Introduction to cosmology (& astroparticle physics)

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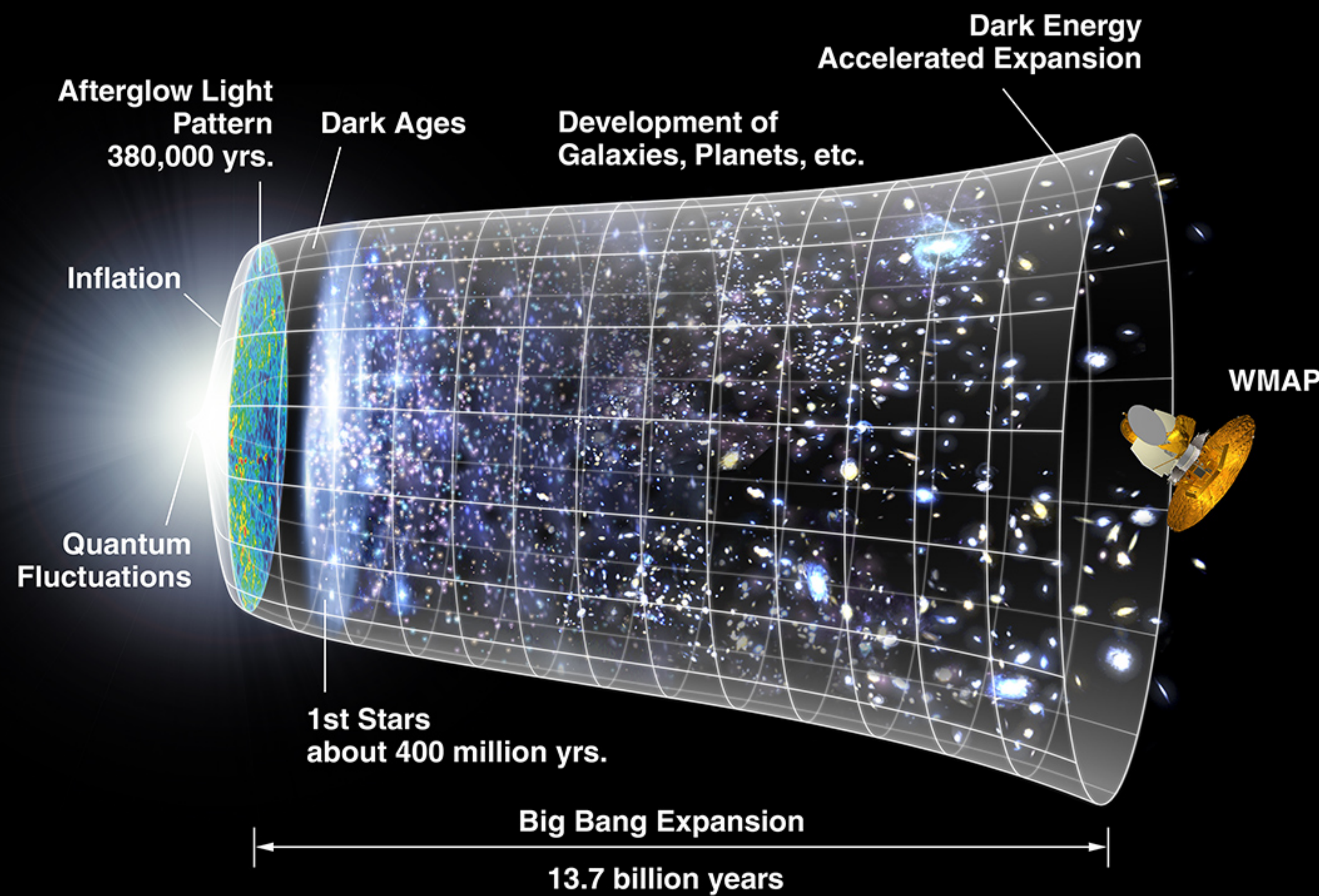
(& the Planck collaboration
& some others)

global outline

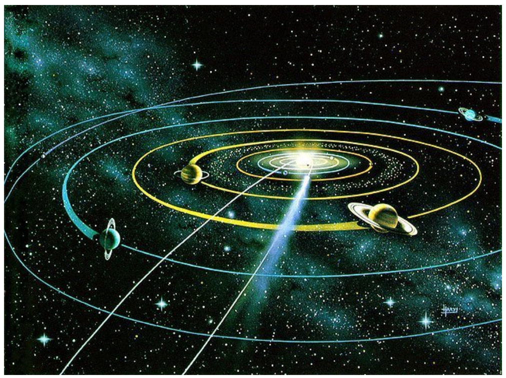
- **background: the FLRW metric**
  - metric, scale factor, redshift, distances
  - Einstein eqn’s, evolution of the universe
  - the cosmological standard model: LCDM
- **the perturbed universe**
  - inflation
  - evolution of the perturbations
  - power spectra: CMB and P(k), observational probes
  - dark energy & modified gravity
- **astro-particle physics**
  - thermal history, neutrinos & WIMPS
  - direct and indirect DM detection
  - cosmic rays
  - multi-messenger: neutrinos, gravitational waves, …

Brief history of the Universe



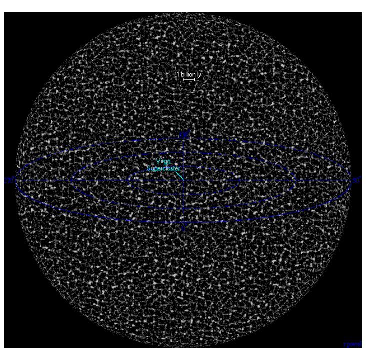
orders of magnitude

cosmology also goes right down to the Planck scale…
… but for now we are more interested in large scales!



solar system:
size: billions of km (10^9 km)
1AU = 1.5×10^8 km
Pluto ~ 40 AU, Voyager 1: 128 AU

galaxies:
size ~ 10 kpc
1pc ≈ 3 light years = 3×10^{13} km
billions of stars (sizes vary!)



(observable) universe
size ~ 10 Gpc (~ 10^{23} km vs $l_P \sim 10^{-38}$ km)
~ 10^{11} galaxies

Outline of part I

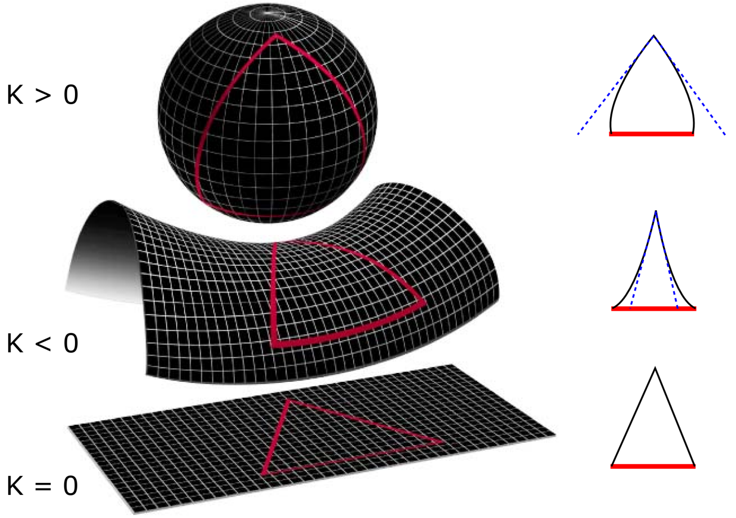
- **metric structure: cosmography**
  - the metric
  - expansion of the universe, redshift and Hubble's law
  - cosmological distances and the age of the universe
- **content and evolution of the universe**
  - Einstein equations and the Bianchi identity
  - the critical density and the Ω's
  - the evolution of the universe
  - contents, the LCDM model

the cosmological space-time

Ingredients:

- the universe looks isotropic around us
- **Cosmological principle**: all observers are equivalent
- some technical assumptions on how stuff behaves
- implication: the universe has a **FLRW metric**

$$ds^2 = dt^2 - \left(\frac{dR^2}{1 - KR^2} + R^2 d\Omega\right)$$



(at least for simply connected spaces)

basic quantities

- Maximal symmetry for spatial sections imposes an even stronger constraint: setting R(t) = a(t) r, the line element has the form

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - \kappa r^2} + r^2 d\Omega \right)$$

  where k = ±1 or 0 is a constant

- For this metric, the curves (r,θ,φ)=const are geodesics for a 4-velocity u=(1,0,0,0) since $\Gamma^{\mu}{}_{00}$=0 [check!] -> **comoving coordinates**

  (geodesic eqn: $\ddot{X}^\mu + \Gamma^\mu_{\alpha\beta} \dot{X}^\alpha \dot{X}^\beta = 0$)

- expansion leads to redshift

$$1 + z = \frac{a(t_0)}{a(t_1)}$$

The Hubble law

for two galaxies at a fixed **comoving** distance r_0:

physical distance $x(t) = a(t)r_0$

-> apparent motion: $$\frac{dx}{dt} = \dot{a}r_0 = \frac{\dot{a}}{a}x \equiv H_0 x$$

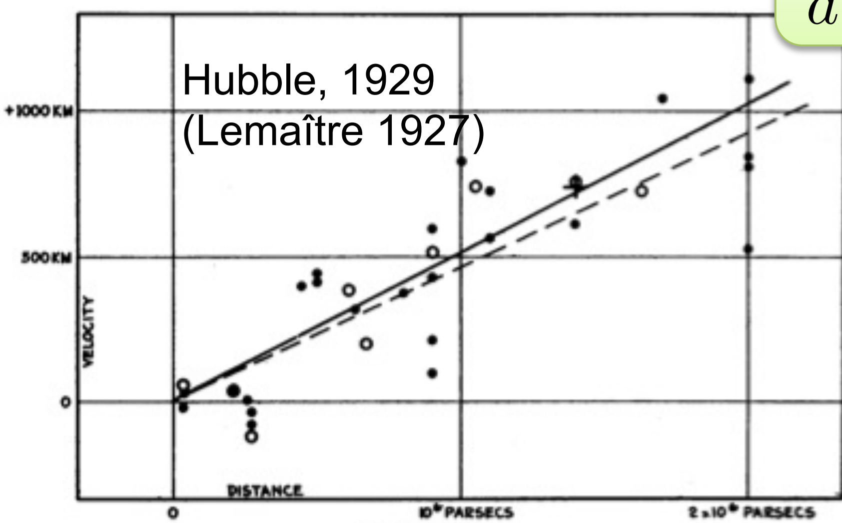
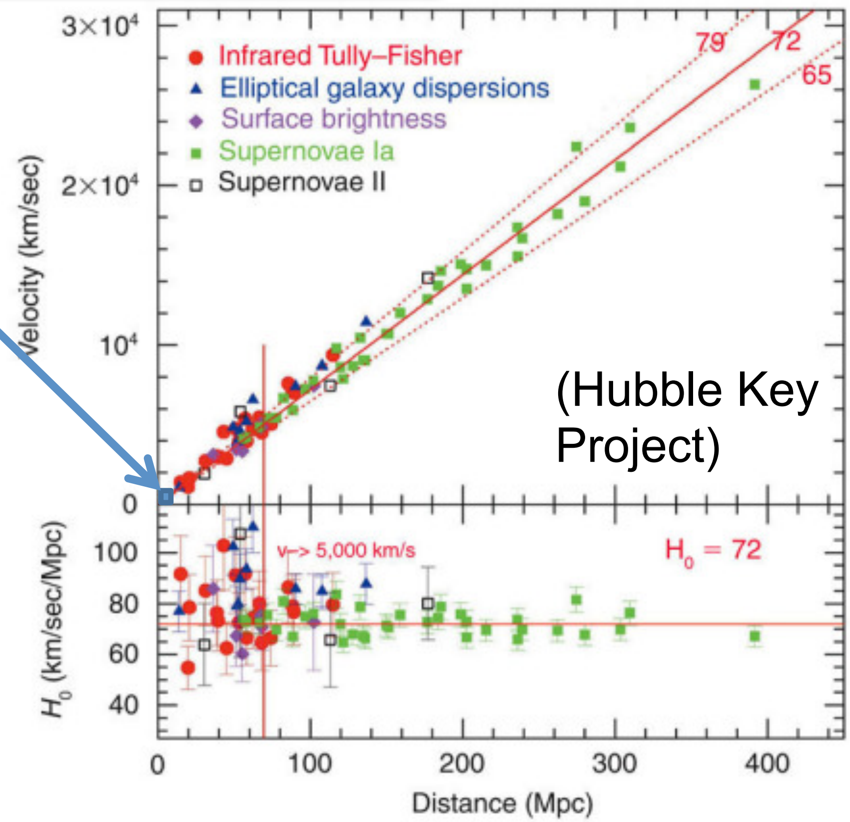


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

Radial velocities, corrected for solar motion, are plotted against distances estimated from involved stars and mean luminosities of nebulae in a cluster. The black discs and full line represent the solution for solar motion using the nebulae individually; the circles and broken line represent the solution combining the nebulae into groups; the cross represents the mean velocity corresponding to the mean distance of 22 nebulae whose distances could not be estimated individually.



philosophical remarks

- The FLRW metric is just picked ‘by hand’

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - \kappa r^2} + r^2 d\Omega \right)$$

- ***This needs to be tested as much as possible!***
- E.g. an even more symmetric possibility would be the de Sitter metric, but observations rule it out!
- We know that the Universe is not exactly FLRW, it’s not entirely clear yet how important this is
- FLRW leads to testable consequences (the ‘3 pillars’ – there are more tests)
- Unfortunately we have only 1 Universe, and we can’t even go everywhere, we can only observe

cosmological distances

simpler to transform the distance variable r to χ:

$$r = S_\kappa(\chi) = \begin{cases} \sin\chi & \kappa = +1 \\ \chi & \kappa = 0 \\ \sinh\chi & \kappa = -1 \end{cases}$$

$$\Rightarrow ds^2 = dt^2 - a^2(t)\left(d\chi^2 + S_\kappa(\chi)^2 d\Omega\right)$$

$$\Rightarrow dV = a_0^2 S_\kappa(\chi)^2 d\Omega d\chi$$ volume element today

we can now *define* a «metric» distance:

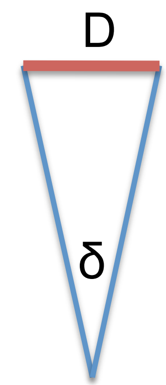
$$\boxed{d_m(\chi) = a_0 S_\kappa(\chi)}$$

$$\chi = \int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_{a_1}^{a_0} \frac{da}{a\dot{a}} = \frac{1}{a_0}\int_0^{z_1} \frac{dz}{H(z)}$$

cosmological distances

but physical distances need to be **observables**!

