q_1|-q_4+p,-p)+\frac{1}{2}W(q_1,-q_1,q_4|p,q_4-p)$$

$$-\frac{1}{4}W(-q_4,q_1,q_4|-q_4+p,q_{14}-p)-\frac{1}{4}W(-q_4,-q_1,q_4|-q_{14}+p,q_4-p)$$

Here $q_1p = q_1+p$, $q_{14} = q_1+q_4$

Final inclusive cross-section

In terms of function

$$F(q|p) \equiv (q|p) = \frac{\eta(q)}{\eta(p)\eta(q-p)} = (q|q-p) = (-q|-p)$$

we have

$$V(q_1, q_2|k_1, k_2) = (k_{12}|k_1) - (q_1|k_1) - (q_2|k_2)$$

$$W(q_1, q_2, q_3|k_1, k_2) = (k_{12}|k_1) - (q_{12}|k_1) - (q_{23}|k_2) + (q_2|k_1 - q_1)$$

where $k_{12} = k_1 + k_2$ etc. It is important to take into account that all contributions which do not depend on the momenta of one of the final pomerons, q_1 or q_4, vanish, since the Pomeron vanishes when its two reggeized gluons are at the same spatial point. After many cancellations we find

$$\mathit{Diff} + \mathit{Nondiff} = 2\Big[(q_{14}|p) + (q_{14}|-p)\Big]$$

With the fixed coupling constant one finds instead

$$(Diff + Nondiff =)^{fix} = \frac{g^2}{\pi}\Big[\frac{q_{14}^2}{p^2(q_{14}-p)^2} + \Big(p \to -p\Big)\Big] \qquad (1)$$

It has been shown that in the fixed coupling case half of this expression comes from the reggeized piece (or the Glauber initial condition in the dipole approach) [Braun,2006]. By the same reasoning the contribution from the triple pomeron vertex is one half of the found contribution. This means that the inclusive cross-section for jet production from the vertex with the running couling is obtained from the fixed coupling case after the substitution of all momenta according to the rule

$$q^2 \to \frac{g^2}{2\pi}\eta(q)$$

The final cross-section from the vertex is

$$-\frac{2N_c}{\eta(p)}\int\frac{d^2q_1d_2q_4}{(2\pi)^4}\eta(q_{14}-p)P(Y-y,q_{14}-p)\eta(q_{14})\Phi(y,q_1)\Phi(y,q_4)$$

The total inclusive cross-section is the sum of emissions from the upper pomeron and the vertex:

$$\frac{(2\pi)^3d\sigma}{dyd^2pd^2b}=\frac{2N_c}{\eta(p)}\int\frac{d^2q_1d^2q_4}{(2\pi)^4}\eta(q_{14}-p)P(Y-y,q_{14}-p)$$

$$\Big[2\eta(q_1)\Phi(y,q_1,b))(2\pi)^2\delta(q_4)-\eta(q_{14})\Phi(y,q_1)\Phi(y,q_4)\Big]$$

IV. Discussion

• Our final expression for the inclusive cross-section has a strong similarity with the one conjectured by W.A.Horowitz and Yu.V.Kovchegov [2011](HK). It contains three factors η, which can be put in correspondence with three coupling constants depending on different arguments in HK. However in our formula the arguments of functions η directly depend on the three momenta involved: that of the observed jet and two of the gluon distributions involved. In contrast in the conjecture of HK the argument of one of the coupling constants depends only on the assumed value of collinearity of the observed jet and the arguments of two others are complex-valued and depend on all three momenta in a very complicated manner. However the literal comparison of the two cross-sections is not possible, since in fact they refer to different processes: to jet production with a given momentum in our case and with additional restriction on jet collinearity in HK. Besides the cross-section of HK is after all only conjecture, whereas ours is more or less consitently derived from the bootstrap condition which demonstrated its validity for the total cross-section.

• It remains to be seen by practical applications to what extent this difference is felt in the actual inclusive cross-sections. To do this consistently one has to previously solve our equation for the unintegrated gluon density. With all its similarity to the currently used equation in the dipole picture, the actual values of the three running couplings involved are not identical, so that already found solutions in the dipole picture cannot be directly used for our inclusive cross-section. We postpone this problem for future studies.

• We stress that that our equations take in account terms of the order $(y\alpha(q))^n$ with $\alpha(q)$ taken in the leading order. Subleading terms of the relative order $1/\ln(q^2/\Lambda^2)$ remain undetermined, since they correspond to the next-to-leading order in the running coupling.