Forward hadron production in saturation formalism

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in collaboration with Bowen Xiao, Feng Yuan and David Zaslavsky
arXiv:1307.4057, arXiv:1405.6311

Outline

- Forward single inclusive hadron production in p(d)-A collisions at low x including saturation

  - LO formalism for particle production at low x including multiple scattering

  - NLO formalism: building blocks, collinear and rapidity divergencies. Factorization formula.

  - Numerical evaluation: comparison with data

  - Matching of small x and collinear formalism

  - Summary

Proton-nucleus collisions

Forward single inclusive hadron production in pA collision

$$p + A \rightarrow h(y, p_\perp) + X$$
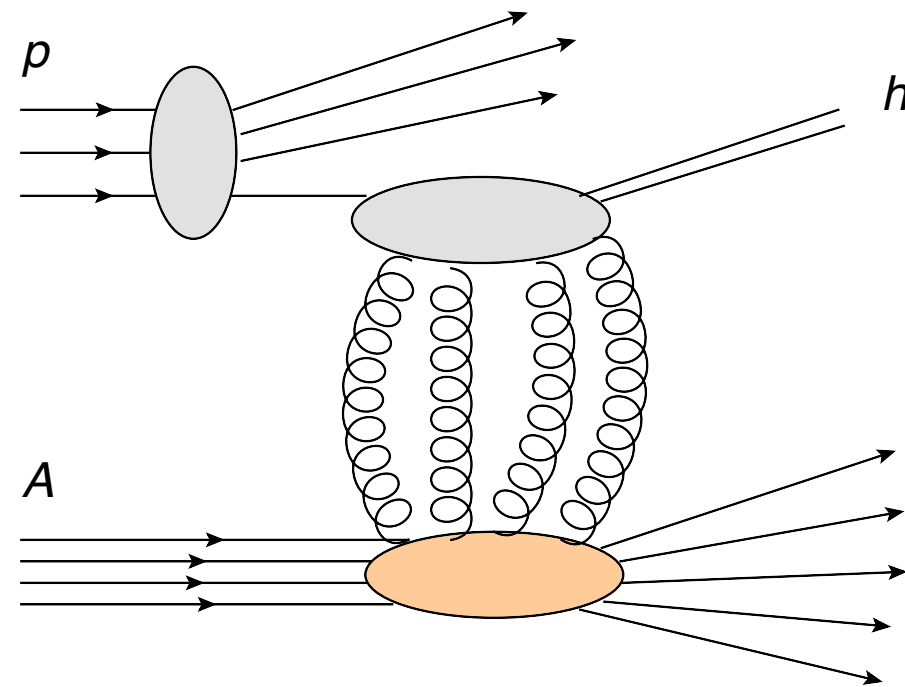
Hadron measured at large rapidity y:

Proton projectile probed at large x

Nucleus target probed at small x

$$p_\perp \sim Q_s(x_g) \gg \Lambda_{\text{QCD}}$$

Saturation effects in the nucleus could be important
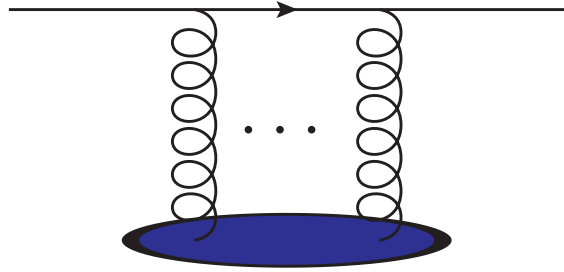Collinear factorization not expected to work

LO formalism

Diagram for leading order quark production with multiple rescattering.

$$U(x_\perp) = \mathcal{P} \exp\left\{ i g_S \int_{-\infty}^{+\infty} \mathrm{d}x^+ T^c A_c^-(x^+, x_\perp) \right\}$$

Multiple rescattering of quark included in the Wilson line.

$$\frac{d\sigma_{\mathrm{LO}}^{pA \to qX}}{d^2 k_\perp dy} = \sum_f x q_f(x) \int \frac{d^2 x_\perp d^2 y_\perp}{(2\pi)^2} e^{-i k_\perp \cdot (x_\perp - y_\perp)} \frac{1}{N_c} \left\langle \mathrm{Tr} U(x_\perp) U^\dagger(y_\perp) \right\rangle_Y$$

$$x = \frac{k_\perp}{\sqrt{s}} e^y$$

$$x_g = \frac{k_\perp}{\sqrt{s}} e^{-y}$$

Leading order cross section for quark production in pA

$$Y = \ln 1/x_g$$

$$\mathcal{F}(k_\perp) = \int \frac{d^2 x_\perp d^2 y_\perp}{(2\pi)^2} e^{-i k_\perp \cdot (x_\perp - y_\perp)} S_Y^{(2)}(x_\perp, y_\perp)$$

$$S_Y^{(2)}(x_\perp, y_\perp) = \frac{1}{N_c} \left\langle \mathrm{Tr} U(x_\perp) U^\dagger(y_\perp) \right\rangle_Y$$

Dipole unintegrated gluon distribution function.

Correlator can be evaluated in the McLerran-Venugopalan model (without energy evolution). To take into account energy evolution one needs to use Balitsky-Kovchegov evolution equation.

Could be evaluated with GBW model

$$S^{(2)}(x_\perp, y_\perp) = \exp\left[-\frac{Q_s^2 (x_\perp - y_\perp)^2}{4} \right]$$

LO formalism

Include the gluon channel. Wilson line correlator in adjoint representation.
Convolution with fragmentation function to obtain final state hadron

$$\frac{d\sigma_{\text{LO}}^{pA \to hX}}{d^2 p_\perp dy_h} = \int_\tau^1 \frac{dz}{z^2} \left[\sum_f x_p q_f(x_p)\mathcal{F}(k_\perp)D_{h/q}(z) + x_p g(x_p)\tilde{\mathcal{F}}(k_\perp)D_{h/g}(z) \right]$$

$$\tau = zx_p$$

$$p_\perp = zk_\perp$$

$$\tilde{\mathcal{F}}(k_\perp) = \int \frac{d^2 x_\perp d^2 y_\perp}{(2\pi)^2} e^{-ik_\perp \cdot (x_\perp - y_\perp)} \tilde{S}_Y^{(2)}(x_\perp, y_\perp)$$

$$x_p = \frac{p_\perp}{z\sqrt{s}} e^{y_h}$$

$$x_g = \frac{p_\perp}{z\sqrt{s}} e^{-y_h}$$

$$\tilde{S}_Y^{(2)}(x_\perp, y_\perp) = \frac{1}{N_c^2 - 1} \left\langle \text{Tr} W(x_\perp) W^\dagger(y_\perp) \right\rangle_Y$$

Wilson lines in adjoint representation representing multiple
interaction between gluon and the nucleus target

$$W^{ab}(x_\perp) = 2\text{Tr}\left[T^a U(x_\perp) T^b U^\dagger(x_\perp) \right]$$

replace Wilson lines in adjoint representation by
Wilson lines in fundamental representation

Using this one can obtain:

$$\tilde{S}_Y^{(2)}(x_\perp, y_\perp) = \frac{1}{N_c^2 - 1} \left[\langle \text{Tr} U(x_\perp) U^\dagger(y_\perp) \text{Tr} U(y_\perp) U^\dagger(x_\perp) \rangle_Y - 1 \right]$$

$$\tilde{\mathcal{F}}(k_\perp) = \int \frac{d^2 x_\perp d^2 y_\perp}{(2\pi)^2} e^{-ik_\perp \cdot (x_\perp - y_\perp)} S_Y^{(2)}(x_\perp, y_\perp) S_Y^{(2)}(y_\perp, x_\perp)$$

in the multicolor limit

No transverse momentum dependence in the incoming parton distribution from the nucleon which is at large x
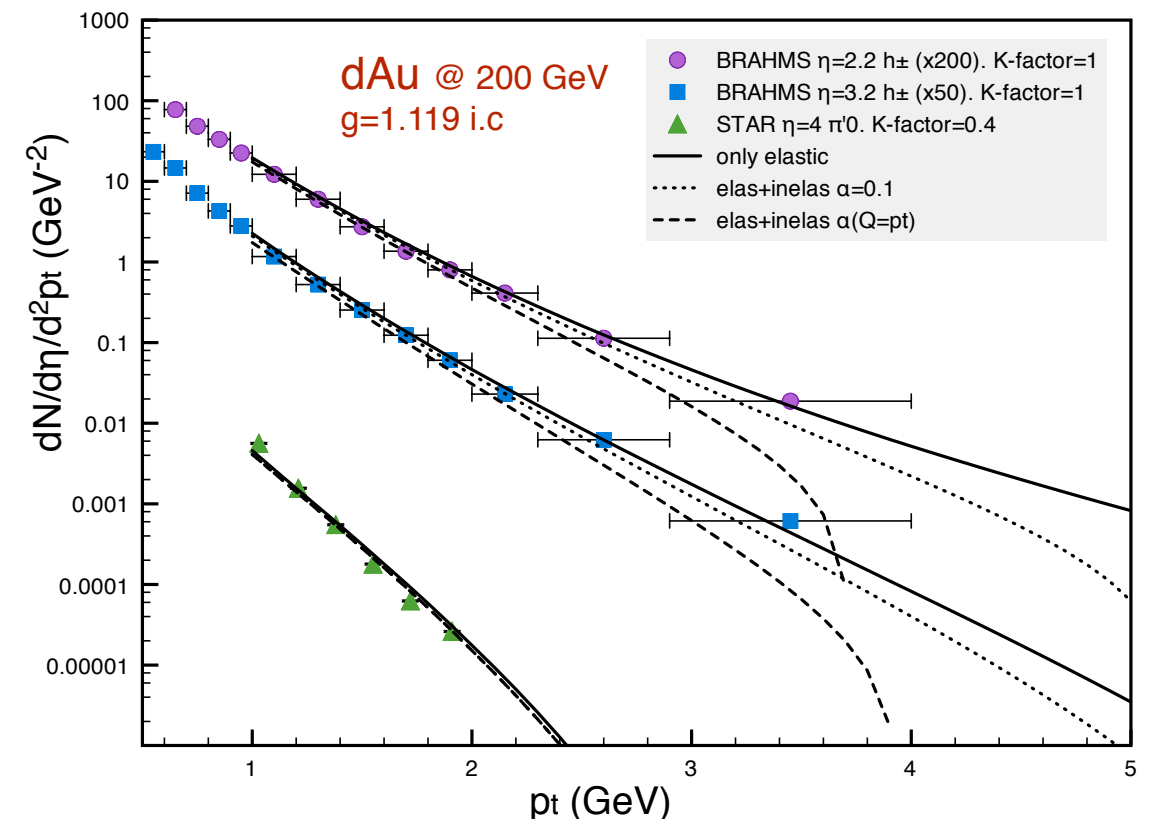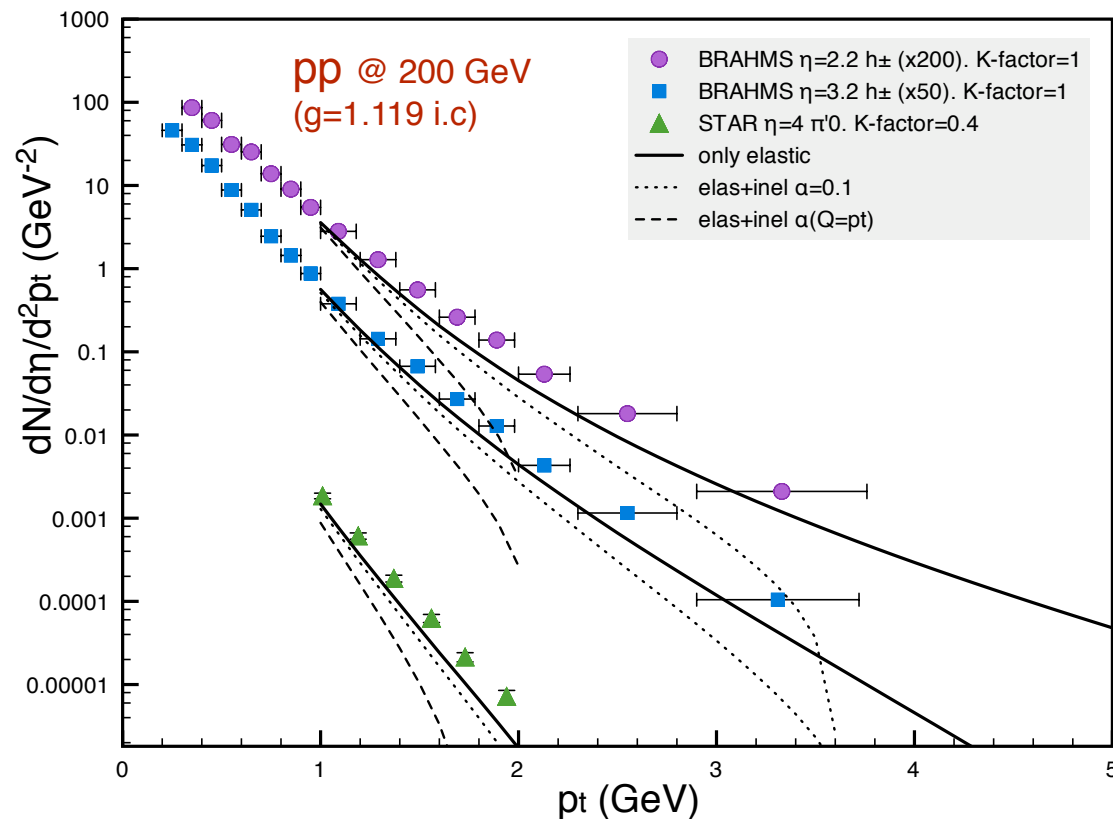Transverse momentum dependence only on the nuclear side inside the gluon distribution.
At LO no scale or rapidity dependence in the derivation. Usually included for phenomenology.

In ... sions

Albacete,Dumitru,Fujii,Nara

dAu @ 200 GeV
MV i.c

- BRAHMS η=2.2 h± (x200). K-factor=1
- BRAHMS η=3.2 h± (x50). K-factor=1
- STAR η=4 π'0. K-factor=0.4
— only elastic
···· elas+inelas α=0.1
- - - elas+inelas α(Q=pt)

dAu @ 200 GeV
g=1.119 i.c

- BRAHMS η=2.2 h± (x200). K-factor=1
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dAu @ 200 GeV
g=1.119 i.c

Partial NLO term included.

