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Flavour Physics (& CP Violation)

Jernej F. Kamenik



Main references:

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Y. Grossman, CERN Yellow Report CERN-2010-002, 111-144 [arXiv:1006.3534 [hep-ph]];
Y. Nir, CERN Yellow Report CERN-2010-001, 279-314 [arXiv:1010.2666 [hep-ph]];
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^{Univerza v Ljubljani} Fakulteta za matematiko in fiziko

19-20/01/2015, Grindelwald

- In SM: fermionic fields (spin 1/2)
- *matter flavours*: several copies of the same gauge representation

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- *matter flavours*: several copies of the same gauge representation
- unbroken SM gauge group $SU(3)_c \times U(1)_{EM}$
 - up-type quarks: $(3)_{2/3} : u, c, t,$
 - down-type quarks: $(3)_{-1/3}$: d, s, b,
 - chrged leptons: $(1)_{-1}$: e, μ, τ ,
 - neutrinos: $(1)_0 : \nu_1, \nu_2, \nu_3,$

\leftrightarrow

differ only in mass

- Ordinary matter essentially first generation:
 - u and d quarks bound within protons & neutrons,
 - electrons form atoms;
 - "electron neutrinos", (admixture of $v_{1,2,3}$) are produced in reactions inside stars.
- 2nd and 3rd generation families decay via weak interactions into first generation particles.

Why there are thee almost identical replicas of quarks and leptons and which is the origin of their different masses?

- Flavour physics
 - Within SM: weak and Yukawa interactions.
- Flavour parameters
 - Within SM: 9 masses of charged fermions & 4 mixing parameters (3 angles + 1 phase)
- Flavour universal (flavour blind)
 - Within SM: QCD & QED
- Flavour diagonal
 - Within SM: Yukawa interaction

- Flavour changing processes
 - Flavour changing charged currents:

$$\mu^- \to e^- \nu_i \bar{\nu}_j \quad K^- \to \mu^- \bar{\nu}_i \ \left(s\bar{u} \to \mu^- \bar{\nu}_i\right)$$

- Within SM: single W exchange at tree-level $(\mathcal{A} \propto G_F)$
- Flavour changing neutral currents:

$$\mu^- \to e^- \gamma \quad K_L \to \mu^+ \mu^- (s\bar{d} \to \mu^+ \mu^-)$$

• Within SM: higher orders in weak expansion (loops) - often highly suppressed

Why is flavour interesting?

•
$$\frac{\Gamma(K_L \to \mu^+ \mu^-)}{\Gamma(K^- \to \mu^- \bar{\nu}_i)} \Rightarrow$$
 prediction of charm quark

- $\Delta m_K \equiv m_{K_L} m_{K_S} \Rightarrow$ prediction of charm mass
- $K_L \to \pi^+ \pi^- (\varepsilon_K) \Rightarrow$ prediction of 3rd generation
- CP Violation
 - Within SM: single CP violating parameter

Why is flavour interesting?

- Electroweak (EW) hierarchy problem
 - requires NP ≤ 1 TeV
 - if generic flavour structure \Rightarrow FCNCs
 - flavour probes NP scales ≤10⁵ TeV
 NP flavour puzzle

Why is flavour interesting?

- Electroweak (EW) hierarchy problem
 - requires NP ≤ 1 TeV
 - if generic flavour structure \Rightarrow FCNCs
 - flavour probes NP scales $\leq 10^5$ TeV NP flavour puzzle
- SM flavour parameters
 - hierarchical: $m_u \ll m_c \ll m_t$
 - most are small: $m_{f \neq t} << m_{W,Z}$ SM flavour puzzle

 $\mathcal{L} = ?$

- i) Symmetries & their spontaneous breaking
- ii) Representations of fermions & scalars

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- ii) Representations of fermions & scalars

i)
$$\mathcal{G}_{\text{local}}^{\text{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y$$

 $\mathcal{G}_{\text{local}}^{\text{SM}} \to SU(3)_c \times U(1)_{\text{EM}}$

ii)
$$Q_L^i \sim (3,2)_{1/6}, \ U_R^i \sim (3,1)_{2/3},$$

 $D_R^i \sim (3,1)_{-1/3}, \ L_L^i \sim (1,2)_{-1/2},$
 $\phi \sim (1,2)_{1/2}, \ \langle \phi^0 \rangle \equiv \frac{v}{\sqrt{2}} \simeq 174 \text{GeV},$

