CONFORMAL REGGE THEORY FROM ADS/CFT AND DIS AT SMALL-X

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Richard Brower, Marko Djurić, Ina Sarcević and Chung-I Tan: *String-Gauge Dual Description of DIS and Small-x*, 10.1007/JHEP 11(2010)051, arXiv:1007.2259

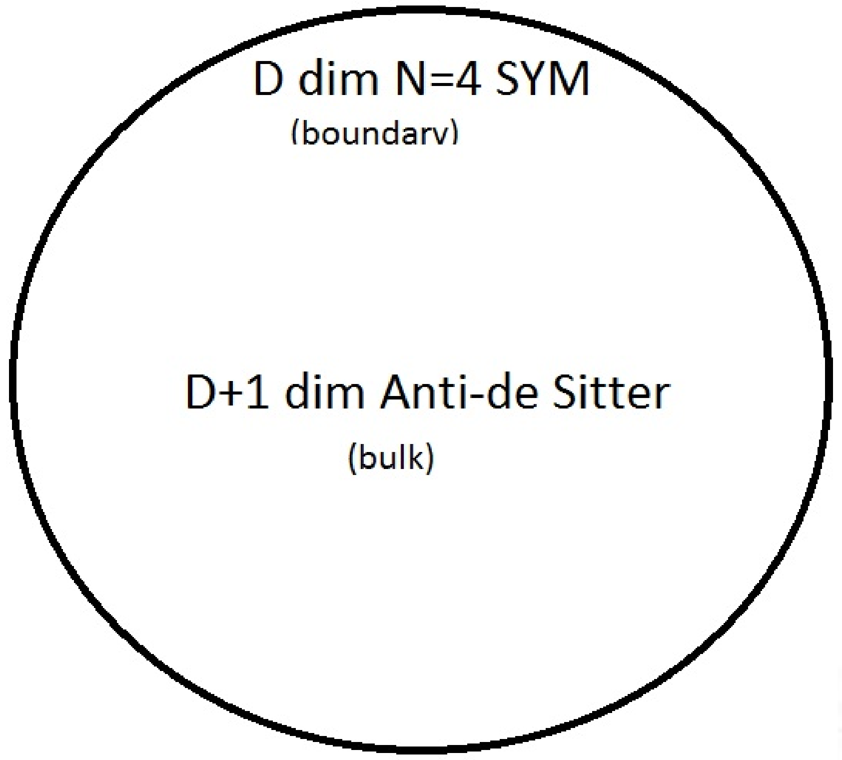
R. Brower, M. Costa, M. Djuric, T. Raben, and C-I Tan: *"Conformal Pomeron and Odderon Intercepts at Strong Coupling"* (to appear.)

R. Brower, M. Djuric, T. Raben and C-I Tan, "DIS, Confinement and Soft-Wall", (to appear)

Outline

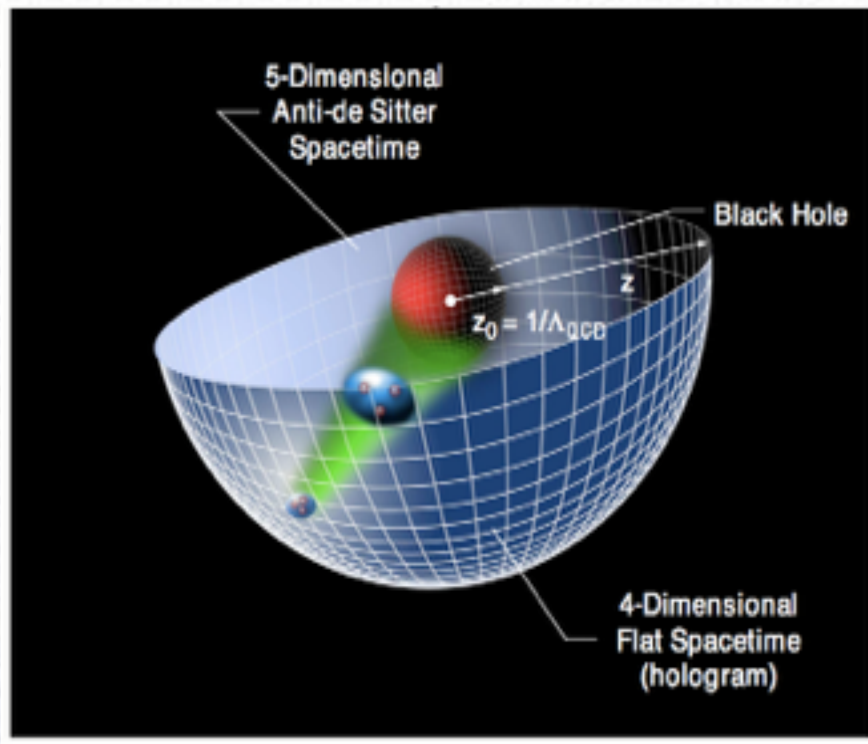
- **Background and Motivation:**
- **Conformal Regge Theory:**
    - Pomeron Spectral Curve in Strong Coupling
- **Saturation, Confinement, etc. and DIS:**
    - **Soft Wall**
- **Pomeron and Odderon Intercepts in strong coupling:**
- **Summary and Outlook:**

Background and Motivation



The AdS/CFT is a holographic duality that equates a string theory (gravity) in high dimension with a conformal field theory (gauge) in 4 dimensions. Specifically, compactified 10 dimensional super string theory is conjectured to correspond to $\mathcal{N} = 4$ Super Yang Mills theory in 4 dimensions in the limit of large 't Hooft coupling:

$$\lambda = g_s N = g_{ym}^2 N_c = R^4/\alpha'^2 >> 1.$$



$$\boxed{ds^2 = \frac{R^2}{z^2}\left[dz^2 + dx \cdot dx\right] + R^2 d\Omega_5 \rightarrow e^{2A(z)}\left[dz^2 + dx \cdot dx\right] + R^2 d\Omega_5}$$

For AdS, $A = -\log(z/R)$. As The function $A(z)$ is changed for z large, the space is "deformed" away from pure AdS

$$\boxed{\text{“Soft-Wall”}: \ A(z) \rightarrow -\log(z/R) + (\Lambda z)^2}$$

Issues: AdS/CFT for QCD ??

- Is strong coupling appropriate?
  - In many regimes, DIS can be treated perturbatively, but at small enough x, (for fixed Q^2), the physics is inclusive and becomes generically non-perturbative.
- Is confinement important?
  - Even for single Pomeron exchange, we will see confinement playing a role in determining the onset of saturation.
- Conformal Pomeron and OPE: Pomeron Spectral Curve and Graviton
- Conformal Pomeron and Odderon Intercepts in strong coupling:

Unification and Universality:

Gauge/String Duality (AdS/CFT) $\longrightarrow$ 2-GLUONS $\simeq$ GRAVITON

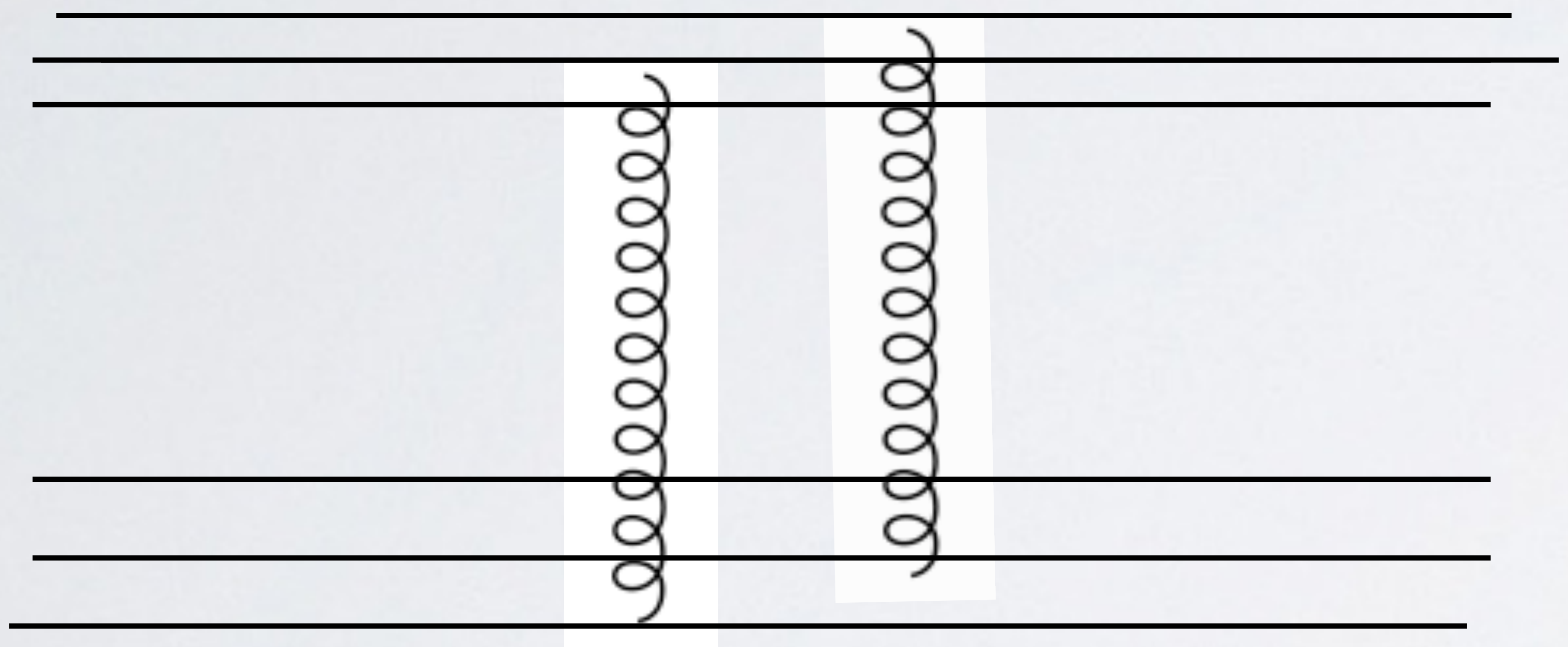
- “Pomeron” in QCD non-perturbatively,

- Unification of Soft and Hard Physics in High Energy Collision

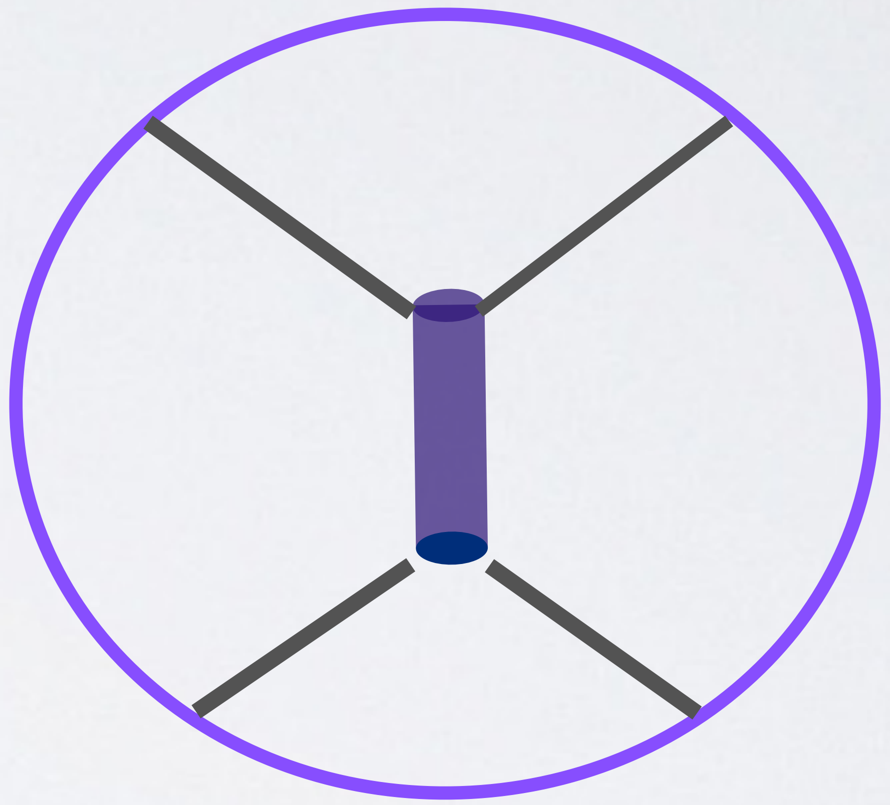
- New phenomenology based on “Large Pomeron intercept”, e.g., DIS at small-x: (DGLAP vs Pomeron), DVCS, Central Diffractive Higgs Production, etc.