1) angular diameter distance: object of physical size D observed under angle δ, but photons were emitted at time $t_1 < t_0$:



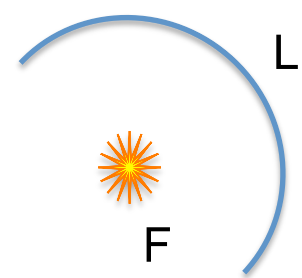
$$D = a(t_1) S_\kappa(\chi)\delta = \frac{a(t_1)}{a_0} a_0 S_\kappa(\chi)\delta \equiv d_A \delta$$

$$\boxed{d_A = \frac{1}{1+z} d_m}$$

2) luminosity distance: consider observed flux F for an object with known intrinsic luminosity L («standard candle»)

$$F \equiv \frac{L}{4\pi d_L^2}$$

surface: $4\pi d_m^2$

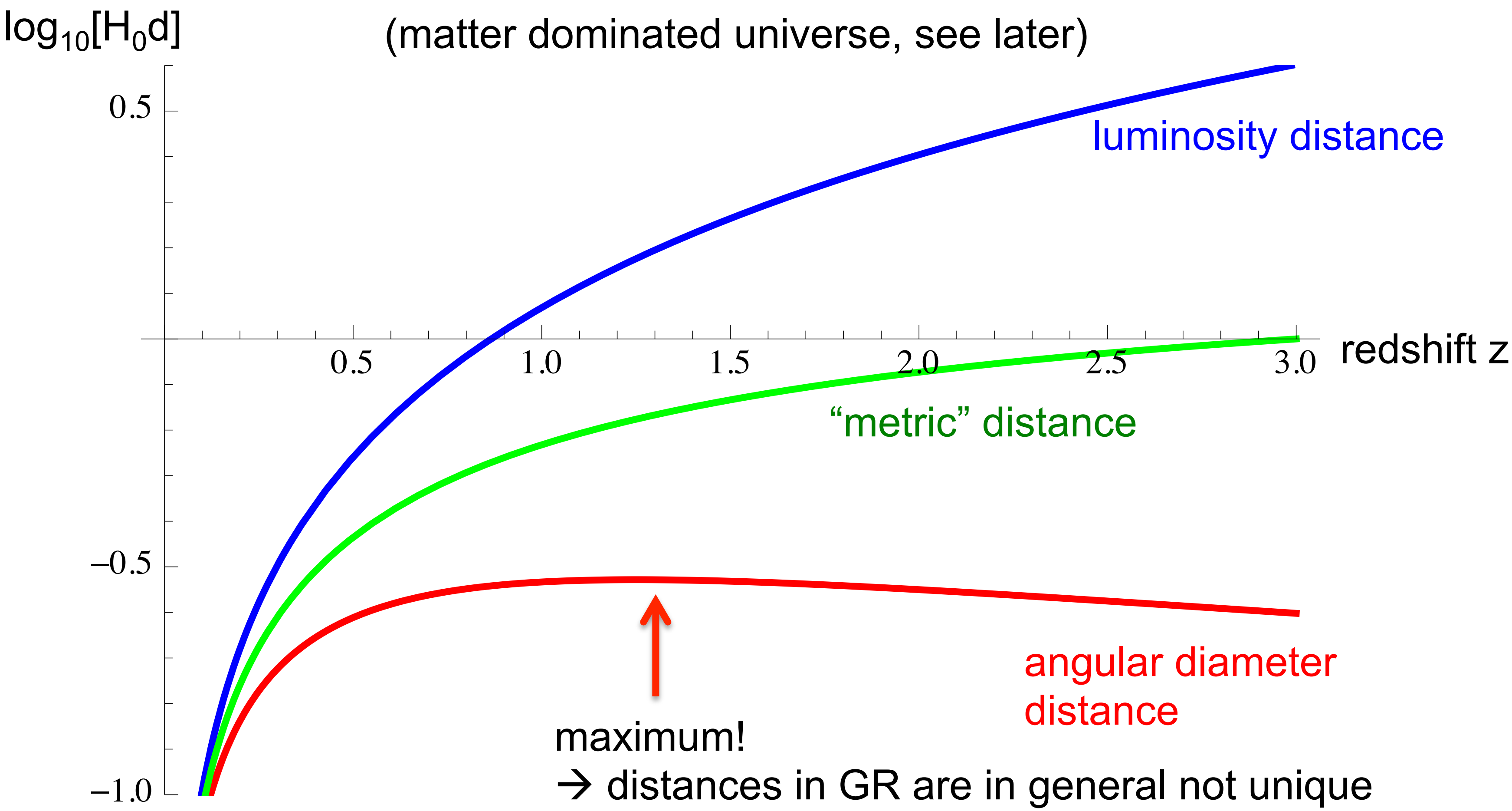


source emitting one photon per second:

1) redshift
2) increased time between arrivals

$$\boxed{d_L = (1+z) d_m}$$

distance example



remark: $d_L = (1+z)^2\, d_A$ is very general

age of the universe

computing the age of the universe is very straightforward:

$$t_0 = \int_0^{t_0} dt = \int_0^{a_0} \frac{da}{\dot{a}} = \int_0^{a_0} \frac{da}{aH(a)} = \int_0^{\infty} \frac{dz}{H(z)(1+z)}$$

but we need to know the evolution of the scale factor a(t). This in turn depends on the contents of the universe…

cue Einstein: $$\underbrace{G_{\mu\nu}}_{\text{geometry}} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N \underbrace{T_{\mu\nu}}_{\text{content}}$$

what is in the universe?

- homogeneous and isotropic metric: matter does also have to be distributed in this way
- in **some** coordinate system the energy momentum tensor has the form:

$$T^i_0 = 0, \quad T^1_1 = T^2_2 = T^3_3$$

and the components depend only on time

$$\boxed{T^\nu_\mu = \mathrm{diag}\left(\rho(t), -p(t), -p(t), -p(t)\right)}$$

- the pressure determines the nature of the fluid, p = w ρ:
  - w = 0 : pressureless 'dust', 'matter'
  - w = 1/3 : radiation
  - what is w for $T_{\mu\nu} = \Lambda g_{\mu\nu}$?

the conservation equation

- Bianchi identity (geometric identity for $G_{\mu\nu}$):

$$T^{\mu\nu}_{;\mu} = 0 = G^{\mu\nu}_{;\mu}$$

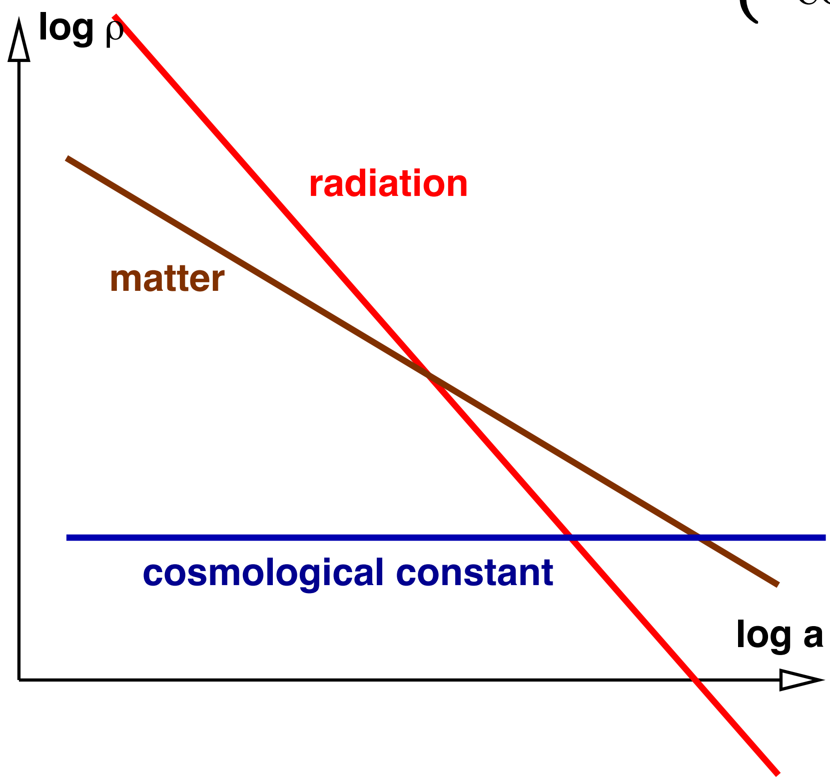
$$T^{\nu}_{0;\nu} = \dot{\rho} + \Gamma^{i}_{i0}(\rho + p) = \boxed{\dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)\underbrace{(\rho + p)}_{(1+w)\rho} = 0}$$

Questions:

- for a constant w, what is the evolution of ρ(a)?
  (eliminate the variable t from the equation)
- for the three cases w = 0, 1/3, -1, what is ρ(a)?
- does the result make sense?

evolution of the energy densities

$$\rho \propto a(t)^{-3(1+w)} \propto \begin{cases} a(t)^{-3} & \text{for } w = 0 \quad \text{(matter)} \\ a(t)^{-4} & \text{for } w = 1/3 \quad \text{(radiation)} \\ \text{const.} & \text{for } w = -1 \quad \text{(vacuum energy)} \end{cases}$$



dust/matter: dilution through expansion of space

radiation: additional redshift

at early times, the energy density in the universe should have been dominated by radiation

Einstein equations

- we now have all necessary ingredients to compute the **Einstein equations**:
  - **metric**
  - **energy-momentum tensor**

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$

$$R_{\mu\nu} \equiv R^{\alpha}_{\mu\alpha\nu} \qquad R \equiv g^{\mu\nu} R_{\mu\nu}$$

$$R^{\alpha}_{\beta\mu\nu} = \Gamma^{\alpha}_{\nu\beta,\mu} - \Gamma^{\alpha}_{\mu\beta,\nu} + \Gamma^{\delta}_{\nu\beta}\Gamma^{\alpha}_{\mu\delta} - \Gamma^{\delta}_{\mu\beta}\Gamma^{\alpha}_{\nu\delta}$$

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \left(g_{\beta\mu,\nu} + g_{\beta\nu,\mu} - g_{\mu\nu,\beta} \right)$$

try to do it yourselves… ☺

Friedmann equations

you should find:

$$R_{00} = -3\frac{\ddot{a}}{a} \qquad R_{ij} = -\left[\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\kappa}{a^2}\right] g_{ij}$$

$$R = -6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2}\right]$$

the **space-time** curvature is non-zero even for k=0!

0-0 component:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = \frac{8\pi G_N}{3}\rho$$

sum of ρ from all types of energy

i-i component:

$$2\left(\frac{\ddot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = -8\pi G_N p$$

Friedmann equations II

three comments:

- you can combine the two equations to find

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G_N}{3}\left(\rho + 3p\right)$$

-> the expansion is accelerating if $p<-\rho/3$

- the two Einstein equations and the conservation equation are not independent
- there are 3 unknown quantities (ρ, p and a) but only two equations, so one quantity needs to be given (normally p) – as well as the constant k.

the critical density

Friedmann eq. $\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = \frac{8\pi G_N}{3}\rho$

$$H \equiv \left(\frac{\dot{a}}{a}\right) \qquad \frac{\kappa}{a^2 H^2} = \frac{8\pi G_N \rho}{3H^2} - 1 \equiv \frac{\rho}{\rho_c} - 1 \equiv \Omega - 1$$

$$\Omega(t) > 1 \quad \Rightarrow \quad \kappa > 0 \Rightarrow \textbf{closed} \text{ universe}$$
$$\Omega(t) = 1 \quad \Rightarrow \quad \kappa = 0 \Rightarrow \textbf{flat} \text{ universe}$$
$$\Omega(t) < 1 \quad \Rightarrow \quad \kappa < 0 \Rightarrow \textbf{open} \text{ universe}$$

and: $\frac{d}{dt}\underbrace{\left(\frac{\Omega - 1}{\kappa}\right)}_{|\Omega - 1| \quad (\kappa \neq 0)} = \frac{d}{dt}\frac{1}{\dot{a}^2} = -2\frac{\ddot{a}}{\dot{a}^3}$

>0 for expanding universe filled with dust or radiation (and k ≠ 0)
-> the universe becomes “less flat”
-> strange (why?)

‘Ω form’ of Friedmann eq.

