Beam Dynamics in Synchrotrons I: transverse

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Just to give a kind of definition ...

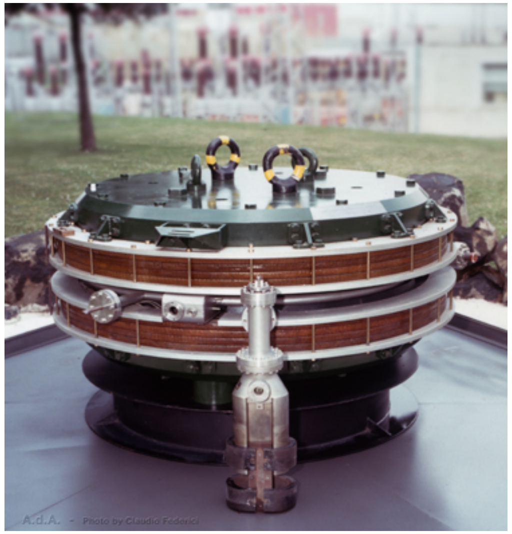
A synchrotron is a type of circular accelerator that needs:

A bending field
to keep the particles on a closed orbit
A mechanism to lock this B-field to the particle energy
→ constant orbit

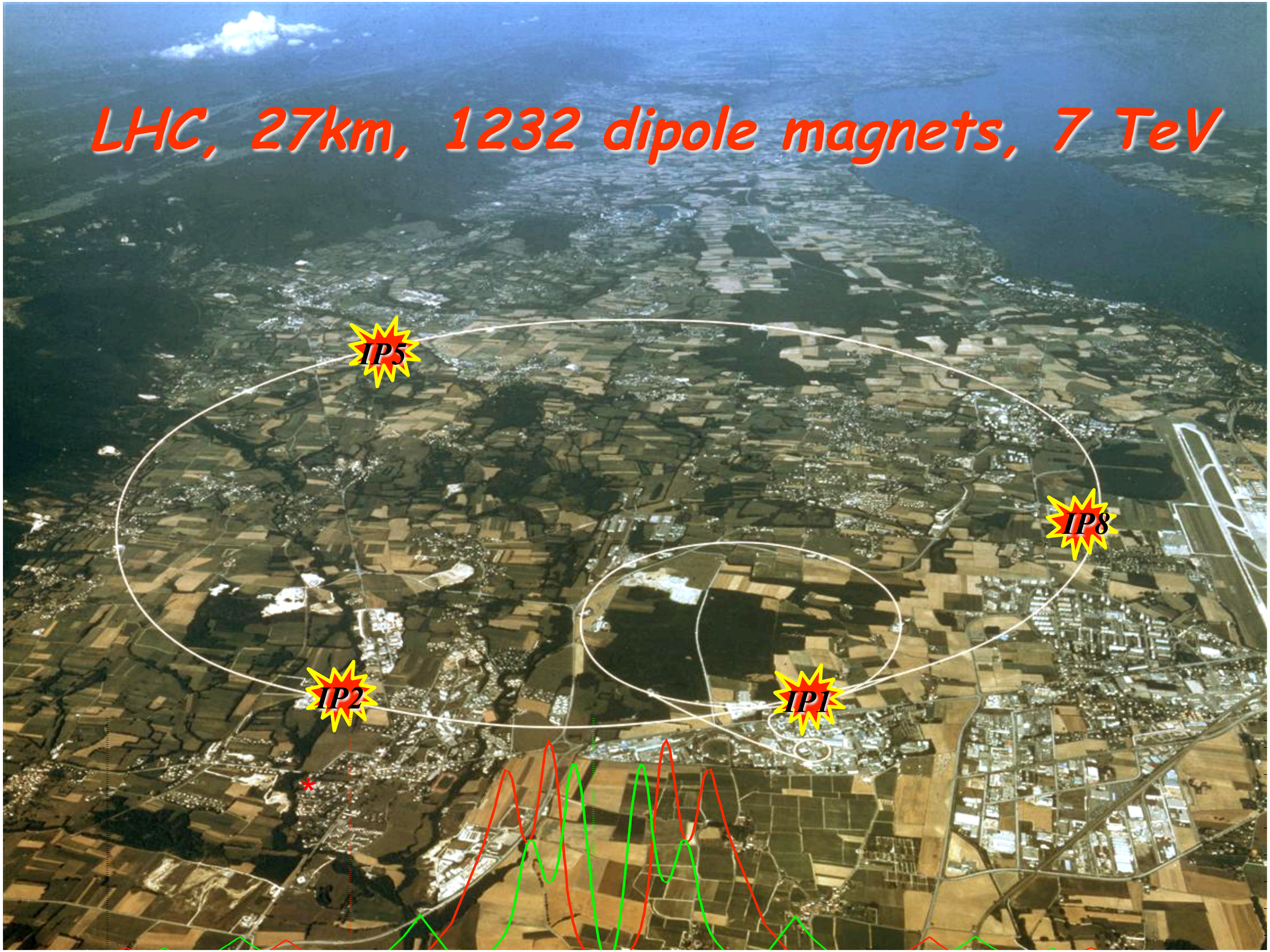
Focusing fields that follow the energy gain to keep the particles together
→ well defined beam size

A RF structure to accelerate the particles
→ energy gain per turn

A mechanism to synchronise the rf frequency to
the particle timing
→ phase focusing / synchrotron principle



ADA, Frascati


LHC, 27km, 1232 dipole magnets, 7 TeV
IP5
IP8
IP2
IP1

Introduction and Basic Ideas

„ … in the end and after all it should be a kind of circular machine“

→ need transverse deflecting force

Lorentz force $\vec{F} = q * (\vec{E} + \vec{v} \times \vec{B})$

typical velocity in high energy machines: $v \approx c \approx 3 * 10^8\ m/s$

Example:

$$B = 1\,T \quad \rightarrow \quad F = q * 3 * 10^8 \frac{m}{s} * 1 \frac{Vs}{m^2}$$

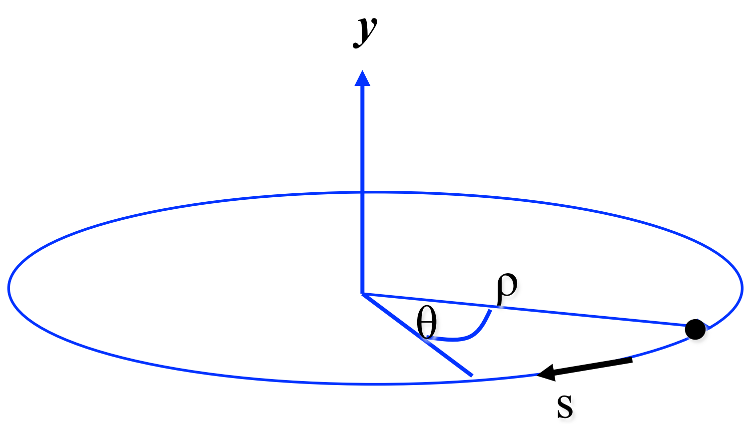
$$F = q * 300 \frac{MV}{m}$$

equivalent el field E

technical limit for a el. field $E \leq 1 \frac{MV}{m}$

Unlike to a cyclotron, we localise & optimise the magnets at the location of the beam … to keep the machine "small".

The ideal circular orbit



condition for circular orbit:

Lorentz force $F_L = e\,v\,B$

centrifugal force $F_{centr} = \frac{\gamma m_0 v^2}{\rho}$

$$\frac{\gamma m_0 v^2}{\rho} = e\,v\,B$$

$$\frac{p}{e} = B\rho$$

$B\rho =$ **"beam rigidity"**

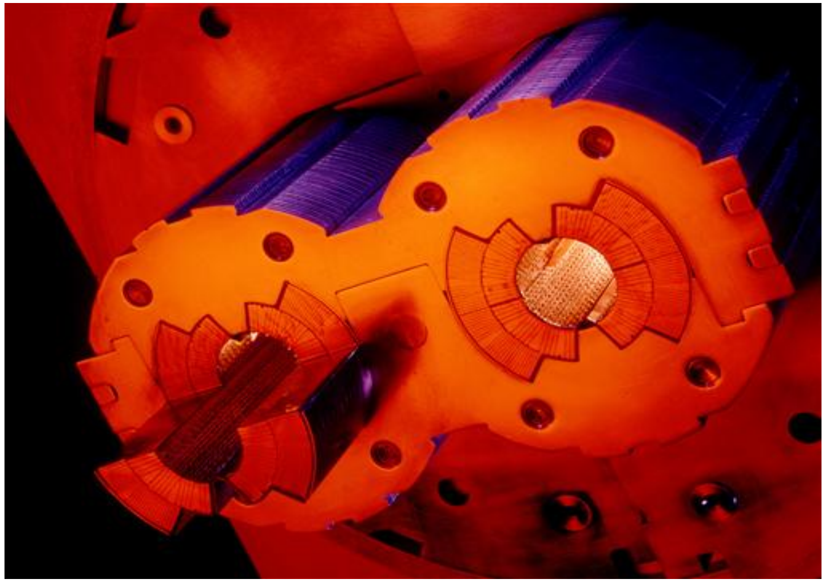
The arrangement and strength of the dipole magnets define the maximum particle momentum that can be carried by the synchrotron.