WHAT IS THE BARE POMERON ? LEADING 1/N TERM CYLINDER EXCHANGE

WEAK: TWO GLUON <=> STRONG: ADS GRAVITON



$$J_{cut} = 1 + 1 - 1 = 1$$

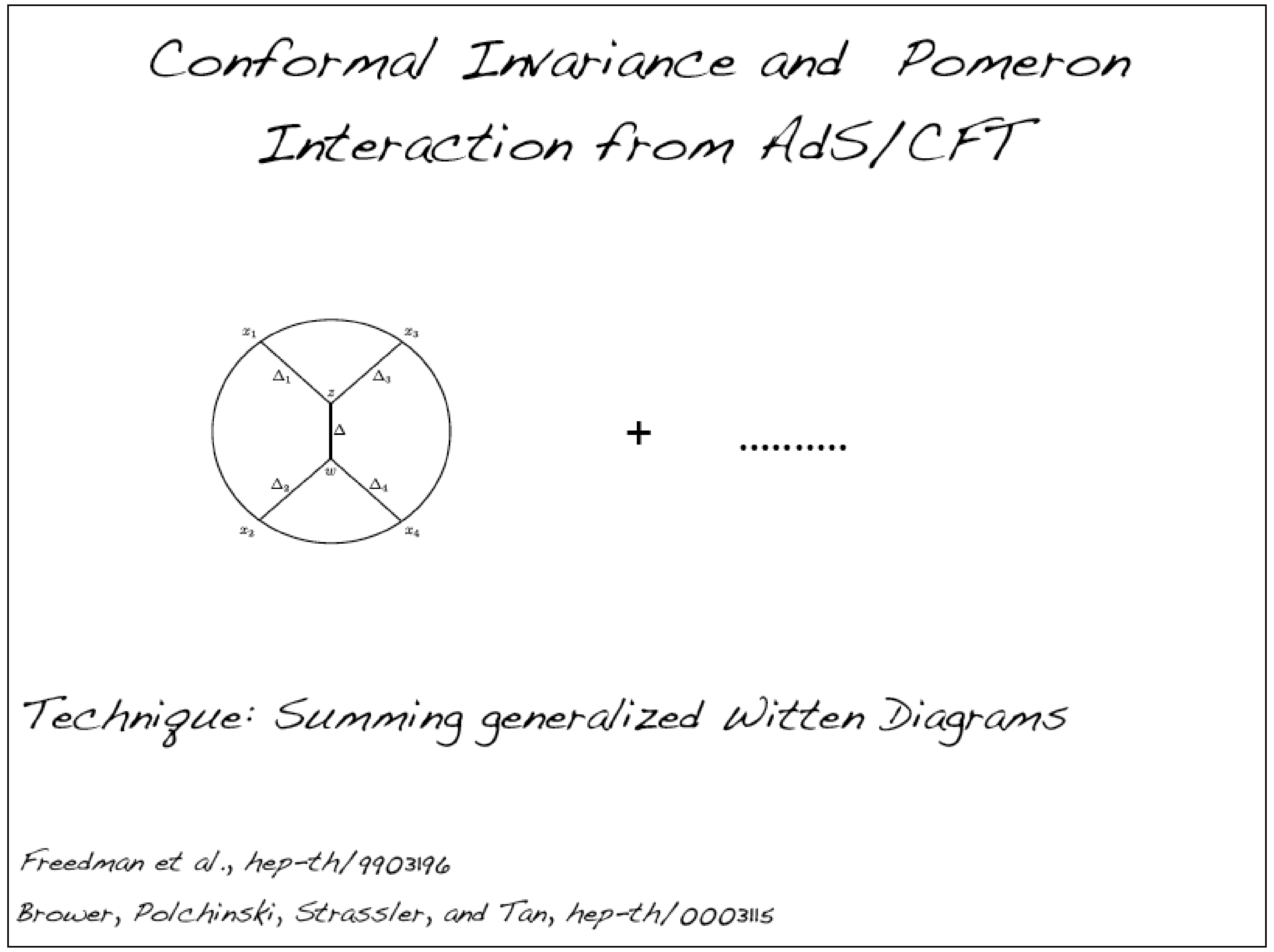


$$J = 2$$

$$S = \frac{1}{2\kappa^2} \int d^4x dz \sqrt{-g(z)} \Big(- \mathcal{R} + \frac{12}{R^2} + \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi \Big)$$

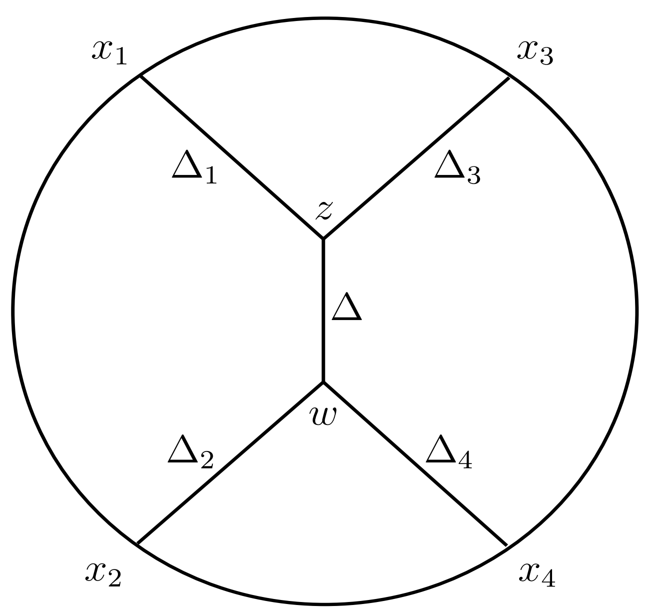
F.E. Low. Phys. Rev. D 12 (1975), p. 163.
S. Nussinov. Phys. Rev. Lett. 34 (1975), p. 1286.

AdS Witten Diagram: Adv. Theor. Math. Physics 2 (1998)253



- Draw all "Witten-Feynman" Diagrams in AdS_5,
- High Energy Dominated by Spin-2 Exchanges:

$$p_1 + p_2 \to p_3 + p_4$$



One Graviton Exchange at High Energy

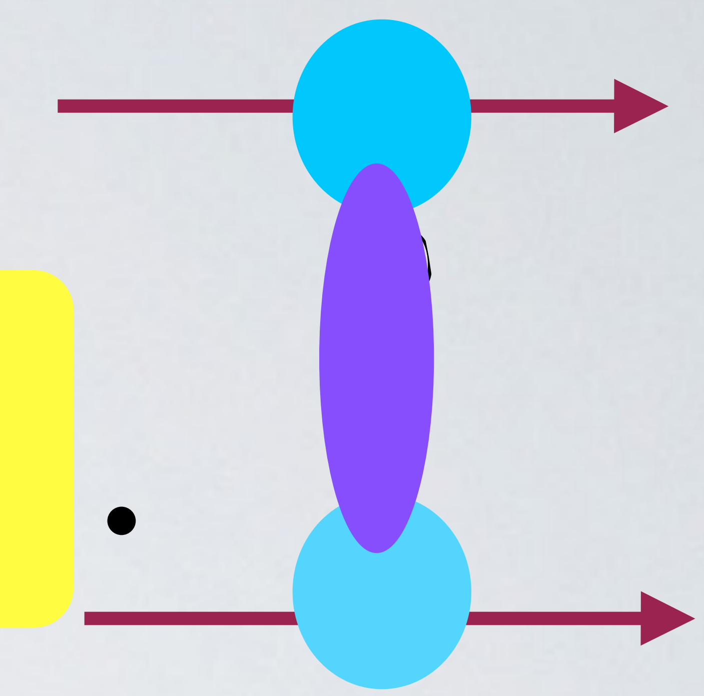
$$T^{(1)}(p_1,p_2,p_3,p_4) = g_s^2 \int \frac{dz}{z^5} \int \frac{dz'}{z'^5}\, \tilde{\Phi}_\Delta(p_1^2,z)\tilde{\Phi}_\Delta(p_3^2,z)\mathcal{T}^{(1)}(p_i,z,z')\tilde{\Phi}_\Delta(p_2^2,z')\tilde{\Phi}_\Delta(p_4^2,z')$$

$$\boxed{\mathcal{T}^{(1)}(p_i,z,z') = (z^2 z'^2 s)^2 G_{++,--}(q,z,z') = (zz's)^2 G^{(5)}_{\Delta=4}(q,z,z')}$$

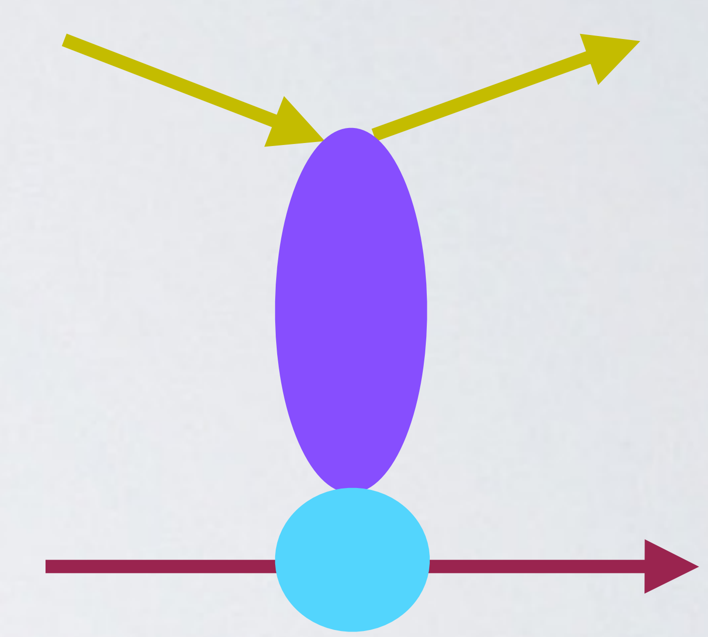
ADS BUILDING BLOCKS BLOCKS

For 2-to-2

$$A(s,t) = \Phi_{13} * \widetilde{\mathcal{K}}_P * \Phi_{24} \, .$$



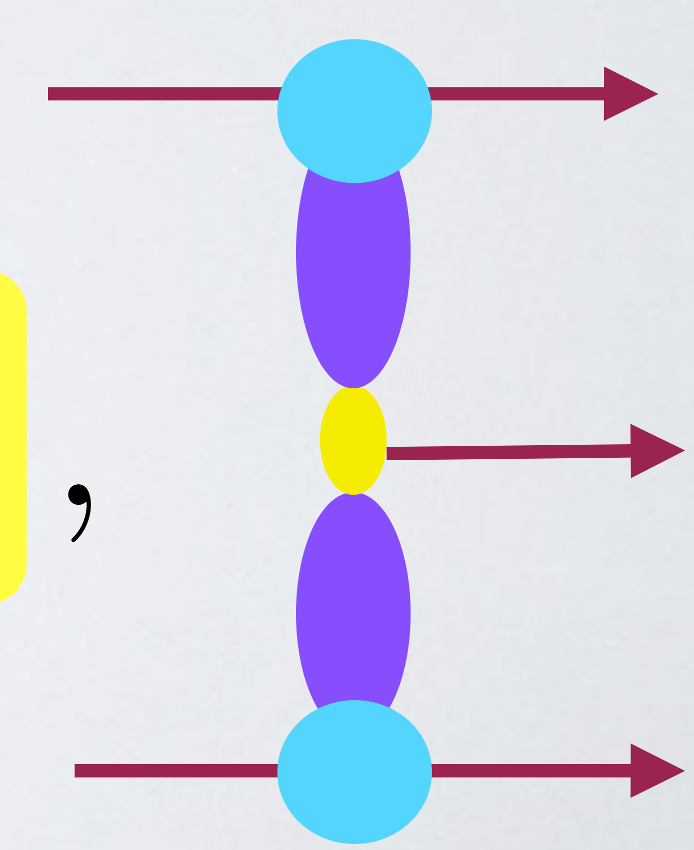
$$A(s,t) = g_0^2 \int d^3\mathbf{b} d^3\mathbf{b}' \, e^{i\mathbf{q}_\perp \cdot (\mathbf{x}-\mathbf{x}')} \, \Phi_{13}(z) \, \mathcal{K}(s, \mathbf{x}-\mathbf{x}', z, z') \, \Phi_{24}(z')$$



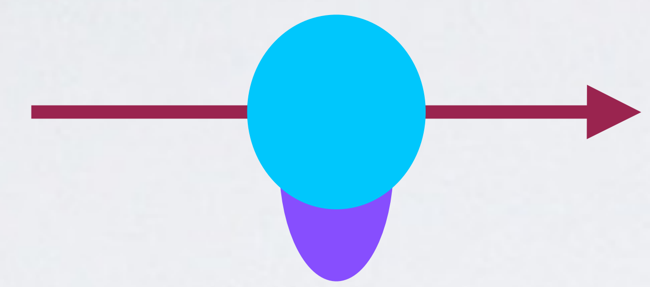
$$d^3\mathbf{b} \equiv dz d^2 x_\perp \sqrt{-g(z)} \quad \text{where} \quad g(z) = \det[g_{nm}] = -e^{5A(z)}$$

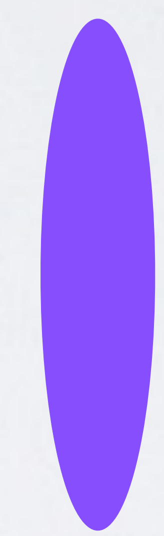
For 2-to-3

$$A(s, s_1, s_2, t_1, t_2) = \Phi_{13} * \widetilde{\mathcal{K}}_P * V * \widetilde{\mathcal{K}}_P * \Phi_{24} \, ,$$



BASIC BUILDING BLOCK

- Elastic Vertex: 