Friedmann eq. $$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = \frac{8\pi G_N}{3}\rho$$

evolution of ρ for the «usual» 4 constituents:

- radiation: a^{-4}
- dust: a^{-3}
- curvature: a^{-2} ($H^2 + k/a^2 \sim \rho$)
- cosmological constant: a^0

notation:

$$\Omega_X = \left.\frac{\rho_X}{\rho_c}\right|_{t_0}$$

$$H^2 = H_0^2 \left[\frac{8\pi G}{3H_0^2}\rho_0 \left(\frac{a}{a_0}\right)^{-n} + \ldots + \frac{\kappa}{H_0^2 a_0^2}\left(\frac{a}{a_0}\right)^{-2}\right]$$

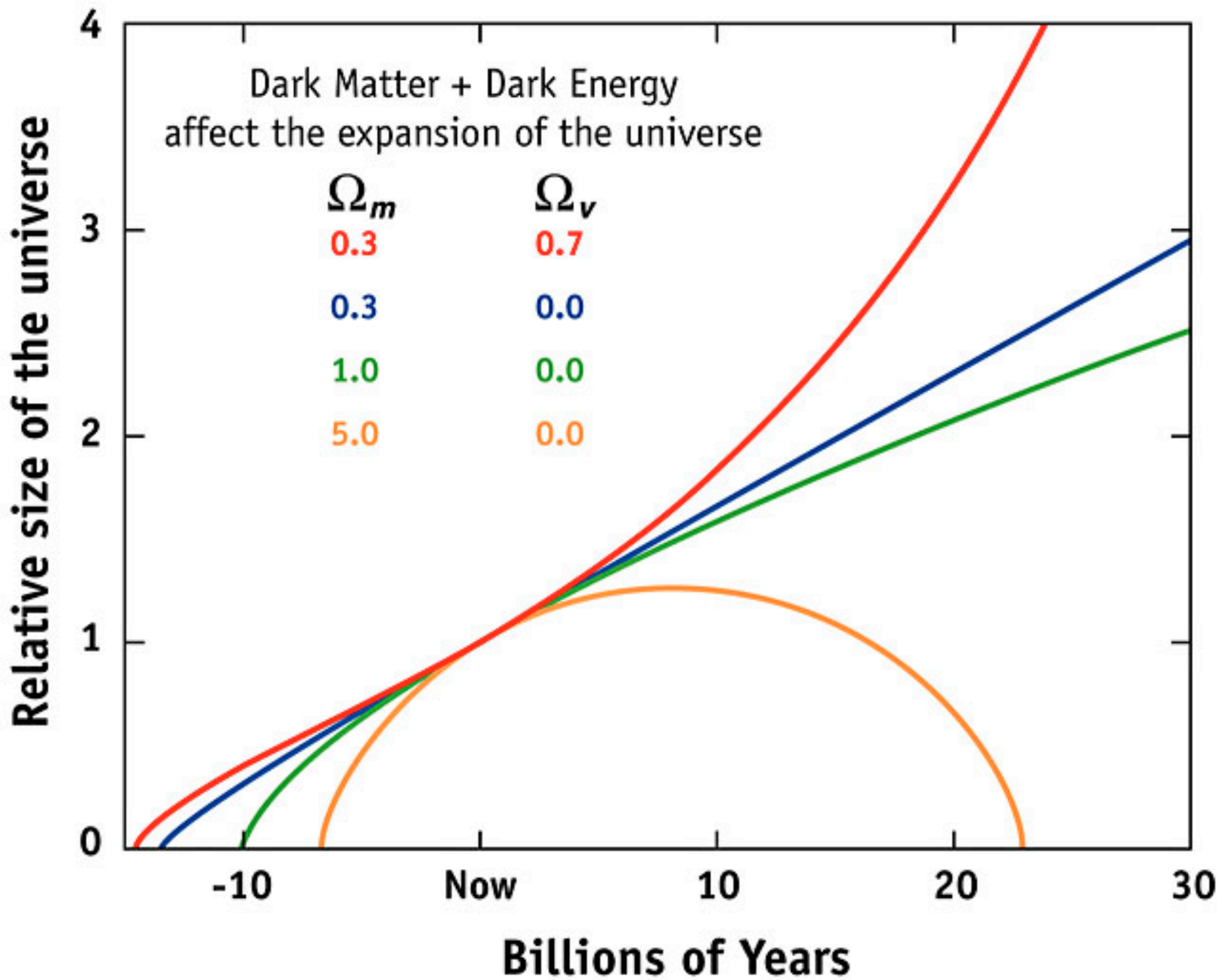
$$H^2 = H_0^2 \left[\Omega_r \left(\frac{a}{a_0}\right)^{-4} + \Omega_m \left(\frac{a}{a_0}\right)^{-3} + \Omega_\Lambda + \Omega_\kappa \left(\frac{a}{a_0}\right)^{-2}\right]$$

$$\Omega_r + \Omega_m + \Omega_\Lambda + \Omega_\kappa = 1$$

evolution of the universe

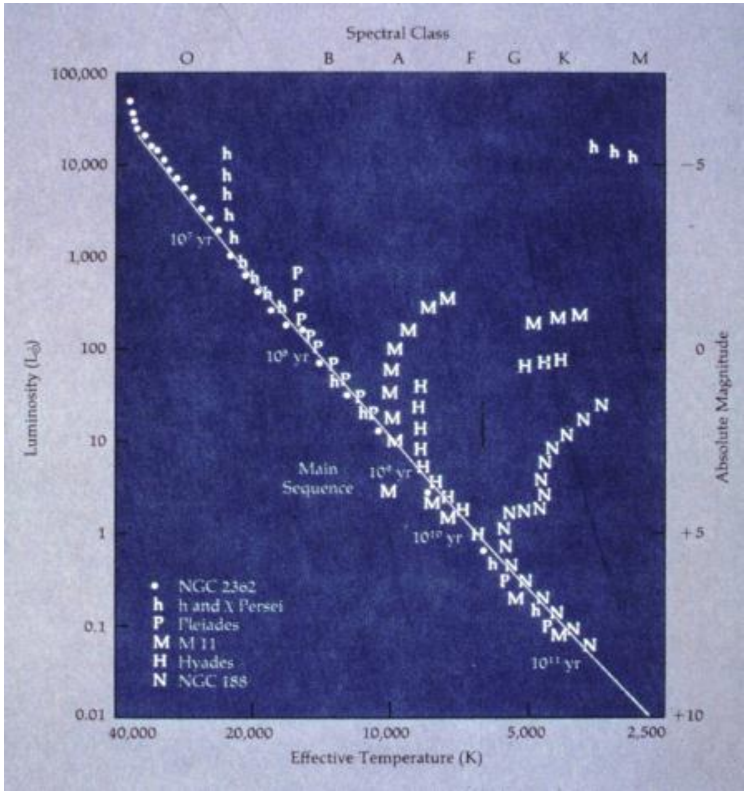
evolution depends on content!

pure radiation: $a \sim t^{1/2}$
pure matter: $a \sim t^{2/3}$
pure Λ: $a \sim e^{Ht}$



age of the universe revisited

we had: $$t_0 = \int_0^\infty \frac{dz}{H(z)(1+z)}$$



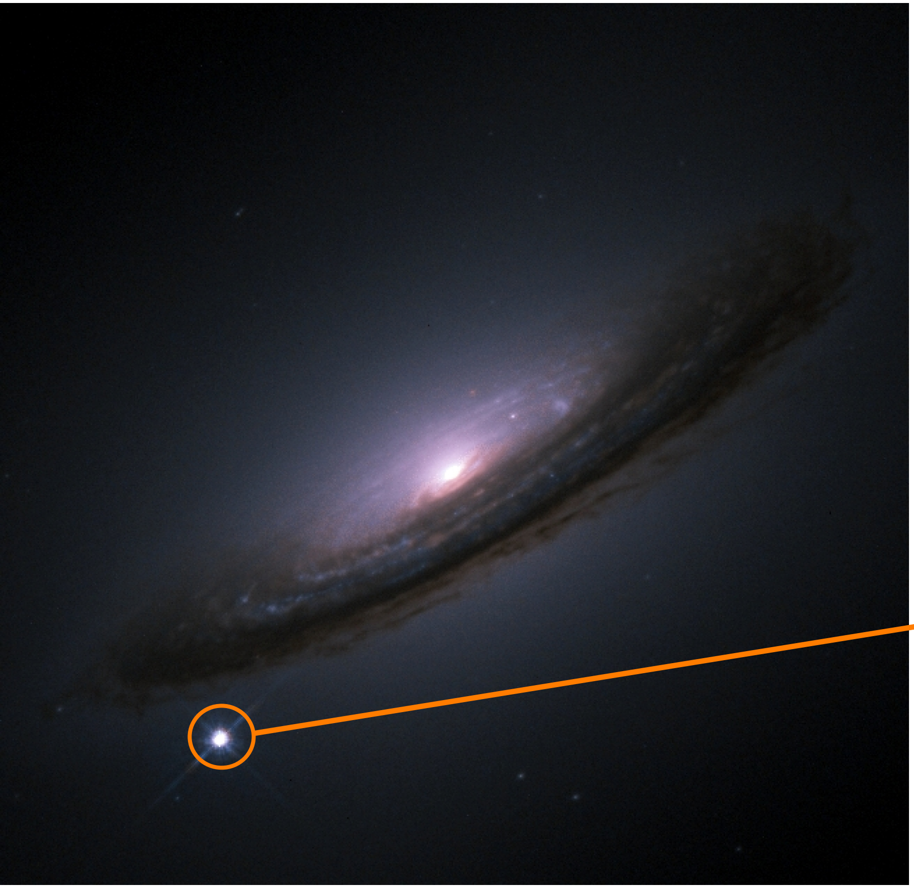
but for a matter-dominated universe:

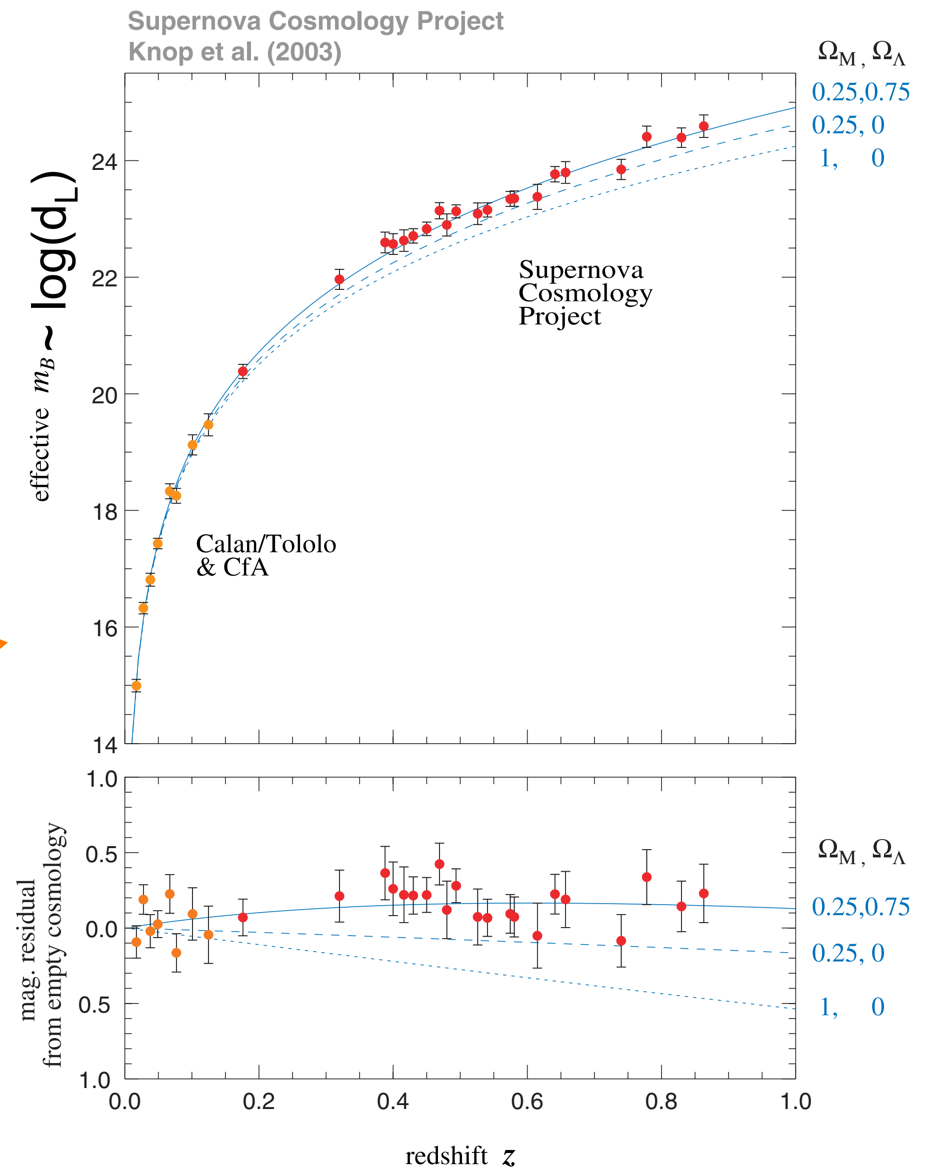
$$H = H_0 \left(\frac{a}{a_0}\right)^{-3/2} = H_0 (1+z)^{3/2}$$

$$H_0 t_0 = \int_0^\infty \frac{dz}{(1+z)^{5/2}} = \int_1^\infty \frac{du}{u^{5/2}} = -\frac{2}{3}\left.\frac{1}{u^{3/2}}\right|_1^\infty = \frac{2}{3}$$

$1/H_0 \sim 9.8$ Gyr/[H_0/100 km/s/Mpc] ~ 13.6 Gyr -> $t_0 \sim 9$ Gyr but oldest globular star clusters are older: 11-18 Gyr …??!!