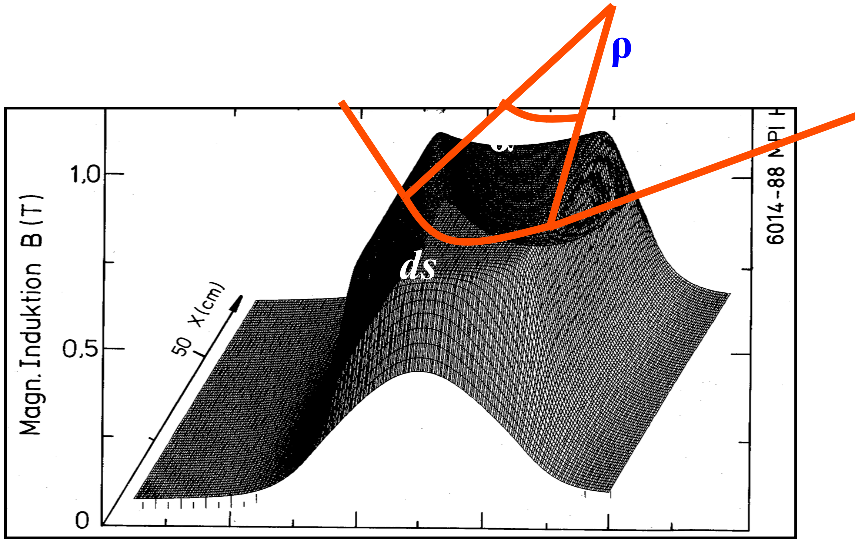
LHC: *7000 GeV Proton storage ring*
dipole magnets $N = 1232$
$l = 15\ m$
$q = +1\ e$

$$\int B\,dl \approx N\,l\,B = 2\pi\ p/e$$

$$B \approx \frac{2\pi\ 7000\ 10^9\, eV}{1232\ 15\,m\ 3\ 10^8 \frac{m}{s}\ e} = 8.3\,Tesla$$

The Magnetic Guide Field





field map of a storage ring dipole magnet

Normalise magnetic field to momentum:

$$\frac{p}{e} = B\rho \longrightarrow \frac{1}{\rho} = \frac{e\,B}{p}$$

convenient units:

$$B = [T] = \left[\frac{Vs}{m^2}\right] \qquad p = \left[\frac{GeV}{c}\right]$$

Example LHC:

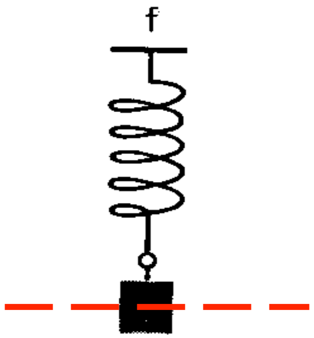
$$\left.\begin{aligned} B &= 8.3\,T \\ p &= 7000\,\frac{GeV}{c} \end{aligned}\right\} \quad \rho = 2.53\,km$$

Dipole Magnets:

define the ideal orbit via their homogeneous field, which is created by two flat pole shoes

Focusing Properties – Transverse Beam Optics

Classical Mechanics:
pendulum



there is a restoring force, proportional to the elongation x:

$$F = m * \frac{d^2x}{dt^2} = -k * x$$

Ansatz $x(t) = A * \cos(\omega t + \varphi)$

general solution: free harmonic oszillation

Storage Ring: we need a Lorentz force that rises as a function of the distance to the design orbit

$$F(x) = q * v * B(x)$$

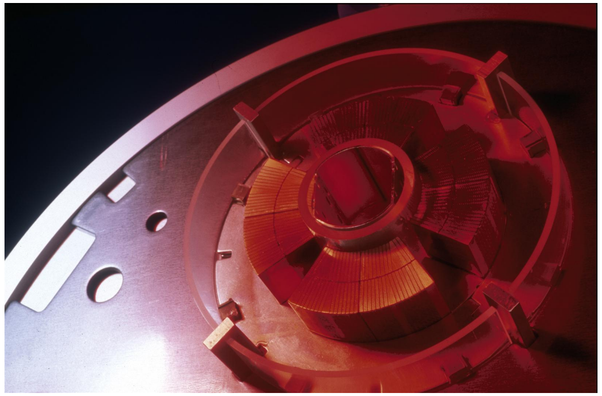
Quadrupole Magnets:

required: ***focusing forces to keep trajectories in vicinity of the ideal orbit***

linear increasing Lorentz force

linear increasing magnetic field $\boldsymbol{B_y = g\ x}$ $\boldsymbol{B_x = g\ y}$

Normalised quadrupole field:



LHC sc quadrupole

Field in a quadrupole $B_y = g\,x \qquad B_x = g\,y$

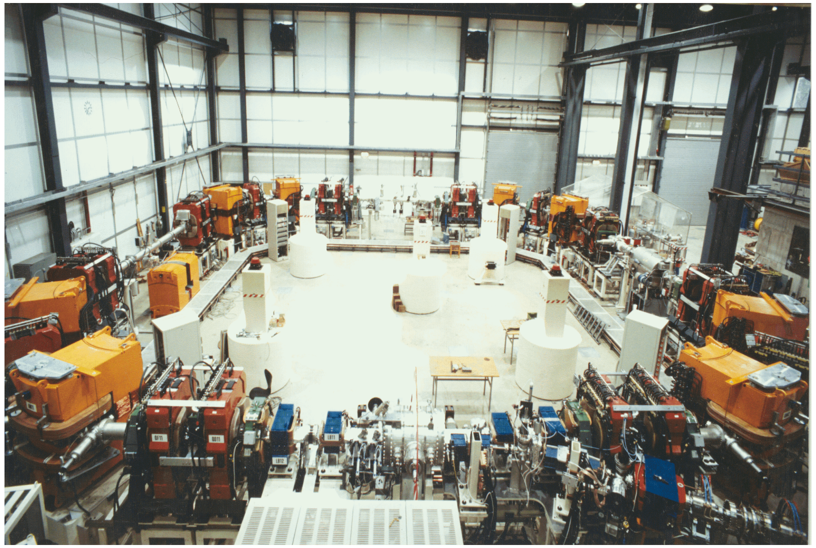
Normalised gradient of a quadrupole → $k = \frac{g}{p/e}$

what about the vertical plane:
... Maxwell

$$\vec{\nabla} \times \vec{B} = \vec{j} + \frac{\partial \vec{E}}{\partial t} \quad \Rightarrow \quad \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y}$$

The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \frac{1}{2!} m\,x^2 + \frac{1}{3!} n\,x^3 + \ldots$$



only terms linear in x, y taken into account

dipole fields

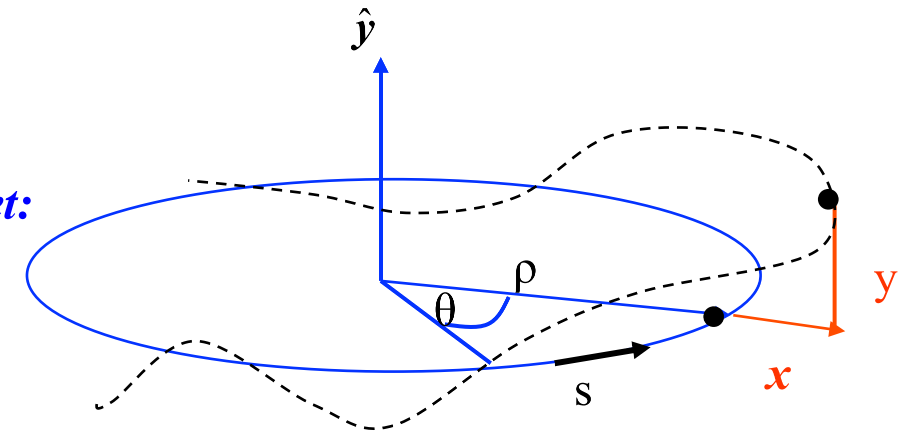
quadrupole fields

Separate Function Machines: Split the magnets and optimise them according to their job

The Equation of Motion:

Equation for the horizontal motion of a particle inside a storage ring magnet:

$$x'' + x\left(\frac{1}{\rho^2} - k\right) = 0$$

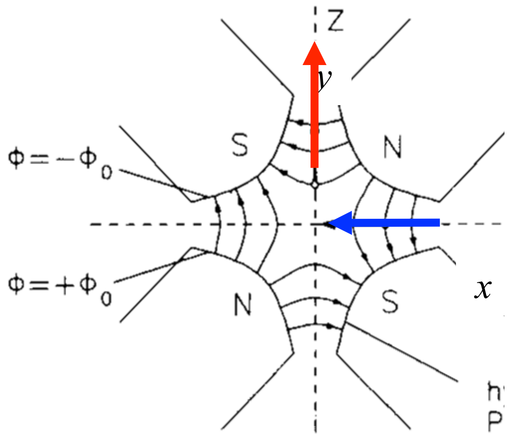


Equation for the vertical motion:

$$\frac{1}{\rho^2} = 0$$ *no dipoles … in general …*

$$k \leftrightarrow -k$$ *quadrupole field changes sign*

$$y'' + k\,y = 0$$



A magnet in a synchrotron that acts as hor. focusing lens, has at the same time, a defocusing effect in the vertical plane.
Et vice versa.

