Renormalization group analysis of reggeon field theory: flow equations

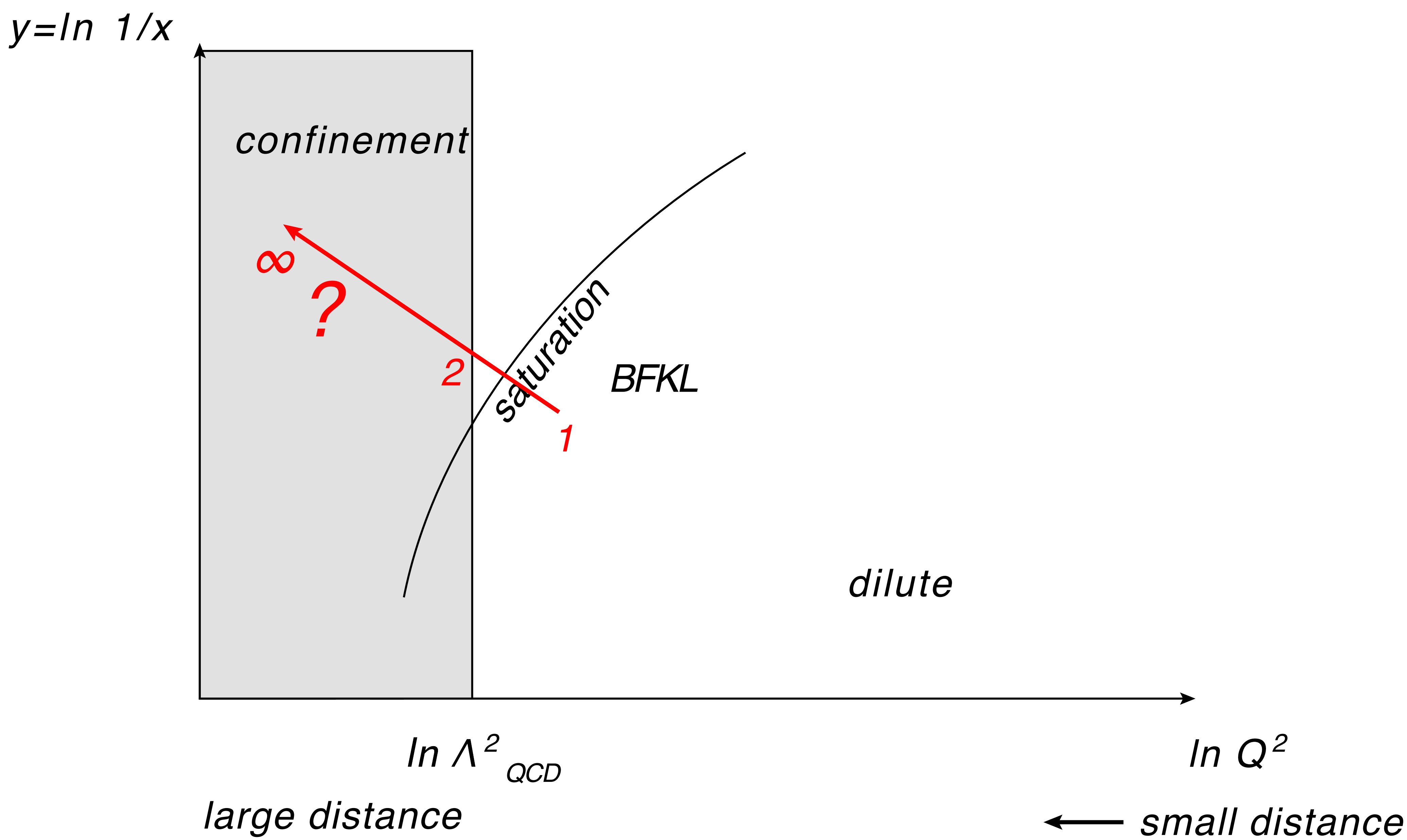
(Project and first results)

Collaboration with C.Contreras and G.P.Vacca

- Motivation and project
- Flow equations
- First results: fixed points

I. Motivation and project

Question: how to continue small-x physics into the nonperturbative region?