Altinoluk, Kovner

$$\frac{dN_h}{d\eta\, d^2k} = \int_{el} \frac{\alpha_s(Q)}{2\pi^2} \int_{x_F}^1 \frac{dz}{z^2} \frac{z^4}{k^4} \int \frac{Q^2 d^2q}{(2\pi)^2} q^2 \tilde{N}_F(x_2, q) x_1 \int_{x_1}^1 \frac{d\xi}{\xi} \sum_{i,j=q,\bar{q},g} w_{i/j}(\xi) P_{i/j}(\xi) f_j\left(\frac{x_1}{\xi}, Q^2\right) D_{h/j}(z, Q^2)$$

Splitting function contains 'plus' prescription. Can lead to negative results in some parts of the phase space.
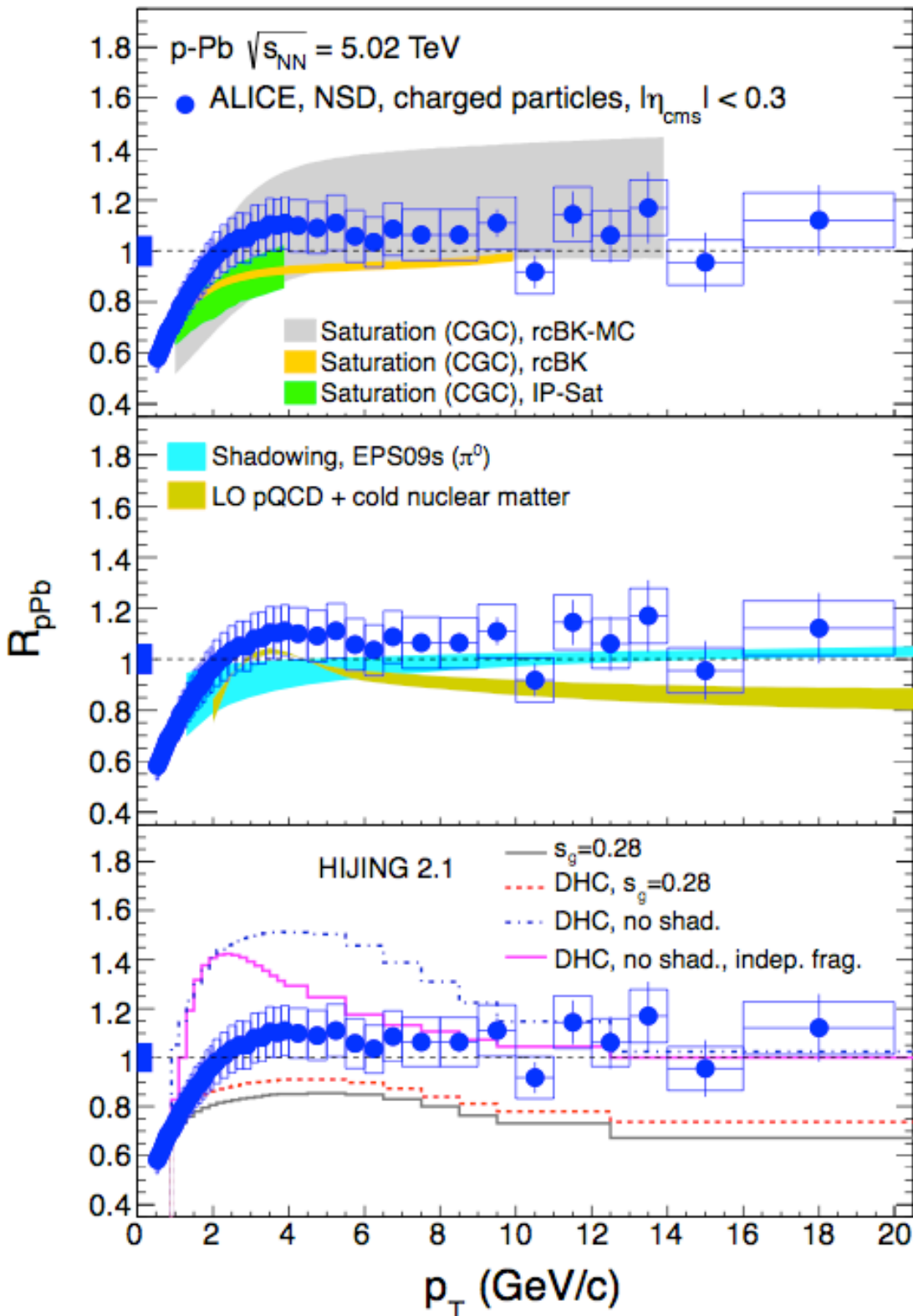
Indeed this NLO correction takes over the LO result for large transverse momenta.
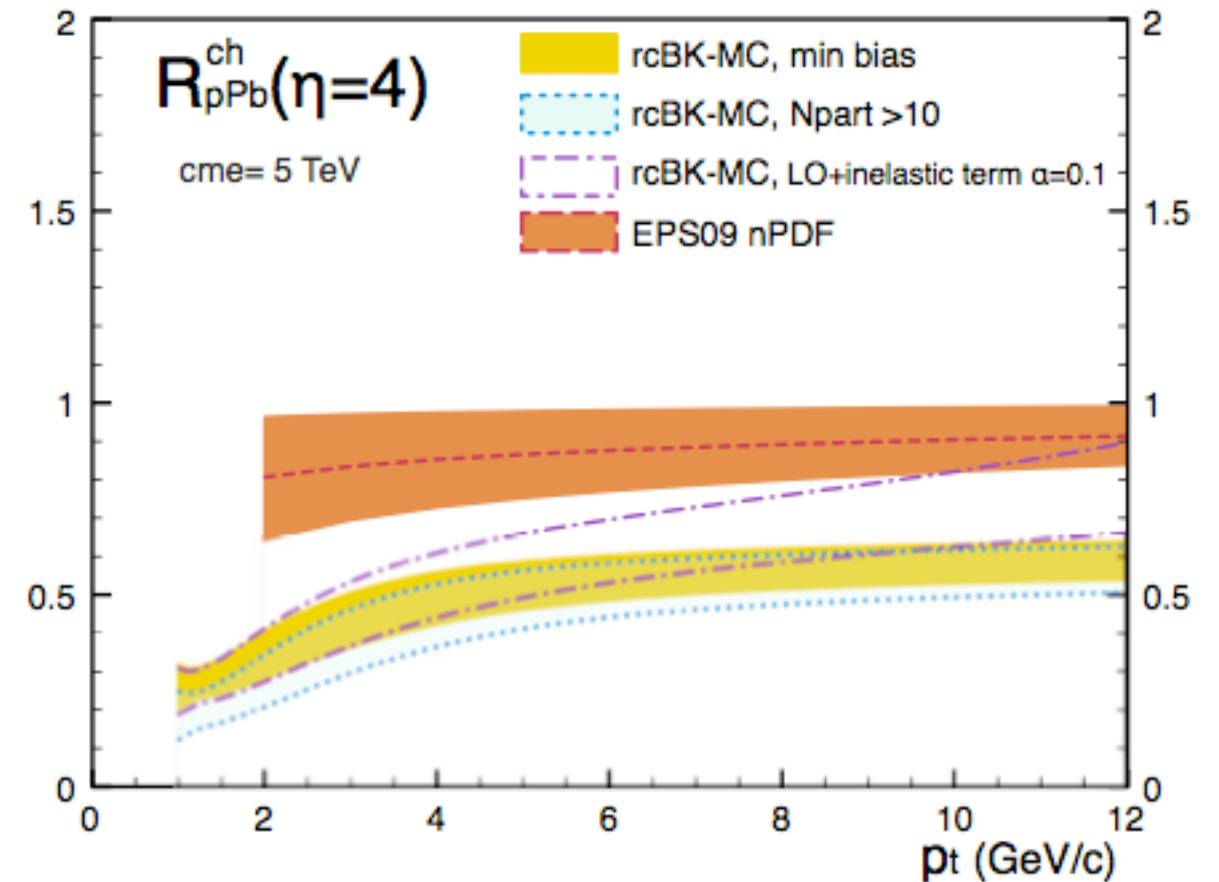
Complete NLO correction might be very large.

Since this is partial NLO correction one needs to evaluate full NLO term.

Nuclear ratios

Albacete-Marquet

$$R_{pA}^h = \frac{dN^{pA \to hX}/dy\,d^2p_\perp}{N_{coll}\;dN^{pp \to hX}/dy\,d^2p_\perp}$$

LO formula stays flat and < 1 for large pT.
Need NLO to match to collinear formula.
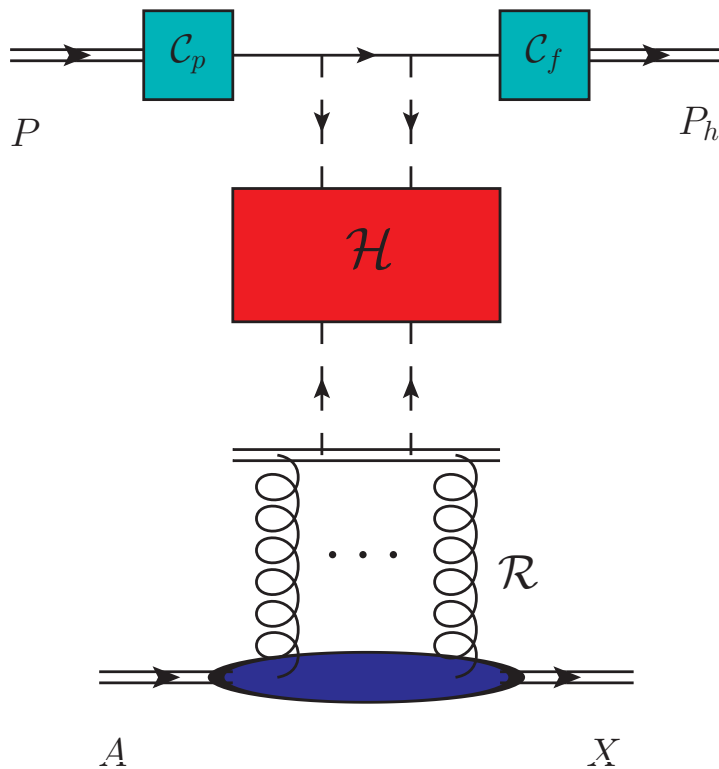
NLO calculation

Partial calculation

Altinoluk, Kovner
&
Dumitru, Hayashigaki, Jalilian-Marian

Full calculation: virtual and real diagrams.
Subtractions performed according to the
renormalization group equations

Chirilli, Xiao, Yuan

Factorization formula in coordinate space at one loop

$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \sum_a \int \frac{dz}{z^2}\frac{dx}{x}\xi x f_a(x,\mu)D_{h/c}(z,\mu)\int[dx_\perp]S^Y_{a,c}([x_\perp])\mathcal{H}_{a\to c}(\alpha_s,\xi,[x_\perp]\mu)$$
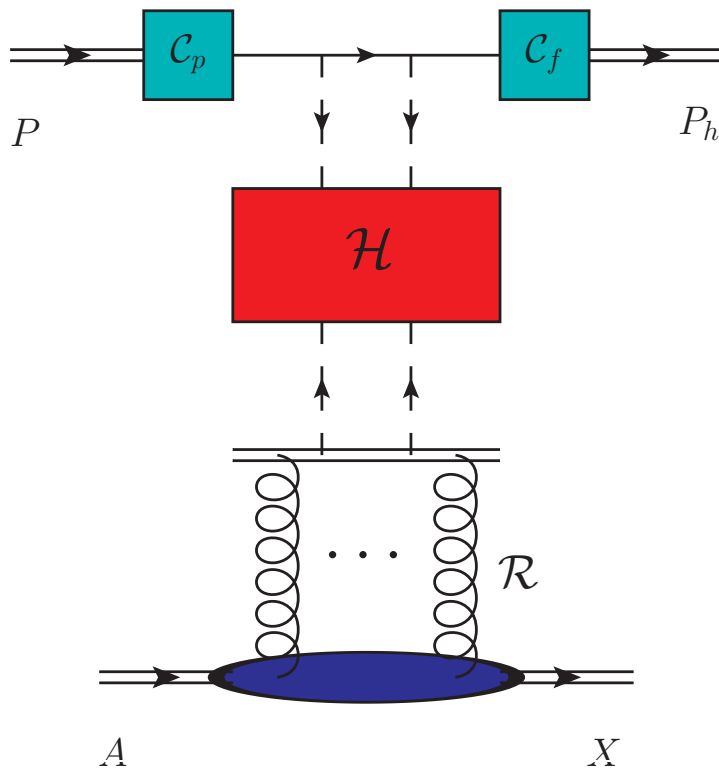
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Collinear divergence: pdfs
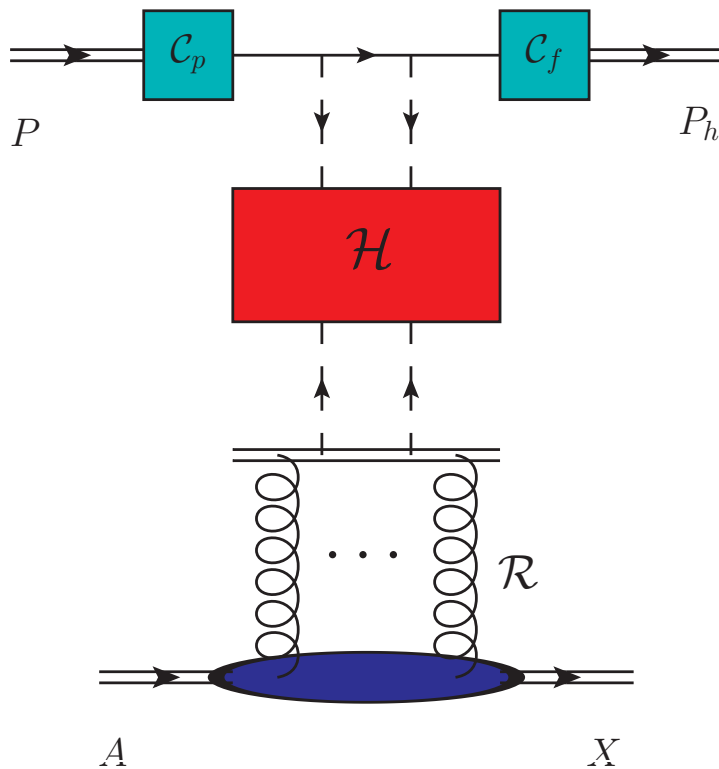
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$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \sum_a \int \frac{dz}{z^2}\frac{dx}{x}\xi\, xf_a(x,\mu)\, D_{h/c}(z,\mu) \int [dx_\perp] S_{a,c}^Y([x_\perp]) \mathcal{H}_{a\to c}(\alpha_s,\xi,[x_\perp]\mu)$$