Solution of Trajectory Equations

Define … hor. plane: $K = 1/\rho^2 - k$

… vert. Plane: $K = k$

$$\boldsymbol{x}'' + \boldsymbol{K}\ \boldsymbol{x} = 0$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: $x(s) = a_1 \cdot \cos(\omega s) + a_2 \cdot \sin(\omega s)$

general solution: linear combination of two independent solutions

$$x'(s) = -a_1 \omega \sin(\omega s) + a_2 \omega \cos(\omega s)$$

$$x''(s) = -a_1 \omega^2 \cos(\omega s) - a_2 \omega^2 \sin(\omega s) = -\omega^2 x(s) \longrightarrow \omega = \sqrt{K}$$

general solution:

$$x(s) = a_1 \cos(\sqrt{K} s) + a_2 \sin(\sqrt{K} s)$$

determine a_1, a_2 by boundary conditions:

$$s = 0 \longrightarrow \begin{cases} x(0) = x_0 \quad , \quad a_1 = x_0 \\ x'(0) = x'_0 \quad , \quad a_2 = \dfrac{x'_0}{\sqrt{K}} \end{cases}$$

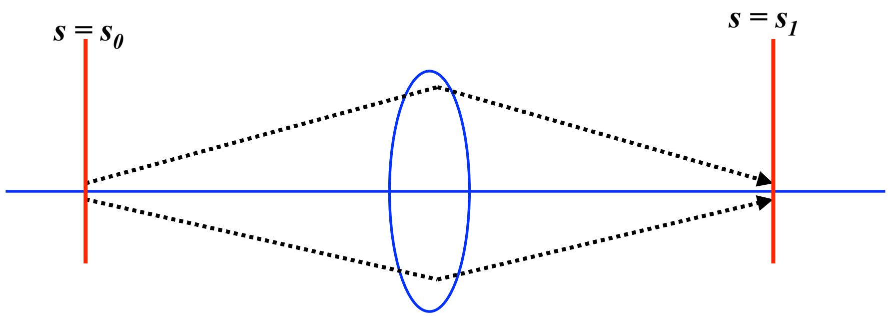
Hor. Focusing Quadrupole $K > 0$:

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x'_0 \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x'_0 \cdot \cos(\sqrt{|K|}s)$$

For convenience expressed in matrix formalism:

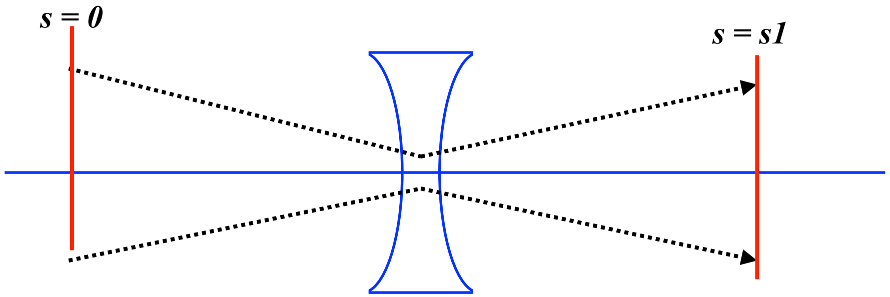
$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$



$$M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|}s) & \dfrac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s \\ -\sqrt{|K|}\sin(\sqrt{|K|}s) & \cos(\sqrt{|K|}s) \end{pmatrix}_0$$

hor. defocusing quadrupole:

$$x'' - K\,x = 0$$



Remember from school:

$$f(s) = \cosh(s) \quad , \qquad f'(s) = \sinh(s)$$

Ansatz: $$x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s)$$

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}\,l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}\,l \\ \sqrt{|K|}\sinh\sqrt{|K|}\,l & \cosh\sqrt{|K|}\,l \end{pmatrix}$$

drift space:

$$K = 0$$

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

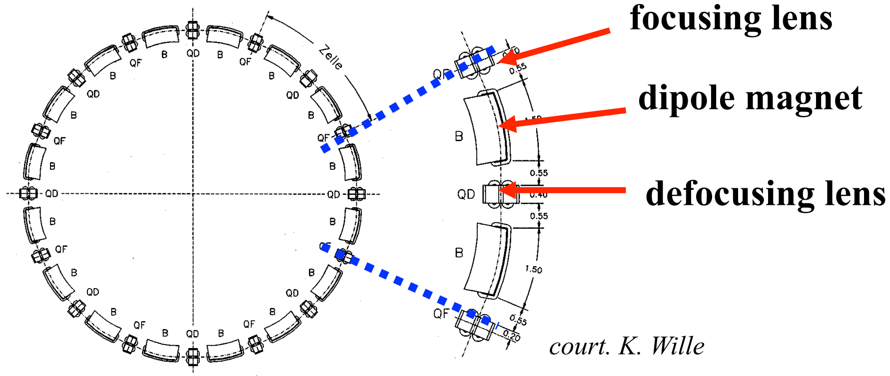
! with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“

Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

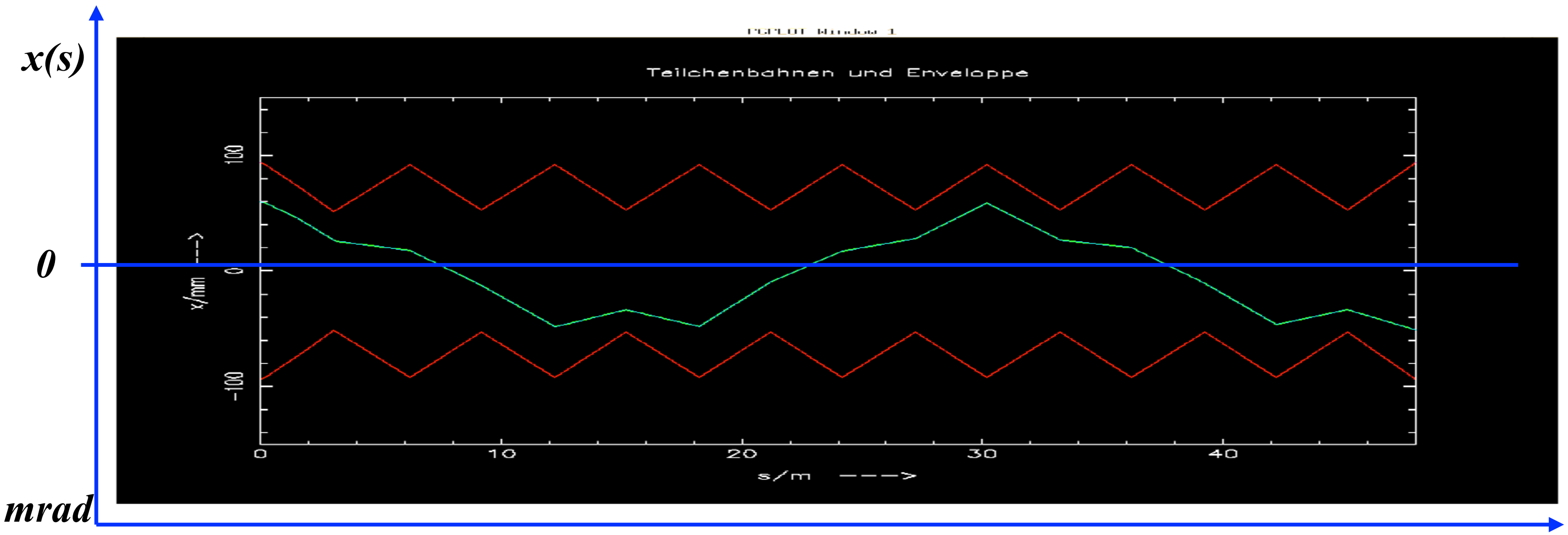
$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D^*} \ldots\ldots$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2, s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$



court. K. Wille

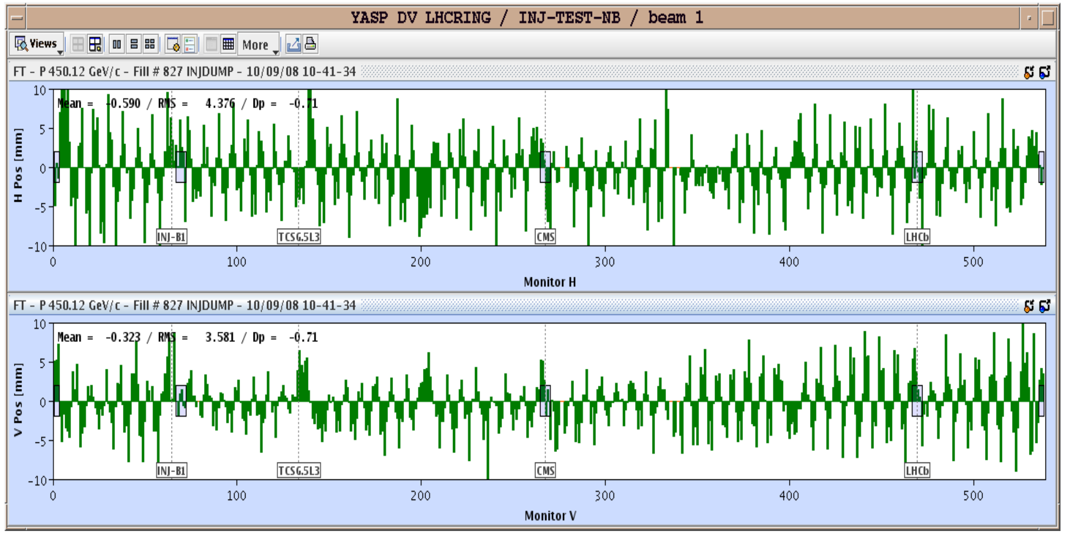
in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „



typical values
in a strong
foc. machine:

x ≈ mm, x´ ≤ mrad

Orbit & Tune:



