Interplay of hard scale and low x dynamics

Krzysztof Kutak

Instytut Fizyki Jądrowej
im. Henryka Niewodniczańskiego
Polskiej Akademii Nauk

Results based on collaboration with:
A van Hameren, P, Kotko, S.Sapeta, D. Toton:

Supported by grant: LIDER/02/35/L-2/10/NCBiR/2011

Notation for gluon densities

Notation I use

$$\mathcal{F}(x,k^2) = \frac{N_c}{\alpha_s(2\pi)^3}\int d^2b \int d^2r e^{ik\cdot r}\nabla_r^2 N(r,b,x)$$

$$\Phi(x,k^2) = \frac{1}{2\pi}\int d^2b \int \frac{d^2r}{r^2} e^{ik\cdot r} N(r,b,x)$$

$$\mathcal{F}(x,k^2) = \frac{N_c}{4\alpha_s\pi^2}k^2\nabla_k^2\Phi(x,k^2)$$

$$\mathcal{F}_A(x,k) = \frac{N_c\, k^2}{2\pi\alpha_s}\tilde{N}_F(x,k)$$

$$f(x,k^2) = k^2\mathcal{F}(x,k^2)$$

Other notations

$$\varphi(k,x,\mathbf{R}) = \frac{C_F}{\alpha_s(k)\,(2\pi)^3}\int d^2\mathbf{r}\; e^{-i\mathbf{k}\cdot\mathbf{r}}\nabla_\mathbf{r}^2\mathcal{N}_A(r,x,\mathbf{R})$$

$$\mathcal{F}(x_g,q_\perp) = \frac{1}{2\pi^2}\int d^2r_\perp e^{-iq_\perp r_\perp}\frac{1}{r_\perp^2}[1-\exp(-\frac{1}{4}r_\perp^2 Q_s^2)]$$

$$\phi(\mathbf{k},Y) = \iint \frac{d^2\mathbf{r}}{2\pi}e^{-i\mathbf{k}\cdot\mathbf{r}}\frac{N(\mathbf{r},Y)}{r^2}$$

$$F_{x_g}(\mathbf{k},\mathbf{b}) = \frac{1}{2\pi}\nabla_\mathbf{k}^2\phi(\mathbf{k},\mathbf{b},Y(x_g))$$

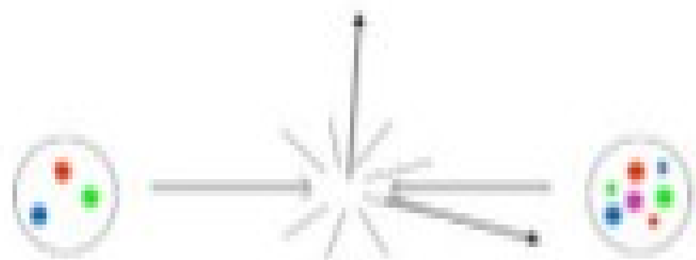
$$\tilde{N}_{F(A)}(x,k) = \int d^2b\int\frac{d^2r}{(2\pi)^2}\; e^{-i\mathbf{k}\cdot\mathbf{r}}\,[1-N_{F(A)}(x,r,\mathbf{b})]$$

High energy prescription and forward-central di-jets

Deak, Jung, Hautmann Kutak
JHEP 0909:121,2009

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \sum_{a,c,d} \frac{1}{16\pi^3 (x_1 x_2 S)^2} \mathcal{M}_{ag\to cd} x_1 f_{a/A}(x_1,\mu^2)\, \mathcal{F}_{g/B}(x_2,k^2,\mu^2) \frac{1}{1+\delta_{cd}}$$

$S = 2P_1 \cdot P_2$



- *Resummation of logs of x and logs of hard scale*
- *Knowing well pdf at large x one can get information about low x physics*
- *Framework goes recently under name "hybride framework"*

$$x_1 = \frac{1}{\sqrt{S}}\left(p_{t1}e^{y_1} + p_{t2}e^{y_2}\right) \xrightarrow{y_1 \sim 0,\, y_2 \gg 0} \sim 1$$

$$x_2 = \frac{1}{\sqrt{S}}\left(p_{t1}e^{-y_1} + p_{t2}e^{-y_2}\right) \qquad \ll 1$$