Collinear divergence: pdfs

Collinear divergence: fragmentation functs
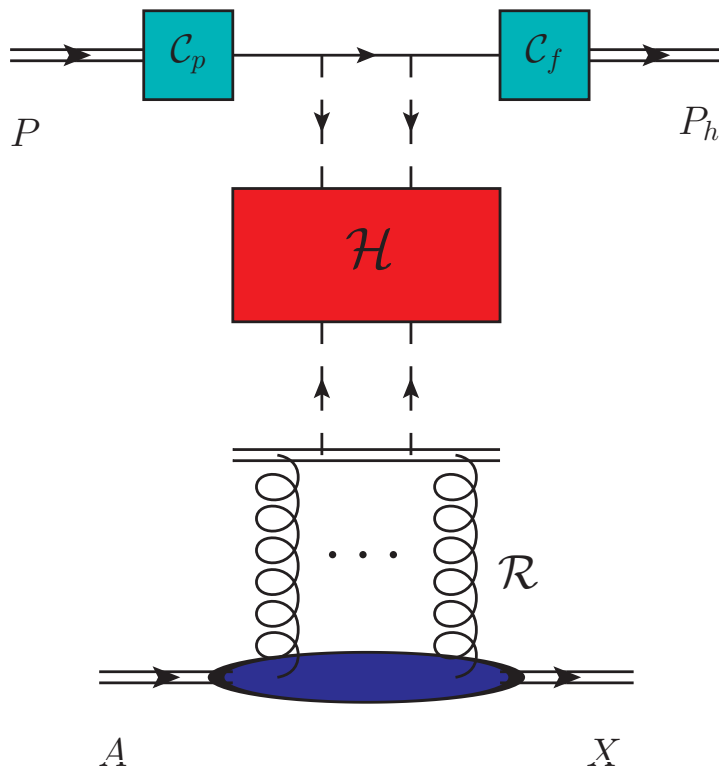
NLO calculation

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Collinear divergence: pdfs

Collinear divergence: fragmentation functs
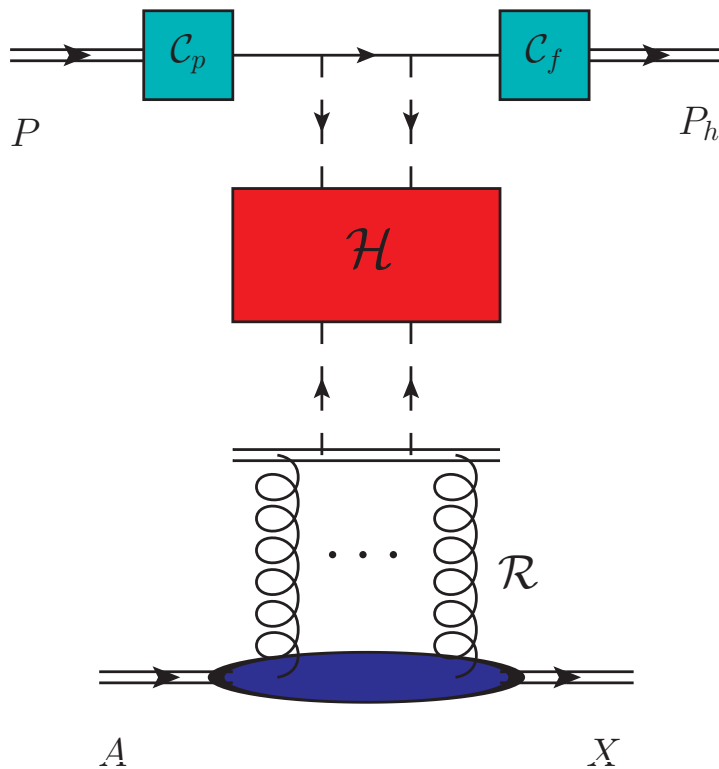
Rapidity divergence: BK evolution

NLO calculation

Partial calculation

Altinoluk, Kovner
&
Dumitru, Hayashigaki, Jalilian-Marian

Full calculation: virtual and real diagrams.
Subtractions performed according to the
renormalization group equations

Chirilli, Xiao, Yuan

Factorization formula in coordinate space at one loop

$$\frac{d^3\sigma^{p+A\to h+X}}{dy\, d^2 p_\perp} = \sum_a \int \frac{dz}{z^2} \frac{dx}{x} \xi\, x f_a(x,\mu)\, D_{h/c}(z,\mu) \int [dx_\perp] S_{a,c}^Y([x_\perp])\, \mathcal{H}_{a\to c}(\alpha_s, \xi, [x_\perp]\mu)$$

Collinear divergence: pdfs

Collinear divergence: fragmentation functs

Rapidity divergence: BK evolution

Finite hard factor

NLO calculation

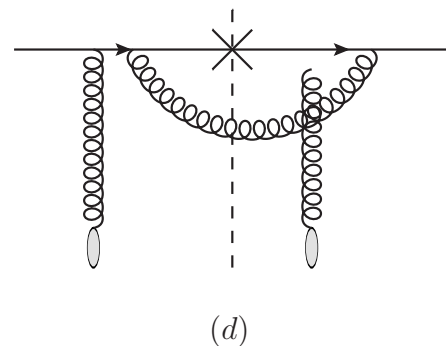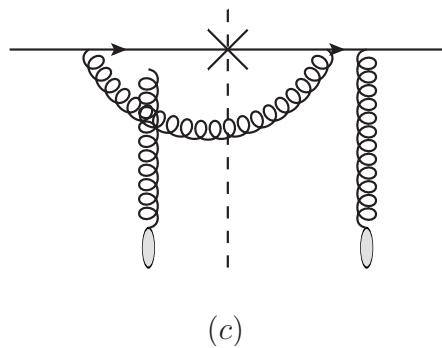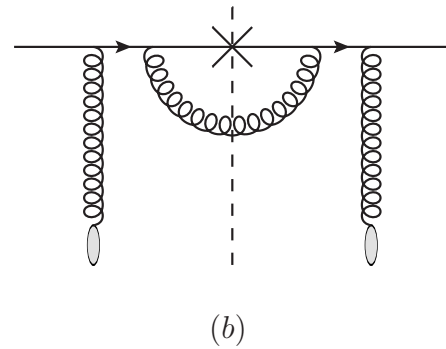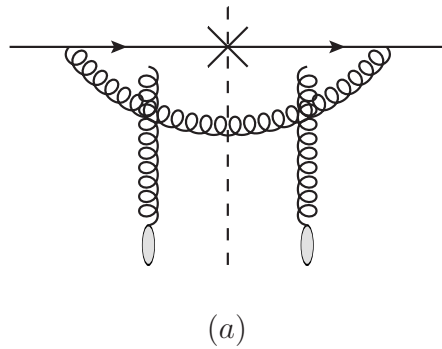There are 4 channels to consider
(diagonal and off-diagonal):

$$q \to q \qquad g \to g$$
$$g \to q \qquad q \to g$$

Example quark to quark at one loop. Real diagrams.

(a)

(b)

(c)

(d)

$$\frac{d\sigma^{qA \to qgX}}{d^3 k_1 d^3 k_2} = \alpha_S C_F \delta(p^+ - k_1^+ - k_2^+) \int \frac{d^2 x_\perp}{(2\pi)^2} \frac{d^2 x'_\perp}{(2\pi)^2} \frac{d^2 b_\perp}{(2\pi)^2} \frac{d^2 b'_\perp}{(2\pi)^2}$$

$$\times e^{-ik_{1\perp} \cdot (x_\perp - x'_\perp)} e^{-ik_{2\perp} \cdot (b_\perp - b'_\perp)} \sum_{\lambda \alpha \beta} \psi_{\alpha\beta}^{\lambda *}(u'_\perp) \psi_{\alpha\beta}^{\lambda}(u_\perp)$$

$$\times \left[S_Y^{(6)}(b_\perp, x_\perp, b'_\perp, x'_\perp) + S_Y^{(2)}(v_\perp, v'_\perp) \right.$$

$$\left. - S_Y^{(3)}(b_\perp, x_\perp, v'_\perp) - S_Y^{(3)}(v_\perp, x'_\perp, b'_\perp) \right].$$

$$u_\perp = x_\perp - b_\perp, \ u'_\perp = x'_\perp - b'_\perp, \ v_\perp = (1-\xi)x_\perp + \xi b_\perp, \ v'_\perp = (1-\xi)x'_\perp + \xi b'_\perp$$

ψ represents the splitting of a quark into quark-gluon pair

$$S_Y^{(6)}, S_Y^{(3)} \qquad \text{higher correlators of Wilson lines}$$

$$S_Y^{(6)}(b_\perp, x_\perp, b'_\perp, x'_\perp) = \frac{1}{C_F N_c} \left\langle \text{Tr}\left(U(b_\perp)U^\dagger(b'_\perp)T^d T^c\right) \left[W(x_\perp)W^\dagger(x'_\perp)\right]^{cd} \right\rangle_Y,$$

$$S_Y^{(3)}(b_\perp, x_\perp, v'_\perp) = \frac{1}{C_F N_c} \left\langle \text{Tr}\left(U(b_\perp)T^d U^\dagger(v'_\perp)T^c\right) W^{cd}(x_\perp) \right\rangle_Y.$$

These diagrams contribute both to quark to quark transitions and quark to gluon transition.
Integrate out the gluon to obtain the contribution to quark to quark channel.
The $S_Y^{(6)}$ term simplifies to $S_Y^{(2)}$

NLO calculation

$q \longrightarrow q$ final factorization formula for quark-quark channel

$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \int \frac{dz}{z^2}\frac{dx}{x}\xi x q(x,\mu)D_{h/q}(z,\mu)\int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2}\left\{S_Y^{(2)}(x_\perp,y_\perp)\left[\mathcal{H}_{2qq}^{(0)}+\frac{\alpha_s}{2\pi}\mathcal{H}_{2qq}^{(1)}\right]\right.$$

$$\left.+\int \frac{d^2b_\perp}{(2\pi)^2}S_Y^{(4)}(x_\perp,b_\perp,y_\perp)\frac{\alpha_s}{2\pi}\mathcal{H}_{4qq}^{(1)}\right\},$$

with hard factors (finite contributions free of any divergencies)

LO $\quad \mathcal{H}_{2qq}^{(0)} = e^{-ik_\perp\cdot r_\perp}\delta(1-\xi)$

NLO $\quad \mathcal{H}_{2qq}^{(1)} = C_F\mathcal{P}_{qq}(\xi)\ln\frac{c_0^2}{r_\perp^2\mu^2}\left(e^{-ik_\perp\cdot r_\perp}+\frac{1}{\xi^2}e^{-i\frac{k_\perp}{\xi}\cdot r_\perp}\right)-3C_F\delta(1-\xi)e^{-ik_\perp\cdot r_\perp}\ln\frac{c_0^2}{r_\perp^2 k_\perp^2}$

$$-\left(2C_F-N_c\right)e^{-ik_\perp\cdot r_\perp}\left[\frac{1+\xi^2}{(1-\xi)_+}\widetilde{I}_{21}-\left(\frac{(1+\xi^2)\ln(1-\xi)^2}{1-\xi}\right)_+\right],$$

$$\widetilde{I}_{21} = \int \frac{d^2b_\perp}{\pi}\left\{e^{-i(1-\xi)k_\perp\cdot b_\perp}\left[\frac{b_\perp\cdot(\xi b_\perp-r_\perp)}{b_\perp^2(\xi b_\perp-r_\perp)^2}-\frac{1}{b_\perp^2}\right]+e^{-ik_\perp\cdot b_\perp}\frac{1}{b_\perp^2}\right\}$$

$$\mathcal{H}_{4qq}^{(1)} = -4\pi N_c e^{-ik_\perp\cdot r_\perp}\left\{e^{-i\frac{1-\xi}{\xi}k_\perp\cdot(x_\perp-b_\perp)}\frac{1+\xi^2}{(1-\xi)_+}\frac{1}{\xi}\frac{x_\perp-b_\perp}{(x_\perp-b_\perp)^2}\cdot\frac{y_\perp-b_\perp}{(y_\perp-b_\perp)^2}\right.$$

$$\left.-\delta(1-\xi)\int_0^1 d\xi'\frac{1+\xi'^2}{(1-\xi')_+}\left[\frac{e^{-i(1-\xi')k_\perp\cdot(y_\perp-b_\perp)}}{(b_\perp-y_\perp)^2}-\delta^{(2)}(b_\perp-y_\perp)\int d^2r'_\perp\frac{e^{ik_\perp\cdot r'_\perp}}{r'^2_\perp}\right]\right\},$$
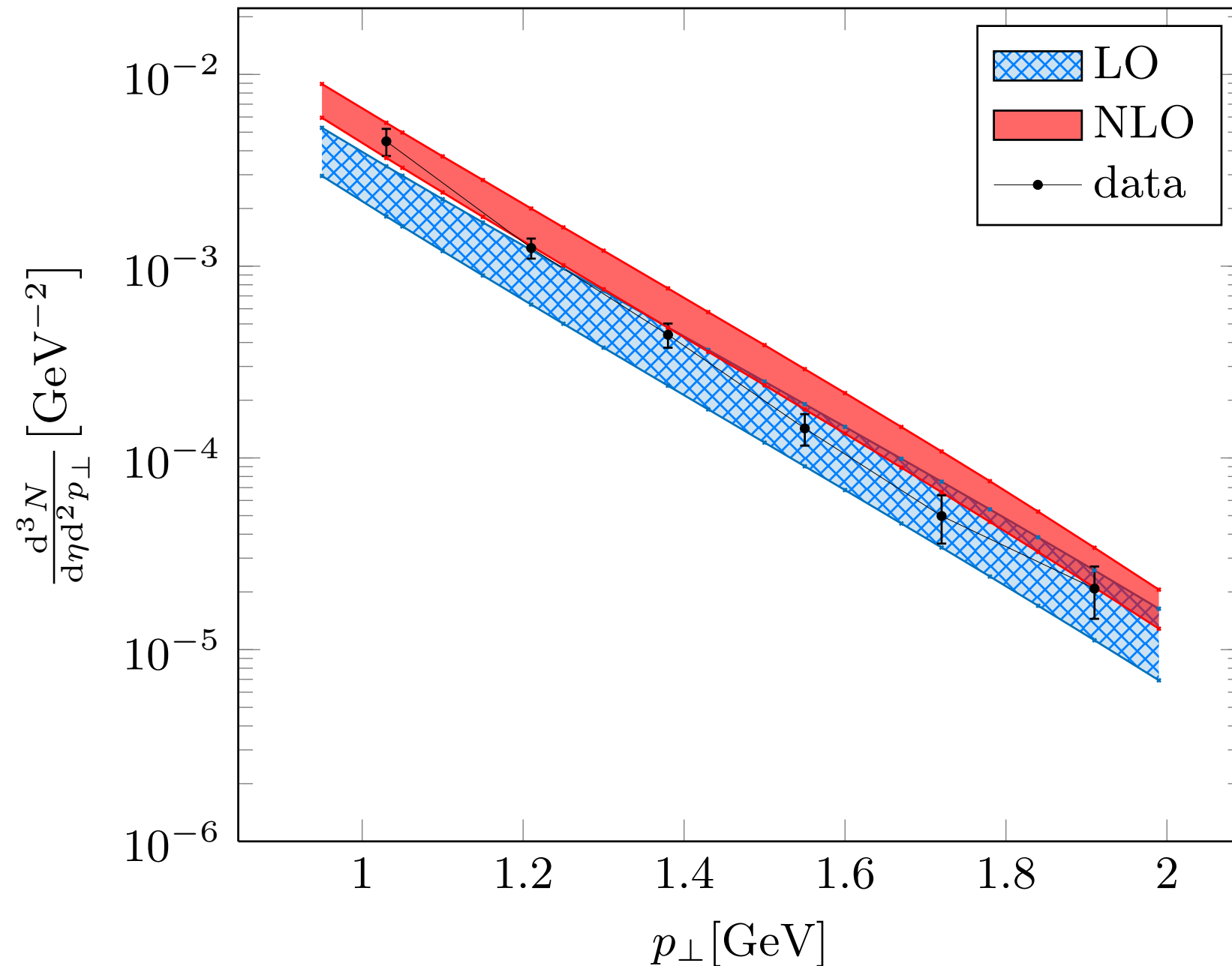
Other channels have similar structure

Numerical evaluation

- Full implementation of NLO hard factors in the numerical code SOLO (Saturation physics at One Loop Order).