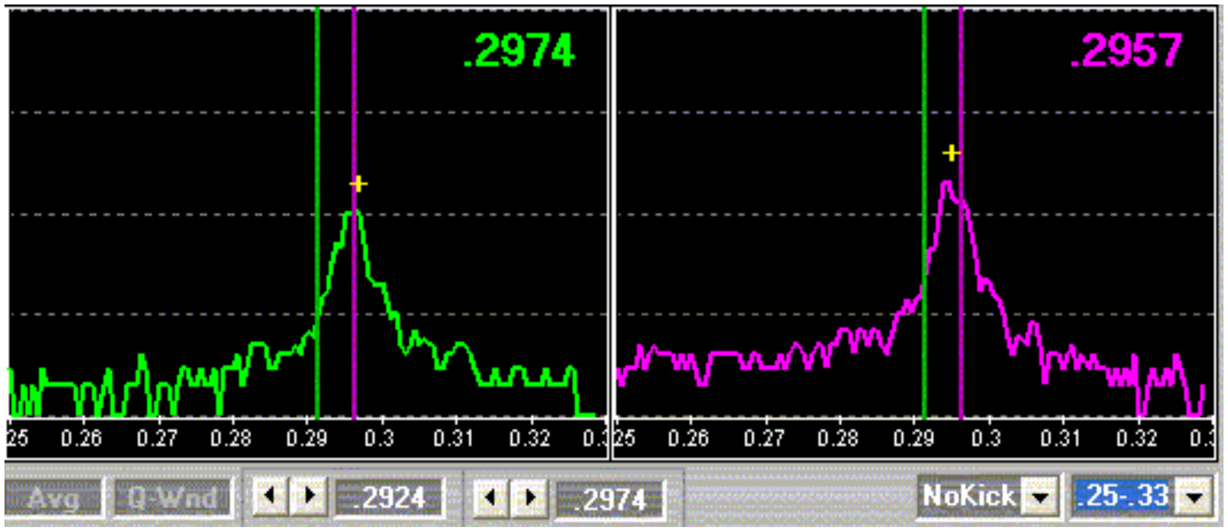
Tune: number of oscillations per turn

64.31
59.32

Relevant for beam stability:
non integer part

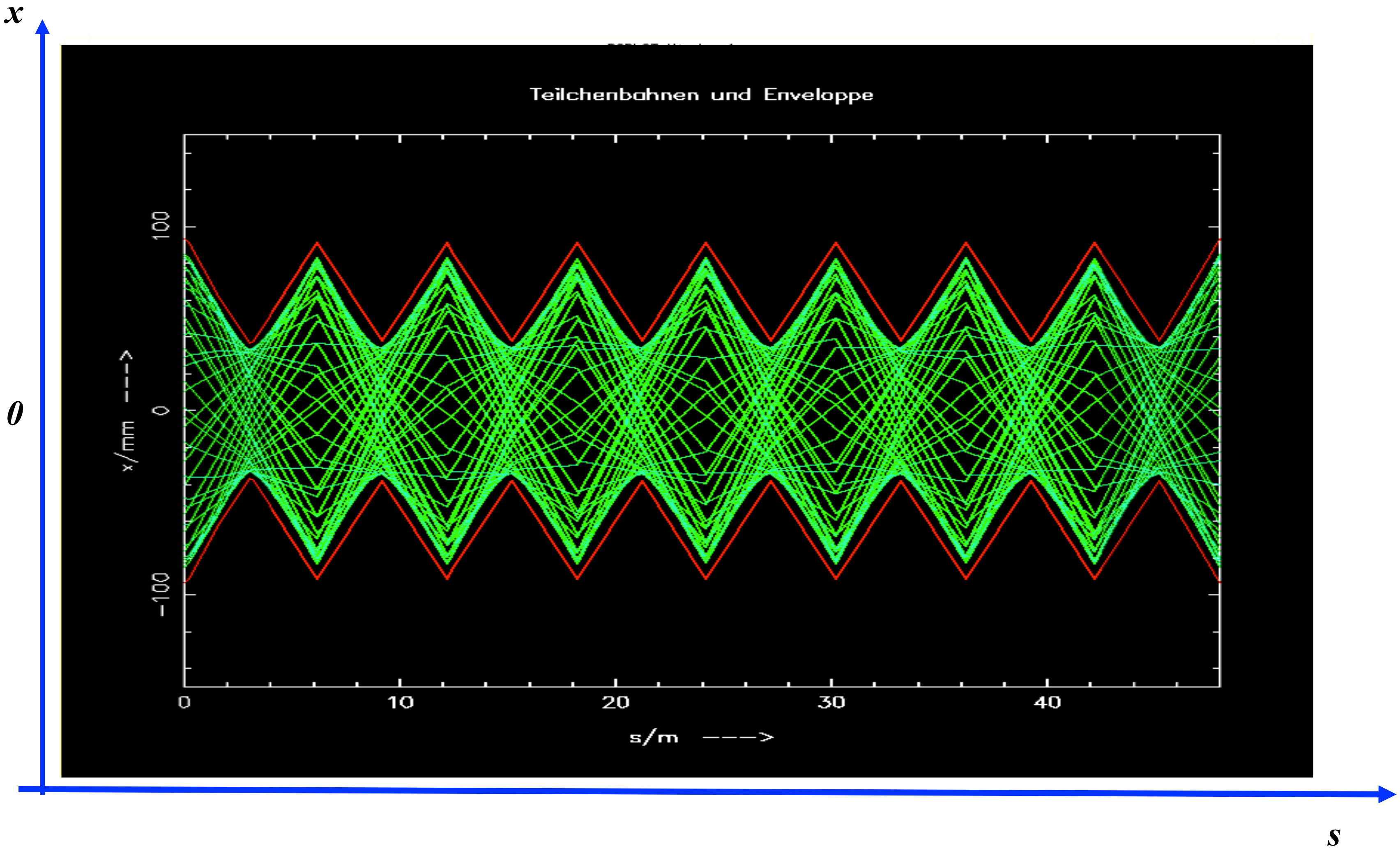
LHC revolution frequency: 11.3 kHz

$$0.31 * 11.3 = 3.5 kHz$$



Question: ***what will happen, if the particle performs a second turn ?***

... or a third one or ... 10^{10} turns



Astronomer Hill:

differential equation for motions with periodic focusing properties
„Hill 's equation"

Example: particle motion with periodic coefficient

equation of motion: $$x''(s) - k(s)x(s) = 0$$

restoring force $\neq$ ***const,***
k(s) = depending on the position s
k(s+L) = k(s), periodic function

we expect a kind of quasi harmonic oscillation: amplitude & phase will depend on the position s in the ring.

The Beta Function

General solution of Hill´s equation:

(i) $$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)} \cdot \cos(\psi(s) + \phi)$$

ε, Φ = integration constants determined by initial conditions

β(s) periodic function given by focusing properties of the lattice ↔ quadrupoles

$$\beta(s+L) = \beta(s)$$

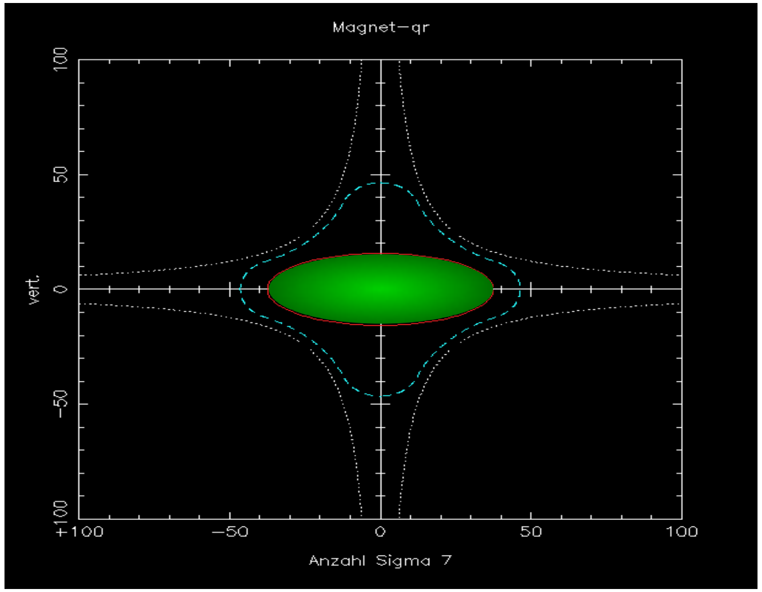
Inserting (i) into the equation of motion …