$$k_1^\mu = x_1 P_1^\mu$$
$$k_2^\mu = x_2 P_2^\mu + k_t^\mu$$

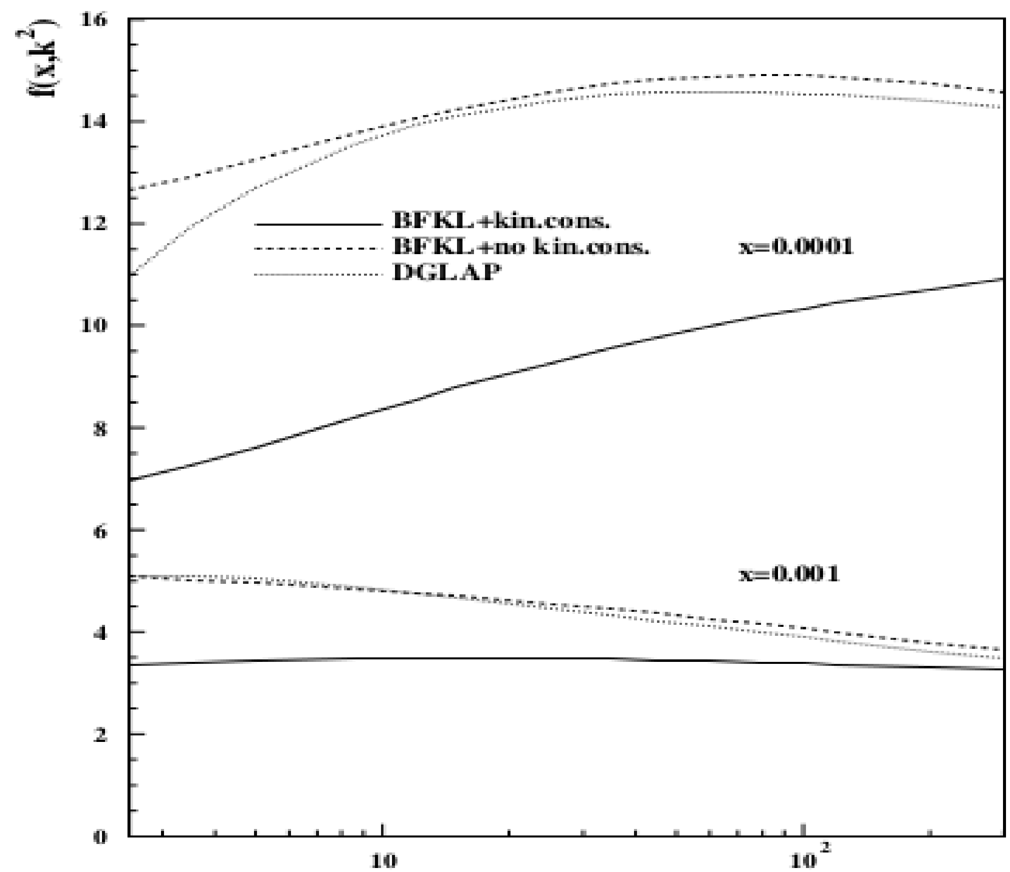
BFKL with subleading corrections
Kwiecinski, Martin, Staśto prescription

Nonsingular pieces of splitting function

Kinematical effects i.e. Momentum of gluon dominated by it's transversal component

Running coupling

In principle not applicable to final states since no hard scale dependence



$$\mathcal{F}_p(x,k^2) = \mathcal{F}_p^{(0)}(x,k^2) + \frac{\alpha_s(k^2)N_c}{\pi}\int_x^1 \frac{dz}{z}\int_{k_0^2}^{\infty}\frac{dl^2}{l^2}\left\{\frac{l^2\mathcal{F}_p(\frac{x}{z},l^2)\,\theta(\frac{k^2}{z}-l^2) - k^2\mathcal{F}_p(\frac{x}{z},k^2)}{|l^2-k^2|} + \frac{k^2\mathcal{F}_p(\frac{x}{z},k^2)}{|4l^4+k^4|^{\frac{1}{2}}}\right\} + \frac{\alpha_s(k^2)}{2\pi k^2}\int_x^1 dz\left(P_{gg}(z) - \frac{2N_c}{z}\right)\int_{k_0^2}^{k^2} dl^2\,\mathcal{F}_p(\frac{x}{z},l^2)$$

From Kwiecinski, Martin, Stasto
Phys.Rev. D56 (1997) 3991-4006

$$f(x,k^2) = k^2\mathcal{F}(x,k^2)$$

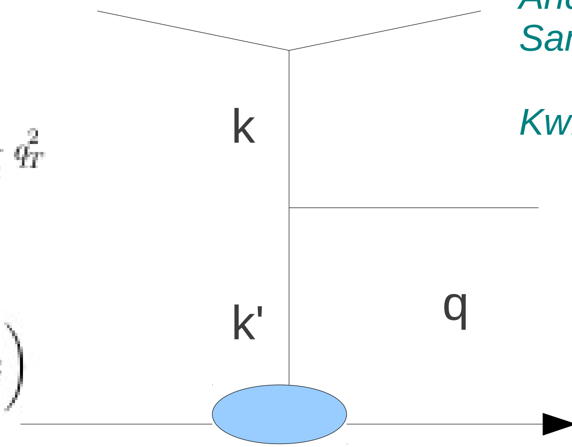
The kinematical constraint effects

$$k^2 = k^+k^- - k_T^2$$

$$k_T^2 > |k^+k^-| \qquad k^+k^- \simeq -\frac{k^+}{q^+}\, q_T^2 = -\frac{k^+}{k'^+ - k^+}\, q_T^2 = -\frac{z}{1-z}\, q_T^2$$

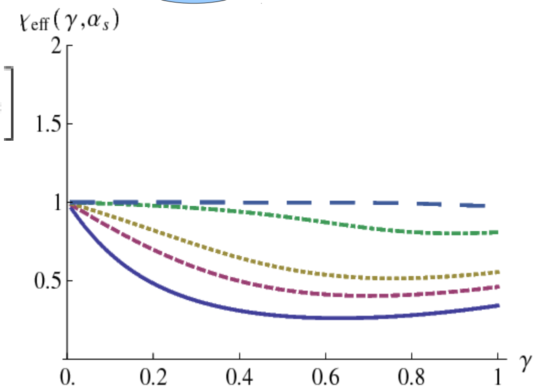
Andersson,Gustafson Samuelsson, Kharraziha '96