- Pomeron/Graviton Propagator: 

$$\mathcal{K}(s,b,z,z') = -\left(\frac{(zz')^2}{R^4}\right)\int \frac{dj}{2\pi i}\left(\frac{1+e^{-i\pi j}}{\sin \pi j}\right)\widehat{s}^j\, G_j(z,x^\perp,z',x'^\perp;j)$$

conformal: $$G_j(z,x^\perp,z',x'^\perp) = \frac{1}{4\pi z z'}\frac{e^{(2-\Delta(j))\xi}}{\sinh\xi}\ ,$$

$$\Delta(j) = 2+\sqrt{2}\,\lambda^{1/4}\sqrt{(j-j_0)}$$

confinement: $G_j(z,x^\perp,z',x'^\perp;j) \longrightarrow$ discrete sum

Holographic Approach to QCD

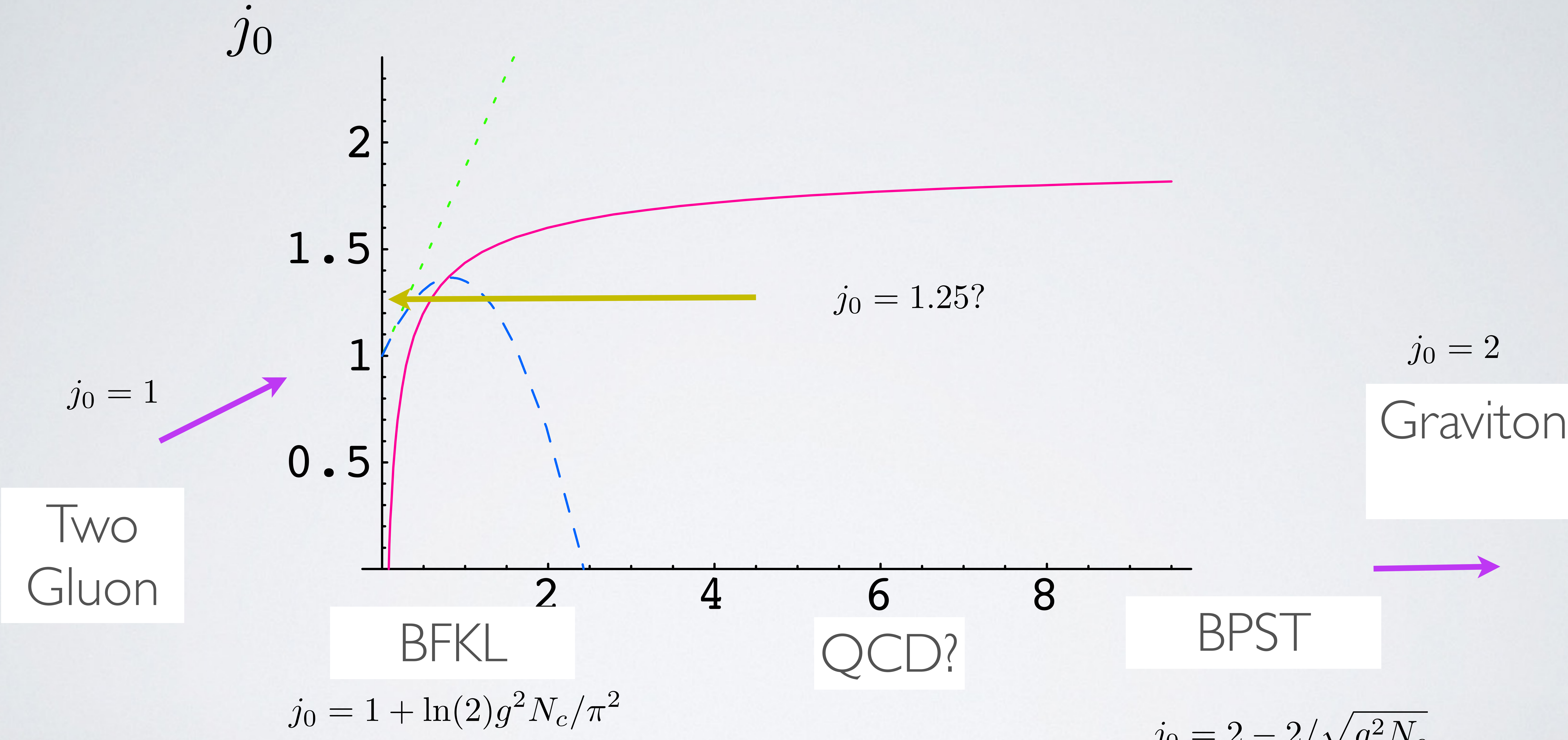
- Spin-2 leads to too fast a rise for cross sections
- Need to consider $\lambda \equiv g^2 N_c$ finite
- Graviton (Pomeron) becomes j-Plane singularity at

$$j_0 : \; 2 \to 2 - 2/\sqrt{\lambda}$$

- Comfinement: Particles and Regge trajectories

- Brower, Polchinski, Strassler, and Tan: “The Pomeron and Gauge/String Duality,” hep-th/063115

$\mathcal{N} = 4$ Strong vs Weak $g^2 N_c$



II: Pomeron in the conformal Limit, OPE, and Anomalous Dimensions

$$G_{mn} = g^0_{mn} + h_{mn}$$

Massless modes of a closed string theory:

Need to keep higher string modes

As CFT, equivalence to OPE in strong coupling: using AdS

CFT correlate function – coordinate representation

$$\langle \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_4(x_4)\rangle$$

OPE: $$\phi(x_1)\phi_2(x_2) \simeq \sum_k C_{1,2;k}(x_{12}, \partial_1)\mathcal{O}_k(x_1)$$

Bootstrap: s-channel OPE = t-channel OPE

unitarity, positivity, locality, analyticity, etc.

Dynamics: $\boxed{\mathcal{O}_{(\Delta,j)_k}(x)}$ Conformal Dimension, Spin

Conformal Partial-Wave Expansion and Regge Limit:

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle_c = \frac{1}{(x_{12}^2 x_{34}^2)^{\Delta_0}}\mathcal{A}(u,v)$$

Conformal inv. cross-ratios $\quad u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$

t-Channel partial-wave $\quad \mathcal{A}(u,v) = \sum_k \sum_{\Delta_k, j} c_{(12,(\Delta_k,j))} c_{(34,\Delta_(k,j))} \mathcal{G}_{(\Delta_k,j)}(u,v)$

Conformal Block $\quad \mathcal{G}_{(\Delta,j)}(u,v)$

Dynamics: $\{(\Delta_k(j),j)\}\,, k = 1,2,\cdots\,,\ j = 0,1,\cdots$ Conformal Data

$\mathcal{N} = 4 \quad SYM$ *Integrability* AdS-Dual, Large-N, etc.

Regge Limit: $u \to 0\,, \quad v \to 1\,, \quad$ with $\sqrt{u}/(1-v)$ fixed

Euclidean vs Minkowski?

Conformal Partial-Wave Expansion and Regge Limit:

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle_c = \frac{1}{(x_{12}^2 x_{34}^2)^{\Delta_0}}\mathcal{A}(u,v)$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

$$\mathcal{A}(u,v) = \sum_k \sum_{\Delta_k,j} c_{(12,(\Delta_k,j))} c_{(34,\Delta_(k,j))} \mathcal{G}_{(\Delta_k,j)}(u,v)$$

Conformal Block $\quad \mathcal{G}_{(\Delta,j)}(u,v)$

Regge Limit:

$u \to 0\,, \quad v \to 1\,, \quad$ with $\quad \sqrt{u}/(1-v) \quad$ fixed

---

Euclidean Regge limit:

$$\mathcal{G}_{(\Delta,j)}(u,v) \sim u^{\Delta/2}\, g(\tilde{b}^2)$$

$$\tilde{b}^2 \sim \frac{1-v}{\sqrt{u}} \sim \cos\theta$$

Minkowski Regge limit:

$$\mathcal{G}_{(\Delta,j)}(u,v) \sim u^{(1-j)/2}\, \mathcal{Y}(\tilde{b}^2)$$

$$\sqrt{u} \sim s^{-1}$$

$$\mathcal{Y}(\tilde{b}^2) \sim \tilde{b}^{-2(\Delta-1)}$$

$$\tilde{b}^2 \sim \frac{1-v}{\sqrt{u}} \quad \text{large}$$

Full $O(4,2)$ Conformal Group

$$\boxed{SO(4,2) = SO(1,1) \times SO(3,1)}$$

$$\mathcal{A}(u,v) \leftrightarrow \int_{d/2-i\infty}^{d/2+i\infty} \frac{d\Delta}{2\pi i} \int_{-1/2-i\infty}^{-1/2+i\infty} \frac{dj}{2\pi i} \quad a(\Delta, j)\, \mathcal{G}(u,v;\Delta,j)$$

Euclidean CFT

$$\boxed{SO(5,1) = SO(1,1) \times SO(4)}$$

$$\mathcal{A}(u,v) \leftrightarrow \int_{d/2-i\infty}^{d/2+i\infty} \frac{d\Delta}{2\pi i} \sum_j \quad a_j(\Delta)\, G_{\Delta,j}(u,v)$$

Dynamics

$$a_j(\Delta) \sim \frac{1}{\Delta - \Delta_j} \quad \rightarrow \quad \frac{1}{\Delta - \Delta(j)}$$

Conformal Partial-Wave Expansion and Regge Limit:

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle_c = \frac{1}{(x_{12}^2 x_{34}^2)^{\Delta_0}}\mathcal{A}(u,v)$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

$$\mathcal{A}(u,v) = \sum_k \sum_{\Delta_k, j} c_{(12,(\Delta_k,j))} c_{(34,\Delta_(k,j))} \mathcal{G}_{(\Delta_k,j)}(u,v)$$

Conformal Block $\quad \mathcal{G}_{(\Delta,j)}(u,v)$

Regge Limit: $\quad u \to 0\,, \quad v \to 1\,, \quad \text{with} \quad \sqrt{u}/(1-v) \quad \text{fixed}$

Minkowski Regge limit:

"Sommerfeld-Watson resummation"

$$\mathcal{A}(u,v) = \sum_{\xi=\pm} \int \frac{d\Delta}{2\pi i} \int \frac{dj}{2\pi i} \frac{1+\xi e^{-i\pi j}}{\sin \pi j} a_\xi(\Delta, j) \mathcal{G}_{(\Delta,j)}(u,v)$$

Conformal Data:

$$a_\pm \sim \sum_k \frac{c_k(j)}{\Delta - \Delta_k^\pm(j)}$$

$$\mathcal{A}(u,v) = \sum_\xi \sum_k \int \frac{dj}{2\pi i} \frac{1+\xi e^{-i\pi j}}{\sin \pi j} c_k(j,\xi) \mathcal{G}_{(\Delta_k^\xi(j),j)}(u,v)$$

Regge Limit:

$$\mathcal{A} \sim u^{(1-j_0)/2}$$

j_0 is the leading singularity of "anomalous dimensions", $\Delta(j) - j - \tau_0$.

Dynamics

$$a_j(\Delta) \sim \frac{1}{\Delta - \Delta_j} \quad \to \quad \frac{1}{\Delta - \Delta(j)}$$

Single Trace Gauge Invariant Operators of $\mathcal{N} = 4$ SYM,

$$Tr[F^2], \quad Tr[F_{\mu\rho}F_{\rho\nu}], \quad Tr[F_{\mu\rho}D_{\pm}^{S}F_{\rho\nu}], \quad Tr[Z^{\tau}], \quad Tr[D_{\pm}^{S}Z^{\tau}], \cdots$$

Super-gravity in the $\lambda \to \infty$:

$$Tr[F^2] \leftrightarrow \phi, \quad Tr[F_{\mu\rho}F_{\rho\nu}] \leftrightarrow G_{\mu\nu}, \quad \cdots$$

Symmetry of Spectral Curve:

$$\Delta(j) \leftrightarrow 4 - \Delta(j)$$

Graviton Spectral Curve:

$$a_j(\Delta) \sim \frac{1}{\Delta - \Delta_j} \quad \to \quad \frac{1}{\Delta - \Delta(j)}$$

Single Trace Gauge Invariant Operators of $\mathcal{N} = 4$ SYM,

$$Tr[F_{\pm\perp} D_{\pm}^{j-2} F_{\perp\pm}], \quad j = 2, 4, \cdots$$

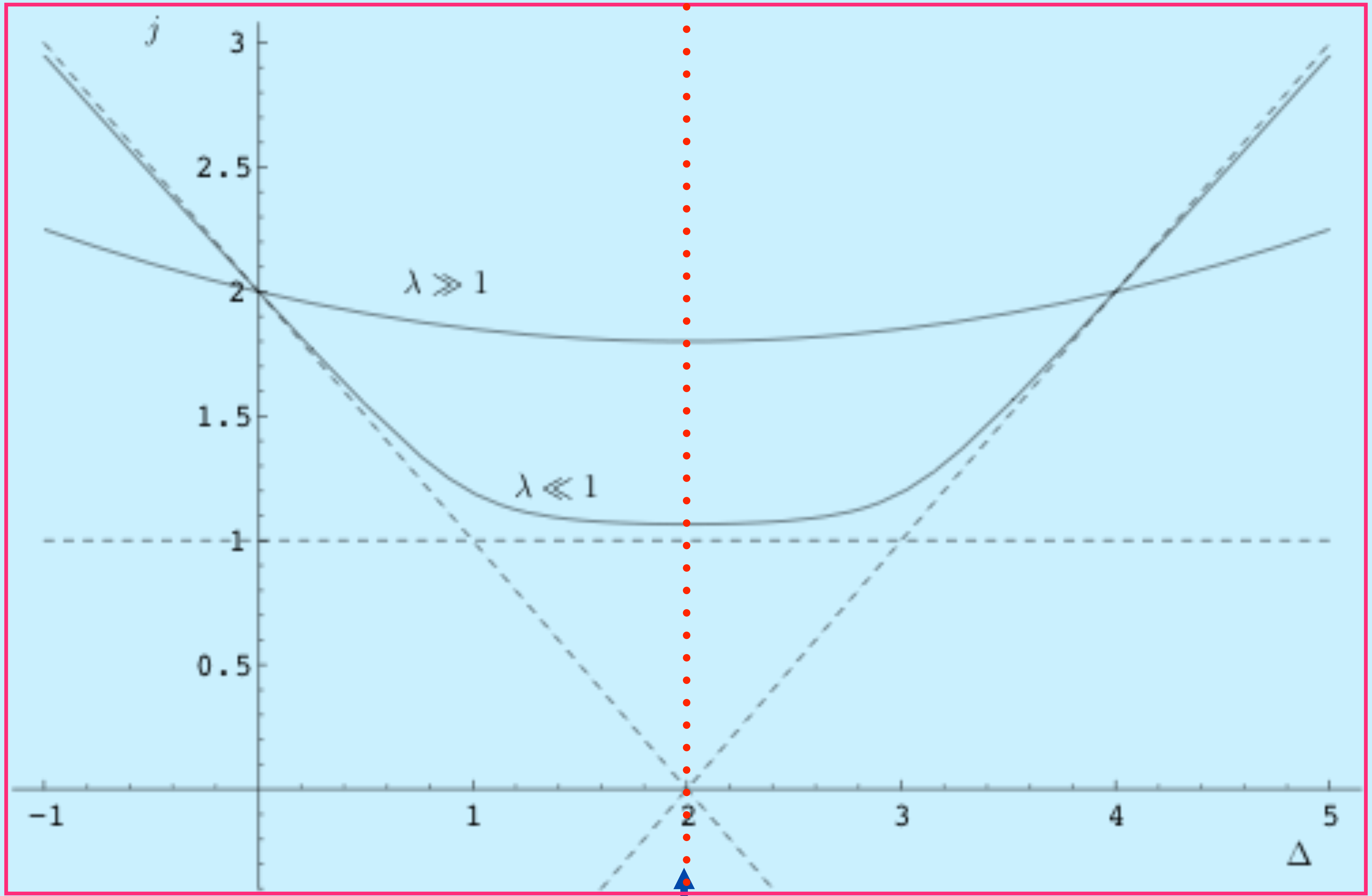
Super-gravity in the $\lambda \to \infty$:

$$\Delta(2) = 4; \quad \Delta(j) = O(\lambda^{1/4}) \to \infty, \quad j > 2$$

Symmetry of Spectral Curve:

$$\Delta(j) \leftrightarrow 4 - \Delta(j)$$

$\mathcal{N} = 4$ SYM Leading Twist $\Delta(J)$ vs J: Anomalous Dimensions



λ = 0 DGLAP (DIS moments)

