Nucear pairing from realistic forces

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23rd January 2015

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Research interests

- Dispersive treatment of πK scattering
  with G. Colangelo (Bern U.),
  work in progress.

- Nuclear pairing problem
  with P. Finelli (Bologna U.) and J. Holt (Washington U.),
  Published on PRC 90 044003 (2014)

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Summary

Nuclear pairing: introduction

Consider a gas (infinite system) of particles with spin $\frac{1}{2}$ (neutrons):
at T=0, density ρ (or Fermi momentum k_F),
the ground state is the Fermi sea $|0\rangle$.

But...

(Cooper theorem)

if we switch on an attractive interaction, $|0\rangle$ is an unstable state.

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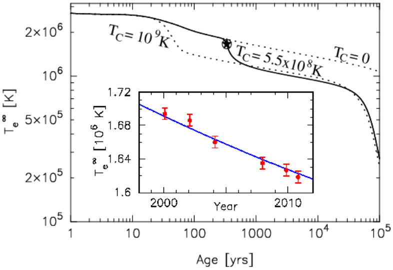
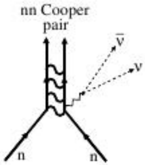
Neutron pairing and neutron stars

Fast cooling of SN 1987A explained by nuclear pairing:





BCS theory: an overview

$$H = \sum_{s=\pm\frac{1}{2}\mathbf{k}} \varepsilon_{\mathbf{k}} c^{+}_{s\mathbf{k}} c_{s\mathbf{k}} + \sum_{\mathbf{k}} V_{\mathbf{k}\mathbf{k}'} c^{+}_{\uparrow\mathbf{k}} c^{+}_{\downarrow-\mathbf{k}} c_{\downarrow-\mathbf{k}'} c_{\uparrow\mathbf{k}'}$$

In the BCS approximation:

$$\begin{pmatrix} b_{s\mathbf{k}} \\ b^{+}_{-s\mathbf{k}} \end{pmatrix} = \begin{pmatrix} u_{s\mathbf{k}} & v_{s\mathbf{k}} \\ -v^{*}_{s\mathbf{k}} & u^{*}_{s\mathbf{k}} \end{pmatrix} \begin{pmatrix} c_{s\mathbf{k}} \\ c^{+}_{-s\mathbf{k}} \end{pmatrix}$$

$$H_{BCS} = \sum_{s=\pm\frac{1}{2}\mathbf{k}} E_{\mathbf{k}} b^{+}_{s\mathbf{k}} b_{s\mathbf{k}}$$

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BCS theory: an overview

To solve the problem, we must satisfy:

$$E_{\mathbf{k}} = \sqrt{\varepsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2} \qquad \Delta_{\mathbf{k}} = \sum_{\mathbf{k}'} \frac{V_{\mathbf{k}\mathbf{k}'}}{2E_{\mathbf{k}'}} \Delta_{\mathbf{k}'}$$

Nonlinear integral equation for the gap $\Delta_{\mathbf{k}}$

$\Longrightarrow$ numerical techniques.

How do we choose the interaction $V_{\mathbf{k}\mathbf{k}'}$?

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Realistic nuclear forces

Recipe for a modern nuclear force (ChPT):

- find out the most appropriate d.o.f. (N,π);
- write down the most general Lagrangian $\mathcal{L}$ allowed by (eventually broken) QCD symmetries;
- expand $\mathcal{L}$ in powers of momenta or masses $(Q/\Lambda_{QCD}) \ll 1$;
- introduce a cutoff to remove UV divergences;
- fit the free parameters to reproduce the low energy experimental data;
- check the cutoff-dependence.