- Running coupling LL BK equation for the unintegrated gluon distribution. Additional runs with fixed coupling BK as well as GBW and MV models.

- MSTW 2008 NLO parton distributions, and DSS NLO fragmentation functions.

- Simulations for RHIC kinematics: Y=2.2, Y=3.2 (BRAHMS), Y=4 (STAR) with $\sqrt{s} = 200 \text{ GeV}$

- Results calculated for different values of factorization scale (1.4-10 GeV).

NLO calculation: RHIC

STAR $\eta = 4$

$\dfrac{\mathrm{d}^3 N}{\mathrm{d}\eta \mathrm{d}^2 p_\perp} \left[\mathrm{GeV}^{-2}\right]$

$p_\perp [\mathrm{GeV}]$
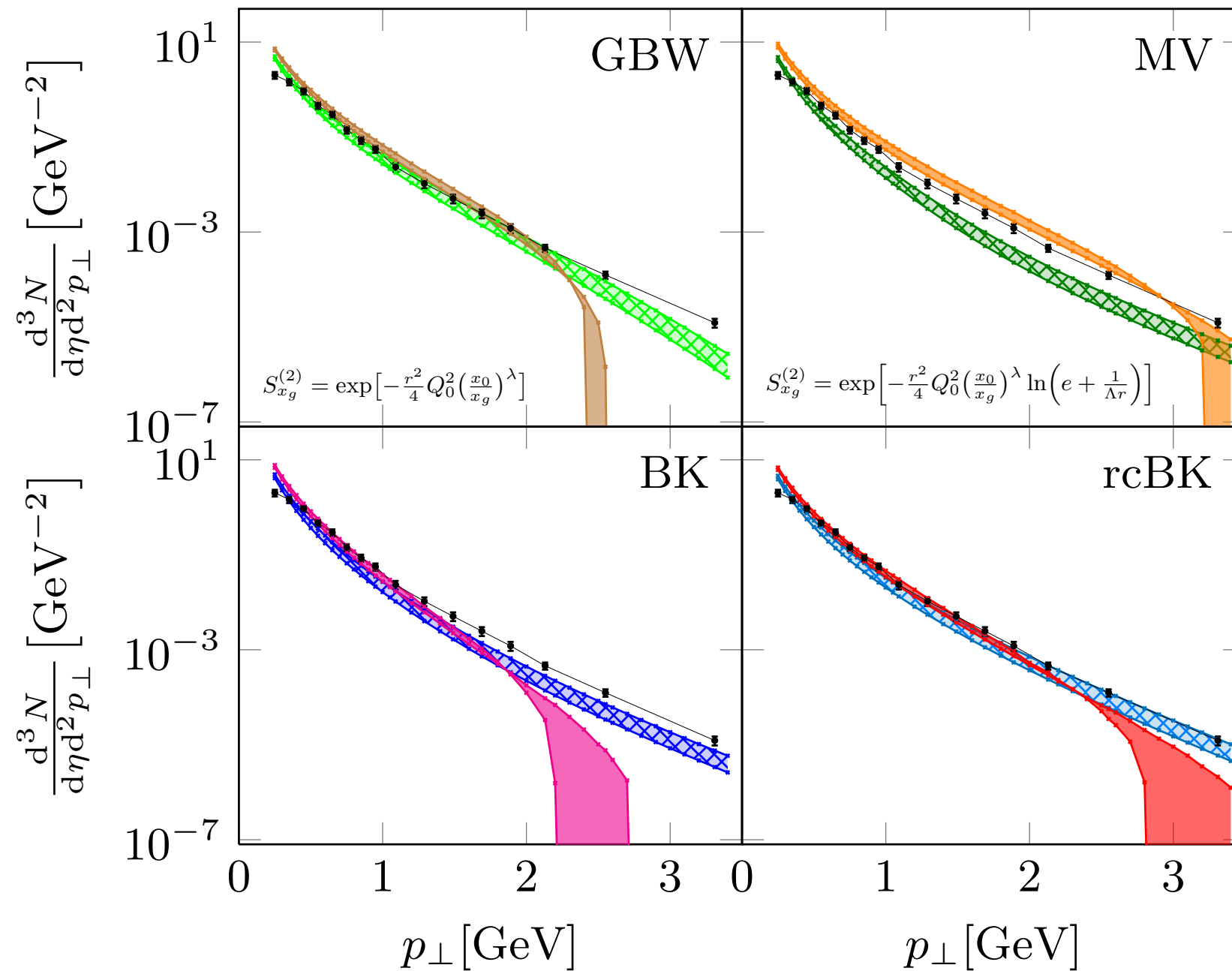
LO
NLO
data

$\eta = 2.2$

$\eta = 3.2$
$(\times 0.1)$

Positive correction for low values of pT, high rapidity.
Shape comparable (within the kinematic range).
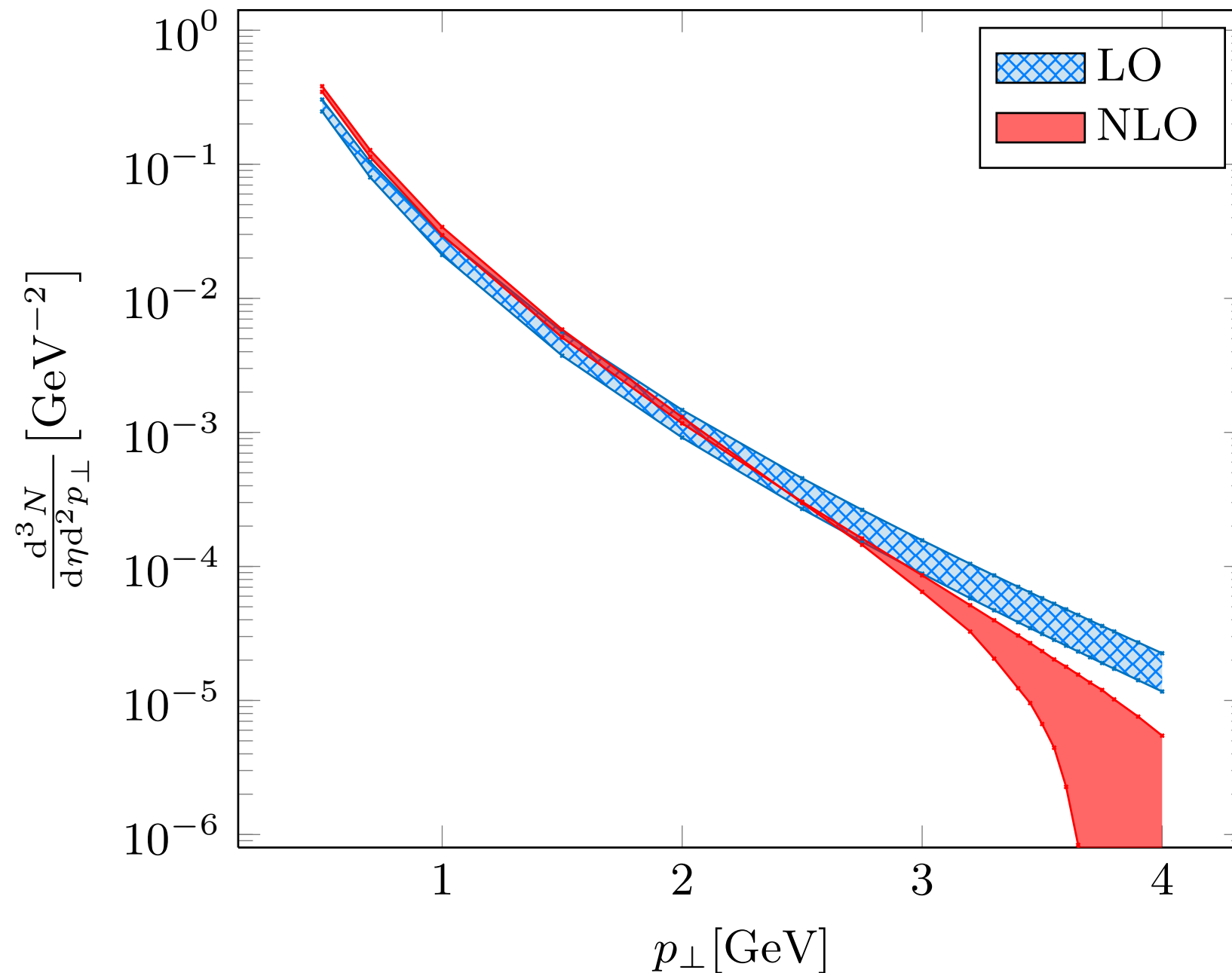Bands indicate range of scale variation, smaller at NLO than at LO.

NLO calculation RHIC

Low rapidity, high pT the NLO correction dominates the cross section which becomes negative. The point at which the calculation breaks down depends on the shape of the gluon distribution.
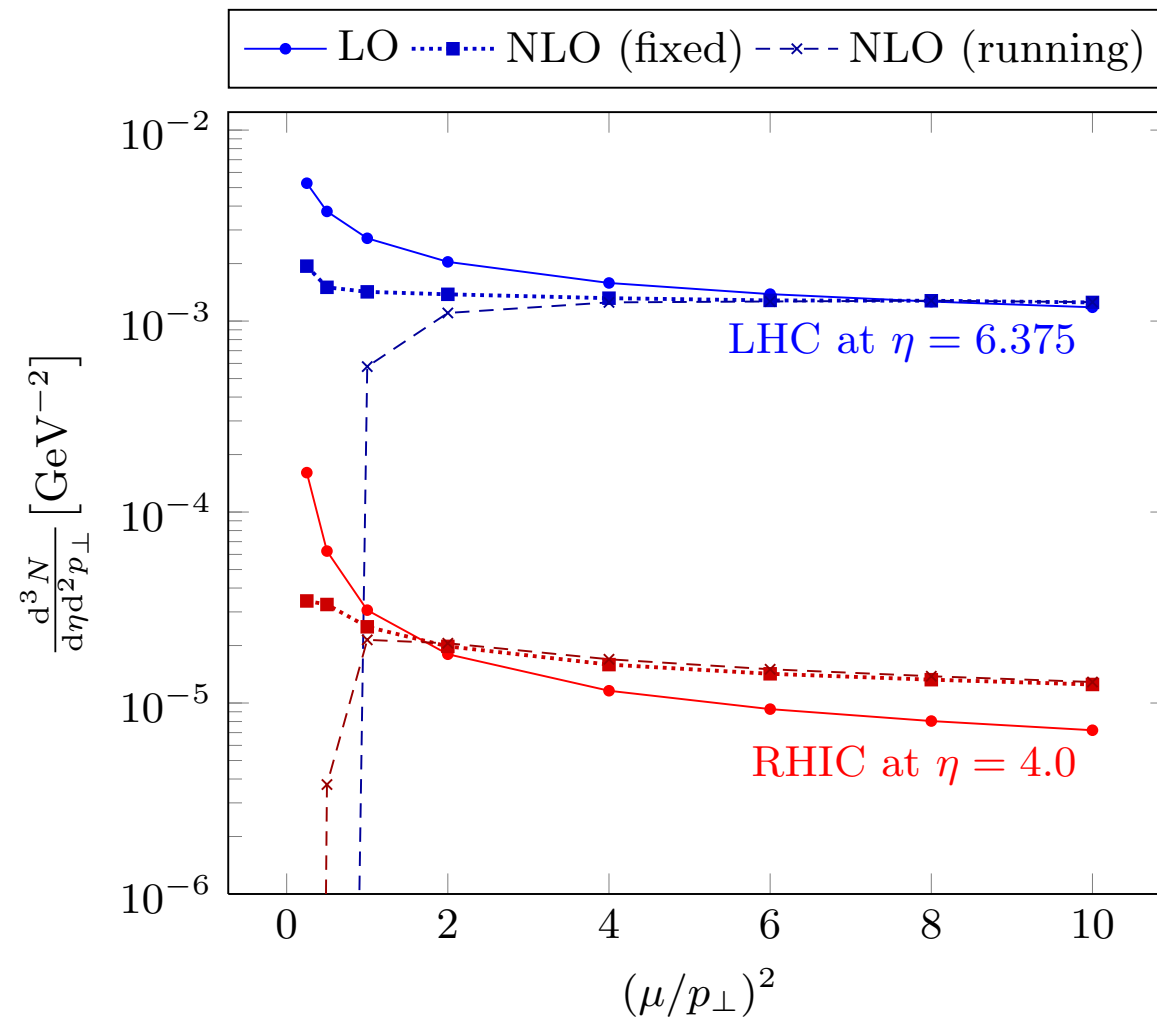
NLO calculation LHC

LHC $\eta = 6.375$

High rapidity, calculation stays positive until moderate values of pT.