$Tr[F_{+\mu} D_{+}^{j-2} F_{+}^{\mu}]$

(0,2) $T_{\mu\nu}$ $\gamma = 0$

λ = 0, BFKL

$\lambda = g^2 N = 0$

$j = j_0$ @ min Δ

Graviton Spectral Curve:

$$a_j(\Delta) \sim \frac{1}{\Delta - \Delta_j} \quad \to \quad \frac{1}{\Delta - \Delta(j)}$$

Flat Space String Theory

$$\frac{1}{j - (2 + \alpha' t/2)}$$

Perturbing about SUGRA of large λ:

$$\frac{1}{j - [2 + (\sqrt{\lambda}/2)\Delta(\Delta - 4)]}$$

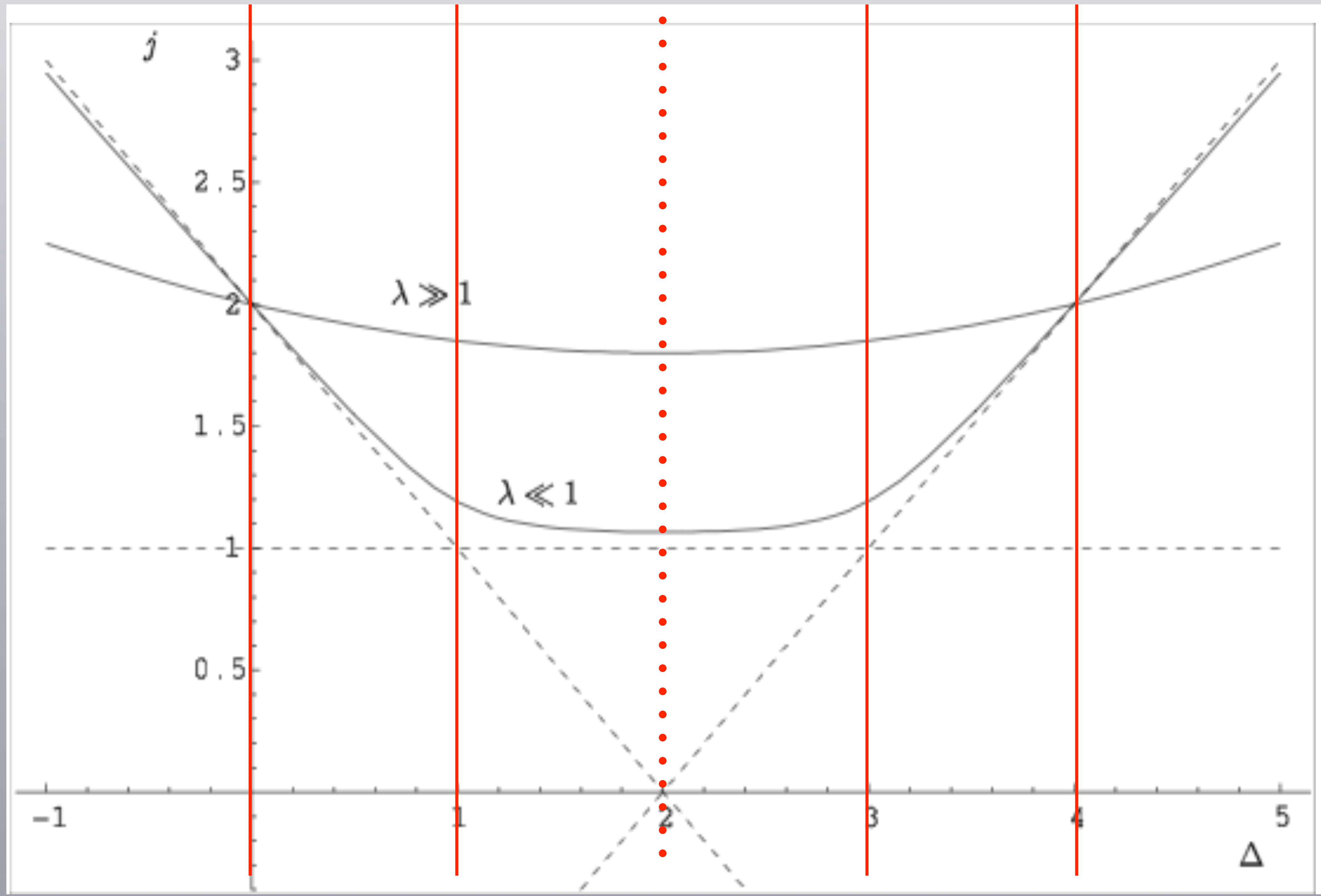
Symmetry of Spectral Curve:

$$\Delta(j) \leftrightarrow 4 - \Delta(j)$$

$$(\Delta(j) - 2)^2 = (2\sqrt{\lambda})(j - j_0), \quad j_0 = 2 - 2/\sqrt{\lambda}$$

ANOMALOUS DIMENSIONS:

$$\gamma(j,\lambda) = \Delta(j,\lambda) - j - 2$$



$$\gamma_2 = 0$$

$$\Delta(j) = 2 + \sqrt{2}\sqrt{\sqrt{g^2 N_c}(j - j_0)}$$

$$\gamma_n = 2\sqrt{1 + \sqrt{g^2 N}(n-2)/2} - n$$

Energy-Momentum Conservation built-in automatically.

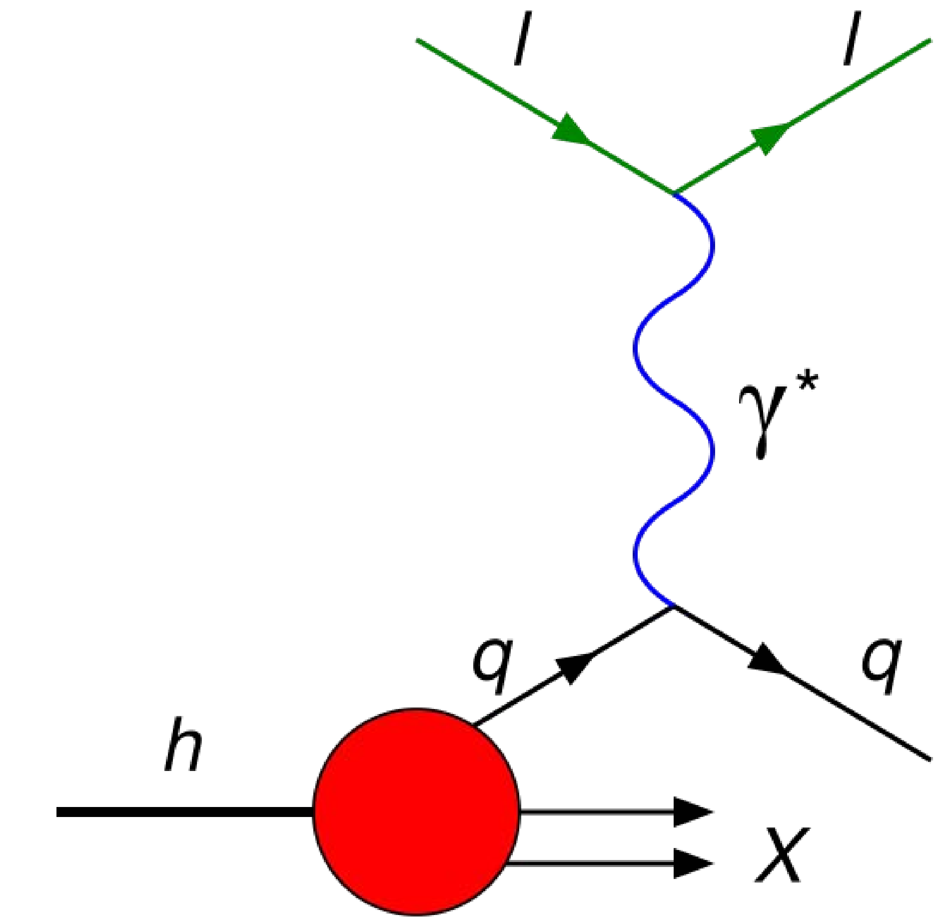
III. Deep Inelastic Scattering (DIS) at small-x:

Confinement ?

Satuation ?

DIS in AdS

We are interested in deep inelastic scattering (DIS) characterize by a virtual photon off a proton ($\gamma^* p$)

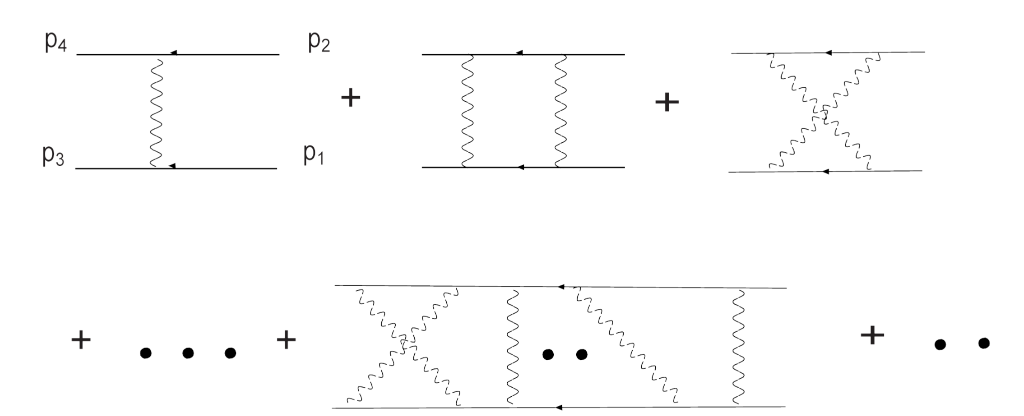


To characterize this process we consider the CM energy $s \approx Q^2/x$ for s large. In the regge limit, with Q^2 fixed, we can treat this process via the exchange of pomerons. (leading order exchange in a sommerfeld-watson decomposition). The primary route to physical relevance is via the opital theorem

$$\sigma_{total} = \frac{1}{s}\text{Im}\left[\mathcal{A}(s,t=0)\right] \sim \frac{1}{s}\text{Im}\left[\chi(s,t=0)\right]$$

We can use this to calculate total cross sections and to determine the proton structure function

$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\left(\sigma_{trans} + \sigma_{long}\right)$



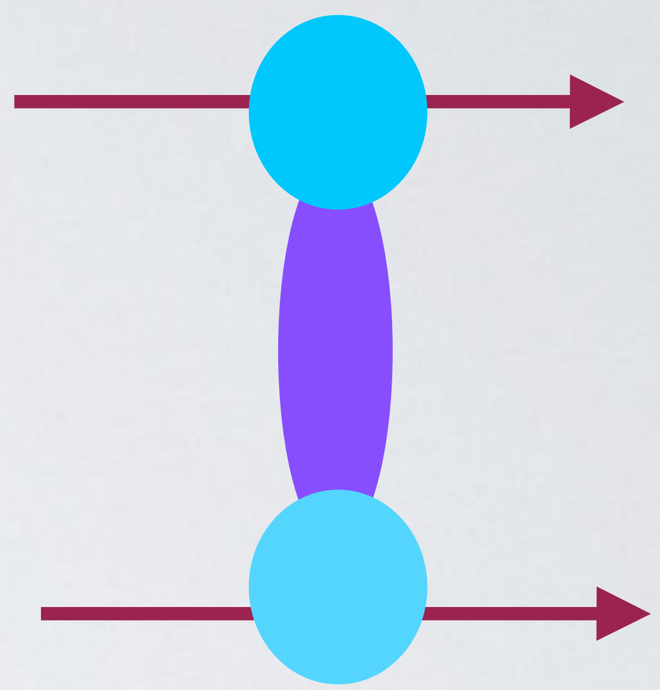
Finally we must be wary of saturation where we must consider multipomeron exchange via eikonalization

$$\chi \to 1 - e^{i\chi}$$

[Cornalba, Costa, Penedones][Brower, Strassler, Tan]