Kwiecinski, Martin, Sutton '97

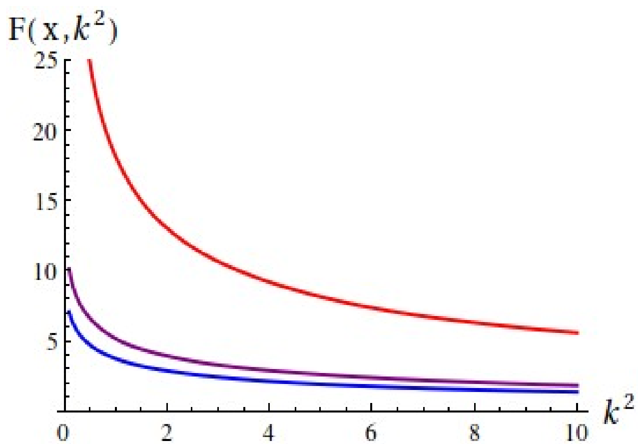


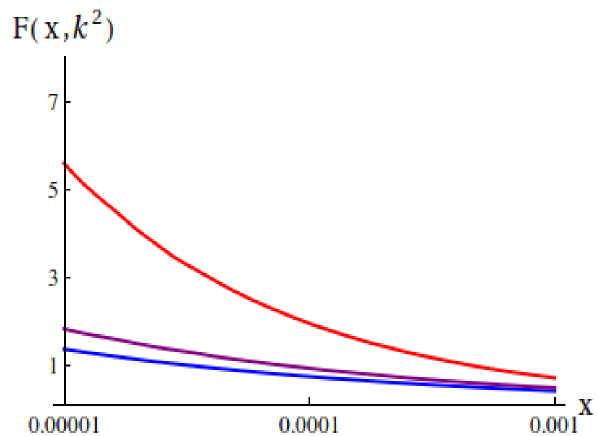
$$\theta\left(\frac{k_T^2}{q_T^2} - z\right)$$ antilollinearl imit $$l \gg k \qquad \theta\left(\frac{k_T^2}{l_T^2} - z\right)$$

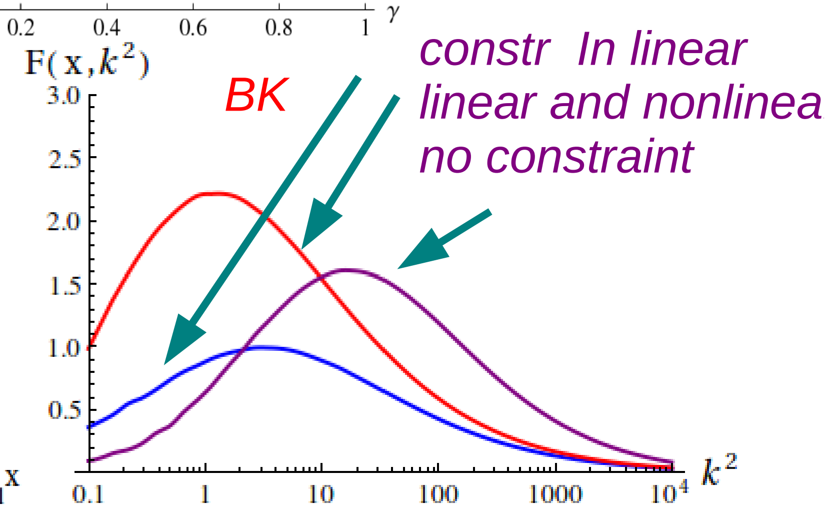
$$f(x,k^2) = f_0(x,k^2) + \bar{\alpha}_s \int_x^1 \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[\frac{\theta(k^2/l^2 - z) f(x/z, l^2) - f(x/z, k^2)}{|k^2 - l^2|} + \frac{f(x/z, k)}{\sqrt{(4l^4 + k^4)}} \right]$$



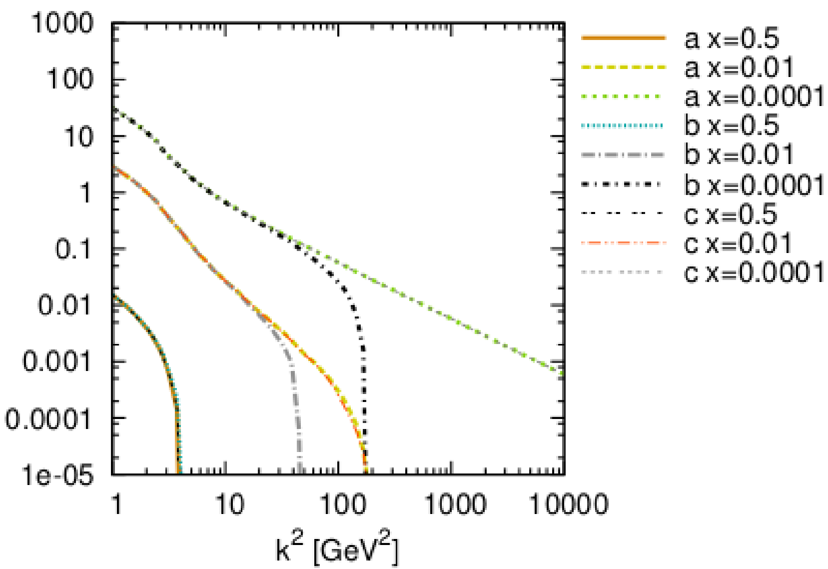
$$f(x,k^2) = \frac{1}{2\pi i} \int d\gamma (k^2)^\gamma \frac{1}{2\pi i} \int d\omega\, x^{-\omega} \frac{\omega \bar{f}_0(\omega,\gamma)}{\omega - \bar{\alpha}_s \chi_{k.c.}(\gamma,\omega,)}$$