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm kinetic}^{\rm SM} + \mathcal{L}_{\rm EWSB}^{\rm SM} + \mathcal{L}_{\rm Yukawa}^{\rm SM}$$
simple and symmetric (g, g', g_s)
EWSB, 2 params

• SM flavour dynamics, flavour parameters

•
$$D_{\mu} = \partial_{\mu} + ig_s G^a_{\mu} L^a + ig W^b_{\mu} T^b + ig' B_{\mu} Y$$

$$\mathcal{L}_{\rm EWSB}^{\rm SM} = \mu^2 \phi^{\dagger} \phi - \lambda (\phi^{\dagger} \phi)^2$$

 $\begin{aligned} \mathcal{G}_{\text{flavour}}^{\text{SM}} &= U(3)^5 = SU(3)_q^3 \times SU(3)_\ell^2 \times U(1)^5 \,, \\ SU(3)_q^3 &= SU(3)_Q \times SU(3)_U \times SU(3)_D \,, \\ SU(3)_\ell^2 &= SU(3)_L \times SU(3)_E \,, \\ U(1)^5 &= U(1)_B \times U(1)_L \times U(1)_Y \times U(1)_{\text{PQ}} \times U(1)_E \,. \end{aligned}$

• <u>Exercise</u>: compute embedding of $U(1)^5$ into $U(3)^5$

$$-\mathcal{L}_{\text{Yukawa}}^{\text{SM}} = Y_d^{ij} \bar{Q}_L^i \phi D_R^j + Y_u^{ij} \bar{Q}_L^i \tilde{\phi} U_R^j + Y_e^{ij} \bar{L}^i \phi E_R^j + \text{h.c.},$$
$$\tilde{\phi} = i\sigma_2 \phi,$$

• in general flavour dependent (unless $Y_F \propto I_{ij}$) & CPV

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- $SU(3)_Q \times SU(3)_U \to U(1)_u \times U(1)_c \times U(1)_t$ is due to $Y_u \not\propto I$,
- $SU(3)_Q \times SU(3)_D \to U(1)_d \times U(1)_s \times U(1)_b$ is due to $Y_d \not\propto I$, ($Y_U \& Y_D$ together break remaining U(1) factors to $U(1)_B$)

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- finally, $SU(3)_L \times SU(3)_E \to U(1)_e \times U(1)_\mu \times U(1)_\tau$ due to $Y_e \not\propto I$

$$\mathcal{G}_{\text{global}}^{\text{SM}}(Y_f \neq 0) = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$$

- Flavour physics: interactions which break $SU(3)_q^3 \times SU(3)_\ell^2$ are flavour violating
- Spurion analysis:

 $Y_u \sim (3, \overline{3}, 1)_{SU(3)_q^3}, Y_d \sim (3, 1, \overline{3})_{SU(3)_q^3}, Y_e \sim (3, \overline{3})_{SU(3)_\ell^2}.$

- parameter counting
- identification of suppression factors
- idea of Minimal Flavour Violation

Counting SM quark flavour parameters

- global symmetry group G_f with N_{total} generators
- $G_f \rightarrow H_f$ with $N_{\text{total}} N_{\text{broken}}$ generators
- $N_{
 m physical} = N_{
 m general} N_{
 m broken}$

Counting SM quark flavour parameters

- global symmetry group G_f with N_{total} generators
- $G_f \rightarrow H_f$ with $N_{\text{total}} N_{\text{broken}}$ generators

$$\bullet \quad N_{\rm physical} = N_{\rm general} - N_{\rm broken}$$

• Within SM: $U(3)_Q \times U(3)_U \times U(3)_D \rightarrow U(1)_B$

 $egin{aligned} N_{ ext{total}} &= 3 imes (3 + 6i), & N_{ ext{broken}} = N_{ ext{total}} - 1i = 9 + 17i, \ N_{ ext{general}} &= 2 imes (9 + 9i) & (Y_U, \ Y_D) \ N_{ ext{physical}} &= N_{ ext{general}} - N_{ ext{broken}} = 9 + 1i \end{aligned}$