ELASTIC VS DIS ADS BUILDING BLOCKS

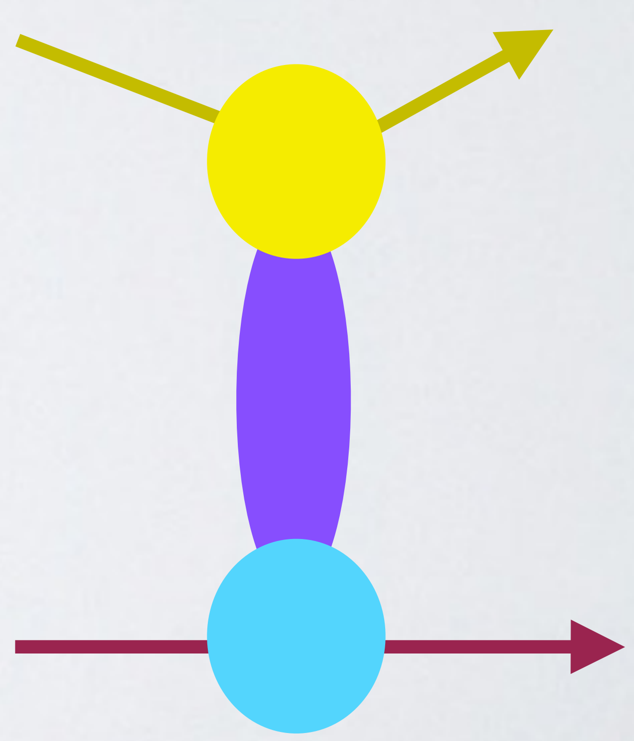
$$A(s, x_\perp - x'_\perp) = g_0^2 \int d^3\mathbf{b} d^3\mathbf{b}' \Phi_{12}(z) G(s, x_\perp - x'_\perp, z, z') \Phi_{34}(z')$$



$$\sigma_T(s) = \frac{1}{s} Im A(s, 0)$$

for $\quad F_2(x, Q)$

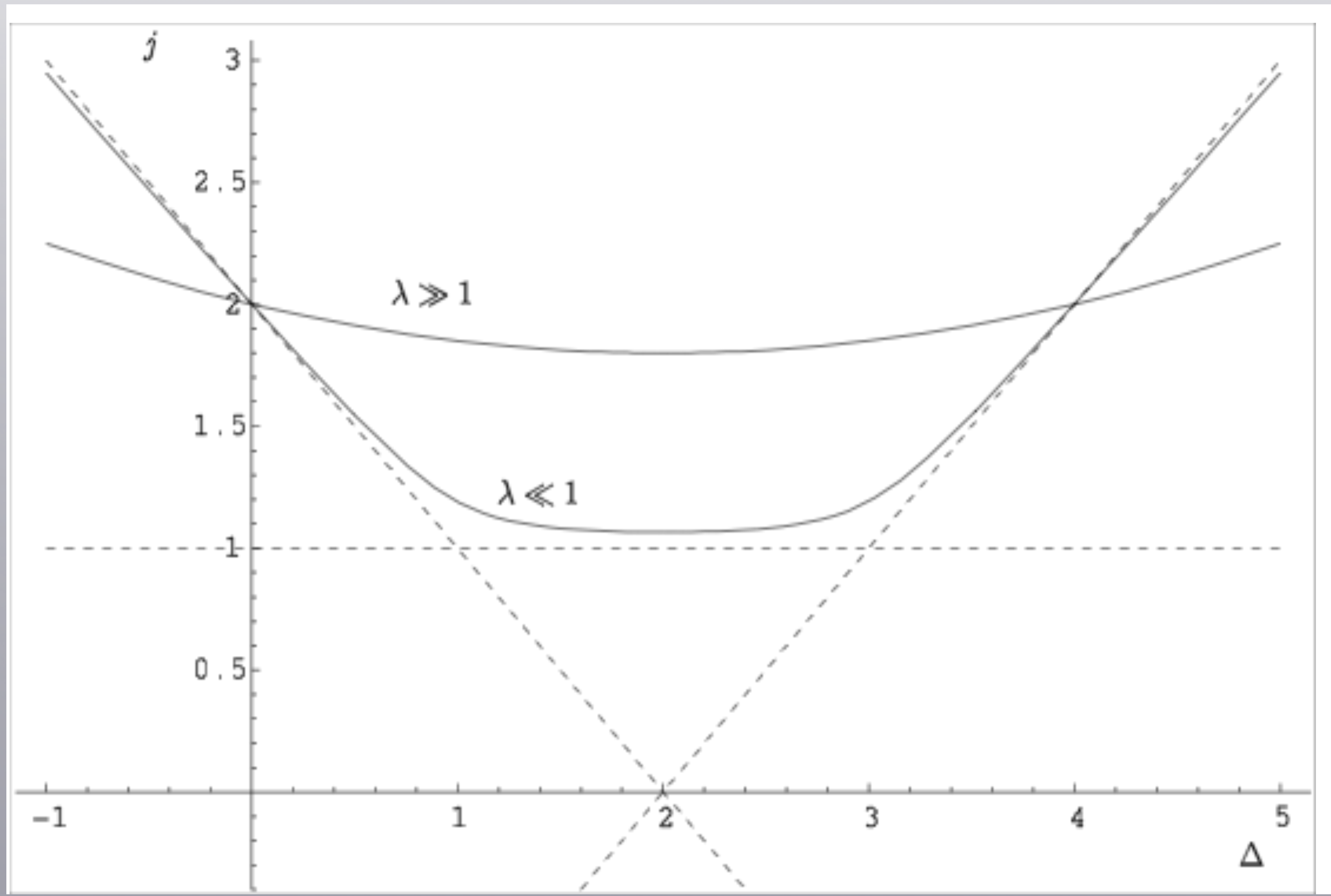
$$\Phi_{13}(z) \to \Phi_{\gamma^*\gamma^*}(z, Q) = \frac{1}{z}[Qz)^4 (K_0^2(Qz) + K_1^2(Qz)]$$



$$d^3\mathbf{b} \equiv dz d^2x_\perp \sqrt{-g(z)} \quad \text{where} \quad g(z) = \det[g_{nm}] = -e^{5A(z)}$$

MOMENTS AND ANOMALOUS DIMENSION

$$M_n(Q^2) = \int_0^1 dx\; x^{n-2} F_2(x, Q^2) \to Q^{-\gamma_n}$$



$$\gamma_2 = 0$$

$$\Delta(j) = 2 + \sqrt{2}\sqrt{\sqrt{g^2 N_c}(j - j_0)}$$

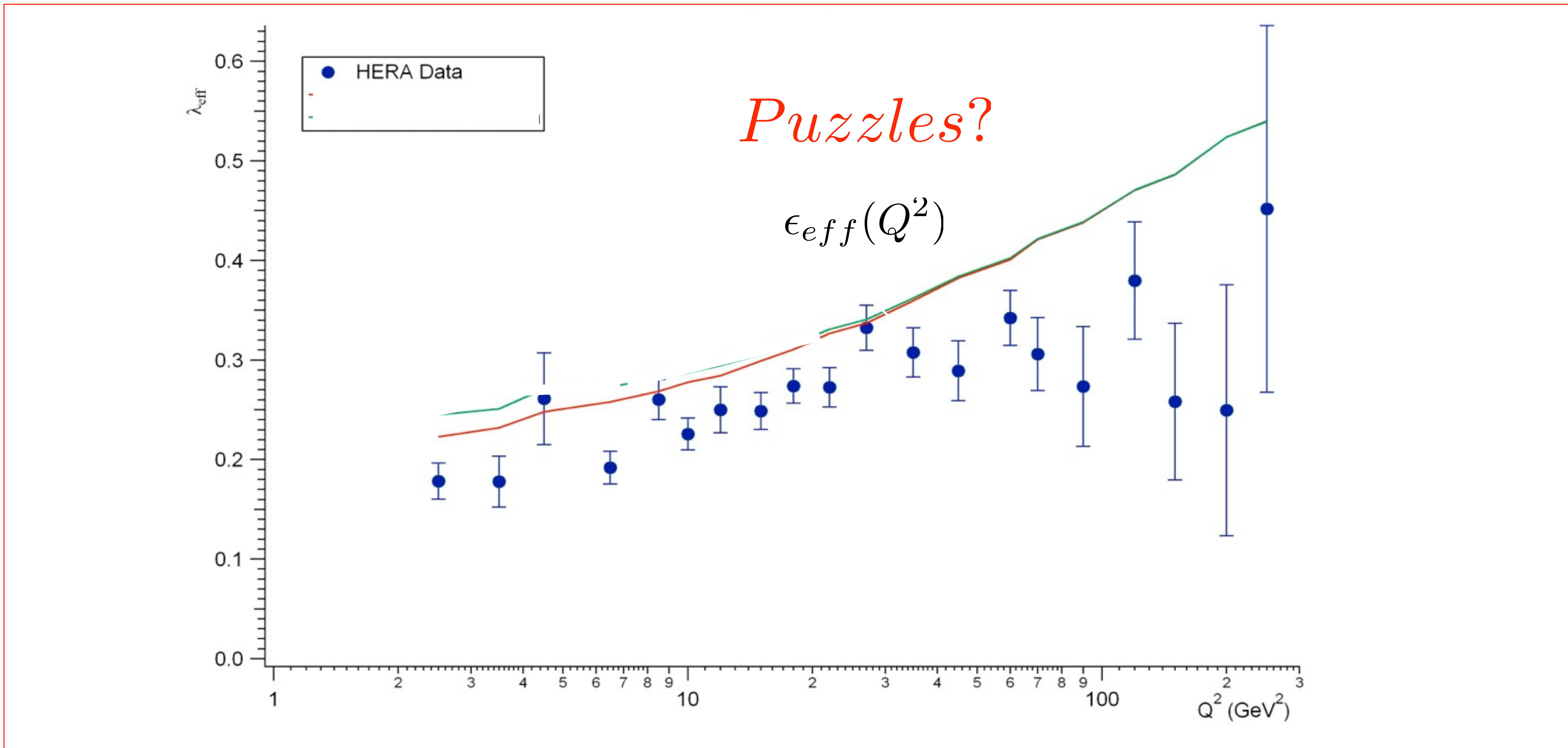
$$\gamma_n = 2\sqrt{1 + \sqrt{g^2 N}(n-2)/2} - n$$

Simultaneous compatible large Q^2 and small x evolutions!

Energy-Momentum Conservation built-in automatically.

Effective Pomeron Intercept from HERA data:

$$F_2 \simeq C(Q^2)\; x^{-\epsilon_{eff}}$$



Questions on HERA DIS small-x data:

- Why $\alpha_{eff} = 1 + \epsilon_{eff}(Q^2)$?
- Confinement? (Perturbative vs. Non-perturbative?)
- Saturation? (evolution vs. non-linear evolution?)

Soft Wall Basics

In order to confine the theory one must effectively deform the AdS geometry. This can be done via:

- Sharp cutoff – $z = z_0 \approx 1/\Lambda_{QCD}$ (Hard Wall Model) [Polchinski, Strassler],[Brower, Djuric, Sarcevic, Tan]
- Gradual increase in length scales / large effective potential boundary for large z leads to possible bound states: confinement

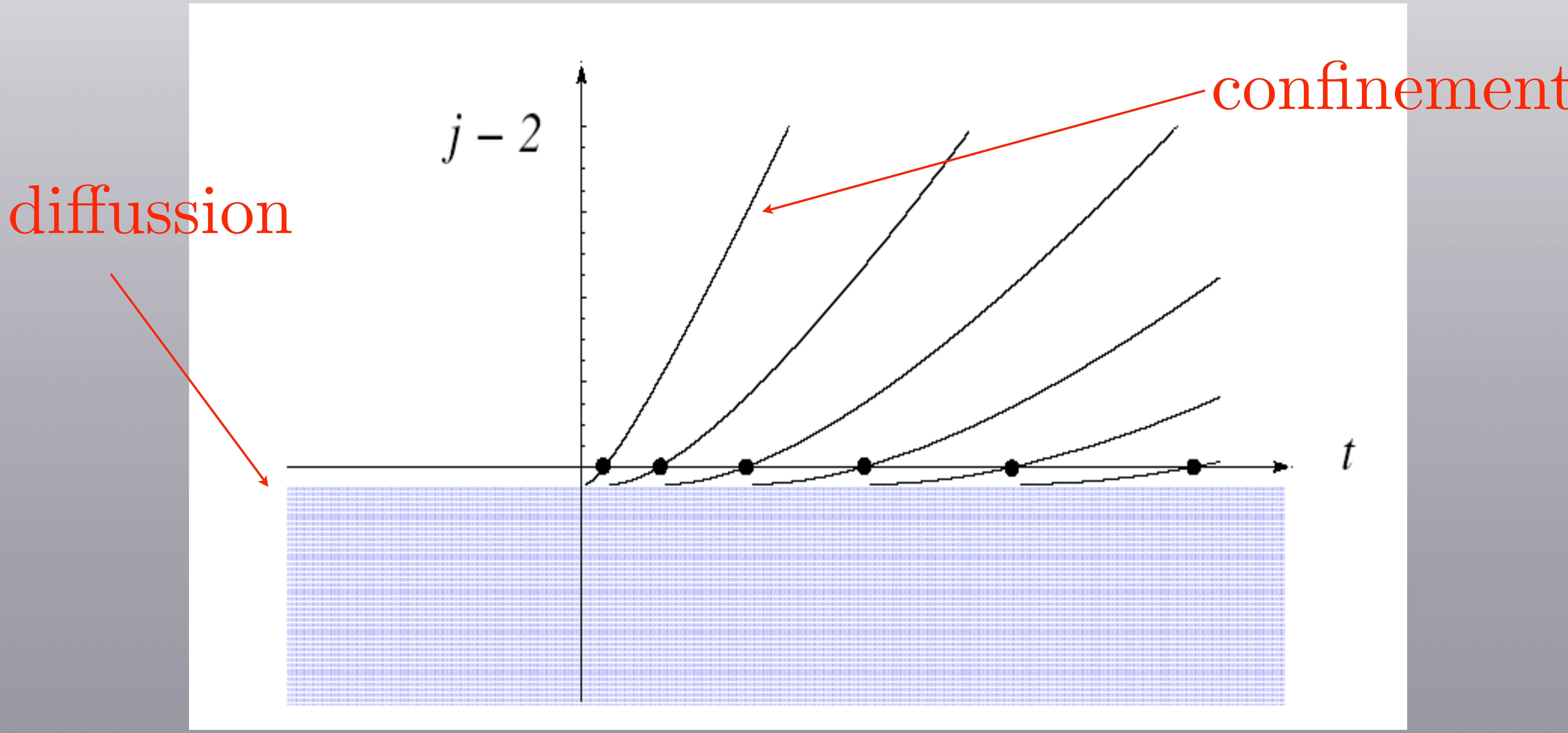
For our geometric softwall, the deformation function becomes $A(z) \to \Lambda^2 z^2 - \mathrm{Log}(z/R)$. This leads to a metric

$$ds^2 \to \frac{e^{2\Lambda^2 z^2} R^2}{z^2} \left[dz^2 + dx \cdot dx\right]$$

We wish to use this soft wall model to describe deep inelastic scattering at leading order in the regge-limit. The object of interest is the AdS-pomeron, which was identified to be the Regge trajectory of the graviton [Brower, Polchinski, Strassler, Tan]. For us, it is sufficient to consider a purely geometric confinement deformation. However, to describe mesons it will be required to consider other dynamical fields in the bulk. [Karch, Katz, Son, Stephanov], [de Teramond, Brodsky], [Batell, Gherghetta]

Unified Hard (conformal) and Soft (confining) Pomeron

At finite λ, due to Confinement in AdS, *at* $t > 0$
aymptotical linear Regge trajectories