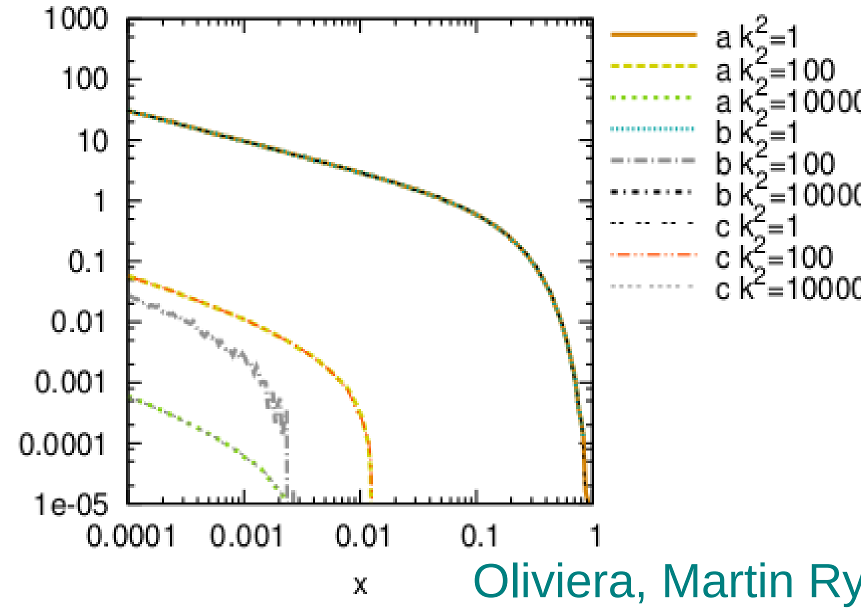






KMS and evolution in angle





Oliviera, Martin Ryskin '14

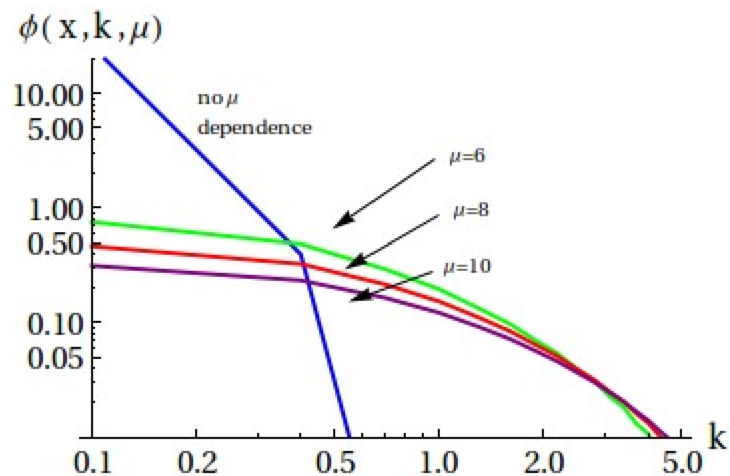
Refined formulation and solution Toton '14

$$\hat{f}(x,\theta) = \hat{f}_0(x,\theta) + \bar{\alpha}_s \Big(\int_{l/p}^{l/(xp)} \frac{\mathrm{d}\theta'}{\theta'} \int_{k_0^2}^{k_{max}^2} \mathrm{d}l^2 \frac{1}{2N_c} \bar{K}(k,l) f(x',l) - \int_0^1 \mathrm{d}z \int_{k_0^2}^{k_{max}^2} \mathrm{d}l^2 \frac{1}{2N_c} \bar{K}(k,l) f(x,l) + \int_x^1 \frac{\mathrm{d}z}{z} \frac{zP(z)}{2N_c} \int_{z^2\theta_0^2(x)}^{(z\theta)^2} \frac{\mathrm{d}\theta'^2}{\theta'^2} \hat{f}\left(\frac{x}{z},\theta'\right) \Big)$$

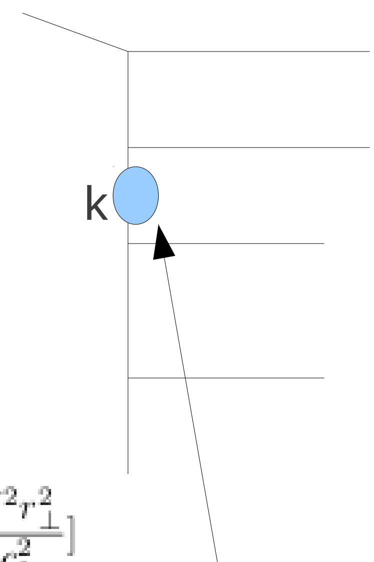
$$\theta = \frac{k}{xp} \qquad \theta_{min}(x,l) = \frac{l}{p} \qquad \theta_{max}(x,l) = \frac{l}{xp}$$

proton's momentum

Final states via Sudakov effects - illustration



Probability of finding no real gluon
Between scales μ



μ hard scale

$$\mathcal{F}(x_g,q_\perp) = \frac{1}{2\pi^2}\int d^2 r_\perp e^{-iq_\perp r_\perp}\frac{1}{r_\perp^2}[1-\exp(-\frac{1}{4}r_\perp^2 Q_s^2)]\exp[-\frac{\alpha_s N_c}{4\pi}\ln^2\frac{K^2 r_\perp^2}{c_0^2}]$$