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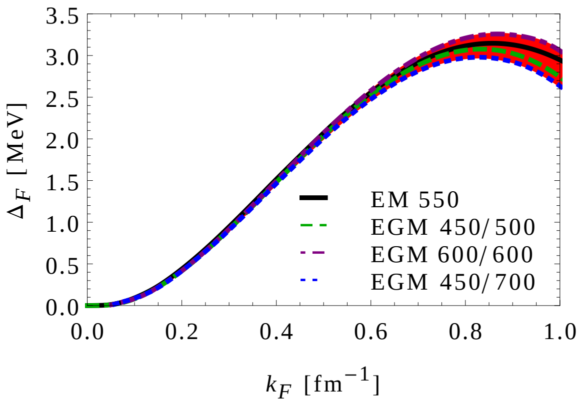
Results for Δ_{k_F} as a function of k_F in neutron matter.

The interaction is decomposed in partial waves
In every density region just one partial wave included.

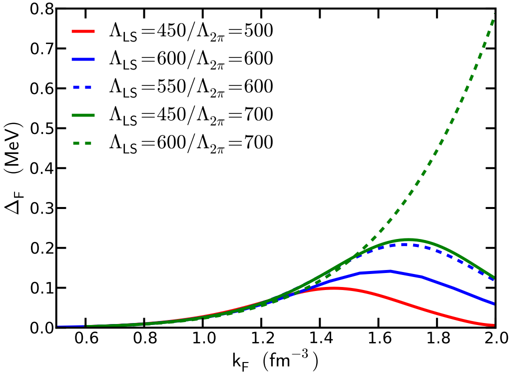
$$V_{\mathbf{k}\mathbf{k}'} = \sum_{ll'mm'} V_{ll'}(|\mathbf{k}-\mathbf{k}'|)Y^*_{lm}(\hat{\mathbf{k}})Y_{l'm}(\hat{\mathbf{k}}')$$

We have used two different interactions (different reguralization schemes and NR reduction).

Low density BCS gap (S wave)



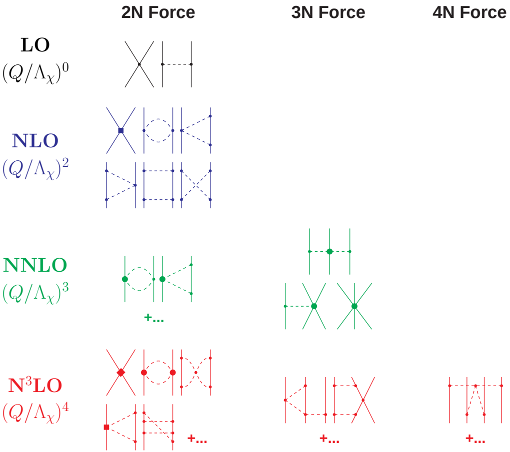
Higher density BCS gap (P-F waves)



Conclusions

- Nuclear pairing is fundamental to understand supernovae physics.
- Thanks to *ChPT* we are now able to obtain informations about nuclear physics from fundamental (QCD) principles + experiment.
- At low densities, we obtained a good cutoff independence.
- Further corrections are needed to better understand the nuclear pairing.
- The finite temperature extension will be soon available.

Structure of the nuclear interaction



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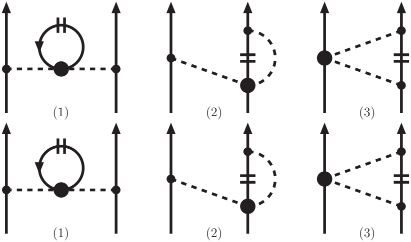
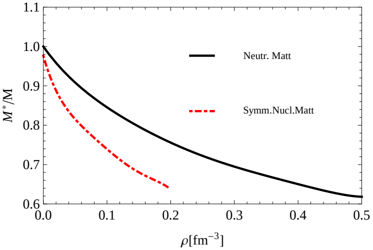
NNN interaction included via den. dep. NN int.:



Holt *et al.* PRC 81 024002 (2010)

Self-energy corrections included via effective mass (density functional):



Holt, J.W. et al. Eur.Phys.J. A47 (2011) 128