Propagators and Wave functions

In this framework the pomeron propagator obeys:

$$\Big[-\partial_z^2+\Lambda^4 z^2+(2\Lambda^2-t)+\frac{\alpha^2(j)-1/4}{z^2}\Big]\chi_P(j,z,z',t)=\delta(z-z')$$

$$\alpha(j)=\Delta(j)-2$$

Where as for a continuous t spectrum the solution becomes a combination of Whittaker's functions (generalized hyper geometric functions)

$$\chi_P \sim ...M_{\kappa,\mu}(z_<)W_{\kappa,\mu}(z_>) \qquad (2)$$

for $\kappa=\kappa(t)$ and $\mu=\mu(j)$

$$\kappa(t)=t/4\Lambda^2-1/2 \qquad \mu(j)=\alpha(j)/2$$

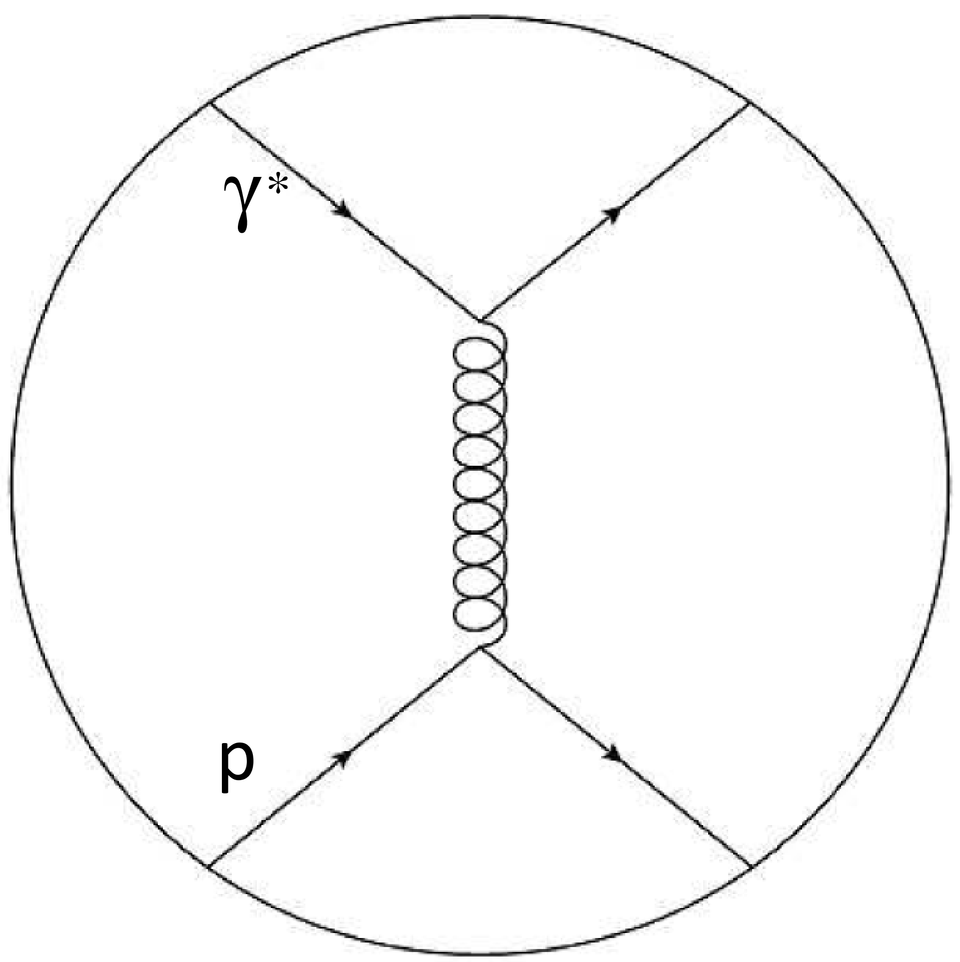
Special Limits, Behavior, and Symmetry

- Λ controls the strength of the soft wall and in the limit $\Lambda \to 0$ one recovers the conformal solution
$$\mathrm{Im}\chi_P^{conformal}(t=0) = \frac{g_0^2}{16}\sqrt{\frac{\rho^3}{\pi}}(zz')\frac{e^{(1-\rho)\tau}}{\tau^{1/2}}exp\left(\frac{-(\mathrm{Log}z - \mathrm{Log}z')^2}{\rho\tau}\right)$$
where $\tau = \mathrm{Log}(\rho z z' s/2)$ and $\rho = 2 - j_0$. Note: this has a similar behavior to the weak coupling BFKL solution where
$$\mathrm{Im}\chi(p_\perp, p'_\perp, s) \sim \frac{s^{j_0-1}}{\sqrt{\pi\mathcal{D}\mathrm{Log}s}}\exp(-(\mathrm{Log}p'_\perp - \mathrm{Log}p_\perp)^2/\mathcal{D}\mathrm{Log}s)$$

- If we look at the energy dependence of the pomeron propagator, we can see a softened behavior in the forward regge limit.
$$\chi_{conformal} \sim \frac{s^{j_0-1}}{\sqrt{\log s}} \to \chi_{HW} \sim \frac{s^{j_0-1}}{(\log s)^{3/2}}$$
Analytically, this corresponded to the softening of a j-plane singularity from $1/\sqrt{j-j_0} \to \sqrt{j-j_0}$. Again, we see this same softened behavior in the soft wall model.

- (Possibly) interesting limit $t = 2\Lambda^2$. Here the EOM simplifies and takes the form of a model with 1+1 dimensional conformal symmetry[Fubini]

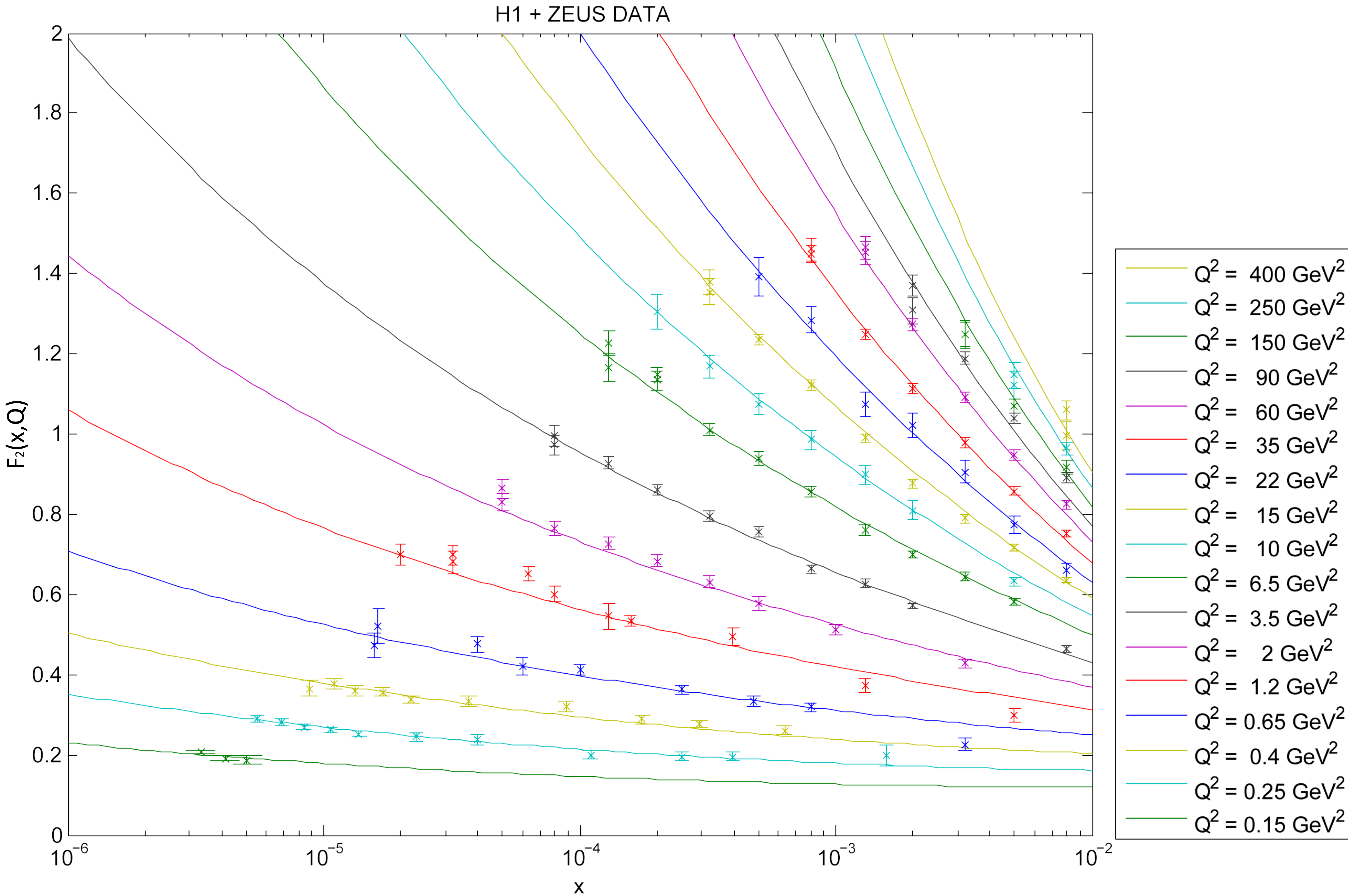
Data Set and Fit



We will consider the combined H1 and Zeus data set published in 2010 [Aaron,et. al.][Chekanov, et. al.], but we restrict ourselves to small- x data, $x < 0.01$. We can write a scattering amplitude as

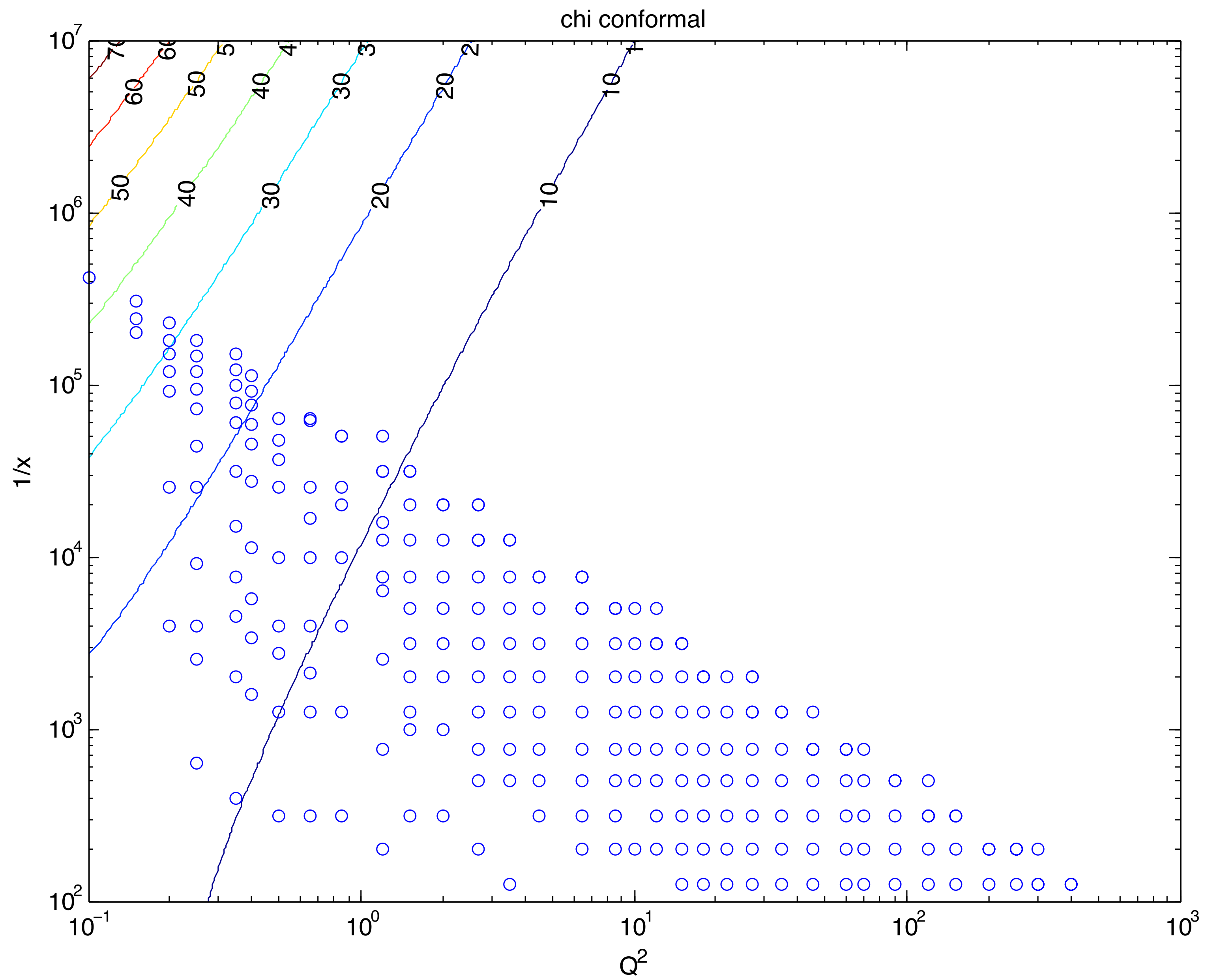
$$\mathcal{A}(s,t) = s\int_{bulk} dzdz' P_{13}(z) P_{24}(z')\chi(s,t,z,z')$$