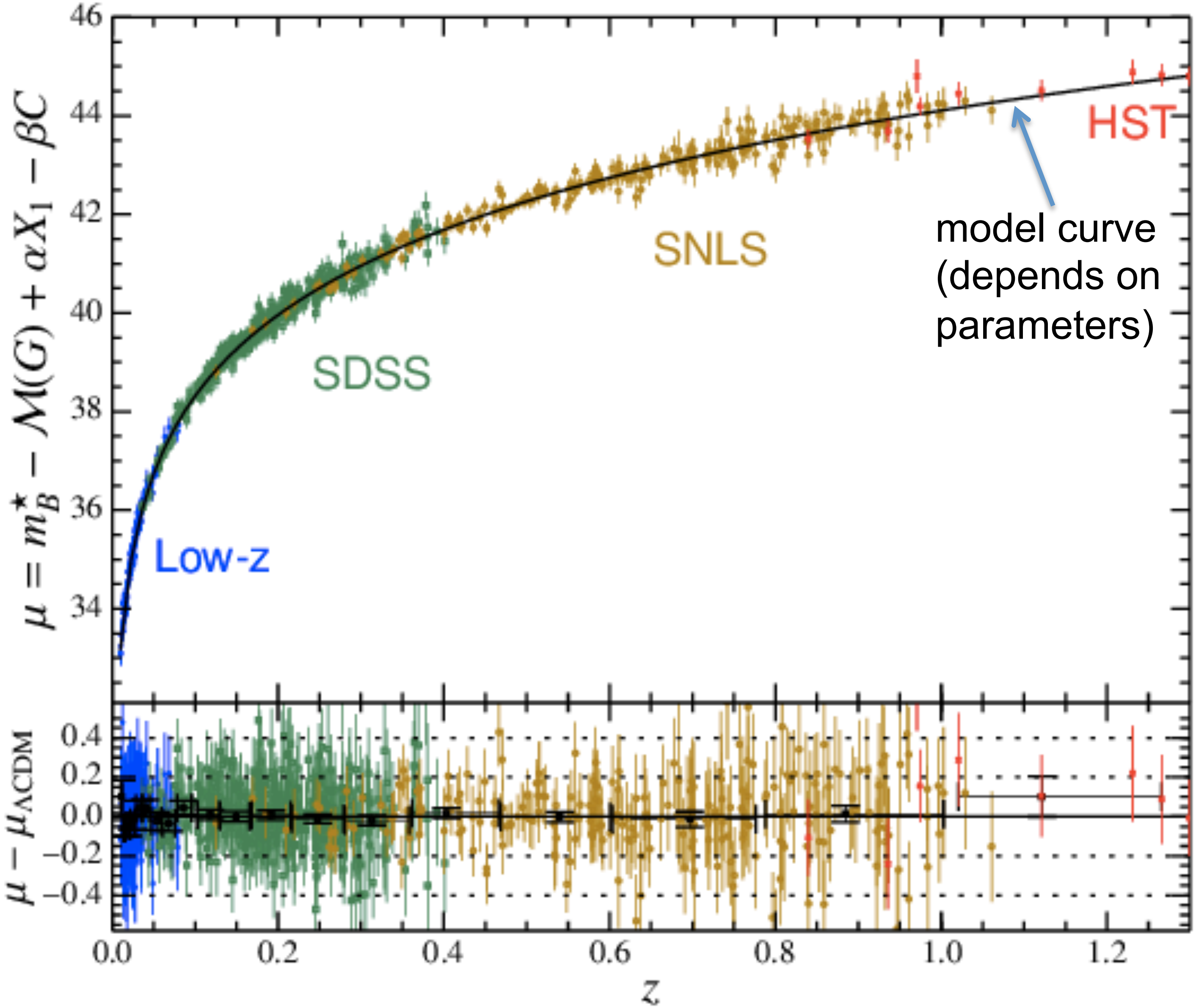
distances revisited





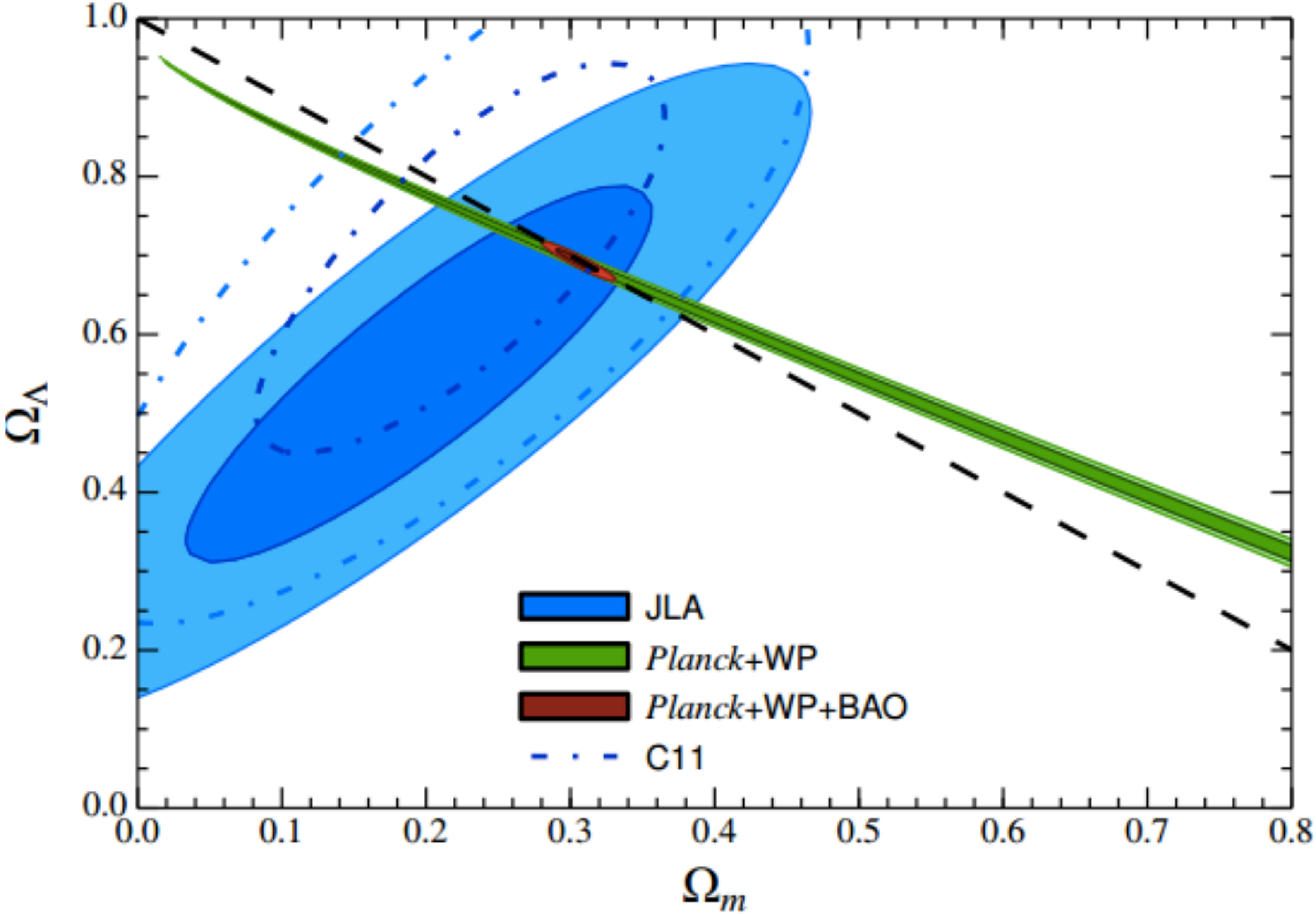
The luminosity distance depends on the contents of the universe through the expansion rate!

current distance diagram



JLA (joint light-curve analysis), SDSS & SNLS

constraints



JLA (joint light-curve analysis), SDSS & SNLS

ingredients for LCDM soup

To explain supernova distances we need:

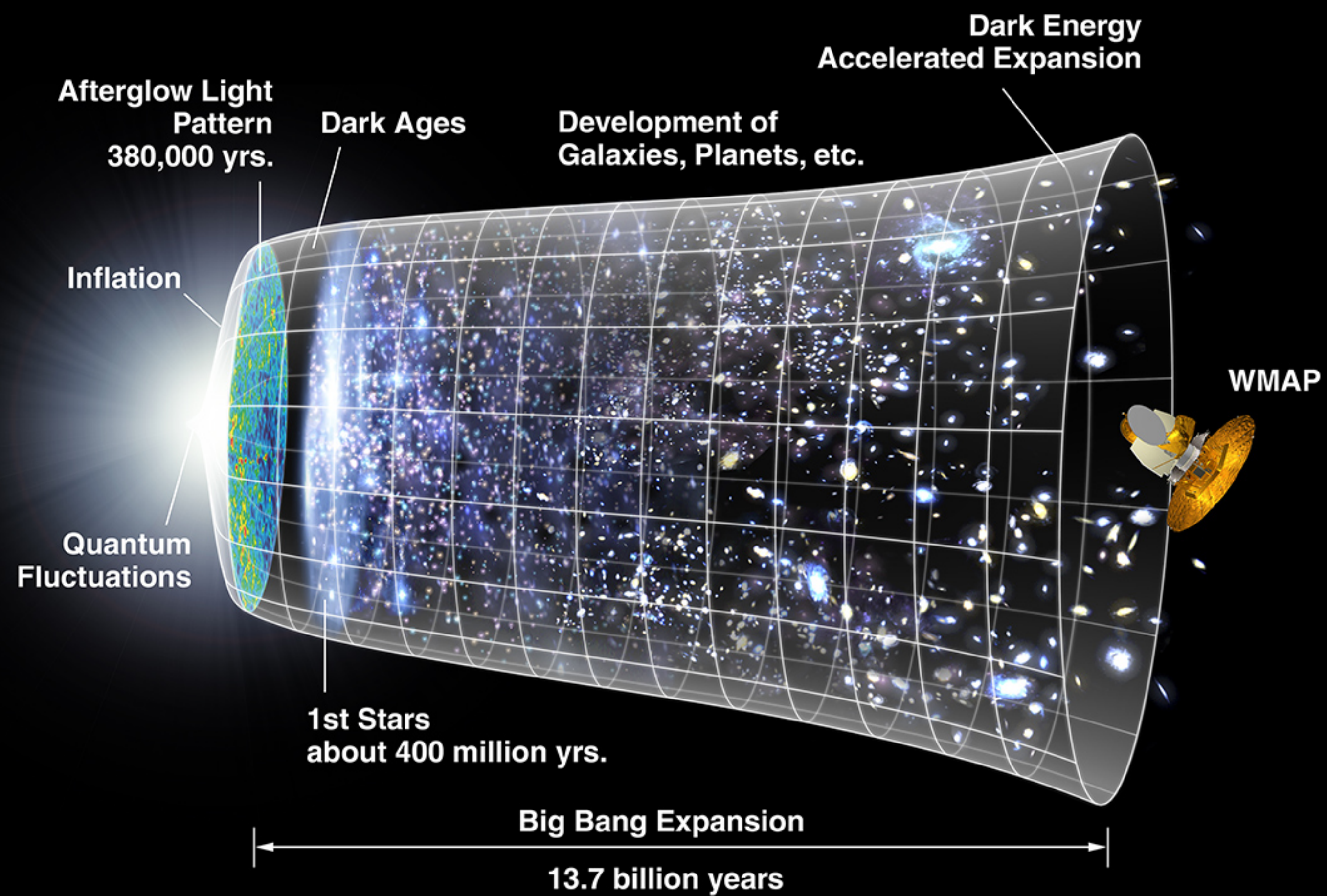
- **(expansion rate: H_0)**
- **(radiation)**
  - given by T_0 through Stefan-Boltzmann
  - includes neutrinos (more later)
- **matter: Ω_m**
  - ‘normal’ and dark
  - “cold” → low velocity and collisionless
- **cosmological constant: Ω_Λ**

→ Lambda-cold-dark-matter model

status report

- **reasonable (?) assumptions → FLRW metric**
- **GR: link of evolution and contents**
  - universe expanding: smaller and hotter in the past
  - age & distance measurements: LCDM model
- **Issues:**
  - universe appears spatially flat
  - where does the structure come from?
  - how do perturbations evolve?
- **Next steps:**
  - inflation with scalar fields
  - creation and evolution of perturbations
  - CMB & the (dark) matter power spectrum
  - dark energy / modified gravity
  - towards particle cosmology & astroparticle physics

Brief history of the Universe

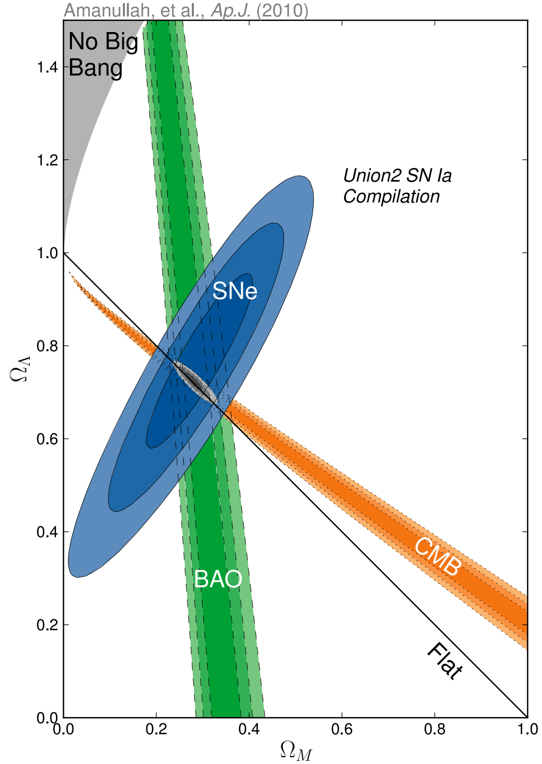


why is the world flat?

we saw:

$$\frac{d}{dt}\underbrace{\left(\frac{\Omega-1}{\kappa}\right)}_{|\Omega-1| \quad (\kappa\neq 0)} = \frac{d}{dt}\frac{1}{\dot{a}^2} = -2\frac{\ddot{a}}{\dot{a}^3}$$

>0 for expanding universe filled with dust or radiation (and k ≠ 0)
-> the universe becomes “less flat”
-> Ω=1 is an unstable fix-point



following the evolution back in time, we find that (during radiation domination, i.e. before t_{eq})

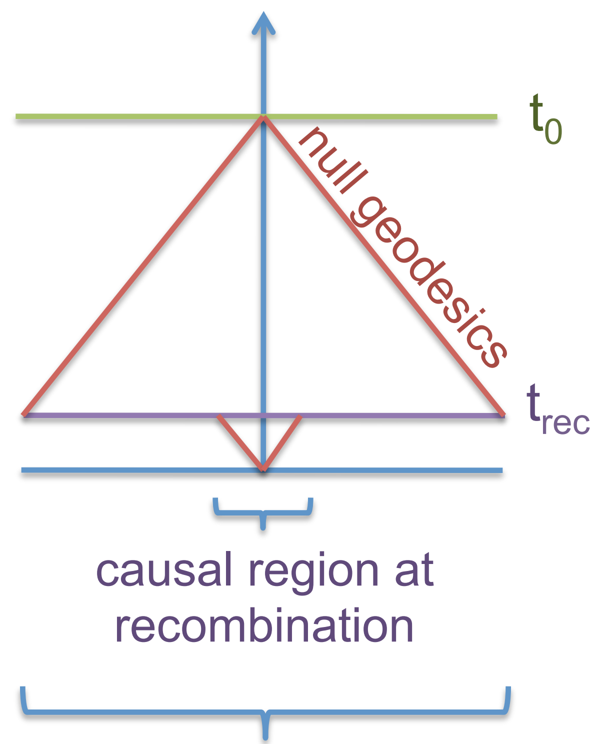
$$\boxed{|\Omega(t)-1| \approx 10^{-4}\left(\frac{1\text{eV}}{T}\right)^2}$$

BBN: T ≈ 1 MeV -> $|\Omega-1| < 10^{-16}$
Planck: T ≈ 10^{19} GeV -> $|\Omega-1| < 10^{-60}$

-> what fine-tuned the initial conditions?

why is the sky uniform?