Along the red line:

rapidity, transverse distance: from low to large scale
reggeon energy (angular momentum), momentum: from large to low scale (IR problem).

1) BFKL with IR boundary conditions (no fixed cut) Lipatov, Ross, Kowalski

2) ‘Pomeron loops’:
corrections, Pomeron = superposition of 2 and 4 reggeized gluons
more corrections, Pomeron= superposition of 2, 4, 6, reggeized gluons

Toy model: BK equations

. . .

∞) (Nonperturbative) Pomeron with (nonlocal?) self-interactions

Important:

- Pomeron field has internal degrees of freedom (BFKL), nonlocal RFT
- Pomeron field changes as function of scale (rapidity and distance)

→ Wilson RG equation, flow equations

The formalism: functional renormalization group

Reminder: Wilson approach

The standard Wilsonian action is defined by an iterative change in the UV-cutoff induced by a partial integration of quantum fluctuations:

$$\Lambda \to \Lambda' < \Lambda$$

$$\int [\mathrm{d}\varphi]^{\Lambda} e^{-S^{\Lambda}[\varphi]} = \int [\mathrm{d}\varphi]^{\Lambda'} e^{-S^{\Lambda'}[\varphi]} \qquad k < \Lambda$$

Alternatively: FRG-approach (Wetterich) IR-cutoff

(successful use in statistical mechanics and in gravity)

define a bare theory at scale Λ .
The integration of the modes in the interval $[k, \Lambda]$ defines a k-dependent average functional.
Letting k flowing down to 0 defines a flow for the functional which leads to full theory. k-dependent effective action:

regulator

$$e^{-\Gamma_k[\phi]} = \int [d\varphi]\mu_k e^{-S[\varphi] + \int_x (\varphi-\phi)_x \frac{\delta\Gamma_k[\phi]}{\delta\phi_x} - \Delta S_k[\varphi-\phi]}$$

Taking a derivative with respect the RG time t=log (k/k_0) one obtains

$$\partial_t \Gamma_k = \frac{1}{2} Tr\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} \partial_t \mathcal{R}_k\right] - \frac{\dot{\mu}_k}{\mu_k}$$

$$\mathcal{R} = \text{regulator operator}$$

which is UV and IR finite.
Quantum fluctuations $\rightarrow$ coupled differential equations

Steps:

1) beta-functions for parameters of the potential:
zeroes determine fixed points, existence of possible theories.
Expand in powers of fields:

$$\partial_t \Gamma_k[\phi] = \mathcal{F}[\Gamma_k[\phi], \Gamma_k'[\phi], \Gamma_k''[\phi]]$$

Equate powers of field on both sides $\rightarrow$ sets of beta functions

2) Vertex functions (physical observables: total cross section)

Taking functional derivatives in the fields:

$$\partial_t \Gamma_k = \frac{1}{2} G_{k;AB}\, \partial_t \mathcal{R}_{k;BA}$$

$$\partial_t \Gamma^{(1)}_{k;A_1} = -\frac{1}{2} G_{k;AB} \Gamma^{(3)}_{k;A_1 BC} G_{k;CD}\, \partial_t \mathcal{R}_{k;DA}$$

$$\begin{aligned}\partial_t \Gamma^{(2)}_{k;A_1 A_2} = & \frac{1}{2} G_{k;AB} \Gamma^{(3)}_{k;A_1 BC} G_{k;CD} \Gamma^{(3)}_{k;A_2 DE} G_{k;EF}\, \partial_t \mathcal{R}_{k;FA} \\ & + \frac{1}{2} G_{k;AB} \Gamma^{(3)}_{k;A_2 BC} G_{k;AB} \Gamma^{(3)}_{k;A_1 BC} G_{k;CD}\, \partial_t \mathcal{R}_{k;DA} \\ & - \frac{1}{2} G_{k;AB} \Gamma^{(4)}_{k;A_1 A_2 BC} G_{k;CD}\, \partial_t \mathcal{R}_{k;DA}\end{aligned}$$