Discrete SM symmetries

- Any local Lorentz invariant QFT conserves CPT
 ⇒ CP violation = T violation
- In SM: C & P violation maximally
 - C & P change chirality
 - Left- & right-handed fields in different gauge reps.

independent of SM parameters

Discrete SM symmetries

- Any local Lorentz invariant QFT conserves CPT
 ⇒ CP violation = T violation
- In SM: CP violation depends on parameters $Y_{ij}\bar{\psi}_L^i\phi\psi_R^j + Y_{ij}^*\bar{\psi}_R^j\phi^\dagger\psi_L^i \xrightarrow{CP} Y_{ij}\bar{\psi}_R^j\phi^\dagger\psi_L^i + Y_{ij}^*\bar{\psi}_L^i\phi\psi_R^j.$
 - CP symmetric if $Y_{ij} = Y_{ij}^*$.
 - Jarlskog invariant

$$J \equiv \operatorname{Im}[\det(Y_d Y_d^{\dagger}, Y_u Y_u^{\dagger})] = 0.$$

Discrete SM symmetries

- Any local Lorentz invariant QFT conserves CPT \Rightarrow CP violation = T violation
- Experimental discovery of CPV in kaon decays



1980

Cronin

•
$$\operatorname{Re}(\phi^0) \to (v+h)/\sqrt{2}, \Rightarrow M_q = \frac{v}{\sqrt{2}}Y_q.$$

• mass basis corresponds to diagonal M_q

•
$$Q_L \to V_Q Q_L$$
, $U_R \to V_U U_R$, $D_R \to V_D D_R$

•
$$Y_u \to V_Q Y_u V_U^{\dagger}, \ Y_d \to V_Q Y_d V_D^{\dagger}$$

• $V_Q^u M_u V_U^{\dagger} = M_u^{\text{diag}} = \frac{v}{\sqrt{2}} \lambda_u; \quad \lambda_u = \text{diag}(y_u, y_c, y_t),$ $V_Q^d M_d V_D^{\dagger} = M_d^{\text{diag}} = \frac{v}{\sqrt{2}} \lambda_d; \quad \lambda_d = \text{diag}(y_d, y_s, y_b).$

- V_{U, V_D} unphysical
- since $[M_u, M_d] \neq 0$, $V_Q^u V_Q^{d\dagger} \equiv V_{\text{CKM}} \neq 1$ Cabibbo, Kobayashi & Maskawa



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- SM flavour Lagrangian

$$\begin{aligned} \mathcal{L}_{m}^{F} &= \left(\bar{q}_{i} \not{D} q^{j} \delta_{ij}\right)_{\mathrm{NC}} + \frac{g}{\sqrt{2}} \bar{u}_{L}^{i} \not{W}^{+} V_{\mathrm{CKM}}^{ij} d_{L}^{j} \\ &+ \bar{u}_{L}^{i} \lambda_{u}^{ij} u_{R}^{j} \left(\frac{v+h}{\sqrt{2}}\right) + \bar{d}_{L}^{i} \lambda_{d}^{ij} d_{R}^{j} \left(\frac{v+h}{\sqrt{2}}\right) + \mathrm{h.c.}, \end{aligned}$$

NC = neutral currents (g, γ, Z) $(u_L^i, d_L^i) \equiv Q_L^T$

• <u>Exercise</u>: Show that NC's are diagonal

• <u>Exercise</u>:Show that in absence of neutrino masses there is no mixing in the leptonic sector

Testing the CKM

- Flavour conversion in SM:
 - fully parametrized by 3 CKM angles
 - mediated by charged current weak interactions
 - these involve left-handed fields only

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

(mass-ordered)

PDG parametrization

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$
$$|V_{ud}| \sim |V_{cs}| \sim |V_{tb}| \sim 1$$
$$|V_{us}| \sim |V_{cd}| \sim 0.22$$
$$|V_{cb}| \sim |V_{ts}| \sim 0.04$$
$$|V_{ub}| \sim |V_{td}| \sim 0.005$$

Wolfenstein parametrization

$$V = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \varrho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda \doteq s_{12}$$
, $A\lambda^2 \doteq s_{23}$, $A\lambda^3(\varrho - i\eta) \doteq s_{13}e^{-i\delta}$