- distance travelled by light: $r = \int \frac{dt}{a(t)}$ (= conformal time)
- distance to last scattering surface: $$r_0 = \int_{t_{\rm rec}}^{t_0} \frac{dt}{a(t)} \approx 3t_0$$
- distance travelled from big bang to recombination: $$r_c = \int_0^{t_{\rm rec}} \frac{dt}{a(t)}$$



in general $r_c << r_0$, unless $a(t) \sim t^a$ with $a \geq 1 \Leftrightarrow w \leq -1/3$!

since $\boxed{a(t) \propto t^{2/(3+3w)}}$

how to solve the problems

all the problems disappear if $\ddot{a} > 0$ for long enough!

Since $\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G_N}{3}(\rho + 3p)$ this needs $p < -\rho/3$

We have seen that for Λ : $p = -\rho$, but forever

-> we need a way to have evolving eq. of state

Solution: use a field … what kind of field? When in doubt, try a scalar field ☺

scalar fields in cosmology

GR + scalar field:

$$S = S_g + S_\phi = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right)$$

gravity e.o.m. (Einstein eq.):

$$\frac{\delta S[g_{\mu\nu},\phi]}{\delta g^{\mu\nu}} = 0 \qquad G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

entries in scalar field EM tensor (FLRW metric)

$$\begin{cases} \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{cases}$$

scalar field e.o.m. :

$$\frac{\delta S[g_{\mu\nu},\phi]}{\delta \phi} = 0 \qquad \ddot{\phi} + 3H\dot{\phi} + dV(\phi)/d\phi = 0$$

- this is the general method to compute Einstein eq., EM tensor and field e.o.m. from any action
- w=p/ρ for scalar fields can vary, as a function of V(ϕ)

the inflaton eq. of state

$$\left.\begin{aligned} \rho_\phi &= \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p_\phi &= \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{aligned}\right\}$$

$\dot{\phi}$ small -> p ≈ -ρ, w ≈ -1 (**slow roll**)

$\dot{\phi}$ large -> p ≈ ρ, w ≈ +1

=> slow roll is just what we need

slow-roll approximation: $3H\dot{\phi} = -V' \quad H^2 = \dfrac{1}{3m_P^2}V$

slow-roll parameters: $\epsilon(\phi) \equiv \dfrac{m_P^2}{2}\left(\dfrac{V'}{V}\right)^2 \; \eta(\phi) \equiv m_P^2 \dfrac{V''}{V}$

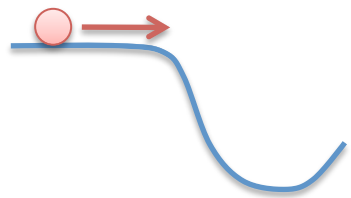
$\epsilon \ll 1\,, \quad |\eta| \ll 1$ for slow-roll -> flat pot.

SR approx => $\dot{\phi}^2 = \dfrac{2}{3}\epsilon V \Rightarrow p = \left(\dfrac{2}{3}\epsilon - 1\right)\rho \;\longrightarrow \ddot{a} > 0 \leftrightarrow \epsilon < 1$

(first order in ε)

prototypical inflation models

- small field



e.g. $V = V_0\,[1-(\phi/\mu)^{\alpha}]$, $\alpha = 2,4,\dots$
original inflation: 1^{st} order phase transition -> exit problem

- chaotic / large field

e.g. $V = m^2\phi^2$ or
$V \sim \phi^4$
also eternal inflation models

- hybrid / multifield

- curvaton, N-flation, cyclic models, …

-> large number of inflation scenarios
-> not all work // initial conditions generally problematic

the duration of inflation

"number of e-foldings": N ~ ln(a)

$$\frac{d}{dt}\left(\frac{\Omega-1}{\kappa}\right)=\frac{d}{dt}\frac{1}{\dot{a}^2}=-2\frac{\ddot{a}}{\dot{a}^3}$$ and SR: $$a(t)=\exp(Ht),\quad H=\sqrt{\frac{\Lambda}{3}}$$

⇒ $|\Omega-1| \sim 1/a^2$ during slow roll inflation

⇒ we need 20 (BBN) to 70 (Planck-scale) e-foldings to achieve necessary flatness (typically 40-60)

⇒ also sufficient to solve horizon problem and to dilute monopoles

(for horizon problem: need $N_{inf} \sim N_{post\text{-}inf}$, which also is between 30 e-foldings (BBN) and 60 e-foldings (GUT))

example

chaotic inflation: $V(\phi) = \frac{1}{2} m^2 \phi^2$

slow-roll equations: $3H\dot{\phi} + m^2\phi = 0 \qquad H^2 = \frac{m^2}{6m_P^2}\phi^2$

slow-roll parameters: $\epsilon = \eta = \frac{2m_P^2}{\phi^2}$ -> $|\phi_f| = \sqrt{2} m_P$

# of e-foldings: $N = \int_{a_i}^{a_f} \frac{da}{a} = \int_{t_i}^{t_f} H dt = \ldots = -\frac{1}{m_P^2} \int_{\phi_i}^{\phi_f} \frac{V}{V'} d\phi = \frac{\phi_i^2}{4m_P^2} - \frac{1}{2}$

solution of SR eqn's $\phi(t) = \phi_i - \sqrt{\frac{3}{2}} m m_P t$

$$a(t) = a_i \exp\left\{ \frac{m}{\sqrt{6} m_P} \left(\phi_i t - \frac{m m_P}{\sqrt{6}} t^2 \right) \right\}$$

reheating the universe

after many e-foldings of inflation, the universe is very empty and cold; but we want a radiation-dominated universe!?

-> **reheating**: convert energy in inflaton field to radiation!

- after end of inflation: inflaton oscillates at bottom of potential -> will decay into other particles if coupling non-zero. Usually modelled as dissipative term $\Gamma\dot{\phi}$

  "cold" inflation: $\Gamma < H$ during inflation, $\rho_\phi \to \rho_\gamma$ when $\Gamma \sim H$

  at that time: $\Gamma^2 \approx H^2 = \frac{1}{3m_P^2}\rho_\phi \Rightarrow \rho_\gamma \approx 3m_P^2\Gamma^2$, ~10^{10} GeV

  (warm inflation: $\Gamma \sim H$ always -> smooth transition)

- however: oscillating inflaton -> oscillating effective mass of coupled fields -> parametric resonance, "**pre-heating**"

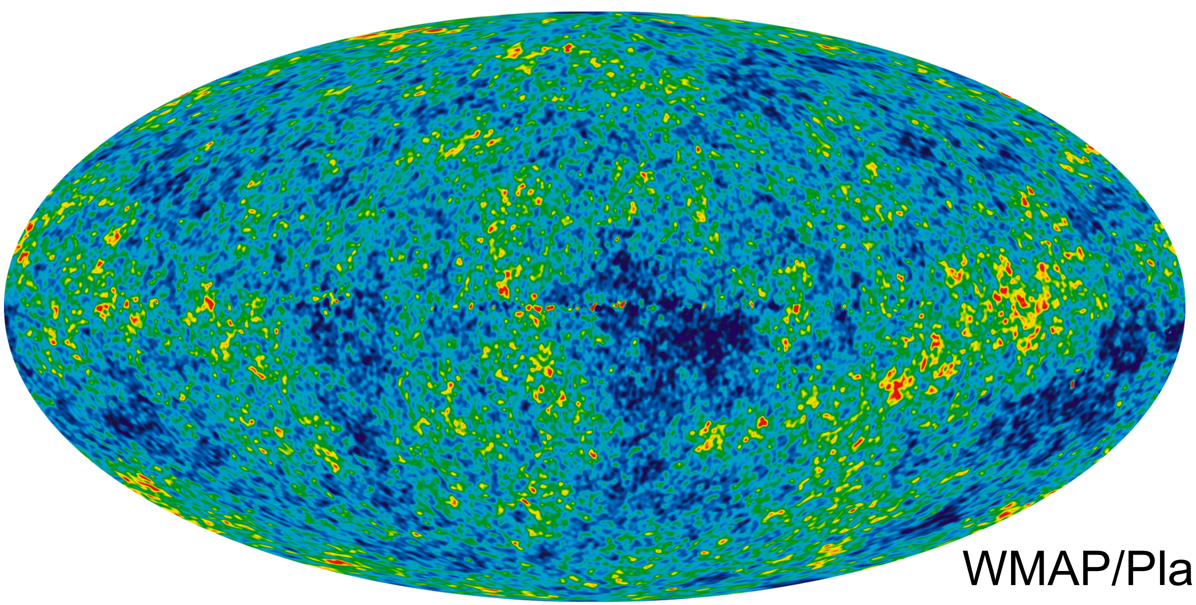
  e.g. coupling $-\frac{1}{2}g^2\chi^2\phi^2$ -> eom $\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\Phi^2\sin^2(m_\phi t)\right)\chi_k = 0$

(lots of nice particle physics to be found here! ☺)

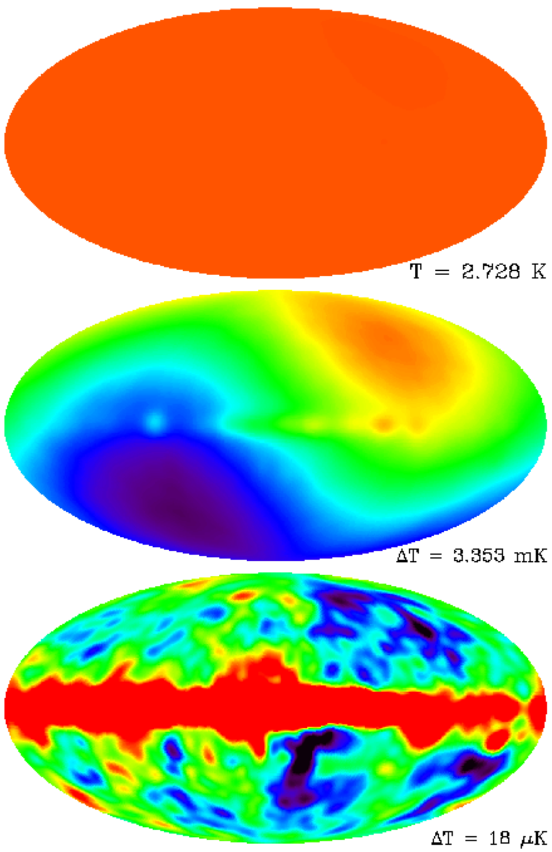
anisotropies in the CMB

Actually, there is another problem in standard cosmology:

- galaxies would not have formed yet from thermal fluctuations alone
- we see fluctuations at high redshift directly in the CMB [later]



WMAP/Planck



COBE

are we quantum fluctuations?

Inflation has another amazing property:

- SR inflation ~ de Sitter space-time -> horizon
- horizon -> Hawking radiation -> particle creation!

$\Rightarrow$ inflation should create perturbations! ☺

$\Rightarrow$ quantum fluctuations are stretched to huge scales and become classical curvature perturbations

$\Rightarrow$ ***the largest structures in the universe are due to quantum fluctuations!!!???***

- what kind of perturbations?
- can we see them?
- what do they tell us about inflation?

inflationary perturbations

- write $\phi(x,t) = \phi(t)+\delta\phi(x,t)$
- linearize eom for $\delta\phi$, $V'(\phi+\delta\phi) \to V'(\phi)+\delta\phi V''(\phi)$
- Fourier-expansion with creation & annihilation op's for $\delta\phi$

$$\ddot{w}(k,t) + 3H\dot{w}(k,t) + \left(\frac{k^2}{a^2} + 3\eta H^2\right) w(k,t) = 0$$

- Horizon: k/a=H: neglect $\eta \ll 1$ during SR (-> corrections)

$$w(k,t) = -H\frac{k\tau - i}{k} e^{-ik\tau}$$

- compute fluctuation spectrum

$$\int d^3x \langle 0|\delta\phi^2(x,t)|0\rangle \to \int \frac{d^3k}{(2\pi)^3 2k}|w(k,t)|^2 = \int \frac{dk}{k} \underbrace{\left(\frac{H}{2\pi}\right)^2}_{k^3 P_{\delta\phi}(k)\text{ : scale invariant spectrum!}} (1 + \overbrace{k^2\tau^2}^{\text{can be neglected}})$$

- phases of $\delta\phi_k$ random -> Gaussian fluctuations

cosmological perturbations

- inflaton will decay, but perturbations are frozen into metric
- can use Poisson eq. $\Delta\Phi = 4\pi G\,\delta\rho_\phi$ for grav. pot. Φ
- in terms of curvature perturbation R:

$$k^3 P_{\mathcal{R}}(k) = \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2 \propto \frac{V}{\epsilon} \quad \text{(eval. at k = aH)}$$

- power-law ansatz $k^3\,P \sim (k/k^*)^{n-1}$ -> n-1 = (d lnP)/(d lnk)

$$\boxed{n_s - 1 = -6\epsilon + 2\eta}$$

- **nearly scale invariant**, models make different predictions!