Originally called N(x,q)

$$T_a(k_t,\mu) = \exp\left(-\int_{k_t^2}^{\mu^2}\frac{\alpha_s(p_t^2)}{2\pi}\frac{dp_t^2}{p_t^2}\sum_{a'}\int_0^{1-\Delta}P_{a'a}(z')\,dz'\right)$$

Two scale dependent gluon density vs. one scale dependent

Survival probability of the gap without emissions

Multiplicative factor inlast step of evolution

Kimber, Martin, Ryskin framework '01

Mueller, Xiao, Huan ;13

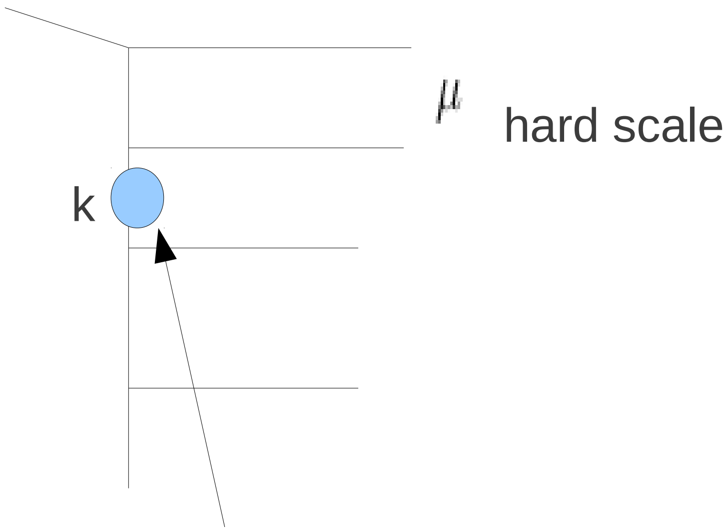
Final states via Sudakov effects - illustration

Motivated by KMR prescription

Probability of finding no real gluon between scales kt and μ

Observable:

$$\overline{\mathcal{O}} = \frac{\sigma}{\overline{W}}\left[\sum_i w_i \Delta(\mu_i, k_{Ti}) F_i^{\mathcal{O}}(X_i)\,\Theta(\mu_i > k_{Ti}) + \frac{\widetilde{W}}{W}\sum_j w_j F_j^{\mathcal{O}}(X_j)\,\Theta(k_{Tj} > \mu_j)\right]$$



σ *total cross section*

$W = \sum_i w_i$ *total weight*

$F_i^{\mathcal{O}}$ *functon defining observable i.e. cuts etc*

Calculated from the condition that the Sudakov f.f. does not change total cross section

$$\Delta(\mu, k_T^2) = \exp\left(-\int_{k_T^2}^{\mu^2}\frac{dk_T'^2}{k_T'^2}\frac{\alpha_s(k_T'^2)}{2\pi}\sum_i\int_0^{1-\epsilon(k_T',\mu)} dz\, P_{ig}(z)\right)$$

Also possible to formulate directly for gluon density

Survival probability of the gap without Fmissions. Formfactor resumes unresolved real and virtual emissions

Kimber, Martin, Ryskin framework '01

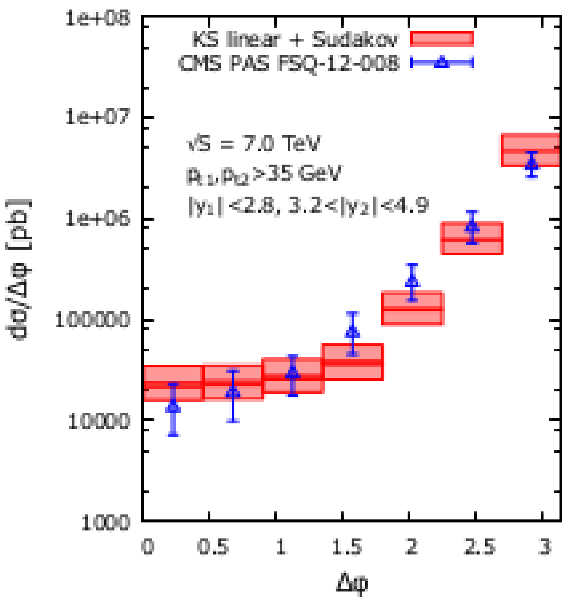
Tools used

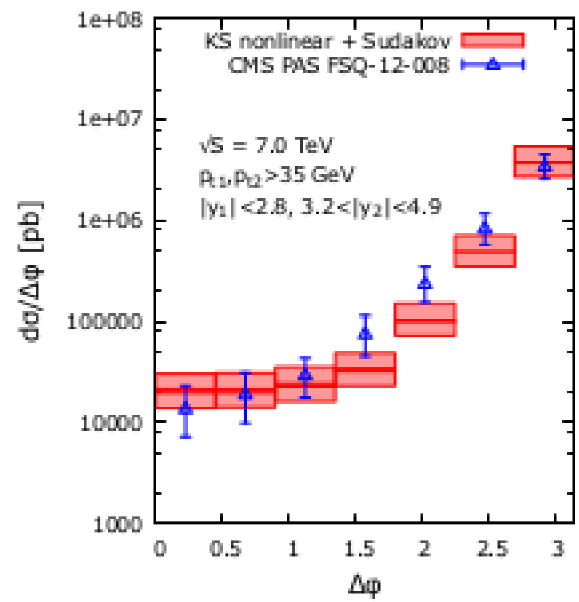
- *General tool for matrix elements within HEF based on spinor helicity method (A. van Hameren)*
- *Gauge link based tool to evaluate matrix elements (OGIME P. Kotko)*
- *Monte Carlo for production of dijets, trijets within HEF LxJet (P. Kotko)*
- *Tool for forward dijets Forward (S. Sapeta)*