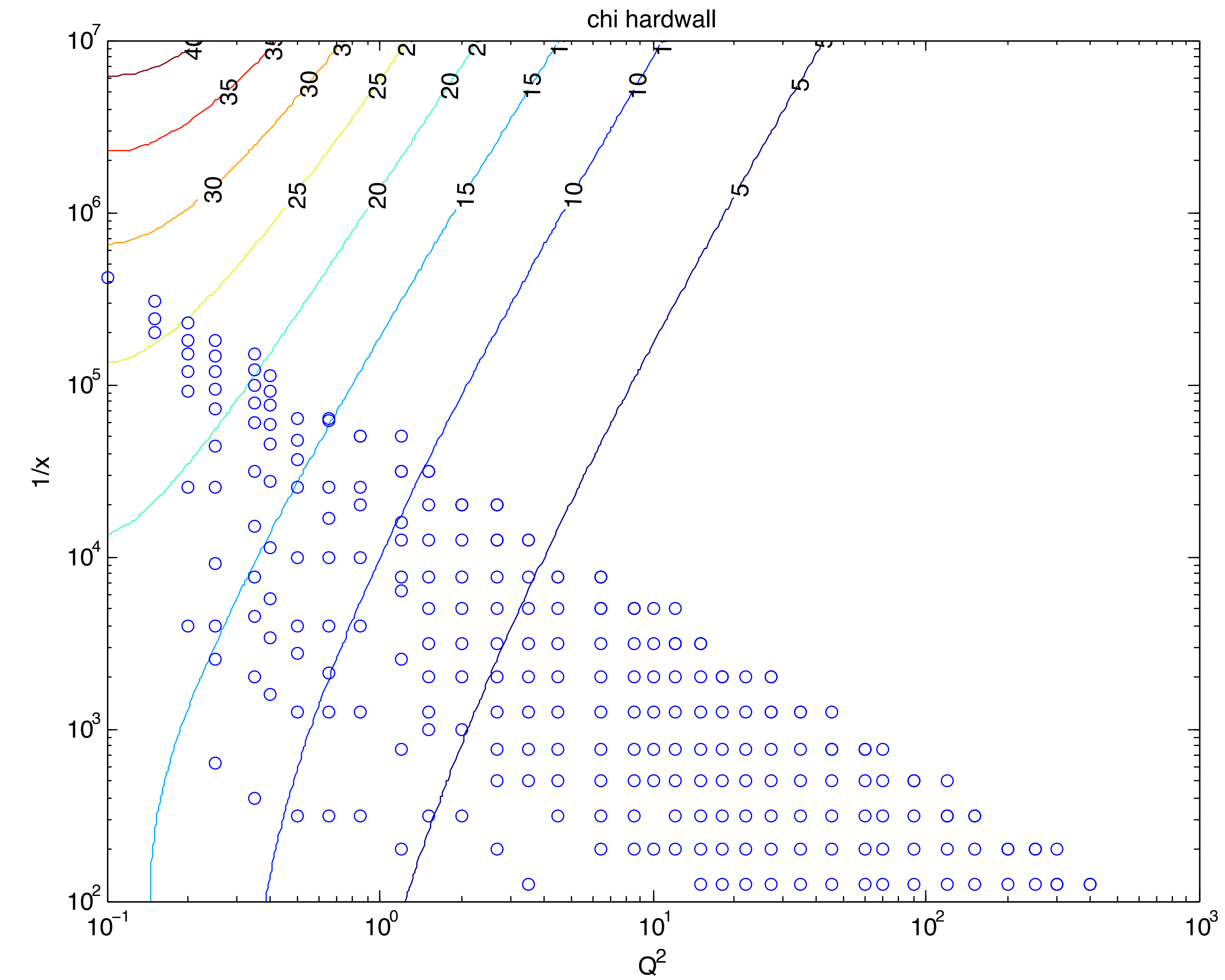
Plots

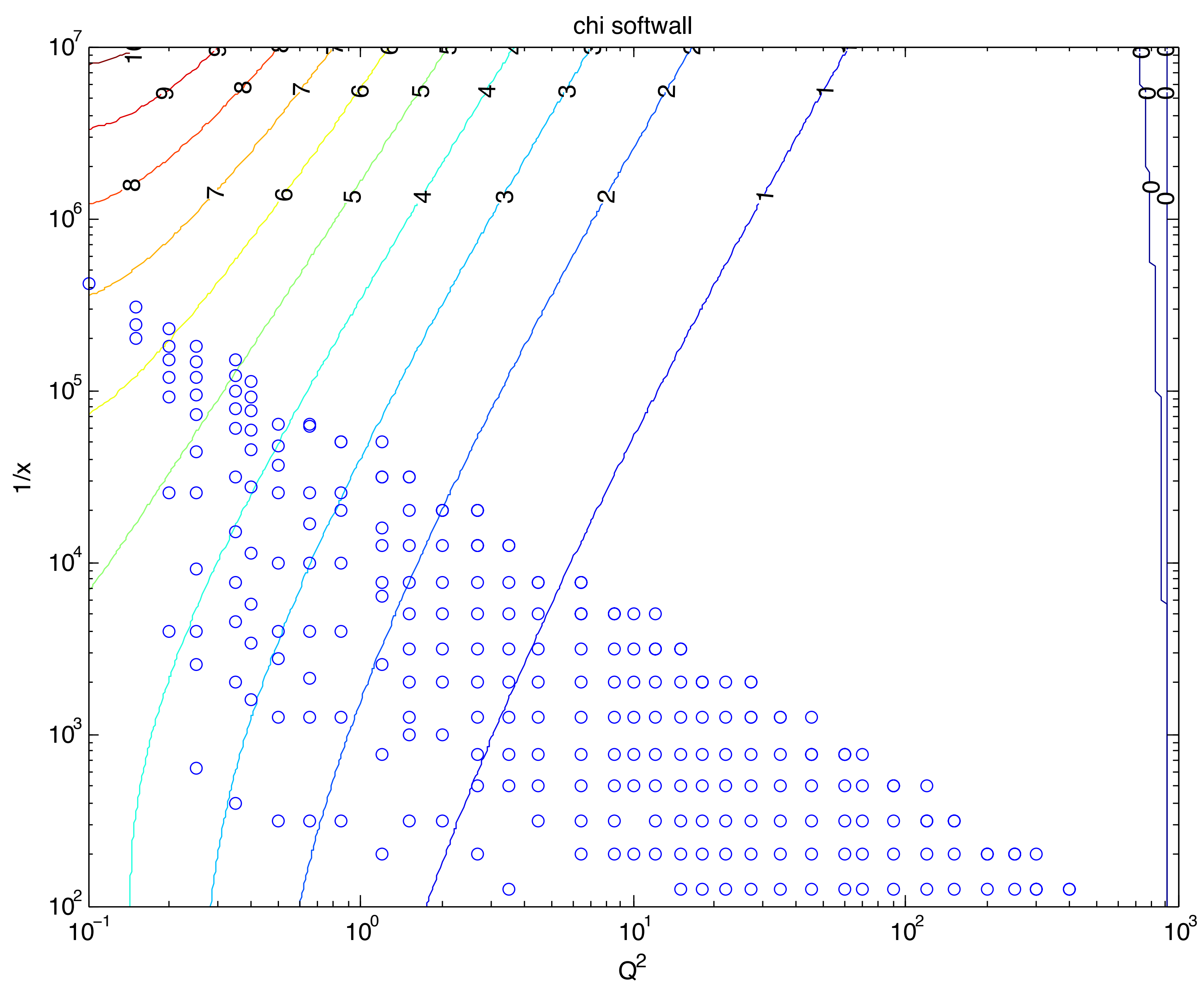


The structure function $F_2(x, Q^2)$ plotted for farious values of Q^2. The data points are from the H1-Zeus collaboration and the solid lines are the soft wall fit values.

Plots Cont.







Contour plots of Im$[\chi]$ as a function of $1/x$ vs Q^2 (Gev) for conformal, hardwall, and softwall models. These plots are all in the forward limit, but the impact parameter representation can tell us about the onset of non-linear eikonal effects. The similar behavior for the softwall implies a similar conclusion about confinement vs saturation.

Comparison With Previous Work

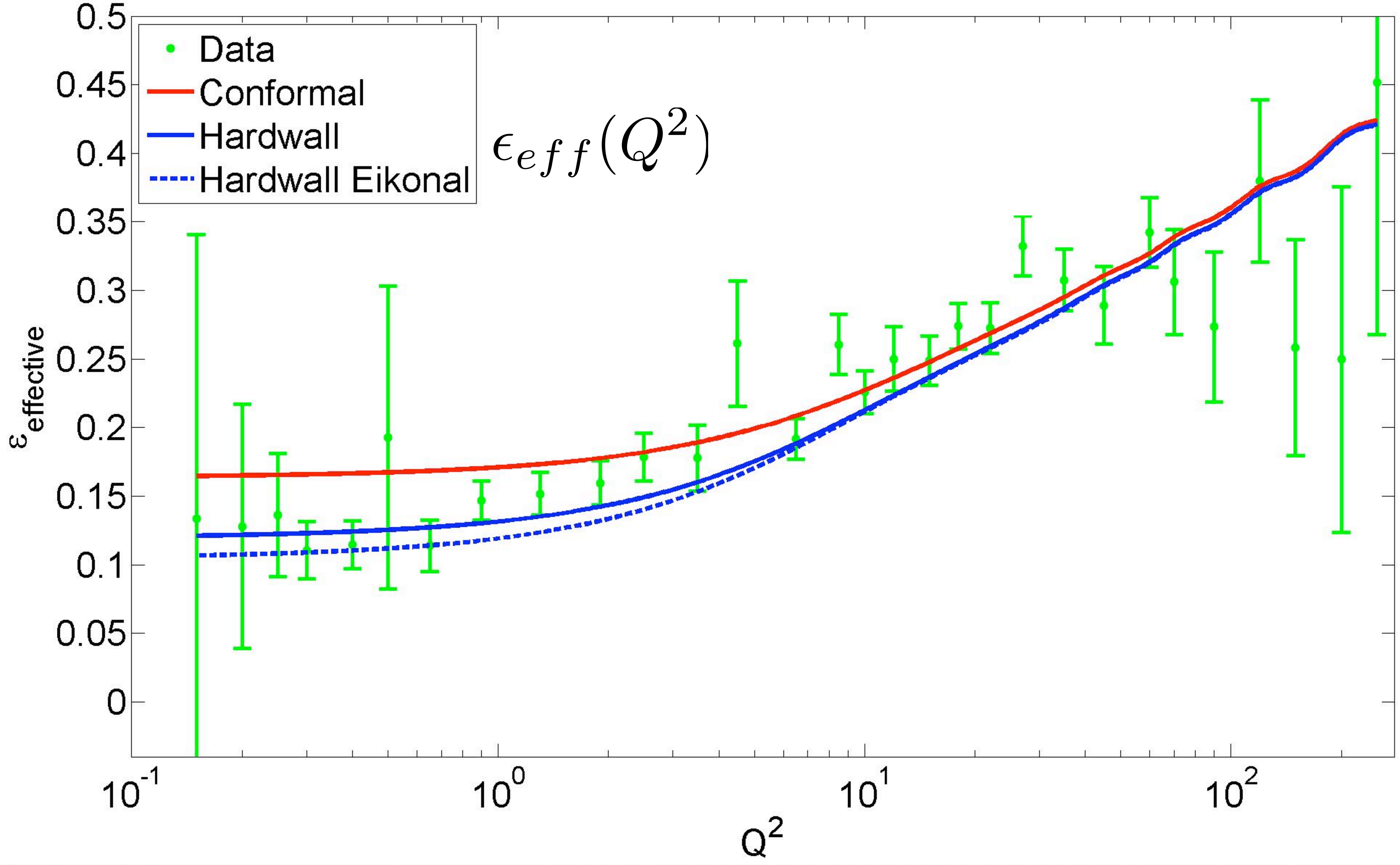
Model	ρ	g_0^2	z_0	Q'	χ^2_{dof}
conformal	0.774*	110.13*	–	0.5575* GeV	11.7 (0.75*)
hard wall	0.7792	103.14	4.96 GeV^{-1}	0.4333 GeV	1.07 (0.69*)
softwall	0.7774	108.3616	8.1798 GeV^{-1}	0.4014 GeV	1.1035
softwall*	0.6741	154.6671	8.3271 GeV^{-1}	0.4467 GeV	1.1245

Comparison of the best fit (including a χ sieve) values for the conformal, hard wall, and soft wall AdS models. The final row includes the soft wall with improved intercept. [Costa, Goncalves, Penedones][Gromov, Levkovich-Maslyuk, Sizov, Valatka]The statistical errors (omitted) are all $\sim 1\%$ of fit parameters.

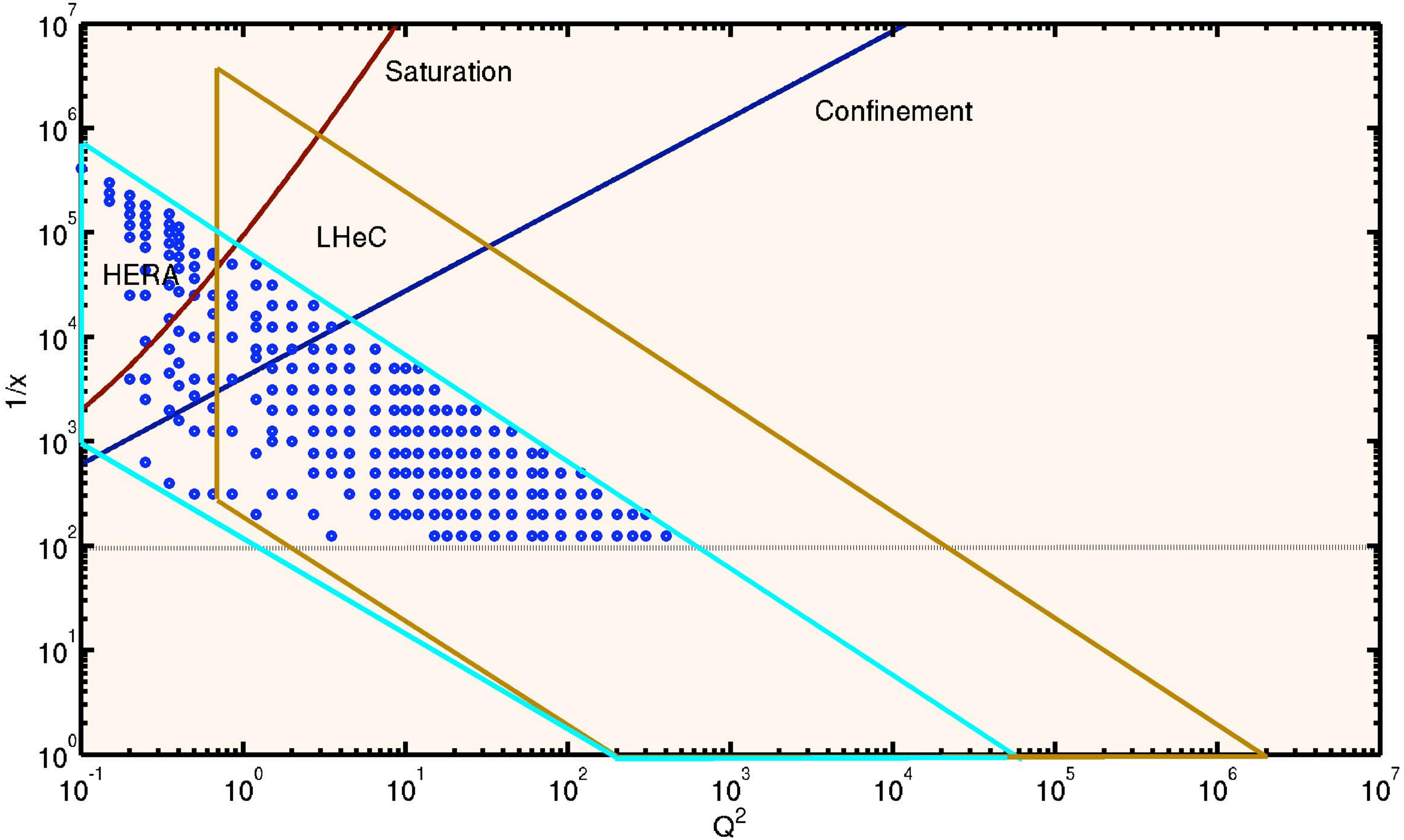
As expected, best fit values imply

$$\rho \to \lambda > 1 \qquad 1/z_0 \sim \Lambda_{QCD} \quad \text{and} \quad Q' \sim m_{proton}$$