- There is another degree of freedom: **gravitational waves!**
- accelerated expansion will ***necessarily*** also create a gravitational wave background!

$$P_g(k) \propto \left(\frac{H}{2\pi}\right)^2 \qquad n_g = -2\epsilon \qquad r = \frac{T}{S} \approx 12.4\epsilon = -6.2 n_g$$

$V \sim \phi^2$ example continued

slow-roll parameters: $$\epsilon = \eta = \frac{2m_P^2}{\phi^2}$$

# of e-foldings: $$N \approx \frac{\phi_i^2}{4m_P^2}$$

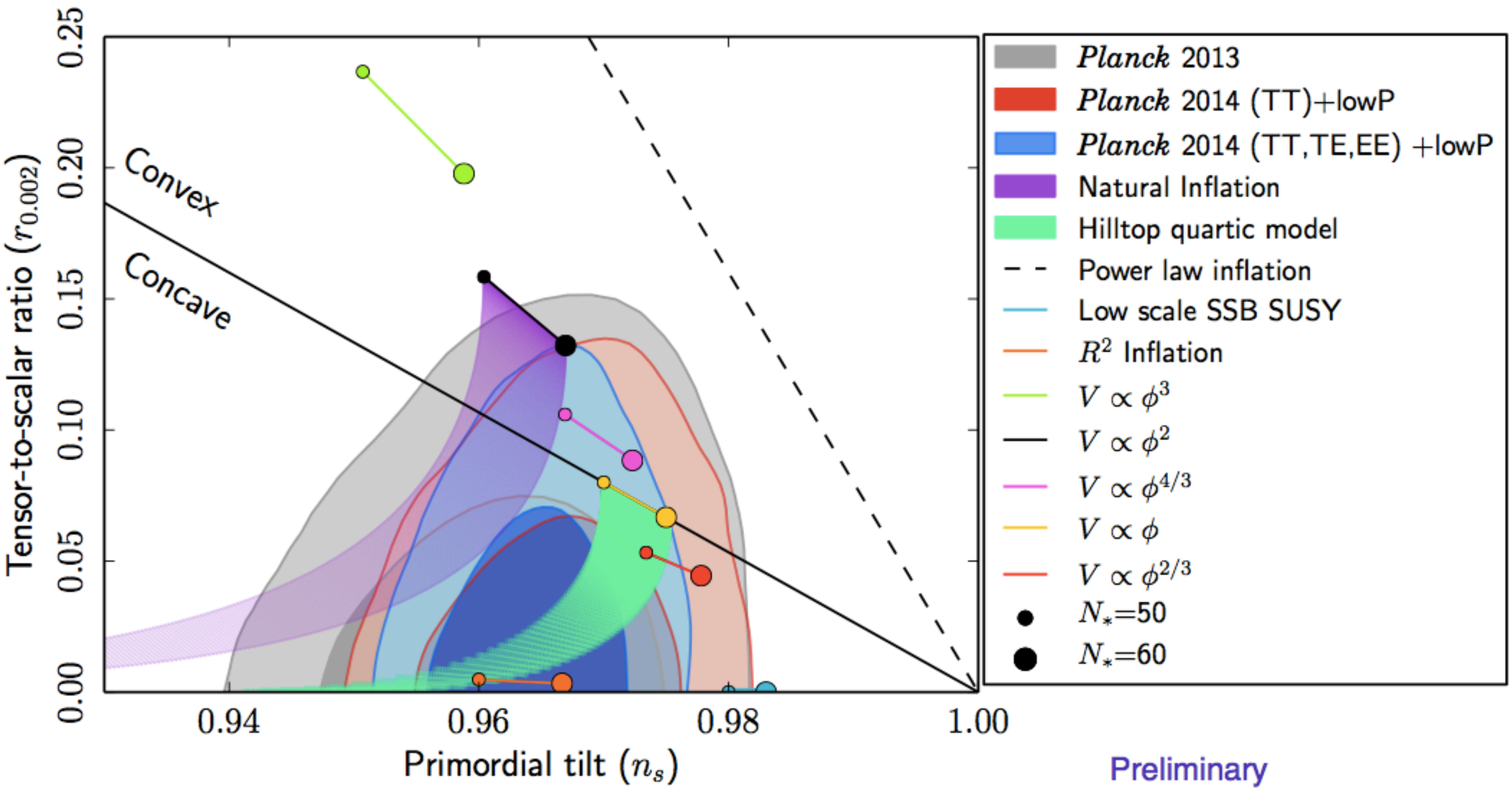
so n_s-1 = -6ε+2η = -4ε = $8m_P^2/\phi$ = -2/N ≈ -0.04

→ $n_s \approx 0.96$

r = T/S ≈ 12.4ε = -3.1 (n_s-1) -> potentially observable

constraints on inflation

As discussed in a bit, the fluctuations visible in the CMB are (believed to be and consistent with) a processed version of the initial fluctuations



Preliminary

generic predictions of inflation

- universe large and nearly flat

  -> **okay**
- nearly (but not quite) scale-invariant spectrum of adiabatic perturbations

  -> **okay** [killed defects]
- (nearly) Gaussian perturbations

  -> **okay** [deviations -> constrain models]
- perturbations on all scales, including super-horizon

  -> **okay** [kills all “causal” sources of perturb.]
- primordial gravitational waves **HOT TOPIC**

  -> ??? (“smoking gun” for acc. exp.)

beyond SR inflation

- **single-field slow roll inflation**: nearly scale invariant adiabatic Gaussian perturbations
- more general models: can create
  - **non-Gaussianity**
  - **isocurvature perturbations**
  - **features in the power spectrum**

  these features usually are correlated
- realistic (multi-field) models often form **cosmic strings** at the end of inflation
- if detected, such signatures would give important information on fundamental physics of inflation!
- Planck: no detection, strong limits

evolution of the perturbations

- From inflation we have a nearly scale invariant spectrum of perturbations…
  – how will they evolve?
  – what do we observe today?

-> **matter power spectrum / galaxy distribution**

– compute evolution of density perturbations of the dark matter and baryons

-> **CMB power spectrum**

– compute evolution of the perturbations in the radiation

k-space, power spectra

We tend to use ‘k’-space (Fourier space):

- only perturbations have spatial dependence, so that linear differential eqn’s -> ODE’s in time
- ‘scales’ instead of ‘location’

physical wavelength vs comoving wave number: $\lambda = \dfrac{2\pi a(t)}{k}$

Fluctuations are random

- need a statistical description -> power spectrum
- power spectra: $P(k) = \langle|\mathrm{perturbations}(k)|^2\rangle$
- <…> : average over realisations (theory) or over independent directions or volumes (observers)
- Gaussian fluctuations -> P(k) has full information

perturbation theory

basic method:

- set $g_{\mu\nu} = \bar{g}_{\mu\nu} + a^2 h_{\mu\nu}$ $\quad T_\mu^\nu = \bar{T}_\mu^\nu + \delta T_\mu^\nu$
- stick into Einstein and conservation equations
- linearize resulting equation (order 0 : "background evol.")

$\Rightarrow$ two 4x4 symmetric matrices -> 20 quantities

$\Rightarrow$ we have 4 extra reparametrization d.o.f. -> can eliminate some quantities ("gauge freedom")

$\Rightarrow$ at linear level, perturbations split into "scalars", "vectors" and "tensors", we will mostly consider scalar d.o.f.

$$\boxed{ds^2 = -(1+2\psi)dt^2 + a^2(1-2\phi)dx^2}$$

$\Rightarrow$ do it yourself as an exercise ☺

scalar perturbation equations

Einstein equations:
r.h.s. summed over "stuff" in universe (index i)

δ = δρ/ρ density contrast
V divergence of velocity field

$$k^2\phi = -4\pi G a^2 \sum_i \rho_i \left(\delta_i + 3Ha\frac{V_i}{k^2}\right)$$

$$k^2(\phi - \psi) = 12\pi G a^2 \sum_i (1+w_i)\rho_i \sigma_i$$

conservation equations:
one set for each type (matter, radiation, DE, …)

$$\delta_i' = 3(1+w_i)\phi' - \frac{V_i}{Ha^2} - \frac{3}{a}\left(\frac{\delta p_i}{\rho_i} - w_i\delta_i\right)$$

$$V_i' = -(1-3w_i)\frac{V_i}{a} + \frac{k^2}{Ha}\left(\frac{\delta p_i}{\rho_i} + (1+w_i)(\psi - \sigma_i)\right)$$

w, δp, σ: determines physical nature, e.g. cold dark matter: w=δp=σ=0

$$\delta_m' = 3\phi' - \frac{V_m}{Ha^2} \quad V_m' = -\frac{V_m}{a} + \frac{k^2}{Ha}\psi$$

perturbation evolution

We can (approximately) eliminate V and obtain a second order eqn for δ,

$$\ddot{\delta}_i = -\alpha_i H \dot{\delta}_i + \left(\mu_i H^2 - \frac{c_{s,i}^2 k^2}{a^2} \right) \delta_i$$

α_i, μ_i depend on w_i, c_s^2 is sound speed (<-> δp), 1/3 for radiation, 0 for matter

- α-term: **expansion damping**, may suppress growth
- last term: gravitational collapse vs pressure support

  -> will prevent growth if c_s k > Ha -> **sound horizon**

  -> with $H^2 = 8\pi G\rho/3$ we have the **Jeans length** $\lambda_J = cs/(\sqrt{G\rho})$
- straightforward to analyze behaviour of matter, radiation, etc as function of scale (horizon, Jeans-length) and of background evolution (radiation or matter dominated).

perturbation evolution

period	scale	CDM	radiation	baryons
$t < t_{eq}$	$k < aH$	grows $\sim a^2$	grows $\sim a^2$	grows $\sim a^2$
$t > t_{eq}$	$k < aH$	grows $\sim a$	grows $\sim a$	grows $\sim a$
$t < t_{eq}$	$k > aH$	~ constant (ln a)	oscillates	oscillates
$t_{eq} < t < t_{dec}$	$k > aH$	grows $\sim a$	oscillates	oscillates
$t_{dec} < t$	$k > aH$	grows $\sim a$	free-streams	grows $\sim a$

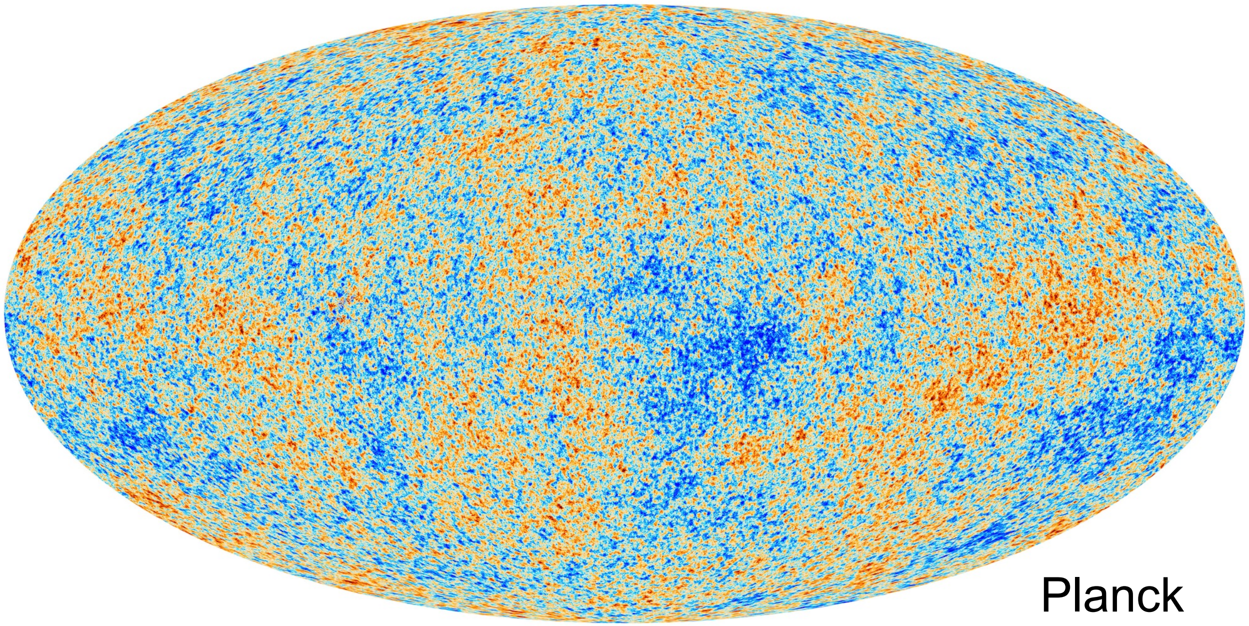
CDM: inside horizon grows only after matter-radiation equality -> scale imprinted in power spectrum where power-law will change!

radiation: oscillates, then free-streams after decoupling -> oscillations remain imprinted in power spectrum -> acoustic oscillations in CMB!

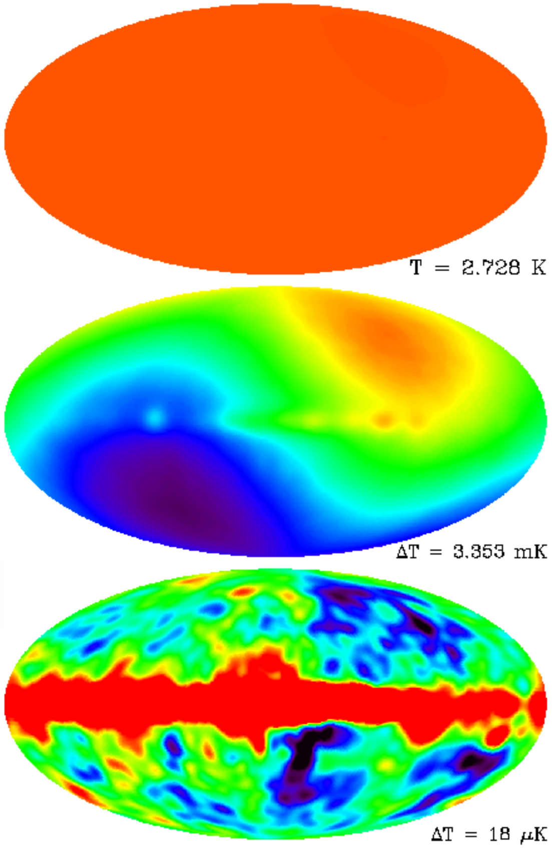
baryons: oscillate with photons until decoupling, then fall into CDM potential wells -> small imprint of acoustic oscillations also in matter power spectrum -> BAO

I. anisotropies in the CMB

You have often seen this picture
- what does it show?
- why?
- what does it tell us about the universe?



Planck



COBE

origin of the CMB

T > 3000 K :

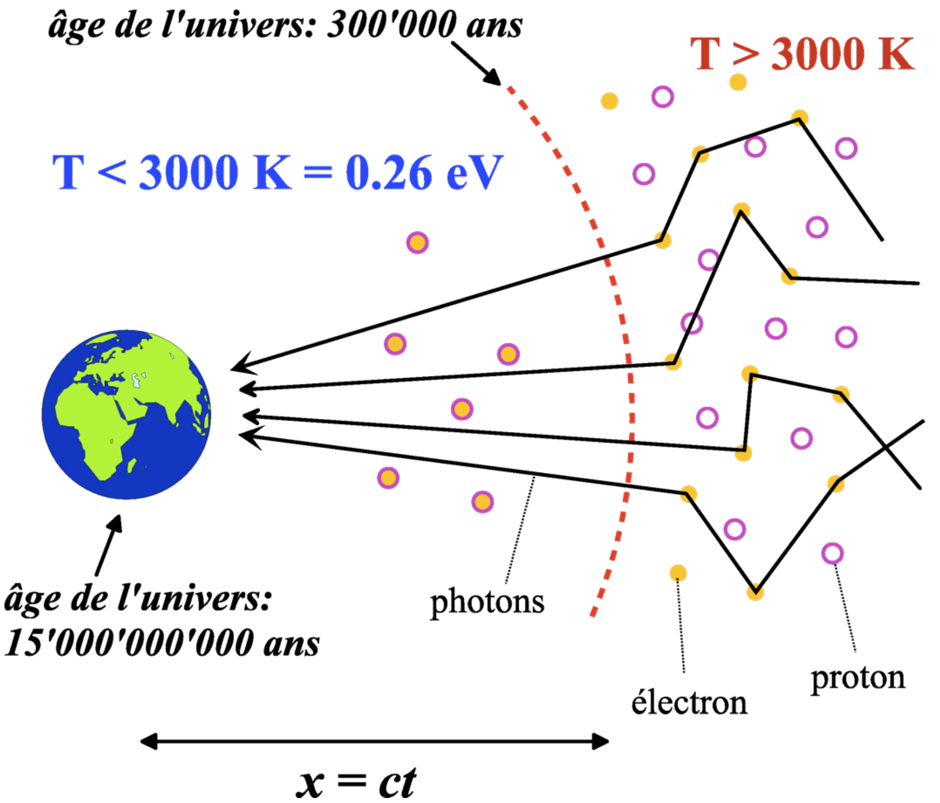
Electrons and protons are free.
Light interacts strongly with the electron (baryon-photon plasma), strong scattering as in fog.