More in talk by
Piotr Kotko

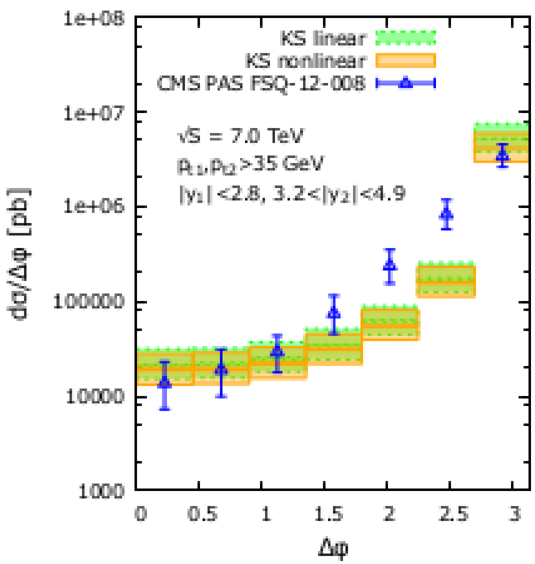
Decorelations inclusive scenario

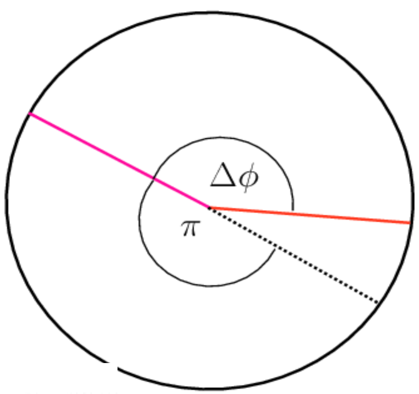
A.v.Hameren, P.Kotko, KK, S.Sapeta '14





pt1,pt2 >35, leading jets
|y1|<2.8, 3.2<|y2|<4.7
No further requirement on jets





In DGLAP approach
i.e 2 →2 + pdf one would
Get delta function at

$$\Delta\phi = \pi$$

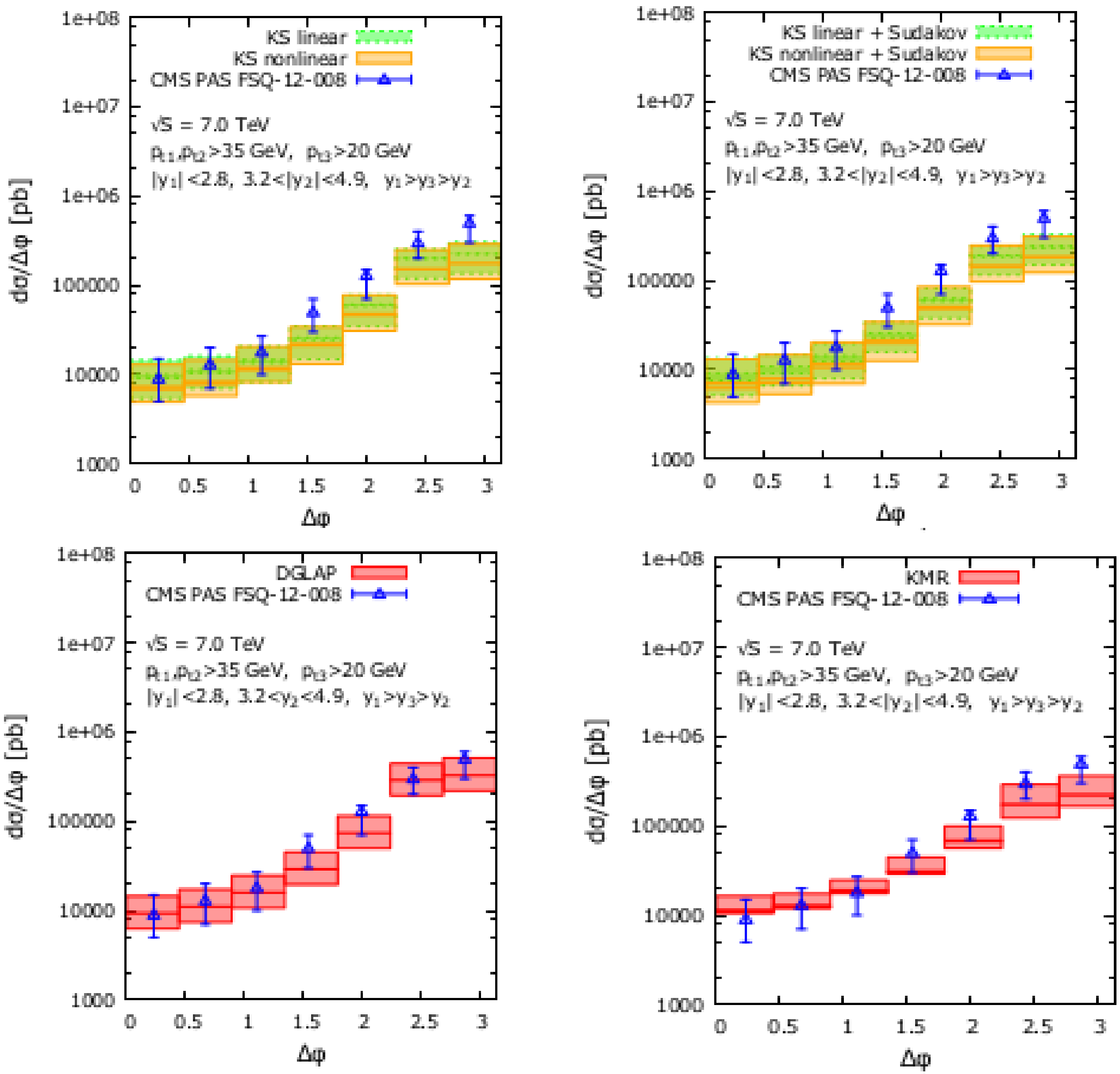
Sudakov effects by reweighting implemented in LxJet Monte Carlo
P. Kotko