Wolfenstein parametrization

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda + \mathcal{O}(\lambda^7) & A\lambda^3(\varrho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\varrho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 + \mathcal{O}(\lambda^8) \\ A\lambda^3(1 - \bar{\varrho} - i\bar{\eta}) & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\varrho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}$$

$$\bar{\varrho} = \varrho(1 - \frac{\lambda^2}{2}) + \mathcal{O}(\lambda^4), \qquad \bar{\eta} = \eta(1 - \frac{\lambda^2}{2}) + \mathcal{O}(\lambda^4).$$
$$\sum_{k} V_{ik}^* V_{jk} = \delta_{ij} , \quad \sum_{k} V_{ki}^* V_{kj} = \delta_{ij} .$$

• most interesting for i=1, j=3

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0,$$

- all three terms on LHS of same order in λ

$$\begin{split} &\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} + 1 = 0 \,, \\ &[\bar{\rho} + i\bar{\eta}] + [(1 - \bar{\rho}) - i\bar{\eta}] + 1 = 0 \,, \end{split}$$

 $|V_{us}|(\lambda) \text{ from } K \to \pi \ell \nu$ $|V_{cb}| (A) \text{ from } B \to X_c \ell \nu$ $\lambda = 0.2253(9)$ A = 0.822(12)

$$|V_{us}|(\lambda) \text{ from } K \to \pi \ell \nu$$
$$|V_{cb}| (A) \text{ from } B \to X_c \ell \nu$$
$$|V_{ub}|^2 \propto \bar{\rho}^2 + \bar{\eta}^2 \text{ from } B \to X_u \ell \nu$$





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$$S_{\psi K_S} = \sin 2\beta = \frac{2\bar{\eta}(1-\bar{\rho})}{(1-\bar{\rho})^2 + \bar{\eta}^2}$$

 $\lambda = 0.2253(9)$ A = 0.822(12)



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$$e^{i\gamma} = \frac{\bar{\rho} + i\bar{\eta}}{\bar{\rho}^2 + \bar{\eta}^2} \quad (B \to D \text{ K rates})$$

 $\lambda = 0.2253(9)$ A = 0.822(12)



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$$\alpha = \pi - \beta - \gamma \quad (B \to \pi \pi, \varrho \pi, \varrho \varrho \text{ rates})$$

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 $\lambda = 0.2253(9)$ A = 0.822(12)



$$\begin{aligned} |V_{us}|(\lambda) \text{ from } K \to \pi \ell \nu \\ |V_{cb}| (A) \text{ from } B \to X_c \ell \nu \\ |V_{ub}|^2 \propto \bar{\rho}^2 + \bar{\eta}^2 \text{ from } B \to X_u \ell \nu \\ S_{\psi K_S} &= \sin 2\beta = \frac{2\bar{\eta}(1-\bar{\rho})}{(1-\bar{\rho})^2 + \bar{\eta}^2} \end{aligned}$$

$$e^{i\gamma} &= \frac{\bar{\rho} + i\bar{\eta}}{\bar{\rho}^2 + \bar{\eta}^2} \quad (B \to D \ K \text{ rates}) \\ \alpha &= \pi - \beta - \gamma \quad (B \to \pi \pi, \varrho \pi, \varrho \varrho \text{ rates}) \\ \frac{\Delta m_d}{\Delta m_s} \propto \left| \frac{V_{td}}{V_{ts}} \right|^2 = \lambda^2 [(1-\bar{\rho})^2 + \bar{\eta}^2] \\ \epsilon_K \quad (\text{CPV in } K \to \pi \pi) \end{aligned}$$

 $\lambda = 0.2253(9)$ A = 0.822(12)



 $\bar{\eta} = 0.362 \pm 0.014$

Legacy of B-factories









• Very likely, CPV in flavour changing processes is dominated by CKM phase & Kobayashi-Maskawa mechanism of CPV is at work