First results

Local reggeon field theory:

Abarbanel, Bronzan;
Migdal, Polyakov, Ter-Martyrosyan

ϵ -expansion around D=4: IR fixpoint

$$\mathcal{L} = (\tfrac{1}{2}\psi^\dagger \overleftrightarrow{\partial}_y \psi - \alpha' \psi^\dagger \nabla^2 \psi) + V(\psi, \psi^\dagger)$$

$$V(\psi, \psi^\dagger) = -\mu \psi^\dagger \psi + i\lambda \psi^\dagger (\psi^\dagger + \psi)\psi$$
$$+ g(\psi^\dagger \psi)^2 + g' \psi^\dagger ({\psi^\dagger}^2 + \psi^2)\psi + \cdots$$

$$\mu = \alpha(0) - 1$$

J. Cardy and R. Sugar noticed in 1980 that the RFT is in the same universality class a Markov process known as Directed Percolation (DP).
Critical exponents can then be accessed also with numerical montecarlo computations.

Effective action with local potential:

$$\Gamma_k = \int \mathrm{d}y\, \mathrm{d}^D x \left[Z_k(\frac{1}{2}\psi^{dagger}\overset{\leftrightarrow}{\partial_y}\psi - \alpha_k' \psi^{dagger}\nabla^2\psi) + V_k(\psi,\psi^\dagger) \right]$$

Propagator of flow equations:

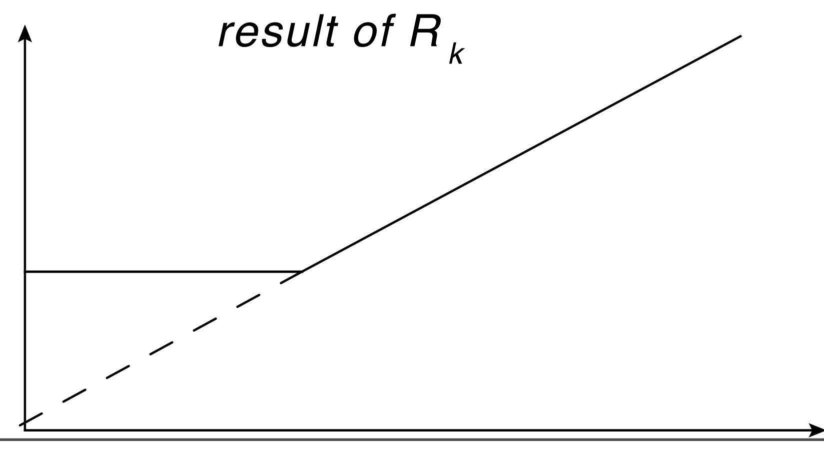
$$\Gamma_k^{(2)} + \mathbb{R} = \begin{pmatrix} V_{k\psi\psi} & -iZ_k\omega + Z_k\alpha_k' q^2 + R_k + V_{k\psi\psi^{dagger}} \\ iZ_k\omega + Z_k\alpha_k' q^2 + R_k + V_{k\psi^{dagger}\psi} & V_{k\psi^{dagger}\psi^{dagger}} \end{pmatrix}$$

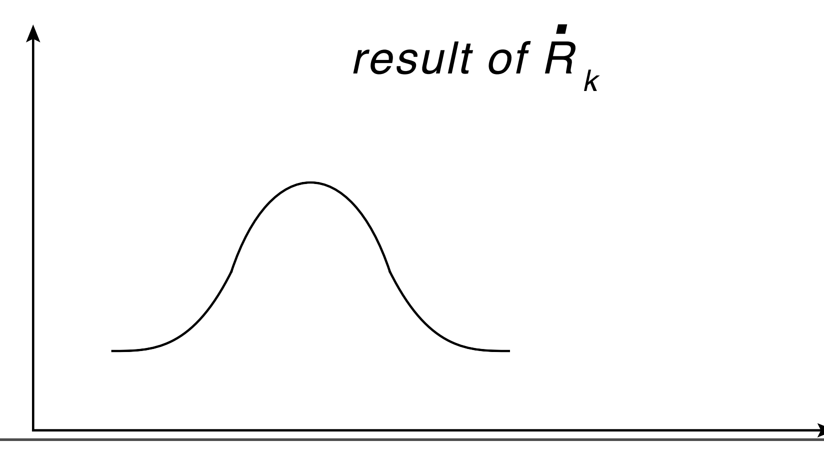
Flow equation for potential:

