## Forward di-jet production in $\mathrm{p}+\mathrm{Pb}$ collisions

## Cyrille Marquet

## Centre de Physique Théorique <br> Ecole Polytechnique \& CNRS

A. van Hameren, P. Kotko, K. Kutak, CM and S. Sapeta, 1402.5065

## Reminder of the context

- Forward particle production in d+Au collisions provided several signals of parton saturation at RHIC: suppression of hadron production and di-hadron correlations in $p+A$ vs $p+p$
- Mid-rapidity at LHC $\neq$ forward rapidity at RHIC
forward rapidities are also needed at the LHC be to sensitive to non-linear effects
- Forward di-jet production in p+Pb collisions
from low- $p_{T}$ hadrons to high- $p_{T}$ jets: the small- $x$ formalism needs to be extended


## LHC di-jet mid-rapidity data

- no sign of nuclear effects on the di-jet imbalance

for small enough values Trigger of $x_{A}$, non-linear effects are expected when $\mathrm{k}_{\mathrm{T}} \sim \mathrm{Qs}$


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for small enough values Trigger of $x_{A}$, non-linear effects are expected when $\mathrm{k}_{\mathrm{T}} \sim \mathrm{Qs}$
the di-jet imbalance is independent of A , and not related to Qs all due to 3-jet final states, and perhaps some non-perturbative intrinsic $\mathrm{k}_{\mathrm{T}}$ one needs to look at forward di-jet systems to see non-linear effects


## Two-particle final-state kinematics

final state: $k_{1}, y_{1} \quad k_{2}, y_{2}$ scanning the wave functions:


$$
x_{p}=\frac{k_{1} e^{y_{1}}+k_{2} e^{y_{2}}}{\sqrt{s}} \quad x_{A}=\frac{k_{1} e^{-y_{1}}+k_{2} e^{-y_{2}}}{\sqrt{s}}
$$

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x_{p} \sim x_{A}<1
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central rapidities probe moderate $x$

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forward/central doesn't probe much smaller x

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\begin{gathered}
\mathrm{x}_{\mathrm{p}} \sim \text { unchanged } \downarrow \mathrm{x}_{\mathrm{A}} \text { decreases } \\
x_{p} \sim 1, x_{A} \ll 1
\end{gathered}
$$

forward rapidities probe small x

## $k_{T}$ factorization for forward di-jets

- a factorization can be established in the small $x$ limit, for nearly back-to-back di-jets $Q_{s},\left|\mathbf{p}_{\mathbf{t} \mathbf{1}}+\mathbf{p}_{\mathbf{t} \mathbf{2}}\right| \ll\left|\mathbf{p}_{\mathbf{t} \mathbf{1}}\right|,\left|\mathbf{p}_{\mathbf{t} \mathbf{2}}\right|$

$$
\begin{aligned}
& \frac{d \sigma^{p A \rightarrow \text { dijets }+X}}{d y_{1} d y_{2} d^{2} p_{1 t} d^{2} p_{2 t}}=\frac{\alpha_{s}^{2}}{\left(x_{1} x_{2} S\right)^{2}} {\left[\sum_{q} x_{1} f_{q / p}\left(x_{1}, \mu^{2}\right) \sum_{i} H_{q g}^{(i)} \mathcal{F}_{q g}^{(i)}\left(x_{2},\left|\mathbf{p}_{\mathbf{1 t}}+\mathbf{p}_{\mathbf{2 t}}\right|\right)\right.} \\
&\text { Dominguez, CM, Xiao and Yuan (2011) } \left.\quad+\frac{1}{2} x_{1} f_{g / p}\left(x_{1}, \mu^{2}\right) \sum_{i} H_{g g}^{(i)} \mathcal{F}_{g g}^{(i)}\left(x_{2},\left|\mathbf{p}_{\mathbf{1 t}}+\mathbf{p}_{\mathbf{2 t}}\right|\right)\right]
\end{aligned}
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with $\quad x_{1}=\frac{1}{\sqrt{S}}\left(p_{1 t} e^{y_{1}}+p_{2 t} e^{y_{2}}\right), \quad x_{2}=\frac{1}{\sqrt{S}}\left(p_{1 t} e^{-y_{1}}+p_{2 t} e^{-y_{2}}\right)$
but it involves several unintegrated gluon densities $\mathcal{F}_{q g}^{(i)}$ and $\mathcal{F}_{g g}^{(i)}$ and their associated hard matrix elements ${ }^{q}$

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- only valid in asymmetric situations

Collins and Qiu (2007), Xiao and Yuan (2010)
does not apply with unintegrated parton densities for both colliding projectiles