- Very likely, CPV in flavour changing processes is dominated by CKM phase & Kobayashi-Maskawa mechanism of CPV is at work
- Reparametrisation invariant measure of CPV $\operatorname{Im}[V_{ij}V_{kj}^*V_{kl}V_{il}^*] = J_{KM}\sum \epsilon_{ikm}\epsilon_{jln},$
- $J_{KM} = \lambda^6 A^2 \eta = \mathcal{O}(10^{-5})$
- Jarlskog determinant in SM

$$J = J_{KM} \prod_{i>j} \frac{m_i^2 - m_j^2}{v^2} = \mathcal{O}(10^{-22}).$$

Continuing the Legacy of B-factories







Continuing the Legacy of B-factories



• Focus on the neutral B meson system: flavour states

 $B^0 \sim \bar{b}d \qquad \qquad \bar{B}^0 \sim b\bar{d}.$

$$CP|B^{0}\rangle = e^{i\xi_{B}}|\bar{B}^{0}\rangle,$$
$$CP|\bar{B}^{0}\rangle = e^{-i\xi_{B}}|B^{0}\rangle.$$

• Time evolution

 $\begin{aligned} |\psi(0)\rangle &= a(0)|B^0\rangle + b(0)|\bar{B}^0\rangle \\ |\psi(t)\rangle &= a(t)|B^0\rangle + b(t)|\bar{B}^0\rangle + c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \dots , \\ & \text{B decay products} \end{aligned}$

• If only interested a(t), b(t):

$$i\frac{d}{dt}\begin{pmatrix}a(t)\\b(t)\end{pmatrix} = H\begin{pmatrix}a(t)\\b(t)\end{pmatrix}, \quad \mathcal{H} = M + i\frac{\Gamma}{2}$$

- $M \& \Gamma$: time-independent, Hermitian 2 × 2 matrices,
- *M*-oscillations (dispersive); Γ -decays (absorptive)

• *H* eigenvectors

$$|B_{L,H}\rangle = p_{L,H}|B^0\rangle \pm q_{L,H}|\bar{B}^0\rangle$$

 $|p_{L,H}|^2 + |q_{L,H}|^2 = 1$

• *H* eigenvectors

$$|B_{L,H}\rangle = p_{L,H}|B^0\rangle \pm q_{L,H}|\bar{B}^0\rangle$$

 $|p_{L,H}|^2 + |q_{L,H}|^2 = 1$

• If CPT: $M_{11} = M_{22}$, $\Gamma_{11} = \Gamma_{22}$, $\Rightarrow p_L = p_H \equiv p, q_L = q_H \equiv q$

• *H* eigenvectors

$$|B_{L,H}\rangle = p_{L,H}|B^0\rangle \pm q_{L,H}|\bar{B}^0\rangle$$

 $|p_{L,H}|^2 + |q_{L,H}|^2 = 1$

- If CPT: $M_{11} = M_{22}, \Gamma_{11} = \Gamma_{22},$ $\Rightarrow p_L = p_H \equiv p, q_L = q_H \equiv q$
- If CP: $\operatorname{Arg}(M_{12}) = \operatorname{Arg}(\Gamma_{12})$ $\Rightarrow |q/p| = 1$
- <u>Exercise</u>: Check!

• CP conserving oscillation parameters:

$$m \equiv \frac{M_L + M_H}{2}, \qquad \Gamma \equiv \frac{\Gamma_L + \Gamma_H}{2},$$
$$\Delta m \equiv M_H - M_L, \qquad \Delta \Gamma \equiv \Gamma_H - \Gamma_L,$$

•
$$(x = \Delta m / \Gamma, y = \Delta \Gamma / 2\Gamma)$$

• Time evolution:

• at
$$t = 0$$
: $|B^0(t)\rangle \Rightarrow |B^0\rangle$
 $|\bar{B}^0(t)\rangle \Rightarrow |\bar{B}^0\rangle$

• Time evolution:

$$|B^{0}(t)\rangle = g_{+}(t)|B^{0}\rangle - \frac{q}{p}g_{-}(t)|\bar{B}^{0}\rangle,$$
$$|\bar{B}^{0}(t)\rangle = g_{+}(t)|\bar{B}^{0}\rangle - \frac{q}{p}g_{-}(t)|B^{0}\rangle,$$
$$g_{\pm} \equiv \frac{1}{2}\left(e^{-m_{H}t - \Gamma_{H}t/2} \pm e^{-m_{L}t - \Gamma_{L}t/2}\right)$$