$$\dot{V}_k(\psi,\psi^\dagger) = \frac{1}{2}\mathrm{tr}\left\{ \int \frac{\mathrm{d}\omega\, \mathrm{d}^D q}{(2\pi)^{D+1}} \left[\left(\Gamma_k^{(2)} + \mathbb{R}\right)^{-1} \dot{\mathcal{R}}_k \right] \right\}$$

Coarse graining in momentum space:

$$R_k(q) = Z_k\alpha_k'(k^2 - q^2)\Theta(k^2 - q^2)$$





Flow equation for the potential:
expand in powers of the fields on both sides,
truncate after some maximal power.

Obtain coupled set of ‘beta functions’, e.g.

$$\dot{\tilde{\mu}} = \tilde{\mu}(-2+\zeta+\eta) + 2N_D A_D(\eta_k,\zeta_k)\frac{\tilde{\lambda}^2}{(1-\tilde{\mu})^2}$$

$$\dot{\tilde{\lambda}} = \tilde{\lambda}\left((-2+\zeta+\frac{D}{2}+\frac{3\eta}{2}) + 2N_D A_D(\eta_k,\zeta_k)\left(\frac{4\tilde{\lambda}^2}{(1-\tilde{\mu})^3}+\frac{(\tilde{g}+3\tilde{g}')}{(1-\tilde{\mu})^2}\right)\right)$$

$$\cdots$$

Fixed points: search for zeroes.

First numerical results on the existence of fixed points: preliminary (3 different truncations)

In all truncations there is always the trivial (zero coupling solution) FP.
The eigenvalues of the stability matrix in this point are:
cubic(-2,-1), quartic (add (0,0)), quintic (add (1,1)), etc

- cubic: $\mathrm{FP}_3 = (\tilde{\mu}^*, \tilde{\lambda}^*) = (0.111, \pm 1.05)$
  eigenvalues:$(2.39, -1.89)$
- quartic: $\mathrm{FP}_4 = (\tilde{\mu}^*, \tilde{\lambda}^*, \tilde{g}^*, \tilde{g}'^*) = (0.27, \pm 1.35, -2.89, -1.27)$
  eigenvalues:$(19.99, 6.08, 2.51, -1.69)$
- quintic: $\mathrm{FP}_5 = (\tilde{\mu}^*, \tilde{\lambda}^*, \tilde{g}^*, \tilde{g}'^*, \tilde{\lambda}_5^*, \tilde{\lambda}_5'^*) = (0.39, \pm 1.35, -4.10, -1.82, -4.83, -1.34)$
  eigenvalues:$(59.11, 33.12, 16.26, 3.99, 2.13, -1.45)$