T < 3000 K :

Electrons and protons (re-)combine to neutral atoms.
The universe becomes transparent for light, which free-streams to us.

We observe:

- 'photo' of last scattering surface
- stuff that happens on the way



statistical description

Temperature T(n) on the sky: Gaussian random field

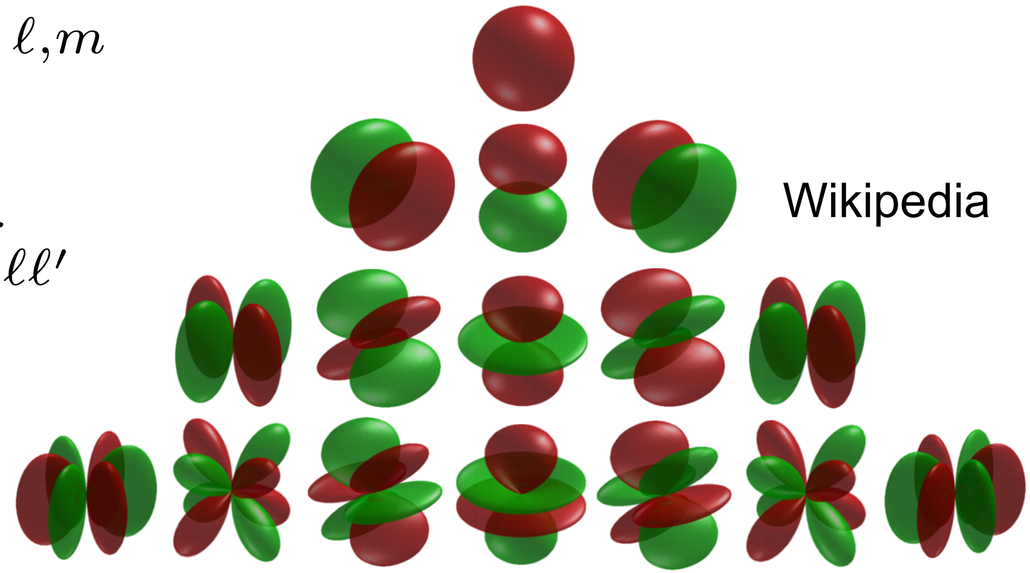
Fourier-analysis on sky sphere: instead of e^{ikt} the basis functions are spherical harmonics $Y_{lm}(n)$

$$\delta T(n) = T(n) - T_0 = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(n)$$

statistical isotropy:

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = C_\ell \delta_{mm'} \delta_{\ell\ell'}$$

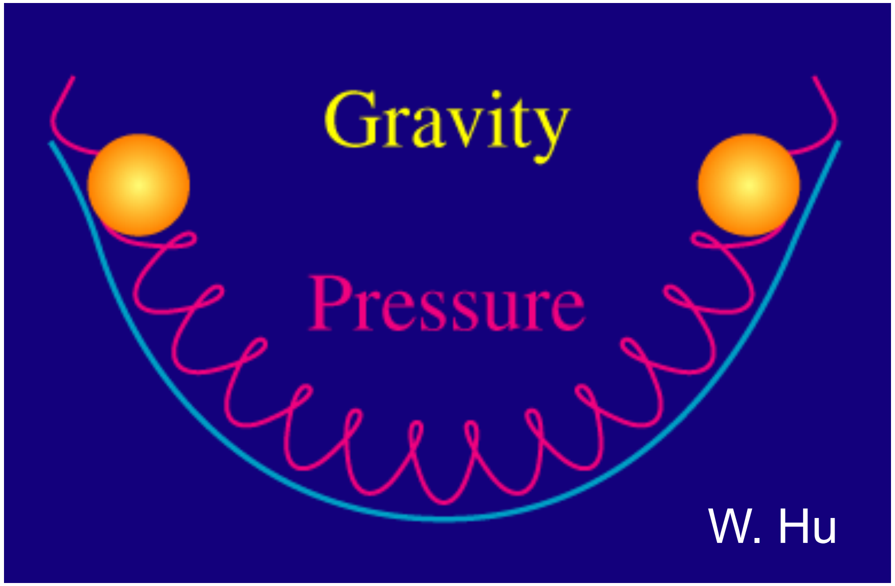
power-spectrum
~ δT^2



Wikipedia

perturbation evolution

The overdensities in the baryon-photon fluid collapse under the influence of gravity, until the pressure is strong enough to resist. Then the plasma starts to oscillate, until recombination.



We therefore see (mostly) the oscillation pattern at t_{rec}!

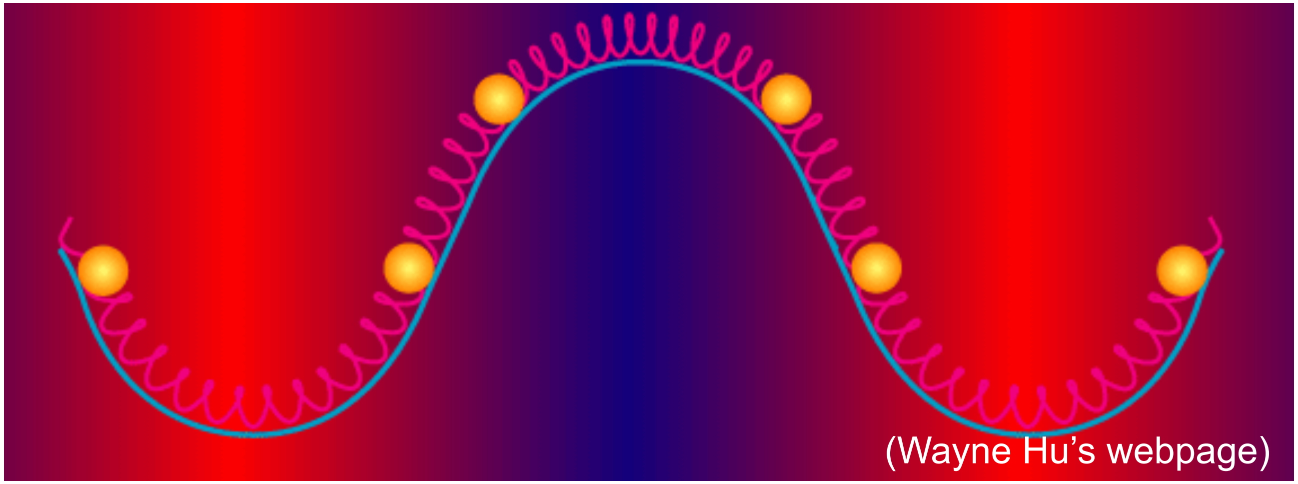
The largest scale that had just time to collapse will create the first peak, the scale that collapsed and re-expanded the second peak, etc.

-> angular diameter distance to z=1100!

density and temperature

Why do we see the density fluctuations as temperature variations?

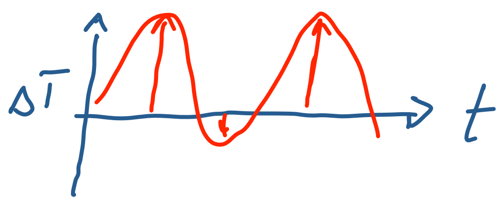
Stefan-Boltzmann: $\rho_\gamma \sim \sigma T^4$ -> $\delta_\gamma = \dfrac{\delta\rho_\gamma}{\rho_\gamma} \approx 4\dfrac{\delta T}{T}$

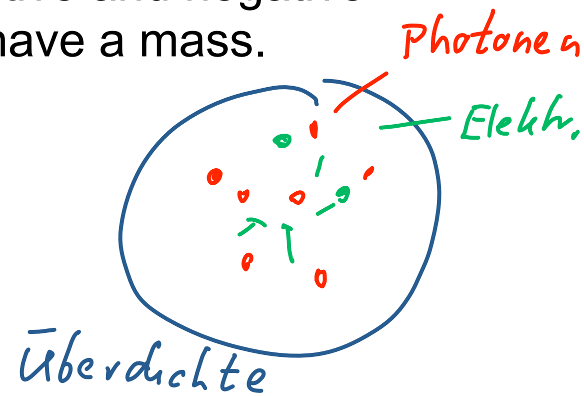


In addition, line-of-sight motion of the "last-scattering" electrons leads to red-/blue shifts $\sim V_b$, out of phase with δ_γ

peak height

A pure radiation „fluid“ would oscillate with equal positive and negative amplitude. But the electrons that are dragged along have a mass.
-> stronger compression (peaks # 1,3,…)
-> reduced rarification (peaks # 2,4,…).





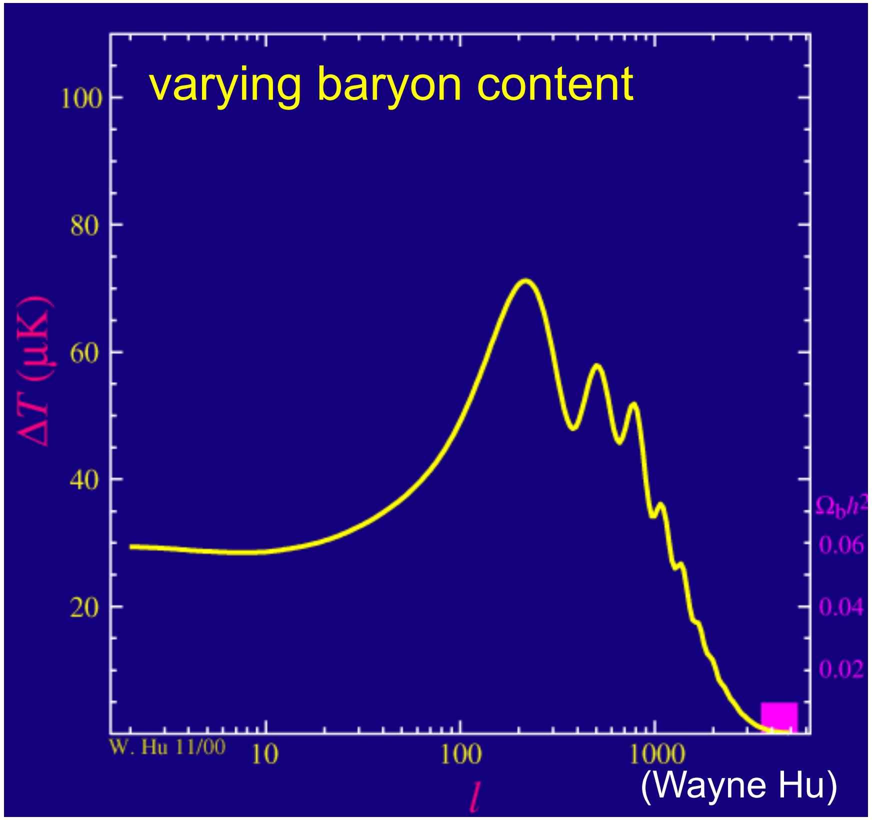
The relative height of the first two peaks thus measures the amount of baryons!

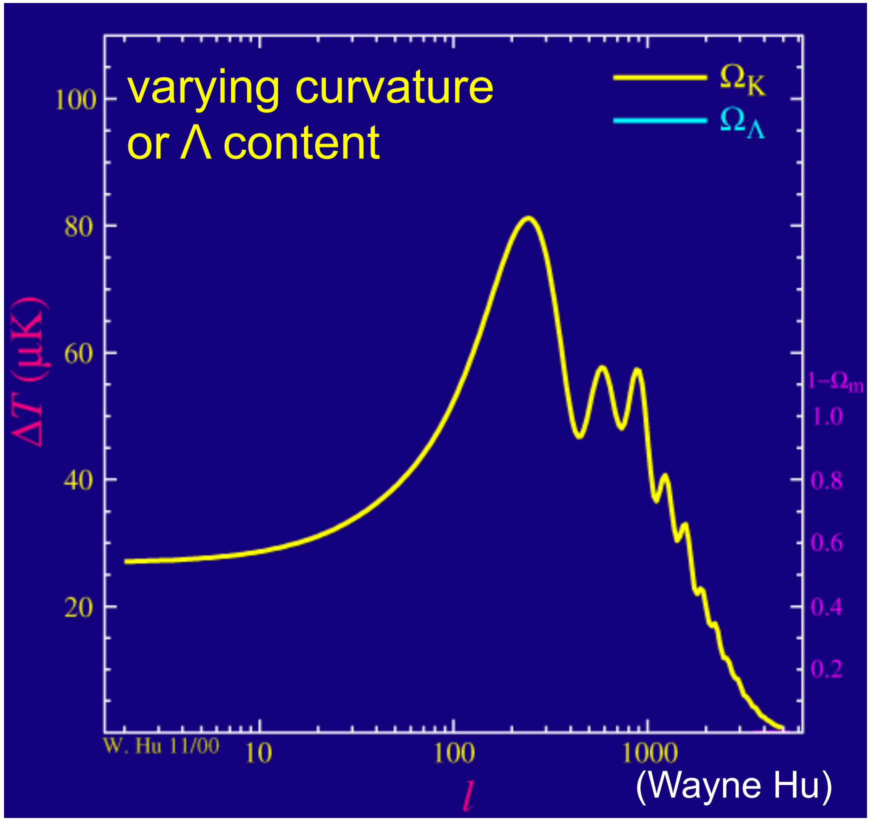
Dark matter doesn‘t feel the radiation pressure and undergoes gravitational collapse. The radiation feels the DM potential wells, which changes the amplitude of the maxima overall.