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)}$$

Ψ(s) = „phase advance" of the oscillation between point „0" and „s" in the lattice.
For one complete revolution: number of oscillations per turn „Tune"

$$Q = \frac{1}{2\pi}\oint \frac{ds}{\beta(s)}$$

The Beta Function

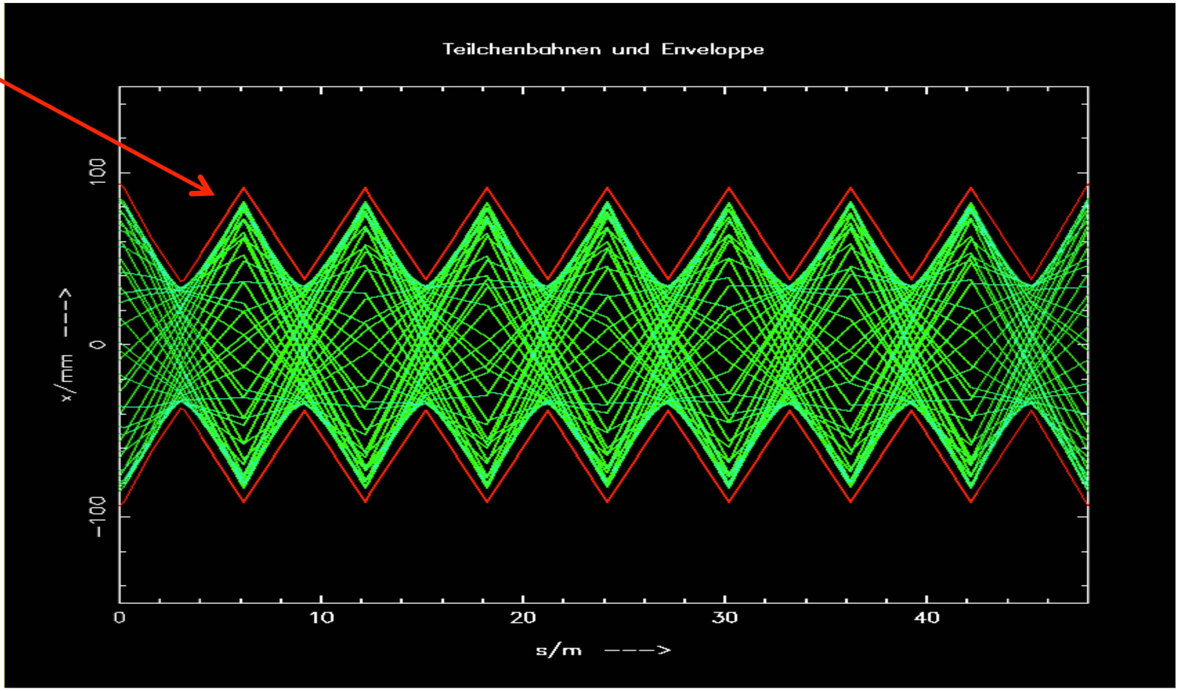


Amplitude of a particle trajectory:

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \varphi)$$

Maximum size of a particle amplitude

$$\hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}$$



β determines the beam size
(... the envelope of all particle trajectories at a given position "s" in the storage ring.

It reflects the periodicity of the magnet structure.

Beam Emittance and Phase Space Ellipse

general solution of Hill equation

(1) $$x(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}\,\cos(\psi(s)+\phi)$$

(2) $$x'(s) = -\frac{\sqrt{\varepsilon}}{\sqrt{\beta(s)}}\,\{\alpha(s)\cos(\psi(s)+\phi)+\sin(\psi(s)+\phi)\}$$

from (1) we get

$$\cos(\psi(s)+\phi) = \frac{x(s)}{\sqrt{\varepsilon}\,\sqrt{\beta(s)}}$$

$$\alpha(s) = \frac{-1}{2}\beta'(s)$$

$$\gamma(s) = \frac{1+\alpha(s)^2}{\beta(s)}$$

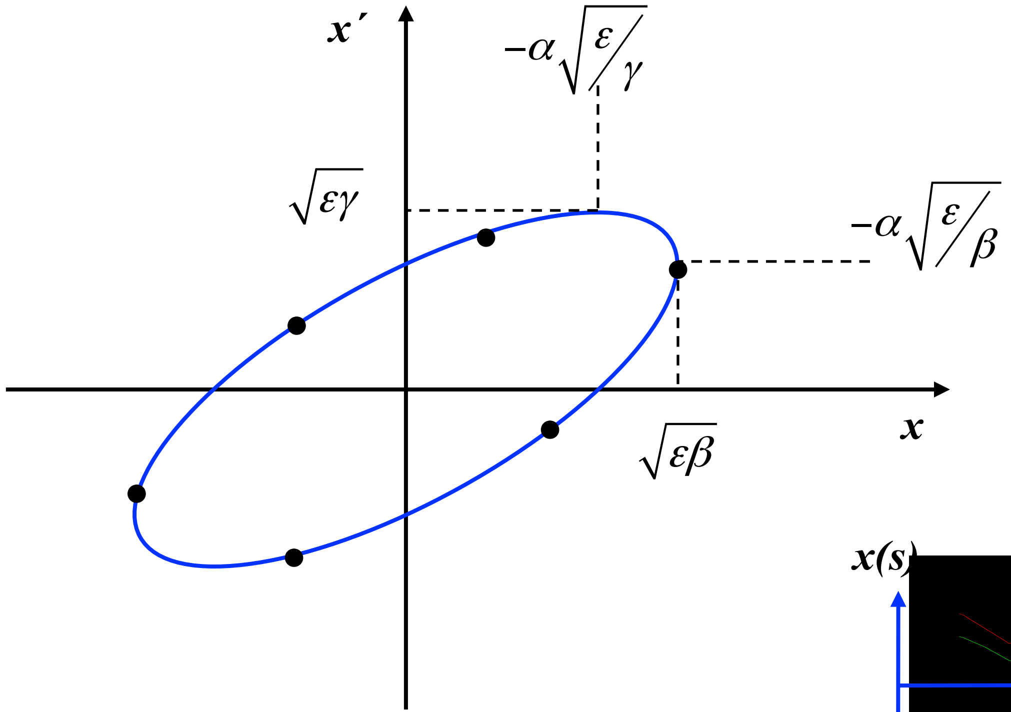
Insert into (2) and solve for ε

$$\varepsilon = \gamma(s)\,x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)\,x'^2(s)$$

***** *ε* ***is a*** ***constant*** ***of the motion … it is independent of „s"***
**** parametric representation of an ellipse in the x x ' space***
**** shape and orientation of ellipse are given by α, β, γ***

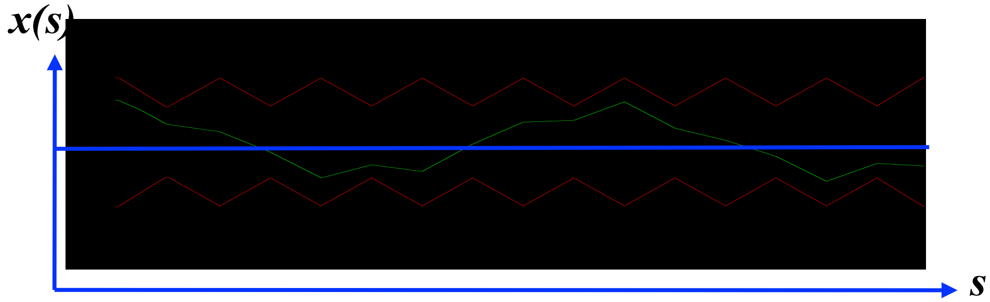
Beam Emittance and Phase Space Ellipse

$$\varepsilon = \gamma(s)\,x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)\,x'^2(s)$$