Scale dependence at NLO

Significant reduction of the scale dependence at NLO as compared with the LO, except at very low values.
Sharp drop in the running coupling case is caused by the breakdown of the calculation at large values of the coupling.

Small x *vs* collinear factorization

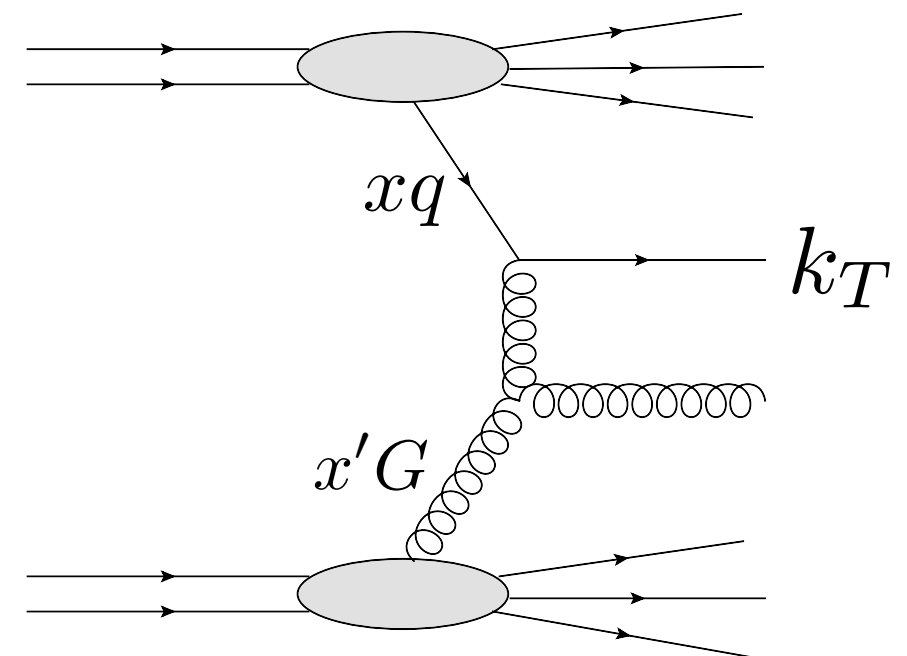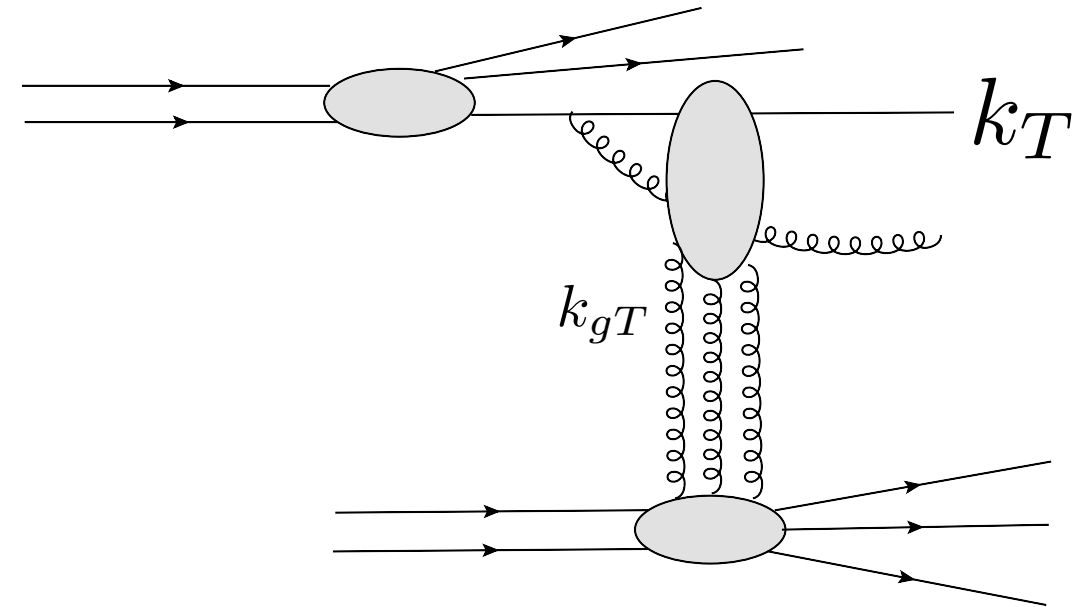$$k_T^2 \simeq Q_s^2(x)$$

Small x calculation should be justified

Transverse momentum comes from multiple scattering off the dense target

$$k_T^2 \gg Q_s^2(x)$$

Collinear approximation works well

Transverse momentum comes from hard scattering

Small x *vs* collinear factorization

$q \to qg$

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}y\mathrm{d}^2\mathbf{p}_\perp} = \frac{\alpha_s}{2\pi^2} \int \frac{\mathrm{d}z}{z^2} D_{h/q}(z) \int_{\tau/z}^1 \mathrm{d}\xi \frac{1+\xi^2}{1-\xi} xq(x) \left\{ C_F \int \mathrm{d}^2\mathbf{k}_{g\perp} \mathcal{I}(\mathbf{k}_\perp, \mathbf{k}_{g\perp}) + \frac{N_c}{2} \int \mathrm{d}^2\mathbf{k}_{g\perp}\mathrm{d}^2\mathbf{k}_{g1\perp} \mathcal{J}(\mathbf{k}_\perp, \mathbf{k}_{g\perp}, \mathbf{k}_{g1\perp}) \right\}$$

$$\mathcal{I}(\mathbf{k}_\perp, \mathbf{k}_{g\perp}) = \mathcal{F}_{x_a}(k_{g\perp}) \left[\frac{\mathbf{k}_\perp - \mathbf{k}_{g\perp}}{(\mathbf{k}_\perp - \mathbf{k}_{g\perp})^2} - \frac{\mathbf{k}_\perp - \xi\mathbf{k}_{g\perp}}{(\mathbf{k}_\perp - \xi\mathbf{k}_{g\perp})^2} \right]^2 ;$$

$$\mathcal{J}(\mathbf{k}_\perp, \mathbf{k}_{g\perp}, \mathbf{k}_{g1\perp}) = \left[\mathcal{F}_{x_a}(k_{g\perp})\delta^{(2)}(\mathbf{k}_{g1\perp} - \mathbf{k}_{g\perp}) - \mathcal{G}_{x_a}(\mathbf{k}_{g\perp}, \mathbf{k}_{g1\perp}) \right] \frac{2(\mathbf{k}_\perp - \xi\mathbf{k}_{g\perp}) \cdot (\mathbf{k}_\perp - \mathbf{k}_{g1\perp})}{(\mathbf{k}_\perp - \xi\mathbf{k}_{g\perp})^2 (\mathbf{k}_\perp - \mathbf{k}_{g1\perp})^2}$$

Expansion for large k_T

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}y\mathrm{d}^2\mathbf{p}_\perp} = \frac{\alpha_s}{2\pi^2} \int \frac{\mathrm{d}z}{z^2} D_{h/q}(z) \int_{\tau/z}^1 \mathrm{d}\xi \frac{1+\xi^2}{1-\xi} xq(x) \left\{ C_F \frac{(1-\xi)^2}{k_\perp^4} + N_c \frac{\xi}{k_\perp^4} \right\} \int \mathrm{d}^2\mathbf{k}_{g\perp} k_{g\perp}^2 \mathcal{F}_{x_a}(k_{g\perp})$$

$$= \frac{\alpha_s^2}{N_c} \int \frac{\mathrm{d}z}{z^2} D_{h/q}(z) \int_{\tau/z}^1 \mathrm{d}\xi \frac{1+\xi^2}{1-\xi} xq(x) \left\{ C_F \frac{(1-\xi)^2}{k_\perp^4} + N_c \frac{\xi}{k_\perp^4} \right\} x'G(x', \mu) ,$$
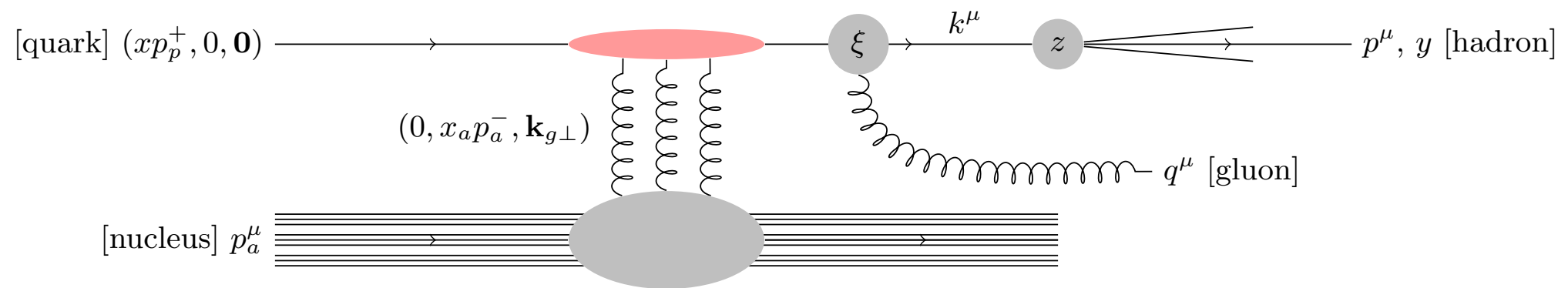
S_T transverse area of the nucleus

In this approximation $\mu \simeq Q_s$

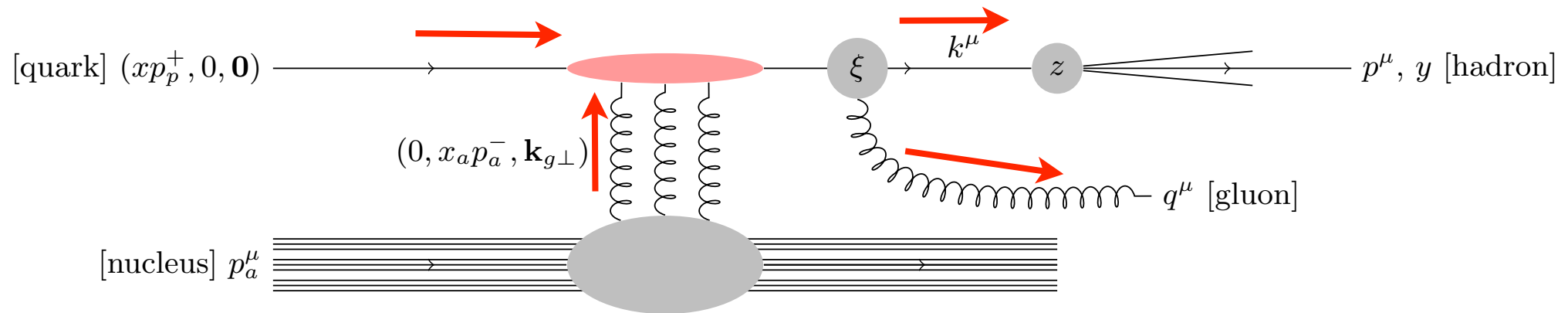$$\int \mathrm{d}^2\mathbf{k}_{g\perp} \mathcal{F}_{x_a}(k_{g\perp}) = S_\perp ,$$

$$\int \mathrm{d}^2\mathbf{k}_{g\perp} k_{g\perp}^2 \mathcal{F}_{x_a}(k_{g\perp}) = S_\perp Q_s^2 \simeq \frac{\alpha_s 2\pi^2}{N_c} x'G(x', \mu) ,$$

Closer look at kinematics

[quark] $(xp_p^+, 0, \mathbf{0})$ ξ k^μ z p^μ, y [hadron]

$(0, x_a p_a^-, \mathbf{k}_{g\perp})$

q^μ [gluon]

[nucleus] p_a^μ

Closer look at kinematics

[quark] $(xp_p^+, 0, \mathbf{0})$ ξ k^μ z p^μ, y [hadron]

$(0, x_a p_a^-, \mathbf{k}_{g\perp})$ q^μ [gluon]

[nucleus] p_a^μ

In exact kinematics for $\quad 2 \to 2$

On-shell outgoing particles: $\quad k^2 = 0 \quad q^2 = 0$

$$xp_p^+ = k^+ + q^+$$

$$x_a p_a^- = k^- + q^-$$

$$\mathbf{k}_{g\perp} = \mathbf{k}_\perp + \mathbf{q}_\perp$$

$$\xi = \frac{k^+}{xp_p^+}$$

$$x = \frac{k_\perp}{\sqrt{s}\xi}e^y$$

Exact:
$$x_a = \frac{k_\perp}{\sqrt{s}}e^{-y} + \frac{(\mathbf{k}_{g\perp} - \mathbf{k}_\perp)^2}{\sqrt{s}k_\perp}\frac{\xi}{1-\xi}e^{-y}$$

$$x' = \frac{k_\perp}{\sqrt{s}}e^{-y} + \frac{k_\perp}{\sqrt{s}}\frac{\xi}{1-\xi}e^{-y} = \frac{k_\perp}{\sqrt{s}}\frac{e^{-y}}{1-\xi}$$

Small x: $\quad x_a = \frac{k_\perp}{\sqrt{s}}e^{-y}$

where $\quad x' = x_a(k_{g\perp} = 0)$

Collinear formalism

Collinear LO formula for qg term:

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}y\mathrm{d}^2\mathbf{p}_\perp} = \int \frac{\mathrm{d}z}{z^2} D_{h/q}(z) \int \mathrm{d}x\mathrm{d}x' q(x) G(x') \frac{|\overline{\mathcal{M}}|^2}{2\hat{s}} \frac{1}{2(2\pi)^3} 2\pi \delta\big(\hat{s} + \hat{t} + \hat{u}\big)$$

LO matrix element:

$$|\overline{\mathcal{M}}|^2 = \frac{g^4}{N_c} \left[-C_F \frac{\hat{u}^2 + \hat{s}^2}{\hat{s}\hat{u}} + N_c \frac{\hat{u}^2 + \hat{s}^2}{\hat{t}^2} \right] \qquad \hat{s} = \frac{k_\perp^2}{\xi(1-\xi)}, \quad \hat{t} = -\frac{k_\perp^2}{\xi}$$