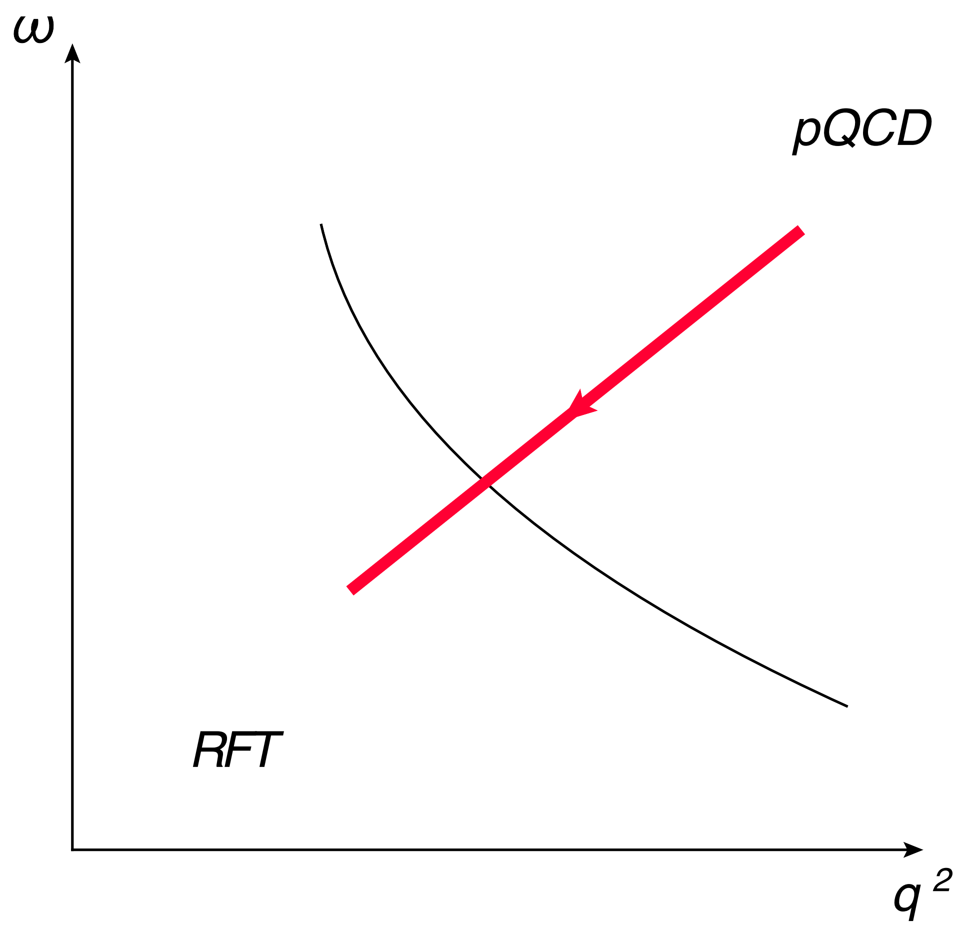
Test: compare with Monte Carlo result.
The leading critical exponent ν (the most negative eigenvalue) (with increasing truncation) is:
$\nu_3 = 0.52, \quad \nu_4 = 0.59, \quad \nu_5 = 0.69$
Compare with the Monte Carlo result for Directed Percolation (same universality class): $\nu = 0.73$.

Our simple estimate in the quintic coupling truncation) is within 5%.

Interpretation:

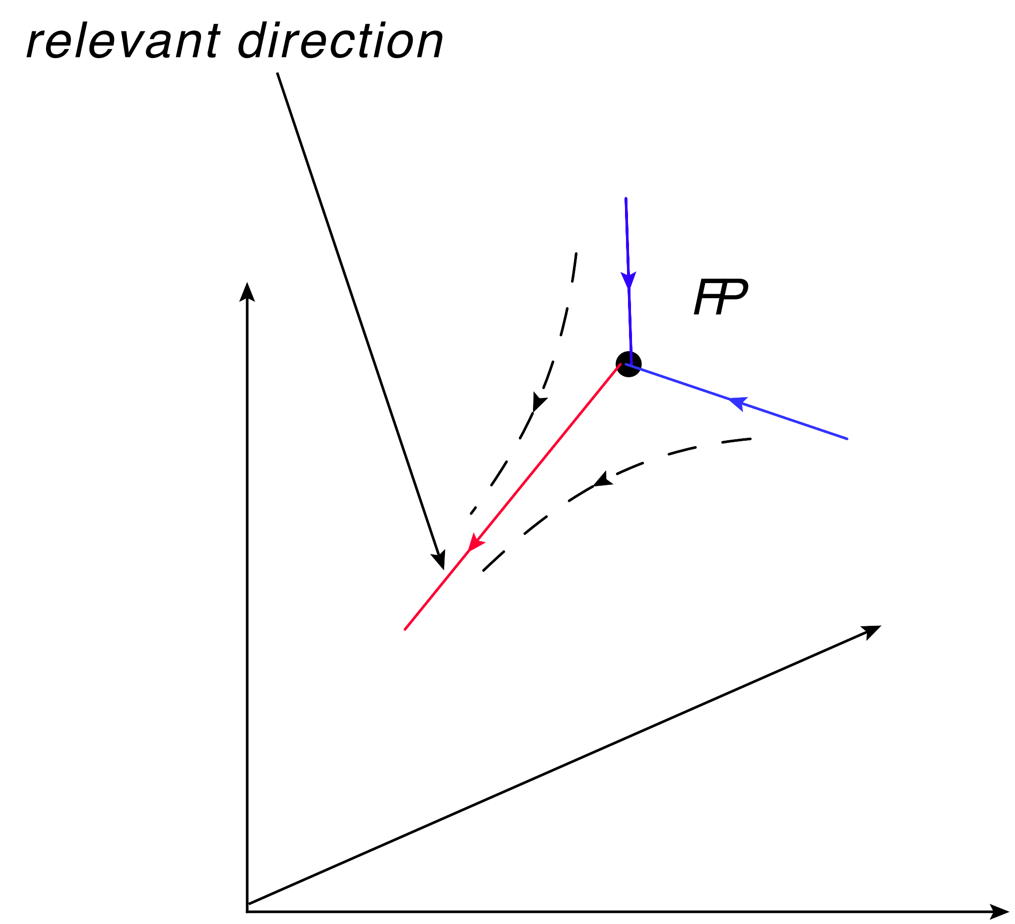
The family of non trivial fixed points is characterized by just one relevant direction.