## Simplified factorization formula

- assuming in addition $Q_{s} \ll\left|\mathbf{p}_{\mathbf{t} 1}+\mathbf{p}_{\mathbf{t} \mathbf{2}}\right|$
one recovers the formula used in the high-energy factorization framework

involving only one unintegrated gluon density, the one also involved in $F_{2}$
Kutak and Sapeta (2012)
it is related to the dipole scattering amplitude $\mathcal{N}(x, r)$

$$
\mathcal{F}_{A}(x, k)=\frac{N_{c}}{\alpha_{s}(2 \pi)^{3}} \int d^{2} b \int d^{2} r e^{-i \mathbf{k} \cdot \mathbf{r}} \nabla_{r}^{2} \mathcal{N}(x, r)
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\frac{d \sigma^{p A \rightarrow d i j e t s}+X}{d y_{1} d y_{2} d^{2} p_{1 t} d^{2} p_{2 t}}=\sum_{a, c, d} \frac{1}{16 \pi^{3}\left(x_{1} x_{2} S\right)^{2}}\left|\overline{\mathcal{M}_{a g \rightarrow c d}}\right|^{2} x_{1} f_{a / p}\left(x_{1}, \mu^{2}\right) \mathcal{F}_{A}\left(x_{2},\left|\mathbf{p}_{\mathbf{1 t}}+\mathbf{p}_{\mathbf{2 t}}\right|\right) \frac{1}{1+\delta_{c d}} .
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saturation effects are expected in the so-called geometric scaling window, when the incoming gluon momenta is not too large compared to $Q_{S}$

- we use two different unintegrated gluons, which both describe $F_{2}$ they are solutions of two small-x evolution equations, reflecting two proposed prescriptions to improve the LL Balitsky-Kovchegov equation


## Running-coupling BK evolution

- the Balitsky-Kovchegov equation

$$
\begin{gathered}
\frac{\partial \mathcal{N}(x, r)}{\partial \ln \left(x_{0} / x\right)}=\bar{\alpha} \int \frac{d^{2} r_{1}}{2 \pi} \frac{r^{2}}{r_{1}^{1} r_{2}^{2}}[\underbrace{\mathcal{N}\left(x, r_{1}\right)+\mathcal{N}\left(x, r_{2}\right)-\mathcal{N}(x, r)}_{\text {linear evolution : BFKL }}-\mathcal{N}\left(x, r_{1}\right) \mathcal{N}\left(x, r_{2}\right)] \\
r_{2}=\left|\mathbf{r}-\mathbf{r}_{1}\right|
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Fourier Transform of dipole amplitude $\mathcal{N}(x, r) \equiv$ unintegrated gluon distribution

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- running-coupling (RC) corrections to the BK equation
taken into account by the substitution

$$
\alpha_{s}\left(\mathbf{r}^{2}\right)=\left[-\frac{11 N_{c}-2 N_{f}}{12 \pi} \ln \left(\mathbf{r}^{2} \wedge_{Q C D}^{2}\right)\right]^{-1}
$$

$$
\frac{\bar{\alpha}}{\left.\left.2 \pi(\mathbf{x}-\mathbf{x})^{2} \mathbf{y}\right)^{2}-\mathbf{y}\right)^{2}} \stackrel{\text { Kovchegov }}{\text { Weigert }} \frac{N_{c}}{2 \pi^{2}}\left[\frac{\alpha_{s}\left((\mathbf{x}-\mathbf{z})^{2}\right)}{(\mathbf{x}-\mathbf{z})^{2}}-2 \frac{\alpha_{s}\left((\mathbf{x}-\mathbf{z})^{2}\right) \alpha_{s}\left((\mathbf{z}-\mathbf{y})^{2}\right)}{\alpha_{s}\left((\mathbf{x}-\mathbf{y})^{2}\right)}+\frac{\alpha_{s}\left((\mathbf{z}-\mathbf{y})^{2}\right)}{(\mathbf{z}-\mathbf{y})^{2}}\right]
$$

Balitsky
(2007)
$\frac{N_{c} \alpha_{s}\left((\mathbf{x}-\mathbf{y})^{2}\right)}{2 \pi^{2}}\left[\frac{(\mathbf{x}-\mathbf{y})^{2}}{(\mathbf{x}-\mathbf{z})^{2}(\mathbf{z}-\mathbf{y})^{2}}+\frac{1}{(\mathbf{x}-\mathbf{z})^{2}}\left(\frac{\alpha_{s}\left((\mathbf{x}-\mathbf{z})^{2}\right)}{\alpha_{s}\left((\mathbf{z}-\mathbf{y})^{2}\right)}-1\right)+\frac{1}{(\mathbf{z}-\mathbf{y})^{2}}\left(\frac{\alpha_{s}\left((\mathbf{z}-\mathbf{y})^{2}\right)}{\alpha_{s}\left((\mathbf{x}-\mathbf{z})^{2}\right)}-1\right)\right]$
RC corrections represent most of the NLO contribution