Observable suggested to study BFKL effects
Sabio-Vera, Schwensen '06

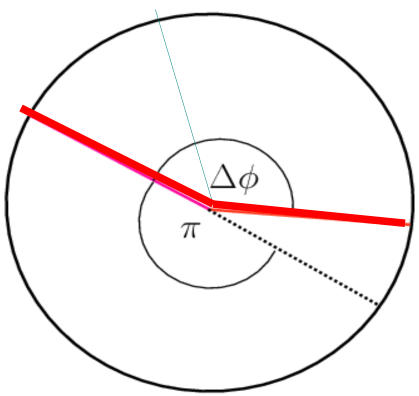
Studied also context of RHIC
Albacete, Marquet '10

Decorelations inside jet tag scenario

A.v.Hameren, P.Kotko, KK, S.Sapeta '14



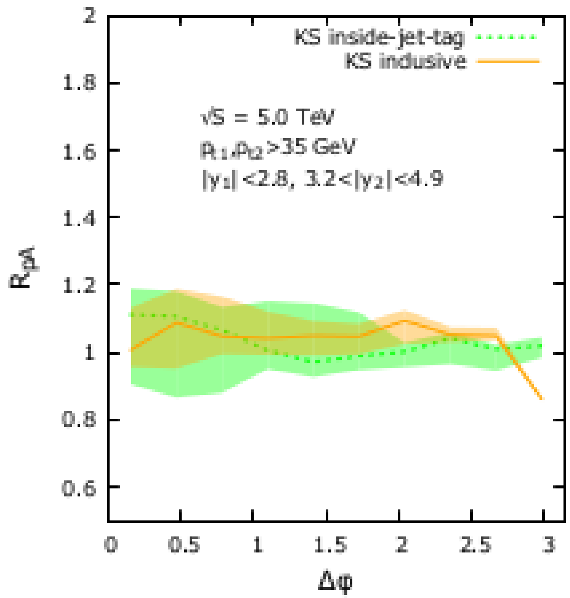
pt1, pt2 >35 GeV, leading jets |y1|<2.8, 3.2<|y2|<4.7 Third jet pt>20GeV. Between the forward and central region

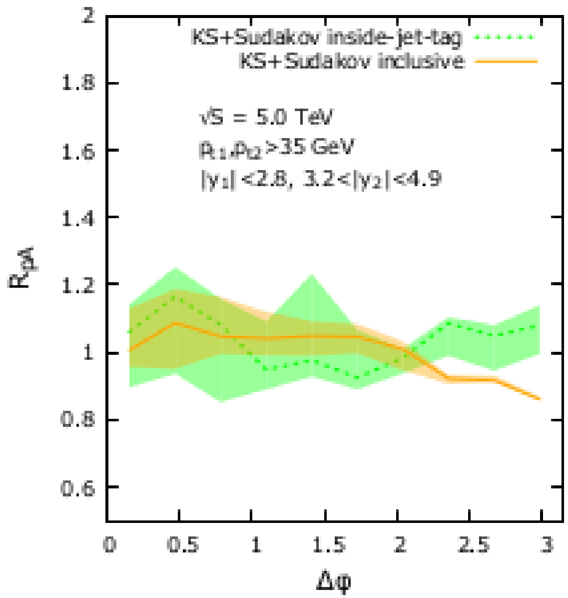


Sudakov effects by reweighting implemented in LxJet Monte Carlo P. Kotko

Predictions for p-Pb

A.v.Hameren, P.Kotko, KK, S.Sapeta '14





- *Sudakov enhance saturation effects*
- *Hawever, satuartion effects are rather weak*

CCFM evolution equation - evolution with observer

$$\mathcal{A}(x,k^2,p) = \mathcal{A}(x,k^2,p) + \bar{\alpha}_s \int \frac{d^2\bar{\mathbf{q}}}{\pi \bar{q}^2} \int_x^{1-Q_0/|\bar{q}|} dz\, \theta(p - z\bar{q}) P_{gg}(z,k^2,\bar{q}) \mathcal{A}(x/z,k',\bar{q})$$

