SATURATION AT STRONG COUPLING

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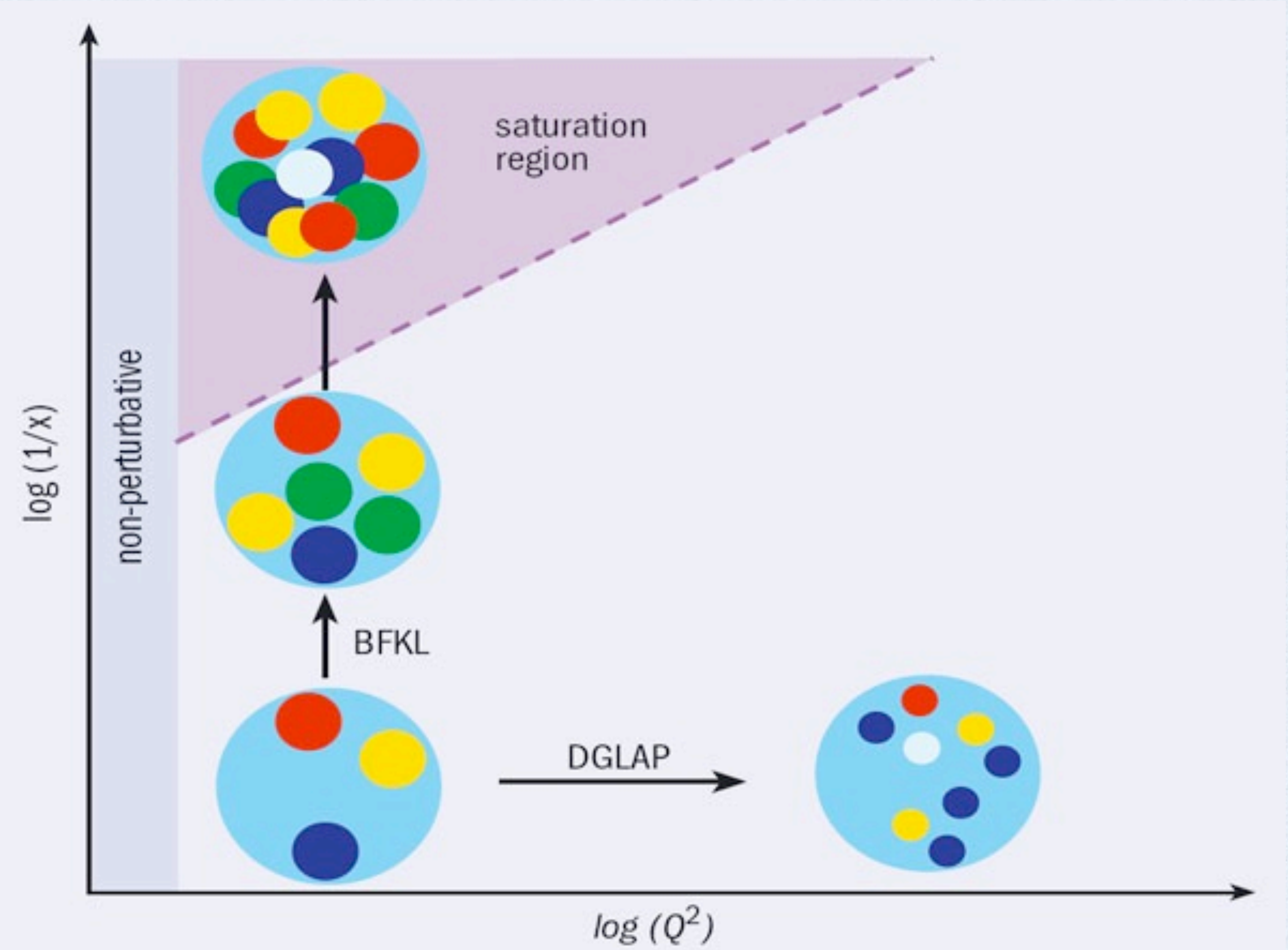
MOTIVATION

Can we calculate the saturation scale from a BFKL/DGLAP model?