## Non-linear CCFM evolution

- a non-linear gluon cascade with coherence effects

Kutak, Golec-Biernat, Jadach and Skrzypek (2012)
solution compatible with F2 data not available yet

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- for now use simpler version Kutak and Stasto (2005)

$$
\begin{aligned}
\mathcal{F}_{p}\left(x, k^{2}\right) & =\mathcal{F}_{p}^{(0)}\left(x, k^{2}\right)+\frac{\alpha_{s}\left(k^{2}\right) N_{c}}{\pi} \int_{x}^{1} \frac{d z}{z} \int_{k_{0}^{2}}^{\infty} \frac{d l^{2}}{l^{2}}\left\{\frac{l^{2} \mathcal{F}_{p}\left(\frac{x}{z}, l^{2}\right) \theta\left(\frac{k^{2}}{z}-l^{2}\right)-k^{2} \mathcal{F}_{p}\left(\frac{x}{z}, k^{2}\right)}{\left|l^{2}-k^{2}\right|}+\frac{k^{2} \mathcal{F}_{p}\left(\frac{x}{z}, k^{2}\right)}{\left|4 l^{4}+k^{4}\right|^{\frac{1}{2}}}\right\} \\
& +\frac{\alpha_{s}\left(k^{2}\right)}{2 \pi k^{2}} \int_{x}^{1} d z\left[\left(P_{g g}(z)-\frac{2 N_{c}}{z}\right) \int_{k_{0}^{2}}^{k^{2}} d l^{2} \mathcal{F}_{p}\left(\frac{x}{z}, l^{2}\right)+z P_{g q}(z) \Sigma\left(\frac{x}{z}, k^{2}\right)\right] \\
& -\frac{2 \alpha_{s}^{2}\left(k^{2}\right)}{R^{2}}\left[\left(\int_{k^{2}}^{\infty} \frac{d l^{2}}{l^{2}} \mathcal{F}_{p}\left(x, l^{2}\right)\right)^{2}+\mathcal{F}_{p}\left(x, k^{2}\right) \int_{k^{2}}^{\infty} \frac{d l^{2}}{l^{2}} \ln \left(\frac{l^{2}}{k^{2}}\right) \mathcal{F}_{p}\left(x, l^{2}\right)\right],
\end{aligned}
$$

this is $B K+$ running coupling + high-pt improvements

- kinematical constraints
- sea quark contributions
- non-singular pieces of the splitting functions
note: this is an equation for the impact-parameter integrated gluon density


## Forward di-jet spectrum in p+p

- obtained with unintegrated gluons constrained from e+p low-x data rcBK: normalization uncentainty due to impact parameter integration

- similar shape at low $p_{t}$
- $K S$ better at large $p_{t}$
$\Delta \varphi$ dependence

- rcBK not plotted away from $\Delta \varphi=\pi$ - kink due to cone radius of 0.5


## Nuclear modification in $\mathrm{p}+\mathrm{Pb}$

- with a free parameter to vary the nuclear saturation scale

$$
\left.Q_{s A}^{2}=d Q_{s p}^{2}(\mathrm{rcBK} \text { case }) \text { or } R^{2} \rightarrow R_{A}^{2}=R^{2} A^{1 / 3} / c \text { (KS case }\right)
$$



- rcBK: not correct at large pt
- KS: reaches unity at large pt

observable very sensitive
to non-linear effects


## Nuclear modification in $\mathrm{p}+\mathrm{Pb}$



potentially big effects depending on the value of the nuclear saturation scale
caveat: near $\Delta \varphi=\pi$, our simplifying assumption $Q_{s} \ll\left|\mathbf{p}_{\mathbf{t} 1}+\mathbf{p}_{\mathbf{t} 2}\right|$ is not valid

## CMS central-forward di-jet data

- non-linear effects are small, as expected

but this is a good test of the formalism, which does a good job describing the data
van Hameren, Kotko, Kutak, and Sapeta (2014)


## Conclusions

- Non-linear evolution of gluon density in Au nucleus at RHIC:
- suppression of single hadron production in $d+A u$ vs $p+p$
- suppression of back-to-back correlations of di-hadrons in d+Au vs p+p
- Our goal: extend di-hadron calculation to di-jets, motivate LHC measurement
- our preliminary results are encouraging
- Several improvements needed:
- implement full factorization formula, to go beyond $Q_{s} \ll\left|\mathbf{p}_{\mathbf{t} 1}+\mathbf{p}_{\mathbf{t} 2}\right|$
- use solution of non-linear CCFM equation when available
- correct treatment of nuclear impact-parameter dependence
- estimate effects of jet fragmentation