• Decay to final state after time *t*:

$$\langle f | \mathcal{H} | B^0 \rangle \equiv A_f ,$$

 $\langle \bar{f} | \mathcal{H} | B^0 \rangle \equiv A_{\bar{f}} .$

• Decay to final state after time *t*:

$$\frac{d\Gamma(|B^{0}(0)\rangle) \rightarrow |f(t)\rangle}{dt} = \mathcal{N}_{0}e^{-\Gamma t}|A_{f}|^{2} \times \left\{\frac{1+|\lambda_{f}|^{2}}{2}\cosh\frac{\Delta\Gamma t}{2} + \frac{1-|\lambda_{f}|^{2}}{2}\cos\Delta m t + \operatorname{Re}\lambda_{f}\sinh\frac{\Delta\Gamma t}{2} - \operatorname{Im}\lambda_{f}\sin\Delta m t\right\},$$
$$\frac{d\Gamma(|\bar{B}^{0}(0)\rangle) \rightarrow |f(t)\rangle}{dt} = \mathcal{N}_{0}e^{-\Gamma t}|\bar{A}_{f}|^{2} \times \left\{\frac{1+|\bar{\lambda}_{f}|^{2}}{2}\cosh\frac{\Delta\Gamma t}{2} + \frac{1-|\bar{\lambda}_{f}|^{2}}{2}\cos\Delta m t + \operatorname{Re}\bar{\lambda}_{f}\sinh\frac{\Delta\Gamma t}{2} - \operatorname{Im}\bar{\lambda}_{f}\sin\Delta m t\right\},$$

•
$$N_0$$
 - flux norm., $\lambda_f \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f}$, $\bar{\lambda}_f \equiv \frac{p}{q} \frac{A_f}{\bar{A}_f} = \frac{1}{\lambda_f}$,

- Terms proportional to $|A_f|^2$, $|\bar{A}_f|^2$ describe a decay without net oscillation.
- Terms proportional to $|\lambda_f|^2$, $|\bar{\lambda}_f|^2$ describe a decays following net oscillations.
- Terms proportional to $\sin(\Delta m t)$, $\sinh(\Delta \Gamma t/2)$ describe interference between the above two cases.
- CP violation in interference is possible only if $\operatorname{Im}(\lambda_f) \neq 0$.

• CP violation in neutral B meson decays to CP eigenstates

$$A_{f_{CP}}(t) \equiv \frac{\frac{d\Gamma}{dt} \left[\bar{B}^0(0) \to f_{CP}(t) \right] - \frac{d\Gamma}{dt} \left[B^0(0) \to f_{CP}(t) \right]}{\frac{d\Gamma}{dt} \left[\bar{B}^0(0) \to f_{CP}(t) \right] + \frac{d\Gamma}{dt} \left[B^0(0) \to f_{CP}(t) \right]}$$

• In the B (& B_s) system experimentally $\Delta \Gamma << \Delta m$ $\Rightarrow |q/p| \approx 1$:

$$A_f(t) = S_f \sin(\Delta m t) - C_f \cos(\Delta m t),$$

$$S_f \equiv \frac{2 \operatorname{Im}(\lambda_f)}{1 + |\lambda_f|^2}, \quad C_f \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}.$$

Phases in decay amplitudes

• $B \to f$: amplitude A_f

- $\overline{B} \to \overline{f}$: amplitude $\overline{A}_{\overline{f}}$.
- complex parameters in L appear complex conjugated after CP \Rightarrow opposite signs
 - CP odd weak phases
- on-shell intermediate states (even for real L) \Rightarrow same signs (CP even) - *strong phases*

Phases in decay amplitudes $A_f = |a_1|e^{i(\delta_1+\phi_1)} + |a_2|e^{i(\delta_2+\phi_2)} + \dots,$ $\bar{A}_{\bar{f}} = |a_1|e^{i(\delta_1-\phi_1)} + |a_2|e^{i(\delta_2-\phi_2)} + \dots,$

- a_{1,2},... contributions to amplitude with different phases
- $\delta_{1,2}$... strong phases
- $\phi_{1,2}$... weak phases

CPV in $B \rightarrow \psi K_S$

$$A_f = |a_f| e^{i(\delta_f + \phi_f)},$$

$$\bar{A}_f = |a_f| e^{i(\delta_f - \phi_f)} \eta_f,$$

$$\lambda_f = \eta_f(q/p) \exp(-2i\phi_f)$$

• In B_d system $|\Gamma_{12}| << |M_{12}|$, due to $O(G_F^2)$ long distance effects, suppressed by small CKM elements