BFKL EQUATION

The BFKL evolution equation for unintegrated gluon density reads:

$$f(x,k^2) = f_0(x,k^2) + \bar{\alpha}_s k^2 \int_{x/x_0}^{1} \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[\frac{f(x/z,l^2) - f(x/z,k^2)}{|l^2-k^2|} + \frac{f(x/z,k^2)}{\sqrt{4l^4+k^4}} \right],$$

one can use Mellin transform to get

$$f(x,k^2) = \frac{1}{2\pi i} \int d\gamma (k^2)^\gamma \frac{1}{2\pi i} \int d\omega x^{-\omega} \frac{\omega \overline{f}_0(\omega,\gamma)}{\omega - \overline{\alpha_s}\chi(\gamma)}.$$

$$\chi(\gamma) = 2\psi(1) - \psi(1-\gamma) - \psi(\gamma).$$

In order to evaluate this integral one needs to know the chcracteristic function along the imaginary axis in the γ plane.

Diffusion Equation

The dimensionful unintegrated gluon density reads

$$\mathcal{F}(x,k^2) = \frac{\mathcal{F}(x_0,1/2)}{\sqrt{4\pi\ln(x_0/x)1/2\lambda'}} e^{\lambda\ln(x_0/x)-1/2\ln(k^2/k_0^2)} e^{\frac{-\ln(k^2/k_0^2)^2}{4\,1/2\lambda'\ln(x_0/x)}},$$

where $\mathcal{F}(x_0,1/2) = f(x_0,1/2)/k^2$ and $\chi(1/2+i\nu) \approx \lambda - \frac{1}{2}\lambda'\nu^2$ with $\lambda = \bar{\alpha}_s 4\ln 2$ and $\lambda' = \bar{\alpha}_s\zeta(3)$.

From this explicit form, one may extract the coefficients of the diffusion equation

$$\partial_Y \mathcal{F}(Y,\rho) = \frac{1}{2}\lambda'\partial_\rho^2\mathcal{F}(Y,\rho) + \frac{1}{2}\lambda'\partial_\rho\mathcal{F}(Y,\rho) + (\lambda+\lambda'/8)\mathcal{F}(Y,\rho).$$

where $Y = \ln\frac{x_0}{x},\ \rho = \ln\frac{k^2}{k_0^2}$.

IMPROVING THE MODEL

Kinematical constraint

$$f(x,k^2)=\frac{1}{2\pi i}\int d\gamma (k^2)^\gamma \frac{1}{2\pi i}\int d\omega x^{-\omega}\frac{\omega \overline{f}_0(\omega,\gamma)}{\omega-\bar{\alpha}_s\chi_{k.c.}(\gamma,\omega,)}.$$

Transcendental equation

$$\omega=\bar{\alpha}_s\chi_{k.c.}(\gamma,\omega)\equiv\chi_{eff\;k.c.}(\gamma,\omega).$$