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}y\mathrm{d}^2\mathbf{p}_\perp} = \frac{\alpha_s^2}{N_c} \int \frac{\mathrm{d}z}{z^2} D_{h/q}(z) \int \frac{\mathrm{d}\xi}{\xi} x q(x) x' G(x') \frac{1+\xi^2}{1-\xi} \frac{\xi}{k_\perp^4} \left[C_F(1-\xi)^2 + N_c\xi \right]$$

with $\qquad x' = \frac{k_\perp}{\sqrt{s}} \frac{e^{-y}}{1-\xi}$
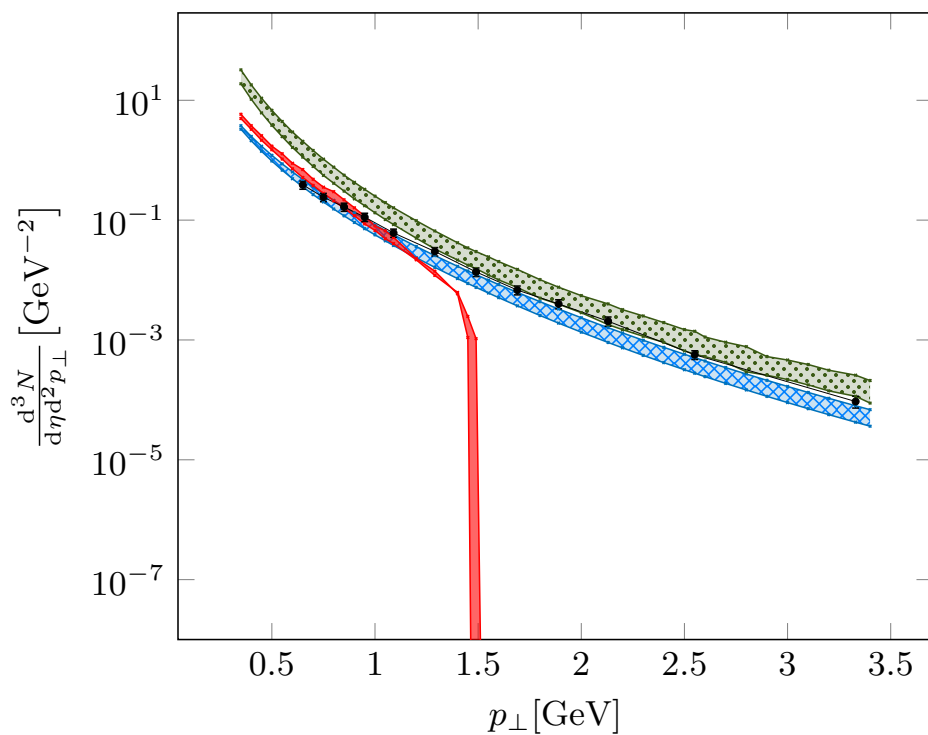
Exact matching of expanded small x formula to collinear formula, when exact kinematics is taken into account (also similar to what happens in kt factorization formula in DIS and collinear limit)
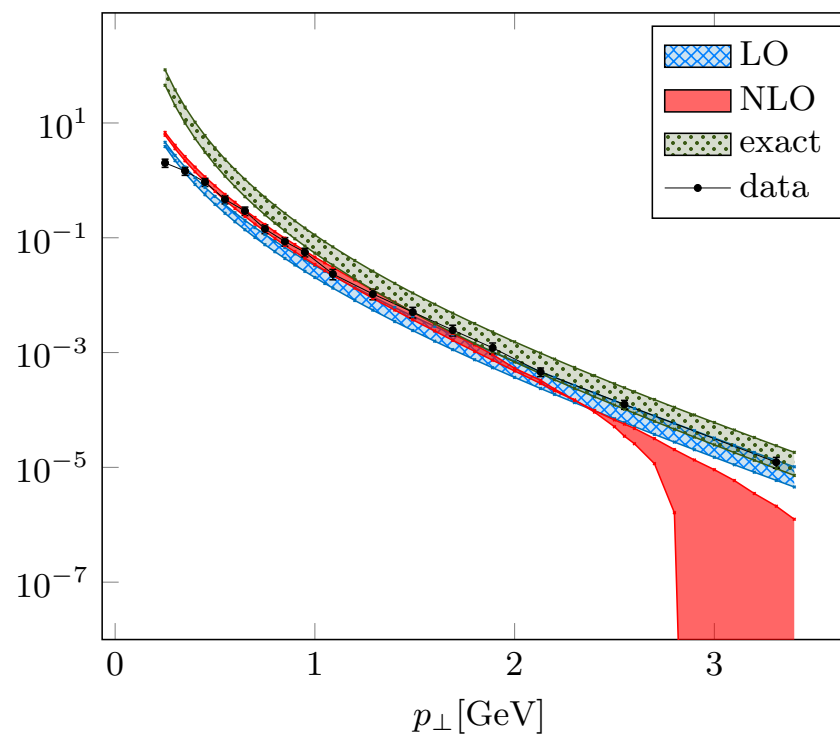
Small x vs collinear

$$\frac{\mathrm{d}\sigma_{q\to q}}{\mathrm{d}y\mathrm{d}^2\mathbf{p}_\perp} = \frac{\alpha_s}{2\pi^2} \int_\tau^1 \frac{\mathrm{d}z}{z^2} D_{h/q}(z) \int_{\tau/z}^1 \mathrm{d}\xi \frac{1+\xi^2}{1-\xi} xq(x) \left\{ C_F \frac{(1-\xi)^2}{k_\perp^4} + N_c \frac{\xi}{k_\perp^4} \right\} \int_{\mathcal{R}} \mathrm{d}^2\mathbf{k}_{g\perp} k_{g\perp}^2 \mathcal{F}_{x_a}(k_{g\perp})$$

$$\frac{\mathrm{d}^3\sigma_{g\to g}}{\mathrm{d}y\mathrm{d}^2\mathbf{p}_\perp} = \frac{\alpha_s N_c}{2\pi^2} \int_\tau^1 \frac{\mathrm{d}z}{z^2} D_{h/g}(z) \int_{\tau/z}^1 \mathrm{d}\xi xg(x) \frac{2[1-\xi(1-\xi)]^2[1+\xi^2+(1-\xi)^2]}{\xi(1-\xi)} \frac{1}{k_\perp^4} \int_{\mathcal{R}} \mathrm{d}^2\mathbf{k}_{g\perp} k_{g\perp}^2 \mathcal{F}_{x_a}(k_{g\perp}).$$
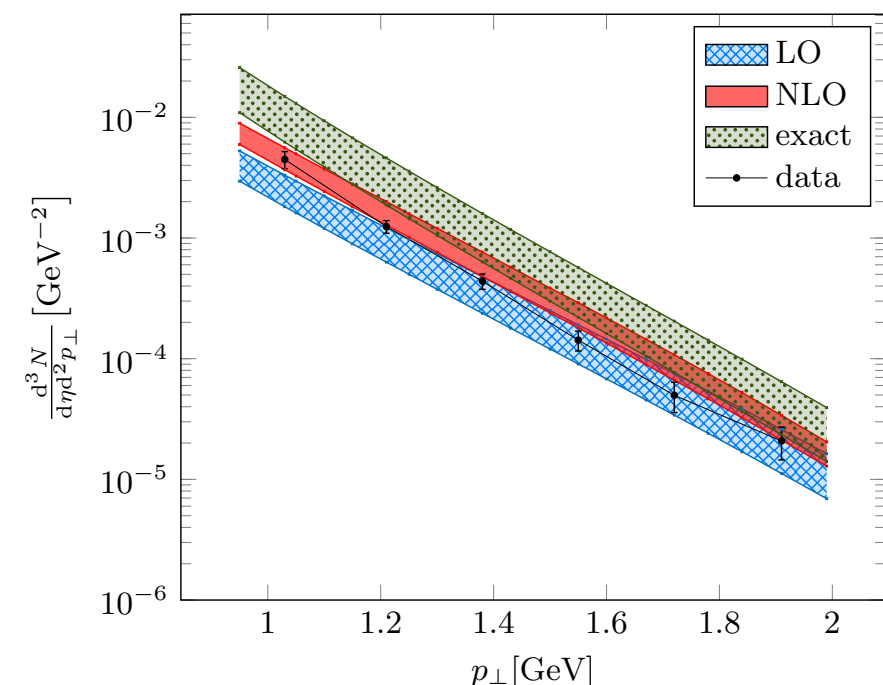

BRAHMS $\eta = 2.2$

BRAHMS $\eta = 3.2$

STAR $\eta = 4$

Matching between small x and collinear approximation with exact kinematics.
The small x gives good prediction up to $p_T \sim Q_s$
Collinear gives good description at large transverse momenta and overpredicts data for low transverse momenta.

Summary

- NLO calculation of the forward single inclusive production in p(d)-A implemented numerically.

- Using running coupling BK and DGLAP evolution for parton distribution functions and fragmentation functions.

- Comparisons with RHIC and calculations for LHC kinematics.

- Generally corrections are quite large, can be positive for low pT, and turn negative for high pT. Results in steeper distribution than LO.

- Scale dependence is smaller at NLO than at LO

- Negativity issue for large pT and not so large rapidities. NLO dominates the cross section.

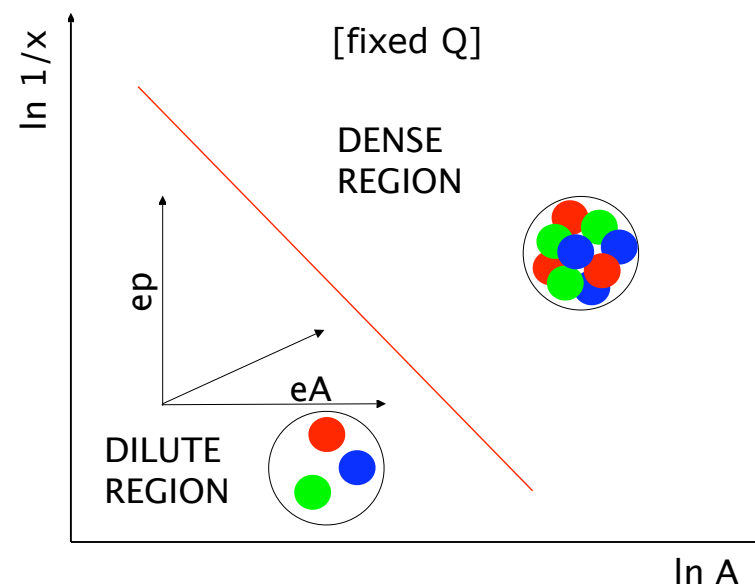- Small x formalism matches collinear approximation when exact kinematics is taken into account.

backup

Proto... ...ions

Can view this process of sca... ...off a dense system (nucleus).
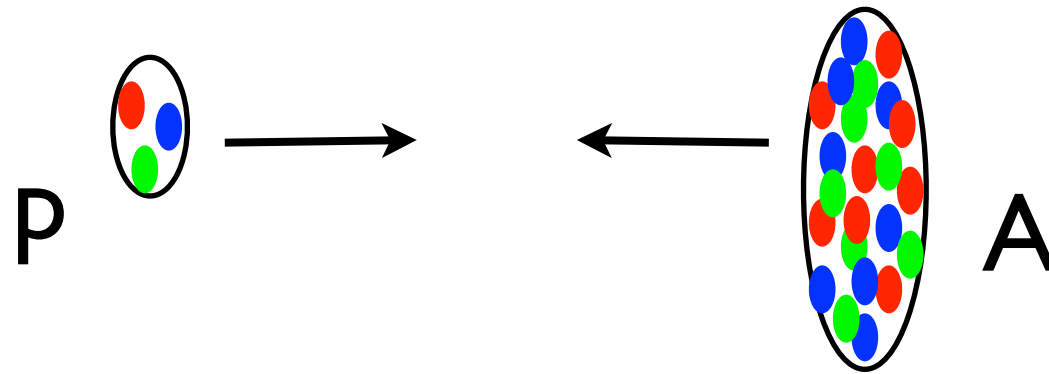
Forward kinematics of the proton: proton structure evaluated at large x, nucleus parton structure evaluated at small x.

Proton-nucleus collisions important for several reasons:

- Initial state for heavy ion collisions, benchmarking for AA collisions
- Nuclear structure, sensitivity to nuclear parton distributions
- At high energy sensitivity to small x components of the nuclear wave functions
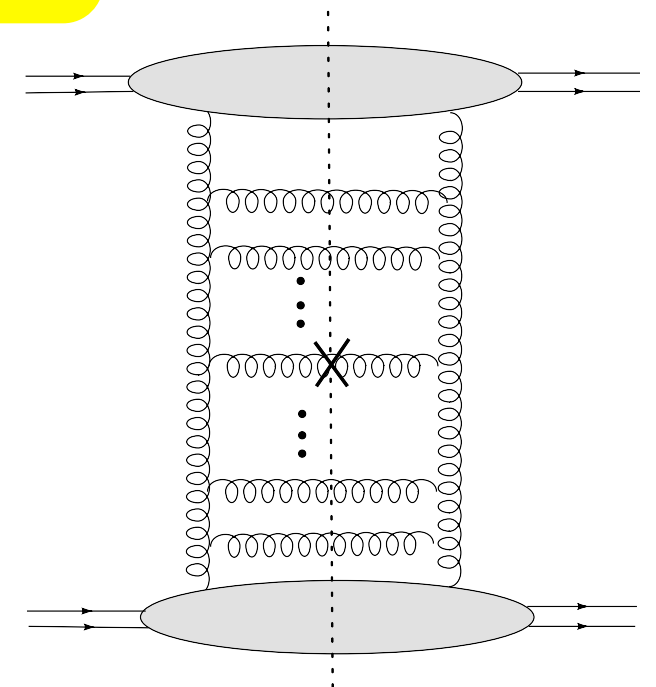
LO formalism-midrapidity

For midrapidity production kT factorization more suitable

$$\frac{d\sigma^{A+B\rightarrow g}}{dy\,d^2p_t\,d^2R} = K^k\,\frac{2}{C_F}\frac{1}{p_t^2}\int^{p_t}\frac{d^2k_t}{4}\int d^2b\,\alpha_s(Q)\,\varphi_P\left(\frac{|p_t+k_t|}{2},x_1;b\right)\,\varphi_T\left(\frac{|p_t-k_t|}{2},x_2;R-b\right)$$

$$\varphi(k,x,\mathbf{R}) = \frac{C_F}{\alpha_s(k)\,(2\pi)^3}\int d^2\mathbf{r}\,e^{-i\mathbf{k}\cdot\mathbf{r}}\,\nabla_\mathbf{r}^2\,\mathcal{N}_A(r,x,\mathbf{R})$$

Gluon production in midrapidity (no quarks in this formalism)

Symmetric treatment of target and projectile

Transverse momentum dependence also included in the projectile

Hadron production obtained through convolution with the fragmentation functions

$$\frac{dN^{A+B\rightarrow hX}}{dy\,d^2p_t} = \int d^2R\int\frac{dz}{z^2}\,D_g^h\left(z=\frac{p_t}{k_t},Q\right)\frac{dN^{A+B\rightarrow g}}{dy\,d^2q_t\,d^2R}$$

Since we focus on forward rapidity we do not consider that case here...