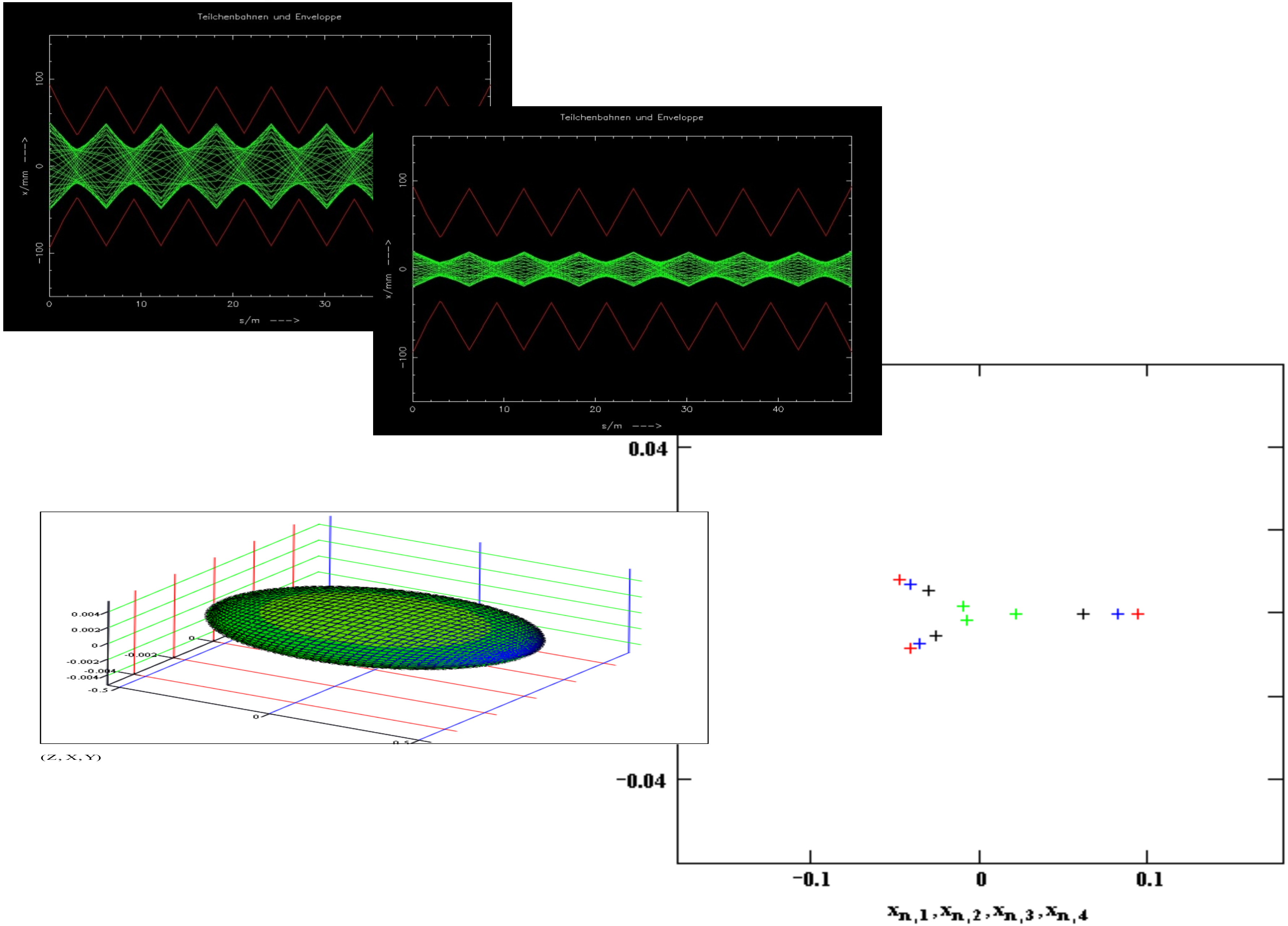
***Liouville:** in reasonable storage rings*
area in phase space is constant.

$A = \pi*\varepsilon = const$



ε beam emittance = woozilycity of the particle ensemble, intrinsic beam parameter,
cannot be changed by the foc. properties.
Scientifiquely speaking: area covered in transverse x, x´ phase space … and it is constant !!!

Emittance of the Particle Ensemble:



Phase Space Ellipse

particel trajectory: $x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}\cos\{\psi(s)+\phi\}$

max. Amplitude: $\hat{x}(s) = \sqrt{\varepsilon\beta}$ → *x´ at that position …?*

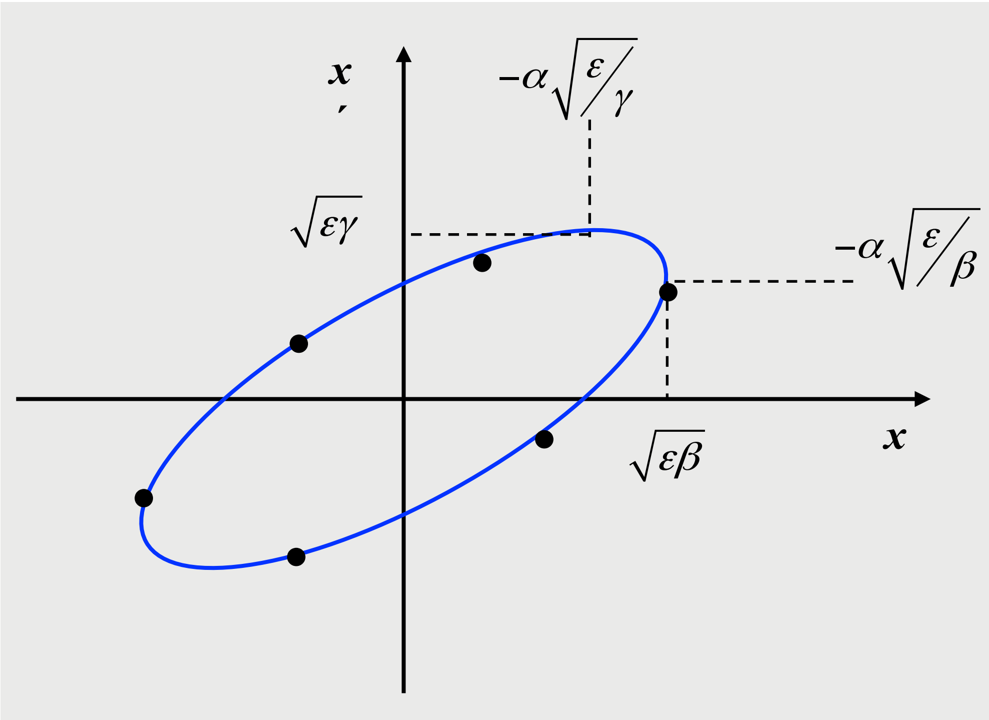
… put $\hat{x}(s)$ *into*

$$\varepsilon = \gamma(s)\, x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)\, x'^2(s)$$

and solve for x´

$$\varepsilon = \gamma\cdot\varepsilon\beta + 2\alpha\sqrt{\varepsilon\beta}\cdot x' + \beta x'^2$$

→ $x' = -\alpha\cdot\sqrt{\varepsilon/\beta}$

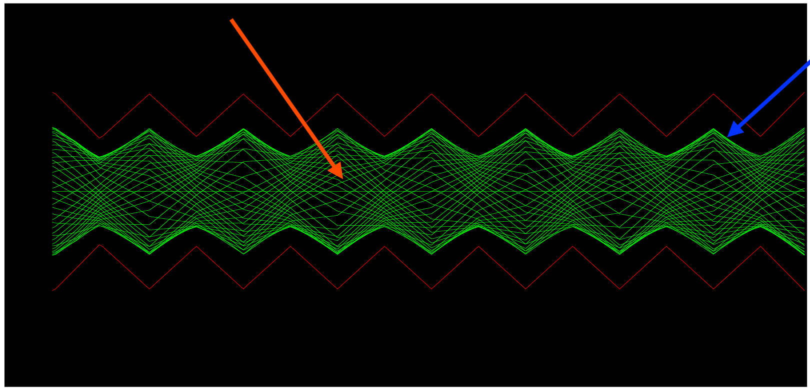


* ***The optical functions determine the shape and orientation of the phase space ellipse.***

* ***A high β-function means a large beam size and a small beam divergence.***
  … et vice versa !!!

Emittance of the Particle Ensemble:

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)} \cdot \cos(\Psi(s) + \phi) \qquad \hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}$$



single particle trajectories, $N \approx 10^{11}$ per bunch

Gauß Particle Distribution: $\rho(x) = \frac{N \cdot e}{\sqrt{2\pi}\sigma_x} \cdot e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}}$

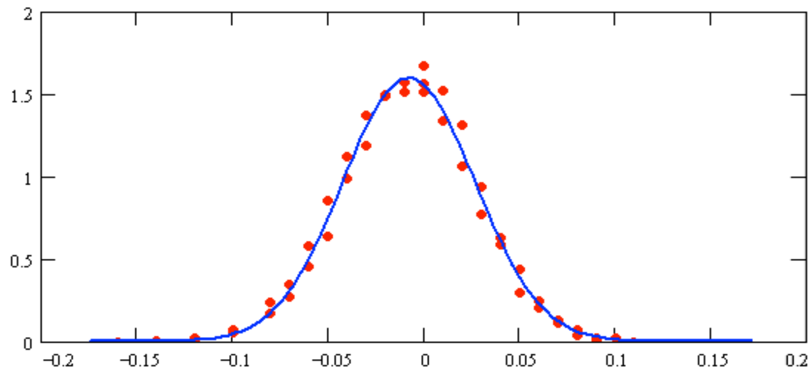
particle at distance 1 σ from centre
↔ 68.3 % of all beam particles

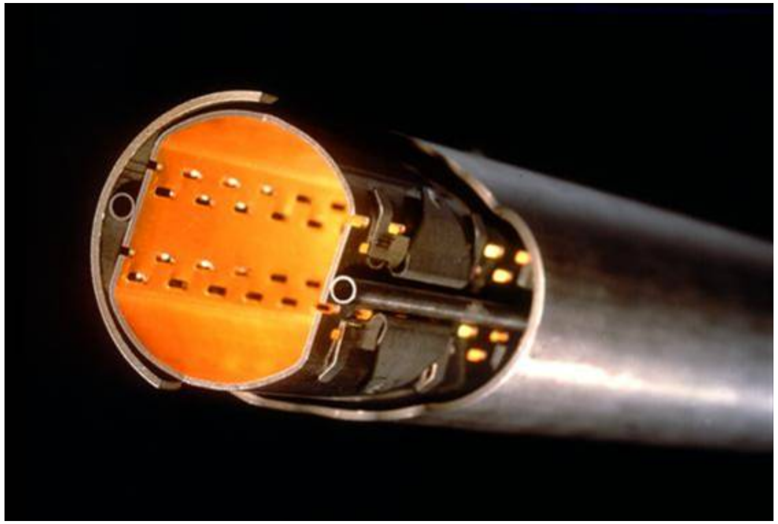
LHC:

$$\beta = 180\, m$$

$$\varepsilon = 5 * 10^{-10}\, m\, rad$$

$$\sigma = \sqrt{\varepsilon * \beta} = \sqrt{5*10^{-10} m * 180 m} = 0.3\, mm$$





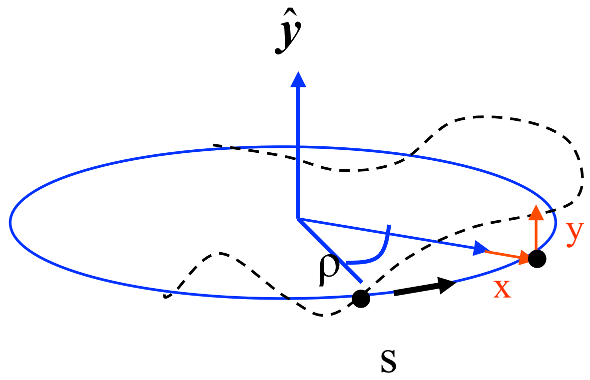
aperture requirements: $r_0 = 12 * \sigma$