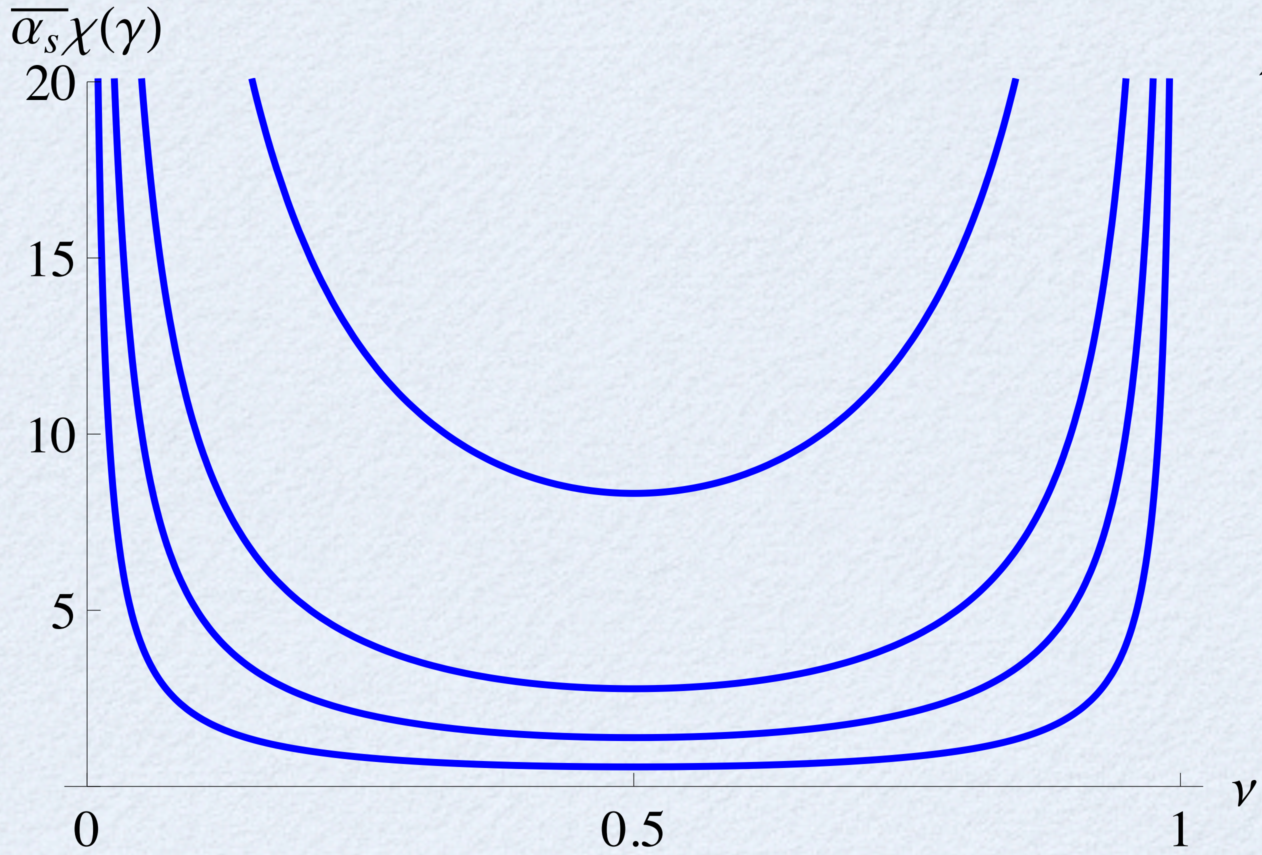
It has been suggested by Staśto that, in order to have a more complete treatment of the contribution of higher orders from the point of view of BFKL, one includes the following modification:

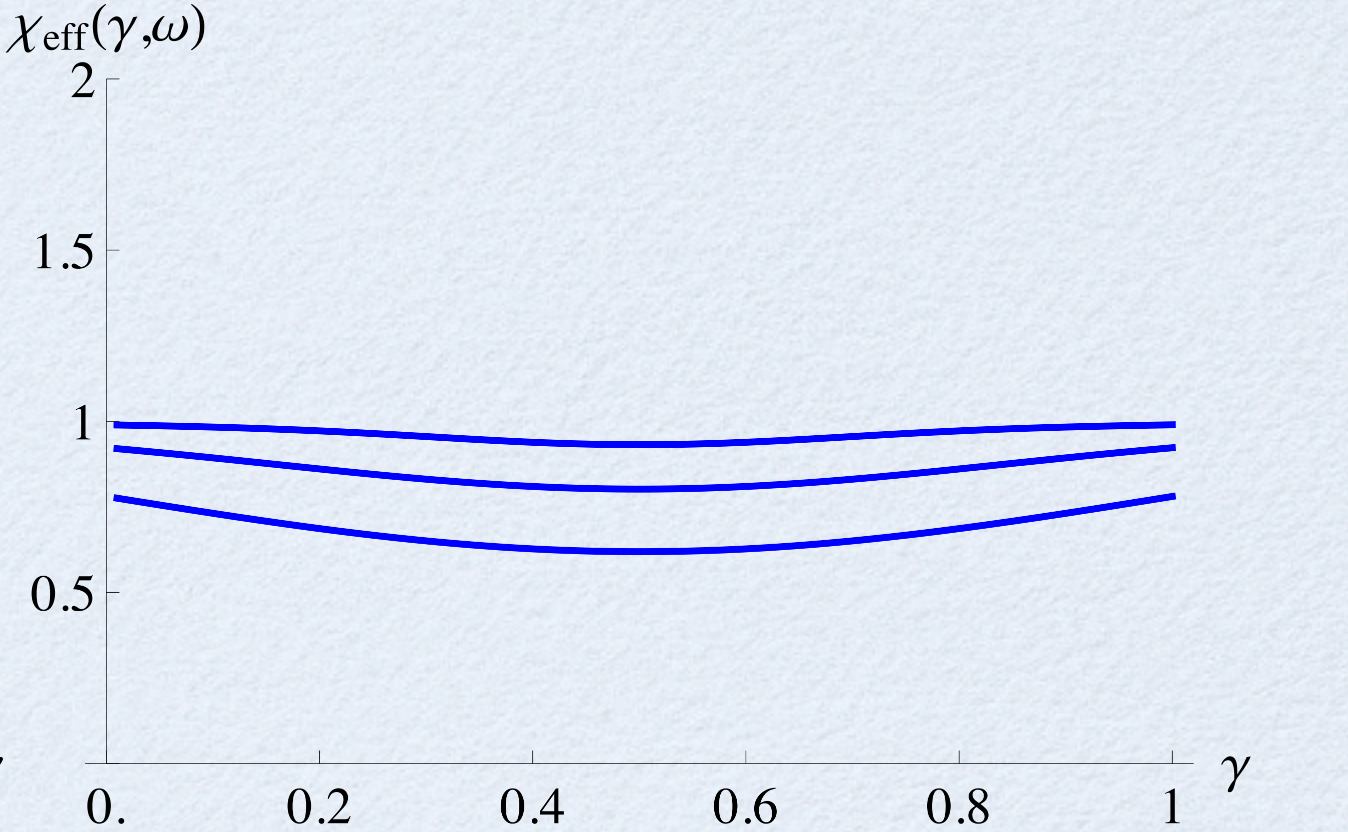
DGLAP anom. dim.