$$F_2(x, Q^2) \sim (1/x)^{\epsilon_{effecti}}$$



HERA vs LHeC region: dots are H1-ZEUS small-x data points



IV: More on Pomeron and Odderon in the conformal Limit

Massless modes of a closed string theory:

metric tensor,	$G_{mn} = g^0_{mn} + h_{mn}$
Kolb-Ramond anti-sym. tensor,	$b_{mn} = -b_{nm}$
dilaton, etc.	$\phi, \chi, \cdots$

ANOMALOUS DIMENSION:

$$\Delta(j) = 2 + \sqrt{2}\sqrt{\sqrt{g^2 N_c}(j - j_0)} \qquad \gamma_n = 2\sqrt{1 + \sqrt{g^2 N}(n-2)/2} - n$$

$$\gamma_2 = 0$$

Energy-Momentum Conservation built-in automatically.

Connection to Spin Chain in $\mathcal{N} = 4$ YM:

$$tr\ D^S\ Z^\tau$$

$$\widetilde{\Delta}(S)^2 = \tau^2 + a_1(\tau,\lambda)S + a_2(\tau,\lambda)S^2 + \cdots$$

$$\tau = 2\,, \quad \widetilde{\Delta}(S) = \Delta(S+2) - 2$$

$$trF_{\mu\nu}D_\nu \cdots D_{\nu'}F_{\nu'\mu'}$$

$$a_1(2,\lambda) = 2\sqrt{\lambda} - 1 + O(1/\sqrt{\lambda})$$

$$a_2(2,\lambda) = 3/2 + O(1/\sqrt{\lambda})$$

B.Basso, 1109.3154v2

$$S = 0 \rightarrow \mathrm{BPS}$$

$$\widetilde{\Delta}(S)^2 \simeq 4 + 2\sqrt{\lambda}\ S$$

Gauge/String Duality: Conformal Limit

- C=+1: Pomeron <===> Graviton

$$j_0^{(+)} = 2 - 2/\sqrt{\lambda} + O(1/\lambda) .$$

- C=-1: Odderon <===> Kalb-Ramond Field

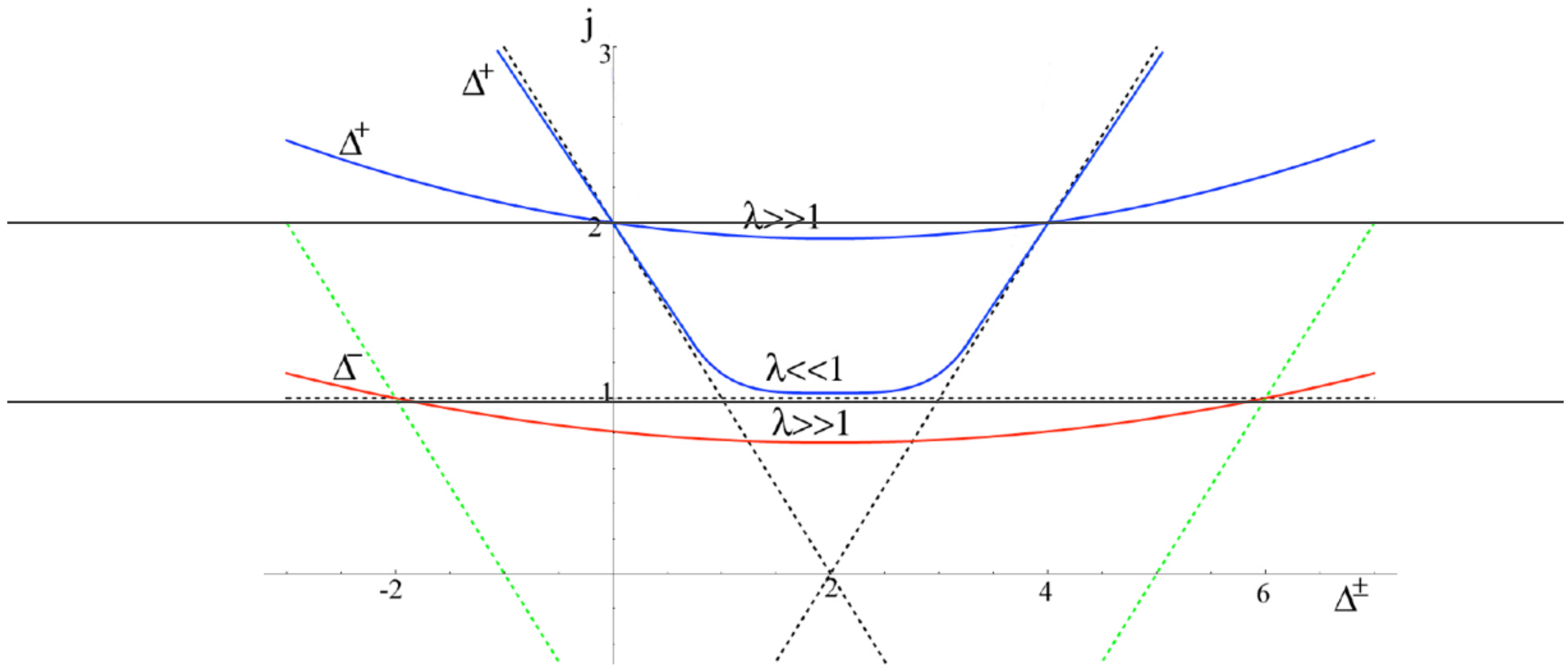
$$j_0^{(-)} = 1 - m_{AdS}^2/2\sqrt{\lambda} + O(1/\lambda) .$$

	Weak Coupling	Strong Coupling
$C=+1$	$j_0^{(+)} = 1 + (\ln 2)\ \lambda/\pi^2 + O(\lambda^2)$	$j_0^{(+)} = 2 - 2/\sqrt{\lambda} + O(1/\lambda)$
$C=-1$	$j_{0,(1)}^{(-)} \simeq 1 - 0.24717\ \lambda/\pi + O(\lambda^2)$ $j_{0,(2)}^{(-)} = 1 + O(\lambda^3)$	$j_{0,(1)}^{(-)} = 1 - 8/\sqrt{\lambda} + O(1/\lambda)$ $j_{0,(2)}^{(-)} = 1 + O(1/\lambda)$

Table 1: Pomeron and Odderon intercepts at weak and strong coupling.

Spectral Curves: J vs Δ

$$\Delta^{(\pm)}(j) = 2 + \sqrt{2}\,\lambda^{1/4}\sqrt{(j - j_0^{(\pm)})}$$



POMERON AND ODDERON IN STRONG COUPLING:

$$\widetilde{\Delta}(S)^2 = \tau^2 + a_1(\tau,\lambda)S + a_2(\tau,\lambda)S^2 + \cdots$$

B.Basso, 1109.3154v2

POMERON

$$\alpha_p = 2 - \frac{2}{\lambda^{1/2}} - \frac{1}{\lambda} + \frac{1}{4\lambda^{3/2}} + \frac{6\zeta(3)+2}{\lambda^2} + \frac{18\zeta(3)+\frac{361}{64}}{\lambda^{5/2}} + \frac{39\zeta(3)+\frac{447}{32}}{\lambda^3} + \cdots$$

Brower, Polchinski, Strassler, Tan

Gromov et al.

ODDERON

Kotikov, Lipatov, et al.

Costa, Goncalves, Penedones (1209.4355)

Kotikov, Lipatov (1301.0882)

Solution-a: $$\alpha_O = 1 - \frac{8}{\lambda^{1/2}} - \frac{4}{\lambda} + \frac{13}{\lambda^{3/2}} + \frac{96\zeta(3)+41}{\lambda^2} + \frac{288\zeta(3)+\frac{1823}{16}}{\lambda^{5/2}} + \frac{720\zeta(5)+1344\zeta(3)-\frac{3585}{4}}{\lambda^3}$$

Solution-b: $$\alpha_O = 1 - \frac{0}{\lambda^{1/2}} - \frac{0}{\lambda} + \frac{0}{\lambda^{3/2}} + \frac{0}{\lambda^2} + \frac{0}{\lambda^{5/2}} + \frac{0}{\lambda^3} + \cdots$$

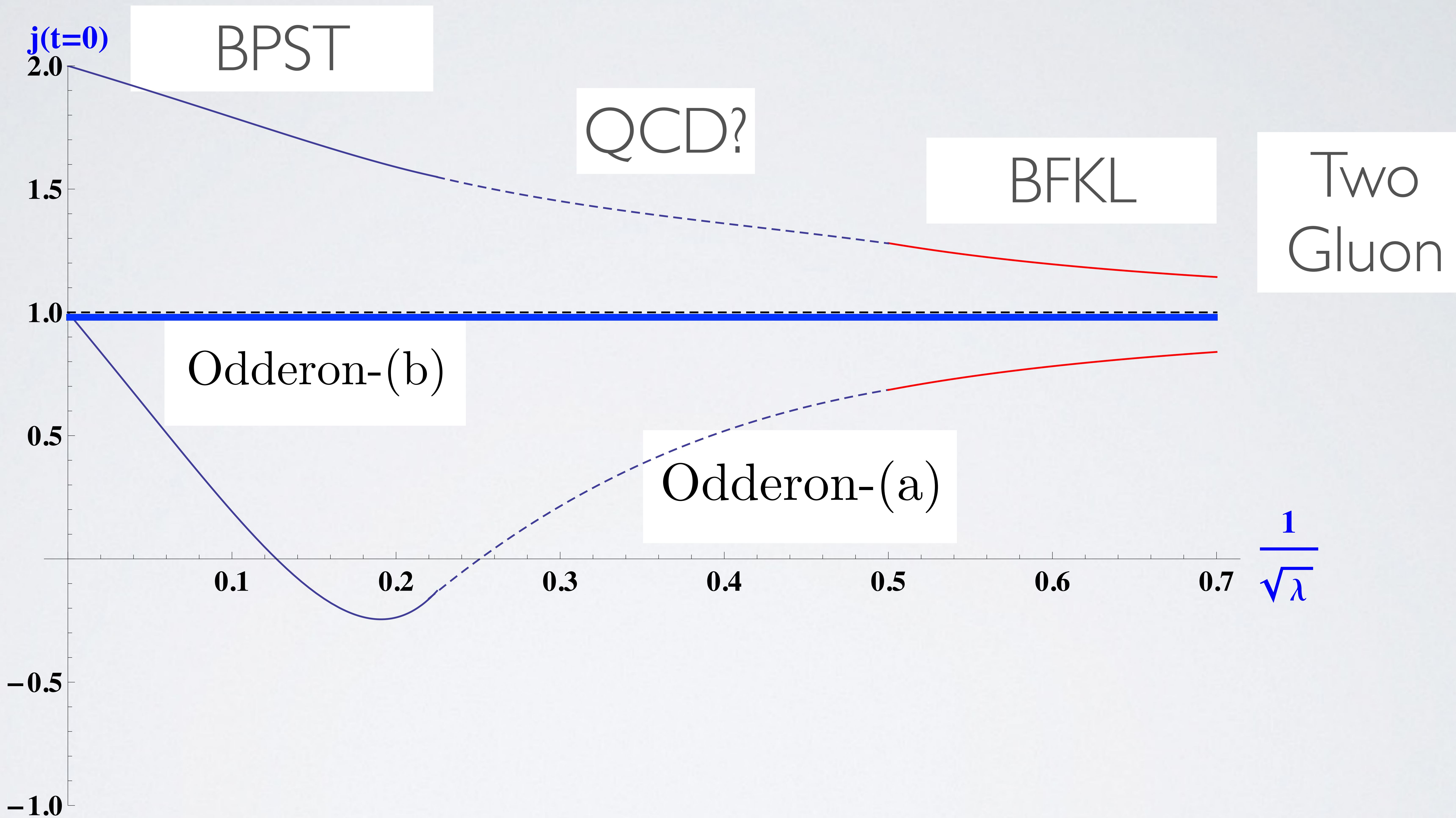
Brower, Djuric, Tan

Avsar, Hatta, Matsuo

Brower, Costa, Djuric, Raben, Tan (to appear shortly.)

$\mathcal{N} = 4$ Strong vs Weak $g^2 N_c$

Graviton



$j_0 = 1$

VIII. Summary and Outlook

- Provide meaning for Pomeron non-perturbatively from first principles.
- Realization of conformal invariance beyond perturbative QCD
- New starting point for unitarization, saturation, etc.
- Phenomenological consequences, DIS at small-x, Diffractive Higgs production at LHC, etc.