13.) Errors in Field and Gradient:
Dispersion: trajectories for $\Delta p / p \neq 0$

Forces acting on the particle

Radial acceleration

$$a_r = \frac{d^2(\rho + x)}{dt^2} - (\rho + x)\left(\frac{d\theta}{dt}\right)^2$$



Is counter vailed by the Lorenz force

$$F = m\frac{d^2}{dt^2}(x+\rho) - \frac{mv^2}{x+\rho} = e\,B_y\,v$$

remember: $x \approx mm$, $\rho \approx m$ … → develop for small x

$$m\frac{d^2x}{dt^2} - \frac{mv^2}{\rho}(1 - \frac{x}{\rho}) = eB_y v$$

consider only linear fields, and change independent variable: $t \to s$

$$B_y = B_0 + x\frac{\partial B_y}{\partial x}$$

$$x'' - \frac{1}{\rho}(1 - \frac{x}{\rho}) = \frac{e\ B_0}{mv} + \frac{e\ x\,g}{mv}$$

… but now take a small momentum error into account !!!

$p = p_0 + \Delta p$

Dispersion:

develop for small momentum error $\Delta p << p_0 \Rightarrow \frac{1}{p_0 + \Delta p} \approx \frac{1}{p_0} - \frac{\Delta p}{p_0^2}$

$$x'' - \frac{1}{\rho} + \frac{x}{\rho^2} \approx \underbrace{\frac{e\ B_0}{p_0}}_{-\frac{1}{\rho}} - \frac{\Delta p}{p_0^2} eB_0 + \underbrace{\frac{xeg}{p_0}}_{k * x} - \underbrace{xeg \frac{\Delta p}{p_0^2}}_{\approx 0}$$

$$x'' + \frac{x}{\rho^2} \approx \frac{\Delta p}{p_0} * \underbrace{\frac{(-eB_0)}{p_0}}_{\frac{1}{\rho}} + k * x = \frac{\Delta p}{p_0} * \frac{1}{\rho} + k * x$$

$$x'' + \frac{x}{\rho^2} - kx = \frac{\Delta p}{p_0} \frac{1}{\rho} \quad \longrightarrow \quad x'' + x\left(\frac{1}{\rho^2} - k\right) = \frac{\Delta p}{p_0} \frac{1}{\rho}$$

Momentum spread of the beam adds a term on the r.h.s. of the equation of motion.
→ inhomogeneous differential equation.

Dispersion:

$$x'' + x\left(\frac{1}{\rho^2} - k\right) = \frac{\Delta p}{p} \cdot \frac{1}{\rho}$$

general solution:

$$x(s) = x_h(s) + x_i(s)$$

$$\begin{cases} x_h''(s) + K(s) \cdot x_h(s) = 0 \\ x_i''(s) + K(s) \cdot x_i(s) = \frac{1}{\rho} \cdot \frac{\Delta p}{p} \end{cases}$$

Normalise with respect to Δp/p:

$$D(s) = \frac{x_i(s)}{\Delta p / p}$$

Dispersion function D(s)

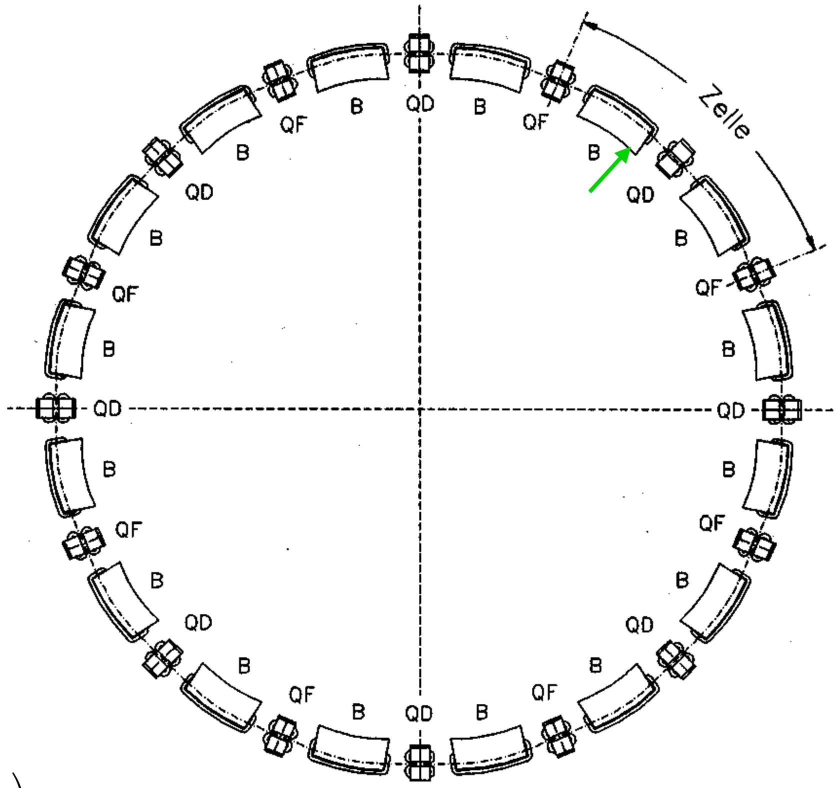
*** is that special orbit, an ideal particle would have for Δp/p = 1**

*** the orbit of any particle is the sum of the well known x_β and the dispersion**

*** as D(s) is just another orbit it will be subject to the focusing properties of the lattice**

Dispersion:

Example: homogenous dipole field



bit for Δp/p > 0

$$= D(s) \cdot \frac{\Delta p}{p}$$

Matrix formalism:

e.g. matrix for a quadrupole lens:

$$M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|}s & \frac{1}{\sqrt{|K|}}\sin(\sqrt{|K|}s \\ -\sqrt{|K|}\sin(\sqrt{|K|}s & \cos(\sqrt{|K|}s \end{pmatrix} = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix}$$

$$x(s) = x_\beta(s) + D(s) \cdot \frac{\Delta p}{p}$$

$$x(s) = C(s) \cdot x_0 + S(s) \cdot x_0' + D(s) \cdot \frac{\Delta p}{p}$$

$$x_\beta = 1 \dots 2\, mm$$

$$D(s) \approx 1 \dots 2\, m$$

$$\Delta p / p \approx 1 \cdot 10^{-3}$$

Amplitude of Orbit oscillation:

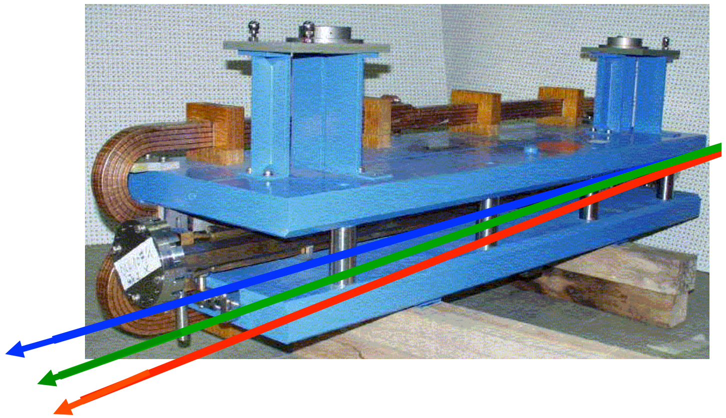
contribution due to dispersion ≈ beam size

16.) Chromaticity:
A Quadrupole Error for $\Delta p/p \neq 0$

Influence of external fields on the beam: *prop. to magn. field & prop. zu 1/p*

dipole magnet

$$\alpha = \frac{\int B\,dl}{p/e}$$



$$x_D(s) = D(s)\,\frac{\Delta p}{p}$$

focusing lens

$$k = \frac{g}{p/e}$$

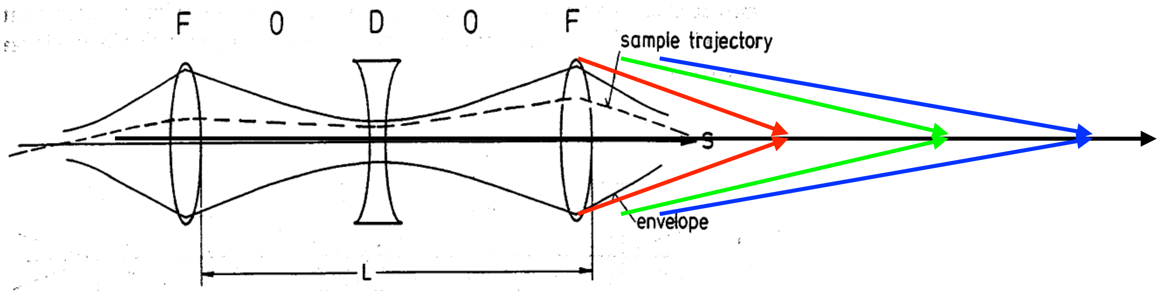


Figure 29: FODO cell

particle having ...