$$\frac{1}{\bar{\alpha}_s}=\gamma^{(0)}(\omega)\chi_{k.c.}(\gamma,\omega),$$

$$\gamma^{(0)}(\omega)=\frac{1}{\omega}+\frac{1}{\omega+1}+\frac{1}{\omega+2}-\frac{1}{\omega+3}-\psi(2+\omega)+\psi(1)+\frac{11}{12}.$$

Characteristic function





We see that the additional contributions stabilize completely the eigenvalue and allow for the investigations of the BFKL in the whole spectrum of the coupling constant. We can extract the characteristic function numerically:

$$\chi_{eff\,\infty}(\omega, 1/2 + i\nu) = 1.02795 - 2.04635\nu^2 \equiv \lambda_{st} - \frac{1}{2}\lambda'_{st}\nu^2.$$

ADS/CFT INTUITION

The main message that comes from our discussion so far is that the diffusive behavior is not particular to the weak coupling regime of the BFKL evolution. In fact it is the dominant contribution in the strong coupling as well. This is not an artifact of the model we are considering. A similar observation was done in the context of gauge and gravity

$$f_{PSBC}(\rho, s) \approx \frac{1}{\sqrt{4\pi \mathcal{D} Y}} e^{j_0 Y} e^{\frac{-\rho^2}{4\mathcal{D}Y}},$$

$$j_0 = 2 - \frac{2}{\sqrt{\lambda}} + O(1/\lambda)\ , \quad \mathcal{D} = \frac{1}{2\sqrt{\lambda}} + O(1/\lambda) \quad .$$

The above gluon distribution function satisfies the diffusion equation

$$\frac{\partial f_{PSCB}(Y,\rho)}{\partial Y} = \mathcal{D} \frac{\partial^2 f_{PSCB}(Y,\rho)}{\partial \rho^2} + j_0 f_{PSCB}(Y,\rho).$$

Balitsky-Kovchegov Eq.