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \simeq e^{2i\xi_B} ,$$
$$\lambda_f \simeq \eta_f \exp[i(\xi_B - 2\phi_f)]$$
$$S_{f_{CP}} \simeq \eta_f \sin(\xi_B - 2\phi_f)$$

CPV in $B \rightarrow \psi K_S$

• In SM:

$$\xi_B = -\operatorname{Arg}(M_{12}) \simeq -\operatorname{Arg}[(V_{tb}^* V_{td})^2] = -\operatorname{Arg}\left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)$$
$$-e^{-2i\phi_f} = \frac{\bar{A}_{\psi K_S}^{(B)}}{A_{\psi K_S}^{(B)}} = -\frac{V_{cb} V_{cs}^* a_T + \dots}{V_{cb}^* V_{cs} a_T + \dots} e^{i\xi_K} \simeq -\frac{V_{cb} V_{cs}^* V_{cd}^* V_{cs}}{V_{cb}^* V_{cs} V_{cs}^* V_{cd}^* V_{cs}}$$
$$K - K \text{ oscillations}$$
forming K_S

CPV in $B \rightarrow \psi K_S$

• In SM:

 $\xi_B = -\operatorname{Arg}(M_{12}) \simeq -\operatorname{Arg}[(V_{tb}^* V_{td})^2] = -\operatorname{Arg}\left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)$ $-e^{-2i\phi_f} = \frac{\bar{A}_{\psi K_S}^{(B)}}{A_{\psi K_S}^{(B)}} = -\frac{V_{cb}V_{cs}^* a_T + \dots}{V_{cb}^* V_{cs} a_T + \dots} e^{i\xi_K} \simeq -\frac{V_{cb}V_{cs}^* V_{cs}^* V_{cd}^* V_{cs}}{V_{cb}^* V_{cs} V_{cs}} \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*}$ K - K oscillations forming K_S $\lambda_{\psi K_S}^{(B)} \simeq \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}} = -e^{-2i\beta}$ CPV in interference $S_{\psi K_{C}}^{(B)} \simeq \sin 2\beta$ (note that $C_{\psi K_{C}}^{(B)} \simeq 0$)

Experimentally measured to an accuracy of \sim 1%

- Golden channel: $B_s \to \psi \varphi$
 - angular analysis required
 - B_s oscillations much faster than those of B_d

$$\frac{\Delta m_s}{\Delta m_d} \sim \frac{|M_{12}^s|}{|M_{12}^d|} \propto \left|\frac{V_{ts}}{V_{td}}\right|^2 \sim 30.$$

• $\Delta \Gamma_s$ effects cannot be neglected

• In SM: $\lambda_{\psi\phi}^{(B_s)} = -\exp[i(\xi_{B_s} - 2\phi_{\psi\phi})]$

$$\left[S_{\psi\phi}^{(B_s)}\right]_{\rm SM} = 2 \operatorname{Arg} \frac{V_{tb}^* V_{ts}}{V_{cb}^* V_{cs}} = 0.036(1) \,,$$



• In SM: $\lambda_{\psi\phi}^{(B_s)} = -\exp[i(\xi_{B_s} - 2\phi_{\psi\phi})]$

$$\left[S_{\psi\phi}^{(B_s)}\right]_{\rm SM} = 2 \text{Arg} \frac{V_{tb}^* V_{ts}}{V_{cb}^* V_{cs}} = 0.036(1) \,,$$

• Experimentally: $S_{\psi\phi}^{(B_s)} = 0.02(4)$

• <u>Exercise</u>: Show that if $B \to \pi\pi$ is dominated by a single (tree) amplitude, then $S_{\pi\pi} = \sin(2\alpha)$