Unintegrated gluon distribution

Unintegrated gluon distribution function usually obtained from the Balitsky-Kovchegov equation.

LO BK equation:

$$\frac{\partial N_{\mathbf{x}_0\mathbf{x}_1}}{\partial Y} = \overline{\alpha}_s \int \frac{d^2\mathbf{x}_2}{2\pi} \frac{(\mathbf{x}_0 - \mathbf{x}_1)^2}{(\mathbf{x}_0 - \mathbf{x}_2)^2(\mathbf{x}_1 - \mathbf{x}_2)^2} \left[N_{\mathbf{x}_0\mathbf{x}_2} + N_{\mathbf{x}_1\mathbf{x}_2} - N_{\mathbf{x}_0\mathbf{x}_1} - N_{\mathbf{x}_0\mathbf{x}_2} N_{\mathbf{x}_1\mathbf{x}_2} \right]$$

$$N_{\mathbf{x}_0\mathbf{x}_1} = \frac{1}{N_c}\mathrm{Tr}\left\langle 1 - U^\dagger(\mathbf{x}_0)U(\mathbf{x}_1)\right\rangle \qquad N = 1 - S^{(2)} \qquad \overline{\alpha}_s \equiv \frac{N_c\alpha_s}{\pi}$$

At LO coupling is fixed, BK kernel is invariant w.r.t. to transformations:

$$\mathbf{x}_i \rightarrow \mathbf{x}_i + C \qquad\qquad \mathbf{x}_i \rightarrow \lambda\,\mathbf{x}_i \qquad\qquad \mathbf{x}_i \rightarrow \frac{\mathbf{x}_i}{|x_i|^2} \qquad\qquad \mathbf{x}_i \rightarrow O(\phi)\mathbf{x}_i$$

NLO BK equation available but difficult to solve even numerically.
Typically only running coupling corrections are taken into account for phenomenological applications.

$$K^{LO}(x_{01}, x_{02}) = \frac{N_c\alpha_s}{2\pi^2}\frac{x_{01}^2}{x_{02}^2 x_{12}^2}$$

$\longrightarrow$

$$K^{Bal}(x_{01}, x_{02}) = \frac{N_c\alpha_s(x_{01}^2)}{2\pi^2}\left[\frac{x_{01}^2}{x_{02}^2 x_{12}^2} + \frac{1}{x_{02}^2}\left(\frac{\alpha_s(x_{02}^2)}{\alpha_s(x_{12}^2)} - 1\right) + \frac{1}{x_{12}^2}\left(\frac{\alpha_s(x_{12}^2)}{\alpha_s(x_{02}^2)} - 1\right)\right]$$

Balitsky prescription. Note nonlinear dependence on the running coupling.

When one dipole size is much smaller than the rest the kernel can be approximated by the LO kernel with running coupling evaluated at this smallest dipole size.

Low x and saturation calculations: examples of higher orders of accuracy

- Linear BFKL evolution at NLO.

- Impact factors for DIS at NLO.

- Jet vertices at NLO.

- Nonlinear BK evolution at NLO.

- Inclusive hadron production at NLO.

Also resummed approaches (for the case of linear evolution) which aim to evaluate/approximate higher orders.

Already large number of calculations at NLO accuracy and beyond (*typically without saturation*) have been applied to phenomenology: DIS inclusive, Mueller-Navelet jets, forward jets, electroproduction of vector mesons, angular decorrelation of jets...

- However, calculations that include saturation are usually done at lowest order of accuracy (with some improvements from the running coupling).
- So there is need to evaluate processes at NLO with saturation and evaluate the impact of higher orders onto the magnitude of the saturation scale.
- Need to do precision higher-order calculations which involve saturation.

NLO calculation

$g \to g$ channel

(a)   *(b)*

(c)   *(d)*

Real diagrams

(a)   *(b)*

(a)   *(b)*
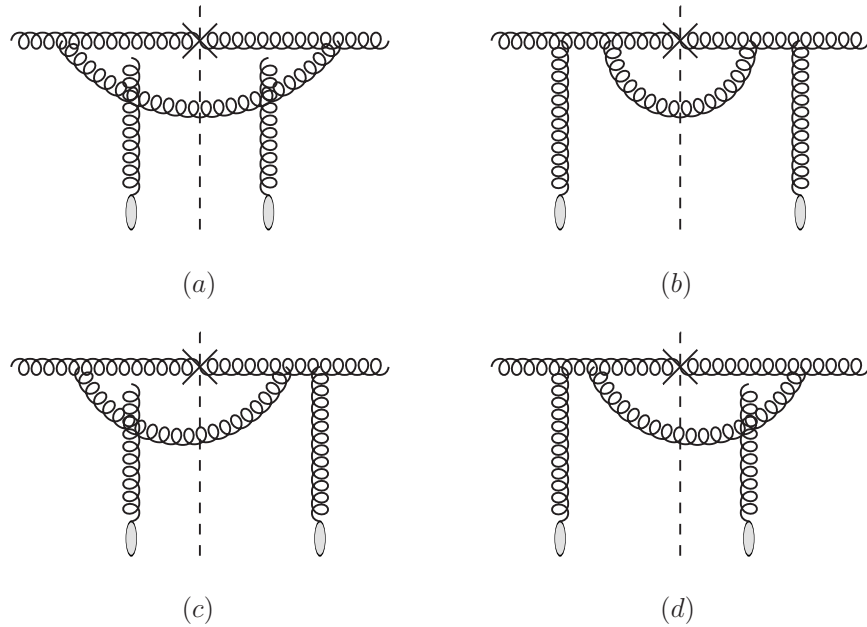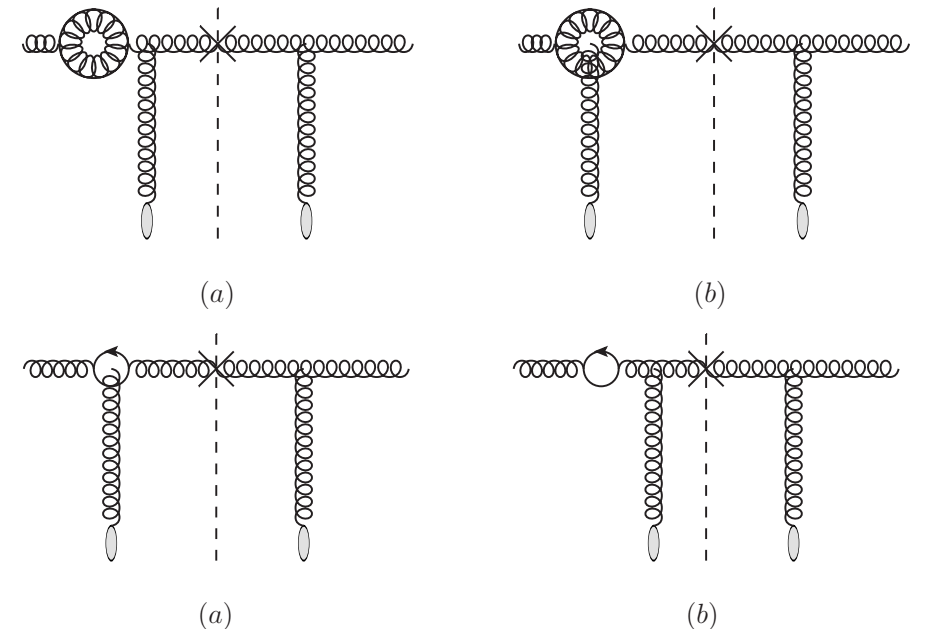
Typical virtual diagrams

Procedure similar to quark-quark channel.
Additional complications arise due to the presence of the sextupole and quadrupole.
Sextupole contribution is suppressed in large color limit.
Quadrupole can be written as a product of dipoles in the large color limit.

Final expression for the
gluon-gluon channel
(large Nc limit taken
here):

$$\frac{d^3\sigma^{p+A\to h/g+X}}{dy d^2 p_\perp} = \int \frac{dz}{z^2}\frac{dx}{x}\xi x g(x,\mu) D_{h/g}(z,\mu)$$

$$\times \left\{ \int \frac{d^2 x_\perp d^2 y_\perp}{(2\pi)^2} S_Y^{(2)}(x_\perp, y_\perp) S_Y^{(2)}(y_\perp, x_\perp) \left[\mathcal{H}_{2gg}^{(0)} + \frac{\alpha_s}{2\pi}\mathcal{H}_{2gg}^{(1)} \right] \right.$$

$$+ \int \frac{d^2 x_\perp d^2 y_\perp d^2 b_\perp}{(2\pi)^4} S_Y^{(2)}(x_\perp, b_\perp) S_Y^{(2)}(b_\perp, y_\perp)\frac{\alpha_s}{2\pi}\mathcal{H}_{2q\bar{q}}^{(1)}$$

$$\left. + \int \frac{d^2 x_\perp d^2 y_\perp d^2 b_\perp}{(2\pi)^4} S_Y^{(2)}(x_\perp, b_\perp) S_Y^{(2)}(b_\perp, y_\perp) S_Y^{(2)}(y_\perp, x_\perp)\frac{\alpha_s}{2\pi}\mathcal{H}_{6gg}^{(1)} \right\}$$
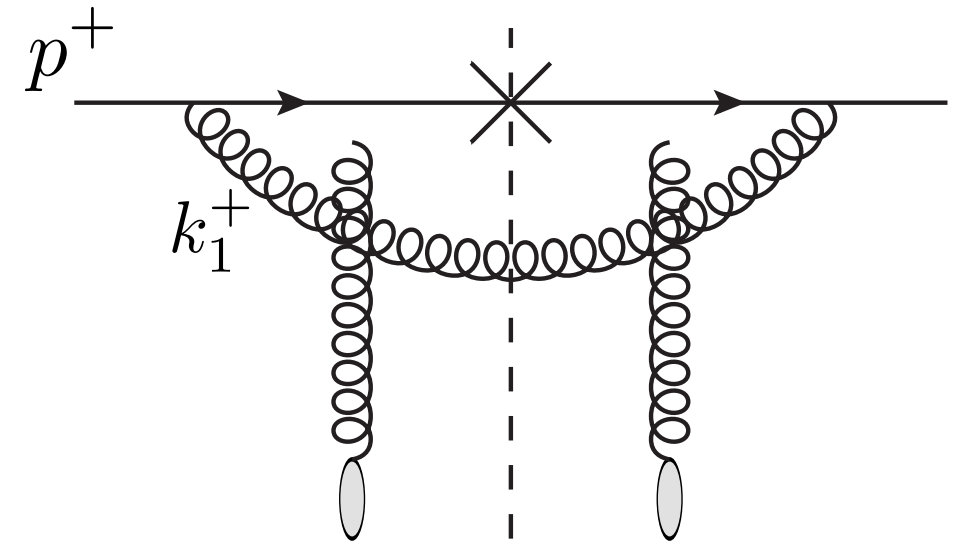
NLO calculation

Rapidity divergence: $\xi \to 1$

$$p^+$$

Momentum fraction of the incoming quark carried by the gluon

$$1 - \xi = \frac{k_1^+}{p^+}$$

$$k_1^+$$

Physical interpretation:

Quark moving with very high positive rapidity.
Longitudinally soft gluon moving with very high negative rapidity.
Include that gluon into the nuclear density.

Subtraction of rapidity divergence into the gluon distribution of the nucleus.

$$\mathcal{F}(k_\perp) = \mathcal{F}^{(0)}(k_\perp) - \frac{\alpha_s N_c}{2\pi^2} \int_0^1 \frac{d\xi}{1-\xi} \int \frac{d^2 x_\perp d^2 y_\perp d^2 b_\perp}{(2\pi)^2} e^{-ik_\perp \cdot (x_\perp - y_\perp)}$$
$$\times \frac{(x_\perp - y_\perp)^2}{(x_\perp - b_\perp)^2 (y_\perp - b_\perp)^2} \left[S_Y^{(2)}(x_\perp, y_\perp) - S_Y^{(4)}(x_\perp, b_\perp, y_\perp) \right],$$

BK evolution at LL in ln1/x

$$\frac{\partial}{\partial Y} S_Y^{(2)}(x_\perp, y_\perp) = -\frac{\alpha_s N_c}{2\pi^2} \int \frac{d^2 b_\perp (x_\perp - y_\perp)^2}{(x_\perp - b_\perp)^2 (y_\perp - b_\perp)^2} \left[S_Y^{(2)}(x_\perp, y_\perp) - S_Y^{(4)}(x_\perp, b_\perp, y_\perp) \right]$$

NLO calculation

Collinear divergence:

Dimensional regularization used and MSbar scheme to remove collinear divergences associated with the gluon collinear either to initial or final state quark.
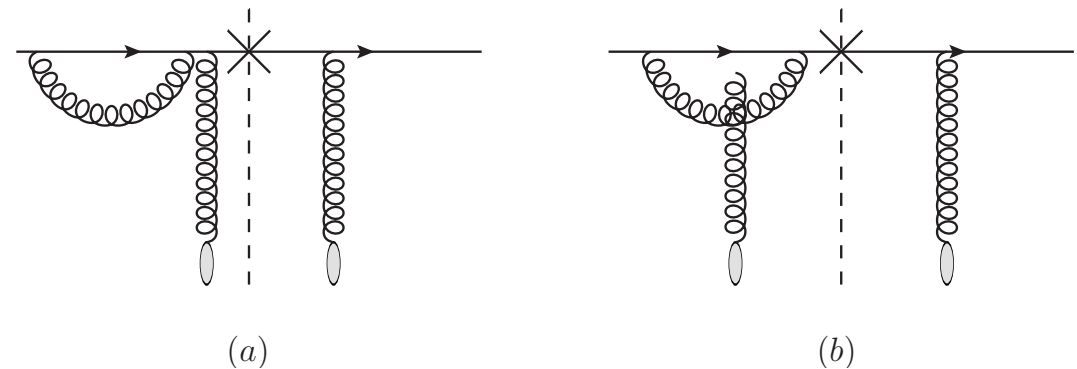
Need to take into account virtual diagrams as well

(a)   (b)

FIG. 4. Typical virtual diagrams for the next-to-leading order quark production $qA \to q + X$.