In DIS $p^2 = \frac{Q^2}{z(1-z)}$ $\bar{q} = q/(1-z)$

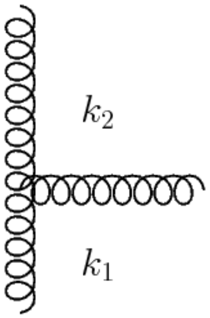
$$P_{gg}(z,k^2,p) = \frac{\alpha_S}{2\pi} 2C_A\, \Delta_S(zq,p) \left(\frac{\Delta_{NS}(z,q,k^2)}{z} + \frac{1}{1-z} \right),$$

non-eikonal emission eikonal emission

regulates 1/(1-z)

regulates 1/z

Double log approx



~ 1/z+1/(1-z)

$$\Delta_{ns}(z_i,q_i,k_i) = \exp\left(- \int_{z_i}^{1} dz' \frac{\bar{\alpha}_s}{z'} \int \frac{dq'^2}{q'^2} \theta(k_i - q')\theta(q' - z'q_i) \right)$$

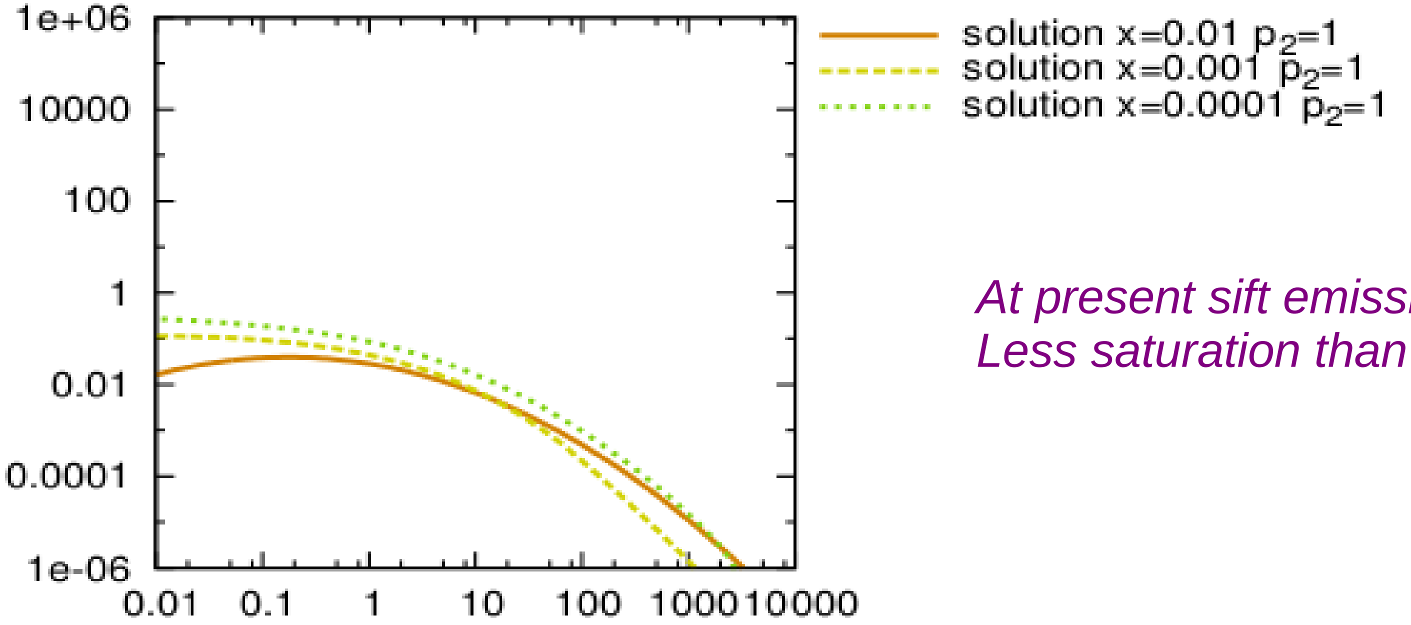
No emission of gluons with $x' = z'x_{i-1}$

in region $x_i < x' < x_{i-1}$

and with momentum q' smaller than k_i

and with angle $\theta' > \theta_i$

Solution of nonlinear equation for unintegrated gluon density with coherence included



At present sift emissions neglected: 1/(1-z)
Less saturation than in BK

p

$$\mathcal{F}(x,k^2,p) = \tilde{\mathcal{F}}_0(x,k^2,p) + \overline{\alpha}_s \int \frac{d^2\mathbf{q}}{\pi q^2} \int_{x/x_0}^{1} \frac{dz}{z} \theta(p - q\,z) \Delta_{ns}(z,k,q) \Big\{ \mathcal{F}(\frac{x}{z}, |\mathbf{k}+\mathbf{q}|^2, q) - \frac{\pi \alpha_s^2}{4 N_c R^2} q^2 \delta(q^2 - k^2) \nabla_q^2 \left[\int_{q^2}^{\infty} \frac{dl^2}{l^2} \ln \frac{l^2}{k^2} \mathcal{F}(x/z, l^2, l) \right]^2 \Big\}$$

KK JHEP 12

Can be further extended to include Sudakov and 1/(1-z) terms

Conclusions

•*Achieved very good description of forward-central jet measurement*

•*Predictions for pPb are robust*

•*MC tool for calculations within HEF – LxJet has been upgraded to include Sudakov effects*

•*Open questions – description of the decorelations within CCFM. It includes Sudakov, and low x dynamics.*

•*Our results suggest that:*

Sudakov effects are important at moderate values of $\Delta\phi$

kt dependent gluon density with k.c and HEF framework works very well

one does not need MPI to have good description of inside jet tag scenario witin tree level DGLAP provided one uses $2 \rightarrow 3$ *ME*