We feel tempted to interpret this as follows:
possible transition from QCD in the Regge limit to a RFT effective description.



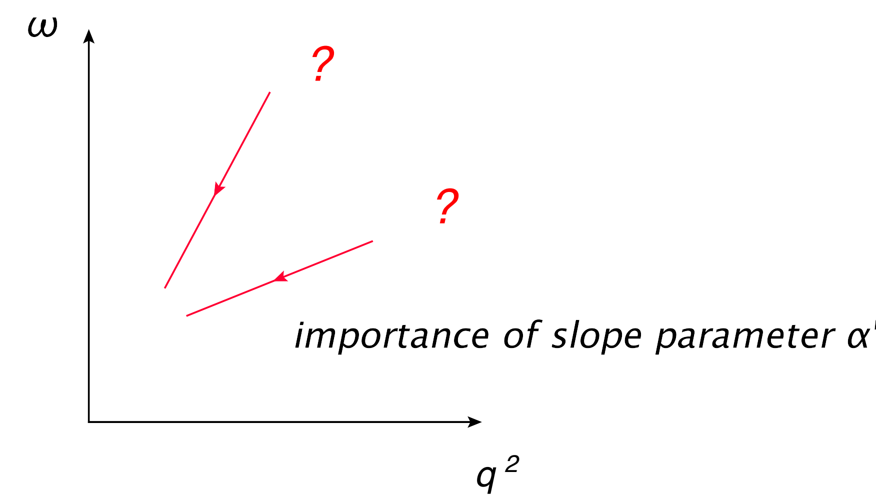
interest in the IR region:

$$\left\{ \begin{array}{l} \text{small } \omega, \text{ small } q^2 \\ \text{large rapidity, large distances} \end{array} \right\}$$



space of couplings

Next steps:



Other course graining (cutoff) schemes:

- controlling integration in energy
- controlling the integration in energy and momentum
  (first indications that results are consistent with first cutoff-scheme)
- expansion around nonzero fields (nonzero vacuum configuration) A.White
- nonlocal interactions

- Compute Green's functions: physical observables

Conclusions

- We have started to study features of RFT, as a candidate for a possible effective theory of the Regge limit of QCD at large rapidities/distances.
- Have used functional renormalization group techniques (flow equations), suitable also for nonperturbative aspects.
- From preliminary results we find a nontrivial fixed point with one relevant direction: this may lead to a possible physical scenario.
- Steps under way: further course graining schemes.
- How important are possible nonzero vacuum configurations? Phase transitions?
- Physical observables: energy dependence of total cross section
- Most difficult: study the transition, from pQCD to RFT?