The linear BFKL evolution equation misses a very important aspect of the high-energy scattering, namely, the saturation physics. Several approaches were constructed in order to include non-linear effects, like multiple scattering and gluon saturation, responsible for the unitarization of the scattering amplitudes. A particularly useful and simple enough approach to unitarize the cross section is the Balitsky-Kovchegov (BK) equation,

$$\Phi(x,k^2) = \Phi_0(x,k^2) + \Phi_1(x,k^2),$$

$$\Phi_1(x,k^2) = \bar{\alpha}_s \int_x^1 \frac{dz}{z} \left\{ \int_0^\infty \frac{dl^2}{l^2} \left[\frac{l^2\Phi(x/z,l^2) - k^2\Phi(x/z,k^2)}{|l^2-k^2|} + \frac{k^2\Phi(x/z,k^2)}{\sqrt{4l^4+k^4}} \right] - \frac{\overline{\alpha}_s}{\pi R^2}\Phi^2(x/z,k^2) \right\}.$$

Saturation Scale

The BK equations belongs to the universality class of Fisher-Kolmogorov-Petrovsky-Piscounov equation

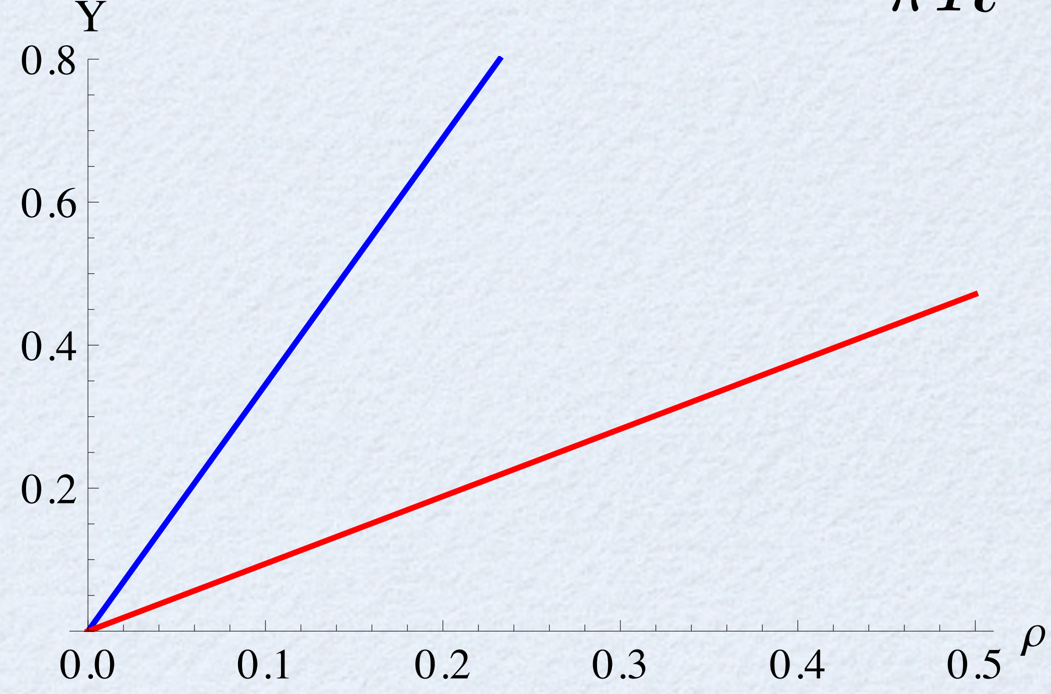
$$\partial_t u(t,x) = \partial_x^2 u(t,x) + u(t,x) - u^2(t,x).$$

One can view this equation as a diffusion equation supplemented with a non-linear term that encodes saturation. We can make an analogous ansatz in the model we analyzed before:

$$\Phi(Y,\rho) = \frac{1}{2}\lambda'_{st}\partial_\rho^2\Phi(Y,\rho) + \frac{1}{2}\lambda'_{st}\partial_\rho\Phi(Y,\rho) + (\lambda_{st}+\lambda'_{st}/8)\Phi(Y,\rho) - \frac{\bar{\alpha}_s}{\pi R^2}\Phi^2(Y,\rho),$$

and extract the saturation scale

$$Q_s^2(Y) \simeq e^{1.06\,Y}.$$



Conclusions

- We postulated a gluon density evolution equation at strong coupling in a model with kinematical constraints and DGLAP anomalous dimension
- We calculated the saturation scale which agrees with the previous predictions in SYM theory

Open problems

- Entropy
- Equation in the full spectrum of the coupling constatnt