to high energy

to low energy

ideal energy

Chromaticity: Q'

$$k = \frac{g}{p/e} \qquad\qquad p = p_0 + \Delta p$$

in case of a momentum spread:

$$k = \frac{eg}{p_0 + \Delta p} \approx \frac{e}{p_0}\left(1 - \frac{\Delta p}{p_0}\right) g = k_0 + \Delta k$$

$$\Delta k = -\frac{\Delta p}{p_0} k_0$$

… which acts like a quadrupole error in the machine and leads to a tune spread:

$$\Delta Q = -\frac{1}{4\pi} \frac{\Delta p}{p_0} k_0 \beta(s) ds$$

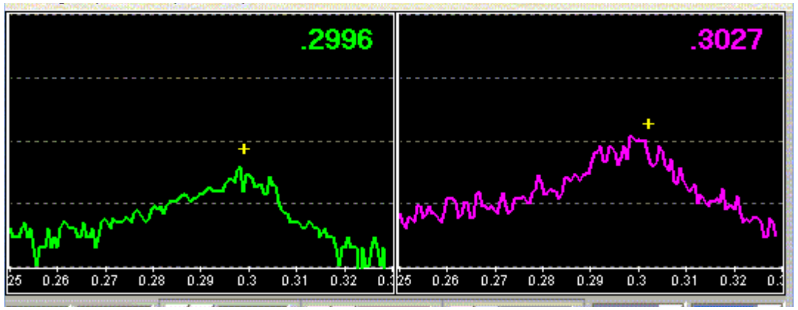
definition of chromaticity:

$$\Delta Q = Q' \frac{\Delta p}{p} \quad ; \quad Q' = -\frac{1}{4\pi} \oint k(s)\beta(s) ds$$

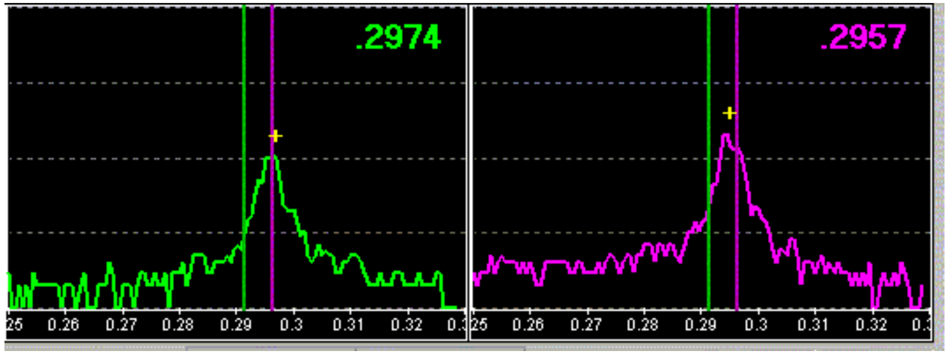
… what is wrong about Chromaticity:

Problem: chromaticity is generated by the lattice itself !!

***Q'* is a *number* indicating the *size of the tune spot* in the working diagram,**
***Q'* is always created if the beam is focussed**
→ it is determined by the focusing strength *k* of all quadrupoles



Tune signal for a nearly uncompensated cromaticity (Q' ≈ 20)



Ideal situation: cromaticity well corrected, (Q' ≈ 1)

Example: LHC

$Q' = 250$
$\Delta p/p = \pm 0.2 \cdot 10^{-3}$
$\Delta Q = 0.256 \ldots 0.36$

→ Some particles get very close to resonances and are lost
in other words: the tune is not a point
it is a pancake

Correction of Chromaticity

1.) sort the particles according to their momentum

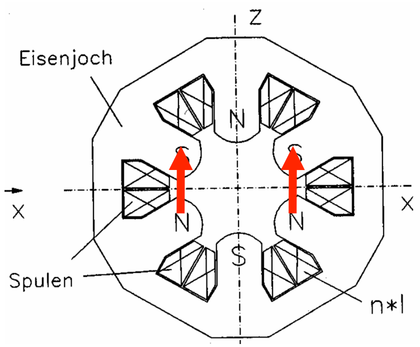
$$x_D(s) = D(s)\frac{\Delta p}{p}$$

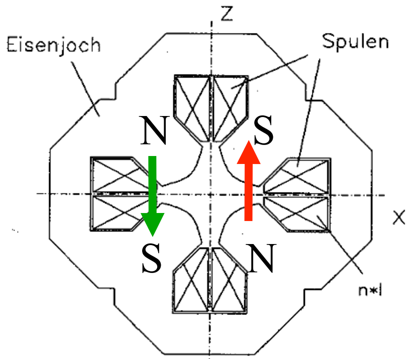
2.) apply a magnetic field that rises quadratically with x (sextupole field)

$$\left.\begin{aligned} B_x &= \tilde{g}xz \\ B_z &= \frac{1}{2}\tilde{g}(x^2 - z^2) \end{aligned}\right\} \quad \frac{\partial B_x}{\partial z} = \frac{\partial B_z}{\partial x} = \tilde{g}x$$

linear rising „gradient“:

Sextupole Magnets:





normalised quadrupole strength:

$$k_{sext} = \frac{\tilde{g}x}{p/e} = m_{sext.}x$$

$$k_{sext} = m_{sext.}D\frac{\Delta p}{p}$$

corrected chromaticity:

$$Q' = -\frac{1}{4\pi}\oint\{K(s) - mD(s)\}\beta(s)\,ds$$

Resume:

The geometry and the maximum momentum of the particles is defined by the dipole strength

$$\frac{p}{e} = B\rho$$

Strong focusing quadrupole lenses lead to a transverse oscillation of the particles

$$x'' + x\left(\frac{1}{\rho^2} - k\right) = 0$$

Focusing properties of a magnet

$$M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|}s) & \frac{1}{\sqrt{|K|}}\sin(\sqrt{|K|}s) \\ -\sqrt{|K|}\sin(\sqrt{|K|}s) & \cos(\sqrt{|K|}s) \end{pmatrix}_0$$

Foc. & defoc. lenses have to be combined to lead to an overall focusing scheme in both planes.

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}l \\ \sqrt{|K|}\sinh\sqrt{|K|}l & \cosh\sqrt{|K|}l \end{pmatrix}$$

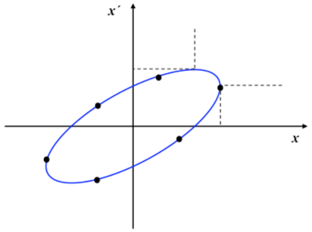
The β-function defines an envelope enclosing the single particle trajectories and together with the emittance it defines the beam size

$$\hat{x}(s) = \sqrt{\varepsilon\beta}$$

Number of oscillations per turn (the tune) depends on the overall focusing fields in the ring

$$Q = \frac{1}{2\pi}\oint\frac{ds}{\beta(s)}$$

The beam emittance is a intrinsic beam property and describes the quality of the particle distribution. It corresponds to the area of an ellipse in x, x‘ phase space and is constant (for a given energy).



Resume:

***Dispersion**: effect of a momentum error (spread) on the particle orbit*

$$x(s) = x_\beta(s) + D(s) \cdot \frac{\Delta p}{p}$$

***Chromaticity**: ... on the focusing properties (tune)*

$$\Delta Q = Q' \quad \frac{\Delta p}{p}$$

Bibliography:

1.) P. Bryant, K. Johnsen: The Principles of Circular Accelerators and Storage Rings
Cambridge Univ. Press

2.) Klaus Wille: Physics of Particle Accelerators and Synchrotron
Radiation Facilicties, Teubner, Stuttgart 1992

3.) Peter Schmüser: Basic Course on Accelerator Optics, CERN Acc.
School: 5th general acc. phys. course CERN 94-01

4.) Bernhard Holzer: Lattice Design, CERN Acc. School: Interm.Acc.phys course,
http://cas.web.cern.ch/cas/ZEUTHEN/lectures-zeuthen.htm
cern report: CERN-2006-002

5.) A.Chao, M.Tigner: Handbook of Accelerator Physics and Engineering,
Singapore : World Scientific, 1999.

6.) Martin Reiser: Theory and Design of Chargged Particle Beams
Wiley-VCH, 2008

7.) Frank Hinterberger: Physik der Teilchenbeschleuniger, Springer Verlag 1997

8.) Mathew Sands: The Physics of e+ e- Storage Rings, SLAC report 121, 1970

9.) D. Edwards, M. Syphers : An Introduction to the Physics of Particle
Accelerators, SSC Lab 1990