CP violation in B decays to CP conjugate states If $B_0 \to \overline{f}$ and $\overline{B}_0 \to f$ forbidden: • $|A_f| = |\bar{A}_{\bar{f}}|$ and $|A_{\bar{f}}| = |\bar{A}_f| = 0$ $\frac{\frac{d\Gamma}{dt}\left[\bar{B}^{0}(0) \to f(t)\right] - \frac{d\Gamma}{dt}\left[B^{0}(0) \to \bar{f}(t)\right]}{\frac{d\Gamma}{dt}\left[\bar{B}^{0}(0) \to f(t)\right] + \frac{d\Gamma}{dt}\left[B^{0}(0) \to \bar{f}(t)\right]} = \frac{\left|\frac{p}{q}\right|^{2} - \left|\frac{q}{p}\right|^{2}}{\left|\frac{p}{q}\right|^{2} + \left|\frac{q}{p}\right|^{2}} \simeq \operatorname{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^{2}\right)$

CP violation in B decays to CP conjugate states • If $B_0 \to \overline{f}$ and $\overline{B}_0 \to f$ forbidden: $|A_f| = |\bar{A}_{\bar{f}}|$ and $|A_{\bar{f}}| = |\bar{A}_f| = 0$ $\frac{\frac{d\Gamma}{dt}\left[\bar{B}^{0}(0) \to f(t)\right] - \frac{d\Gamma}{dt}\left[B^{0}(0) \to \bar{f}(t)\right]}{\frac{d\Gamma}{dt}\left[\bar{B}^{0}(0) \to f(t)\right] + \frac{d\Gamma}{dt}\left[B^{0}(0) \to \bar{f}(t)\right]} = \frac{\left|\frac{p}{q}\right|^{2} - \left|\frac{q}{p}\right|^{2}}{\left|\frac{p}{q}\right|^{2} + \left|\frac{q}{p}\right|^{2}} \simeq \operatorname{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^{2}\right)$

- Time-independent measurement
- CP violation in mixing, (indirect CP violation)
- <u>Example</u>: $a_{SL}^{(d)} = \frac{\Gamma(\bar{B}^0 \to X\ell^+\nu) \Gamma(B^0 \to X\ell^-\bar{\nu})}{\Gamma(\bar{B}^0 \to X\ell^+\nu) + \Gamma(B^0 \to X\ell^-\bar{\nu})}$ • In SM: $a_{SL}^{(d)} = -8(2) \times 10^{-4}$
CP violation in B decays to CP conjugate states



CP violation in charged B decays

• Interesting example: $B^{\pm} \to DK^{\pm}$

 $B^- \to D^0 K^- : b \to c \bar{u} s ,$ $B^- \to \bar{D}^0 K^- : b \to \bar{c} u s .$

• Particularly transparent in D decays to CP eigenstates

$$D^0 \to f_{CP}: c \to d\bar{d}u, s\bar{s}u,$$

 $\bar{D}^0 \to f_{CP}: \bar{c} \to d\bar{d}\bar{u}, s\bar{s}\bar{u}.$

CP violation in charged B decays

• In SM:

$$\frac{A^B_{(D\to f)K}}{A^B_{(\bar{D}\to f)K}} = \frac{V^*_{cb}V_{us}a^B_{DK}}{V^*_{ub}V_{cs}a^B_{\bar{D}K}}e^{i(\delta^B_{DK}-\delta^B_{\bar{D}K})}\eta_f \frac{V_{cd}V^*_{ud}}{V^*_{cd}V_{ud}}\frac{a^D_f}{a^{\bar{D}}_f}e^{i(\delta^D_f-\delta^{\bar{D}}_f)} \simeq \eta_f r_B e^{i(\delta_B-\gamma)}$$

•
$$\gamma \equiv \operatorname{Arg}(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*) \simeq 70^\circ$$

• Several decay rates: CPV in decay (direct CPV)

$$\begin{aligned} A(B^{-} \to f_{+}K^{-}) &= A_{0} \left[1 + r_{B}e^{i(\delta_{B} - \gamma)} \right] ,\\ A(B^{-} \to f_{-}K^{-}) &= A_{0} \left[1 - r_{B}e^{i(\delta_{B} - \gamma)} \right] ,\\ A(B^{+} \to f_{+}K^{+}) &= A_{0} \left[1 + r_{B}e^{i(\delta_{B} + \gamma)} \right] ,\\ A(B^{+} \to f_{-}K^{+}) &= A_{0} \left[1 - r_{B}e^{i(\delta_{B} + \gamma)} \right] .\end{aligned}$$