Coefficient of the collinear divergence is equal to the splitting function

$$\mathcal{P}_{qq}(\xi) = \left(\frac{1 + \xi^2}{1 - \xi} \right)_+ = \frac{1 + \xi^2}{(1 - \xi)_+} + \frac{3}{2}\delta(1 - \xi).$$

Collinear divergence absorbed into renormalization of parton distribution and fragmentation functions

$$q(x, \mu) = q^{(0)}(x) - \frac{1}{\hat{\epsilon}} \frac{\alpha_s(\mu)}{2\pi} \int_x^1 \frac{d\xi}{\xi} C_F \mathcal{P}_{qq}(\xi) q \left(\frac{x}{\xi} \right),$$

$$D_{h/q}(z, \mu) = D_{h/q}^{(0)}(z) - \frac{1}{\hat{\epsilon}} \frac{\alpha_s(\mu)}{2\pi} \int_z^1 \frac{d\xi}{\xi} C_F \mathcal{P}_{qq}(\xi) D_{h/q} \left(\frac{z}{\xi} \right)$$

Now we have the scale dependence inside pdfs and ffs

NLO calculation

All channels put together:

$$\frac{d^3\sigma^{p+A\to h+X}}{dy\,d^2p_\perp} = \int \frac{dz}{z^2}\frac{dx}{x}\xi\,[xq(x,\mu), xg(x,\mu)]\begin{bmatrix} S_{qq} & S_{qg} \\ S_{gq} & S_{gg} \end{bmatrix}\begin{bmatrix} D_{h/q}(z,\mu) \\ D_{h/g}(z,\mu) \end{bmatrix}$$

$$S_{qq} = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} S_Y^{(2)}(x_\perp, y_\perp) e^{-ik_\perp\cdot r_\perp}\delta(1-\xi)\left[1 - \frac{\alpha_s}{2\pi}3C_F\ln\frac{c_0^2}{r_\perp^2 k_\perp^2}\right]$$

$$+ \int \frac{d^2x_\perp d^2y_\perp d^2b_\perp}{(2\pi)^4} S_Y^{(4)}(x_\perp, b_\perp, y_\perp)\frac{\alpha_s}{2\pi}\mathcal{H}_{4qq}^{(1)},$$

$$S_{qg} = \frac{\alpha_s}{2\pi}\int \frac{d^2x_\perp d^2y_\perp d^2b_\perp}{(2\pi)^4} S_Y^{(4)}(x_\perp, b_\perp, y_\perp)\mathcal{H}_{4gq}^{(1)},$$

$$S_{gq} = \frac{\alpha_s}{2\pi}\int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} S_Y^{(2)}(x_\perp, y_\perp)\left[\mathcal{H}_{2qg}^{(1,1)} + S_Y^{(2)}(x_\perp, y_\perp)\mathcal{H}_{2qg}^{(1,2)}\right]$$

$$+\frac{\alpha_s}{2\pi}\int \frac{d^2x_\perp d^2y_\perp d^2b_\perp}{(2\pi)^4} S_Y^{(4)}(x_\perp, b_\perp, y_\perp)\mathcal{H}_{4qg}^{(1)},$$

$$S_{gg} = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} S_Y^{(2)}(x_\perp, y_\perp)S_Y^{(2)}(y_\perp, x_\perp)e^{-ik_\perp\cdot r_\perp}\delta(1-\xi)\left[1 - \frac{\alpha_s}{2\pi}N_c\left(\frac{11}{3} - \frac{4N_f T_R}{3N_c}\right)\ln\frac{c_0^2}{r_\perp^2 k_\perp^2}\right]$$

$$+ \int \frac{d^2x_\perp d^2y_\perp d^2b_\perp}{(2\pi)^4} S_Y^{(2)}(x_\perp, b_\perp)S_Y^{(2)}(b_\perp, y_\perp)\frac{\alpha_s}{2\pi}\mathcal{H}_{2q\bar{q}}^{(1)}$$

$$+ \int \frac{d^2x_\perp d^2y_\perp d^2b_\perp}{(2\pi)^4} S_Y^{(2)}(x_\perp, b_\perp)S_Y^{(2)}(b_\perp, y_\perp)S_Y^{(2)}(y_\perp, x_\perp)\frac{\alpha_s}{2\pi}\mathcal{H}_{6gg}^{(1)},$$

Chirilli, Xiao, Yuan

Factorizable structure in coordinate space

$$xq(x,\mu) \quad xg(x,\mu) \quad D_{h/q}(z,\mu) \quad D_{h/g}(z,\mu)$$

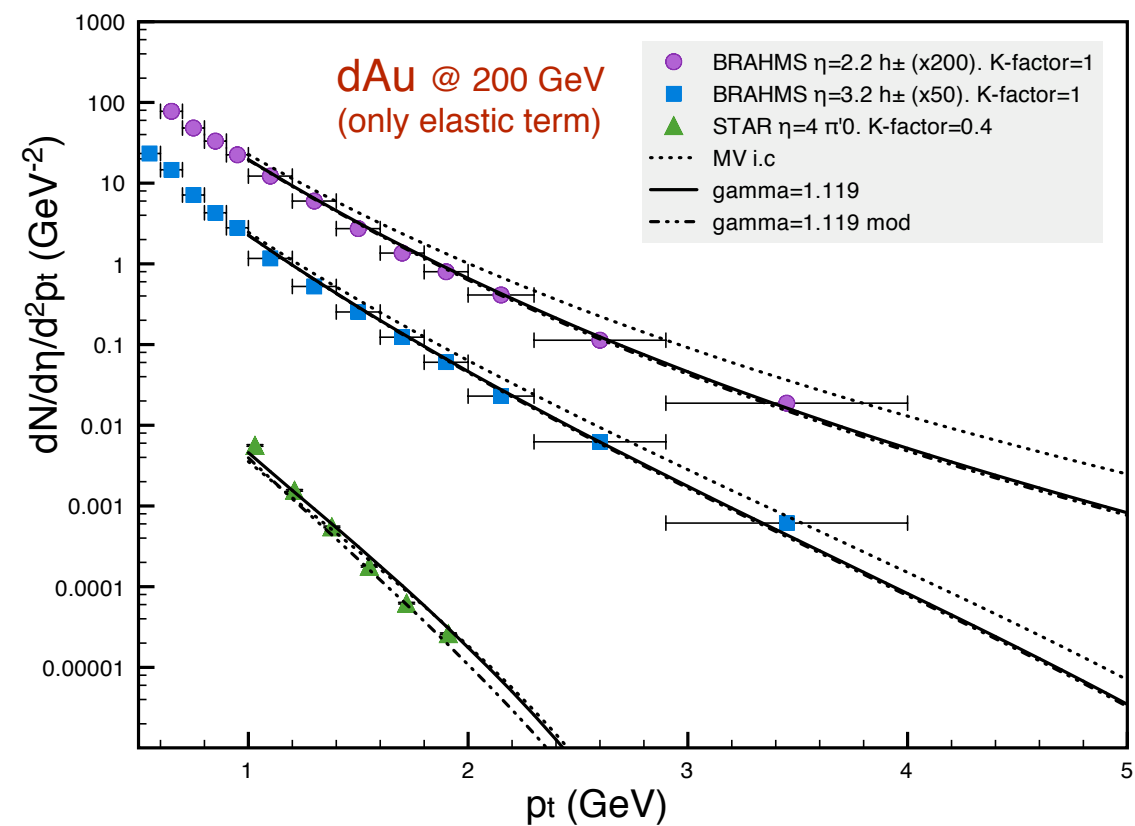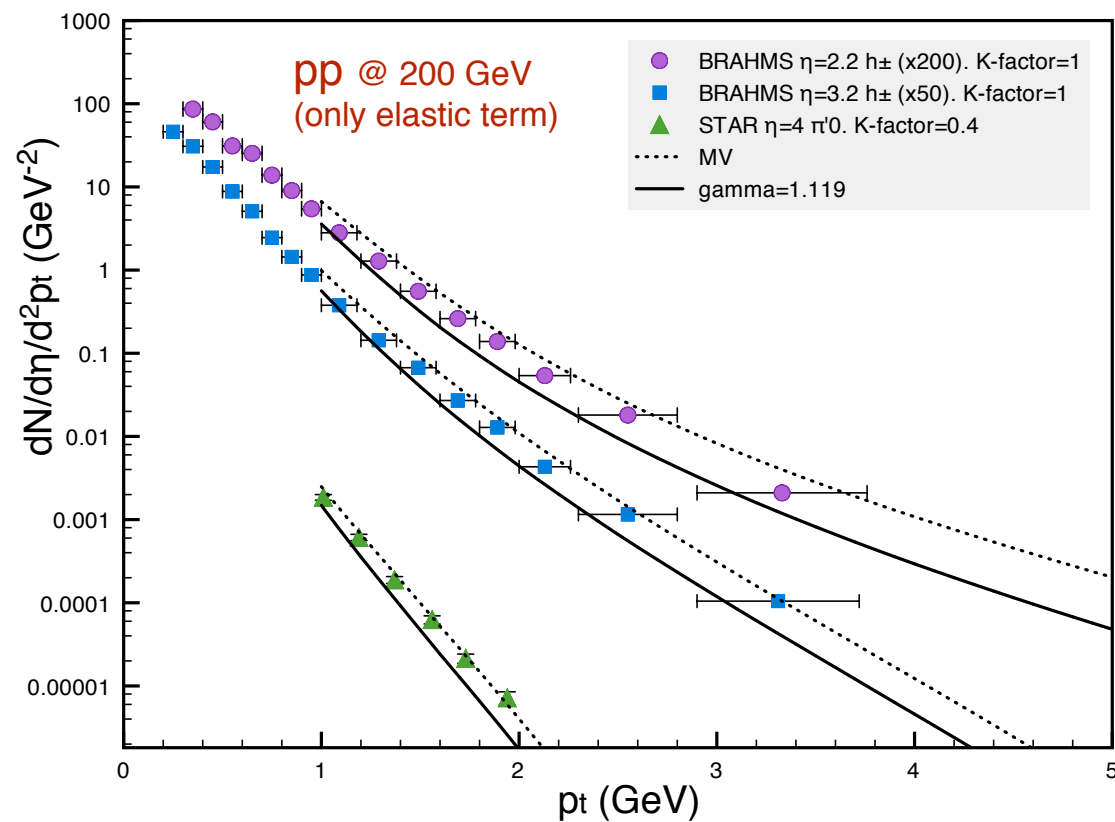evolving according to the DGLAP equation

$$S_Y^{(2,4)}$$

evolving according to the small x evolution equation

Relevant for the forward rapidity kinematics, no transverse momentum dependence on the proton side.

Phenomenology with LO formalism

Inclusive charged hadron in pp and dA collisions
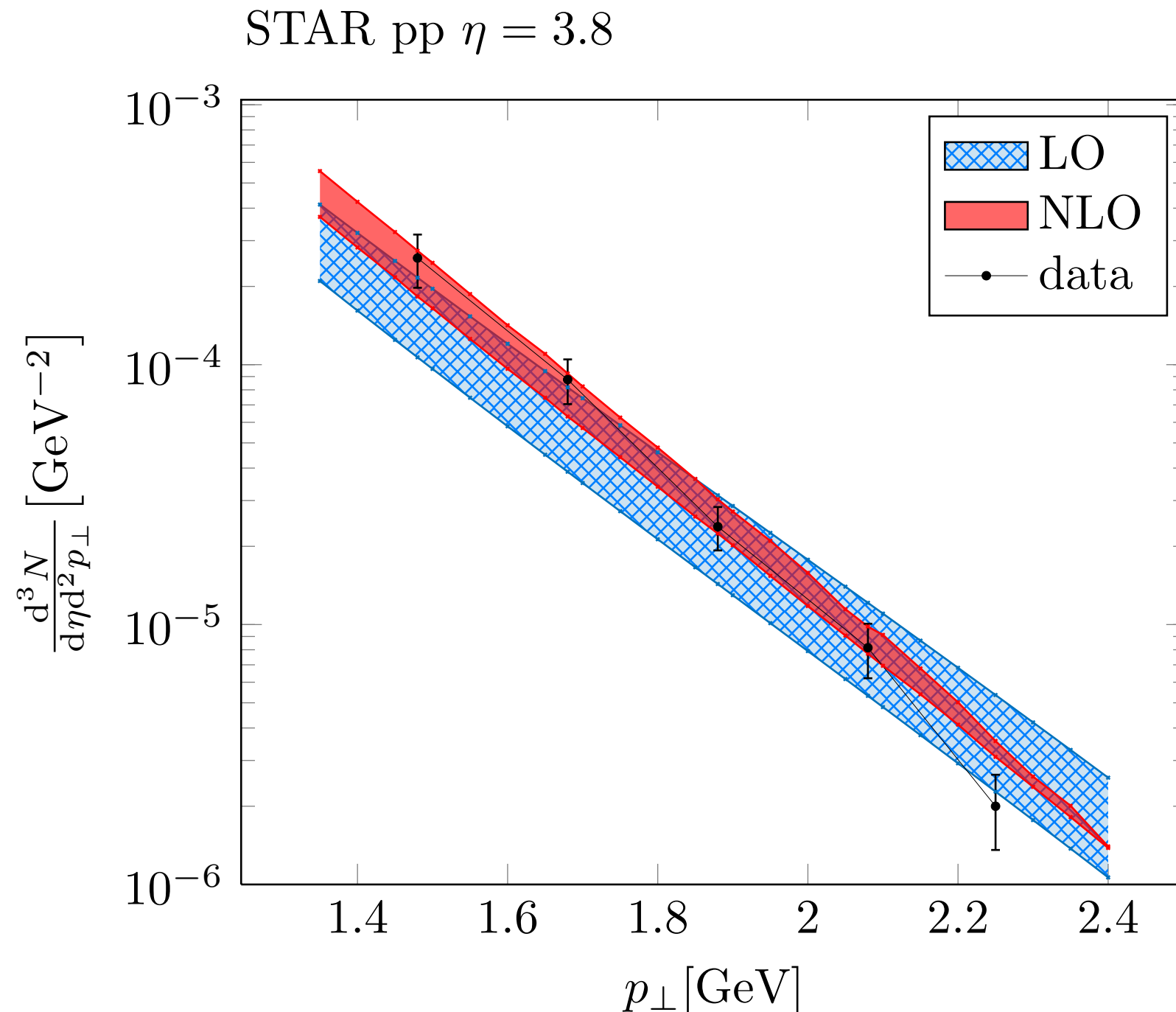
Albacete,Dumitru,Fujii,Nara

Successful description of the data within LO and using rcBK evolution.
Different initial conditions are allowed.
K-factor of K=0.4 for STAR data is needed. Very forward rapidities, end of kinematic phase space?

Forward production in pp

Unintegrated gluon density from rcBK with MV initial conditions.
The slope of NLO matches better the data than LO.

Forward production in pp