measuring cosmological parameters

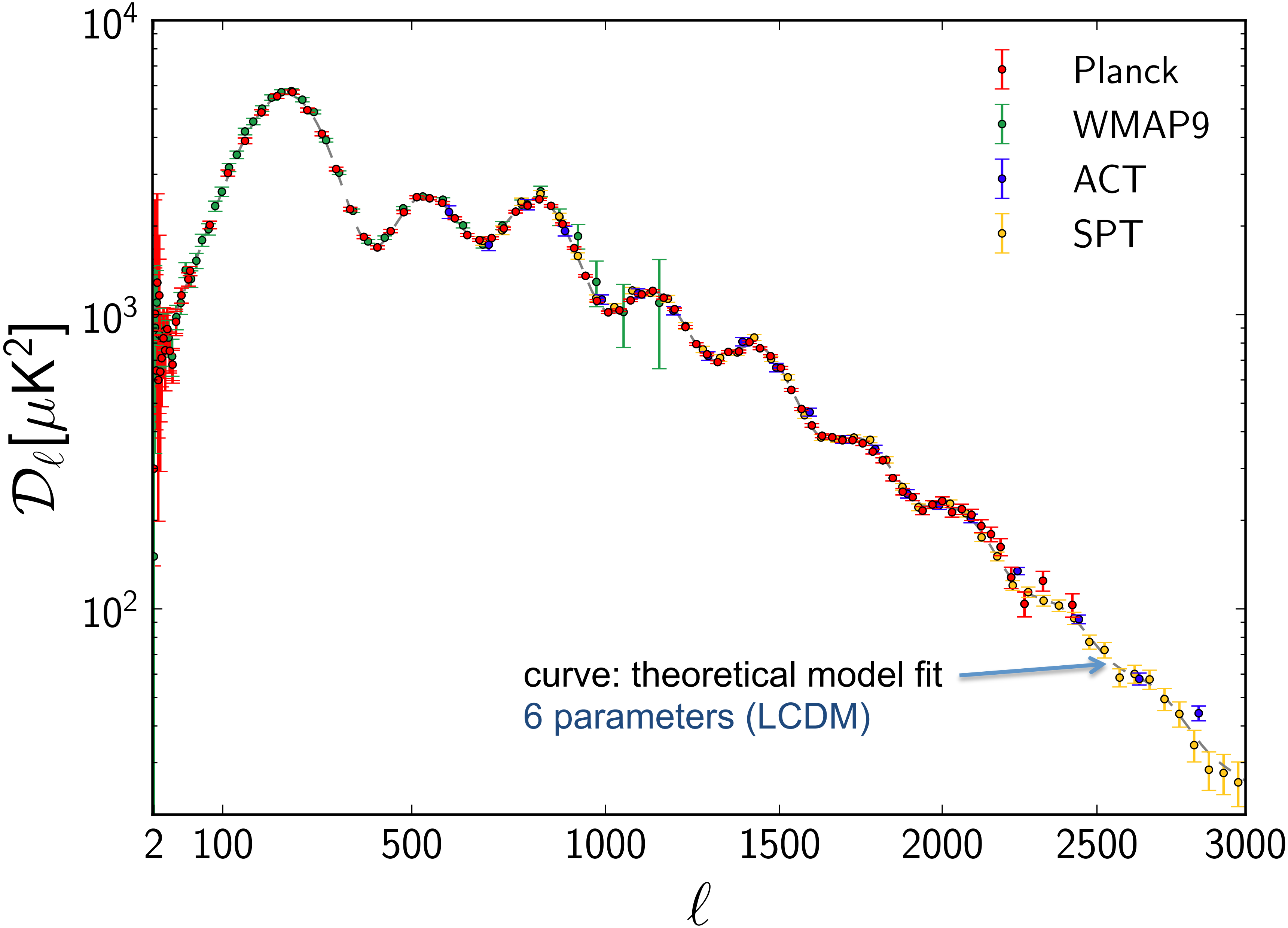
The CMB fluctuations depend on the values of the parameters
→ we just vary all of them to find the best values
(there are public codes for this, e.g. CAMB and CLASS)

CMB physics is mostly linear -> very clean probe!





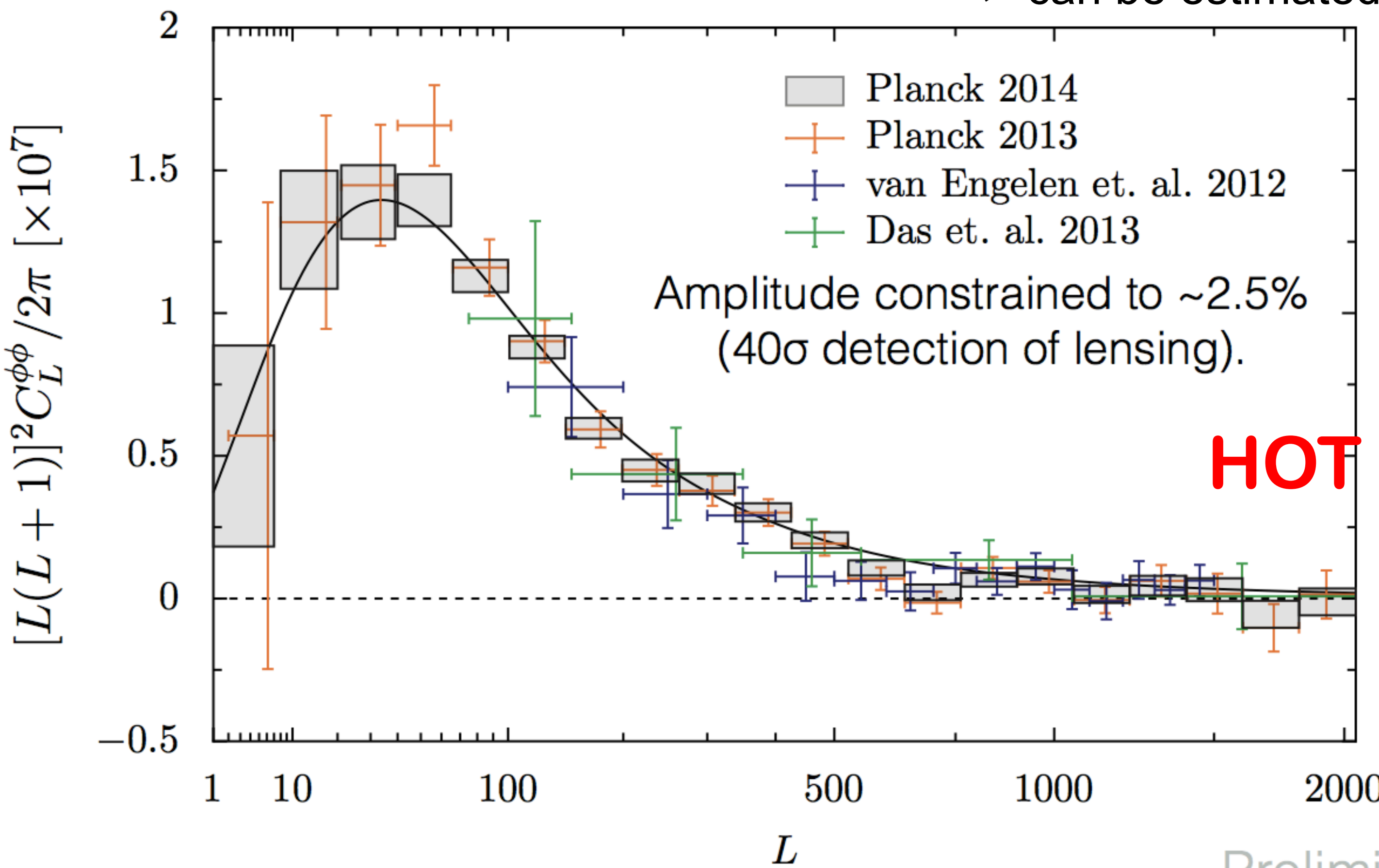
the CMB in 2013



gravitational lensing of CMB

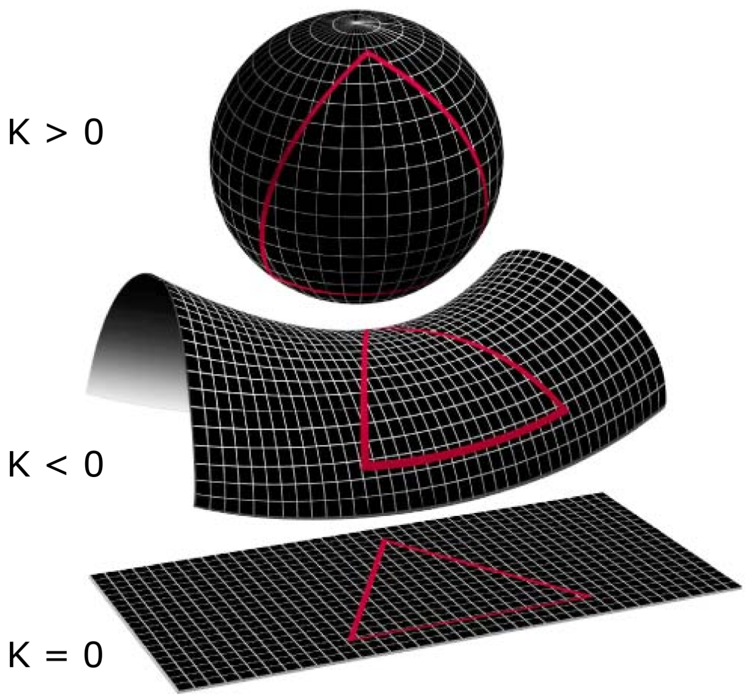
Light is deflected by gravitational perturbations along photon path.

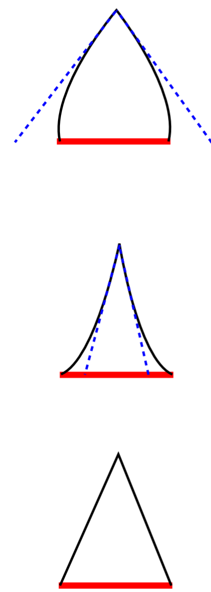
Also true for CMB
-> shifts power around in C_l
-> introduces non-Gaussianity
-> changes polarisation
$\Rightarrow$ can be estimated!



HOT TOPIC

CMB and curvature

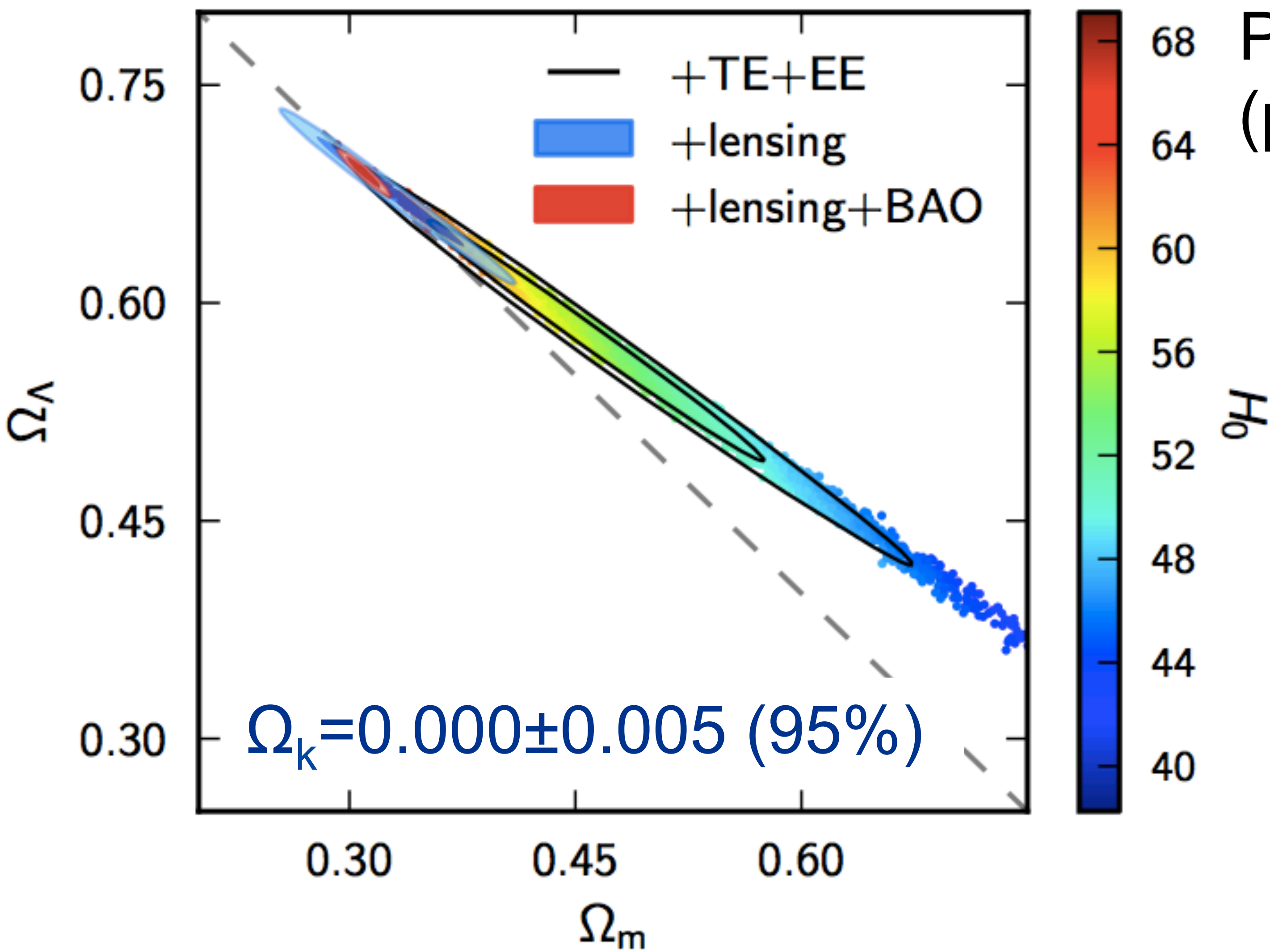




The Planck satellite provides ~ 0.03% measurement of the angular scale of the first peak!

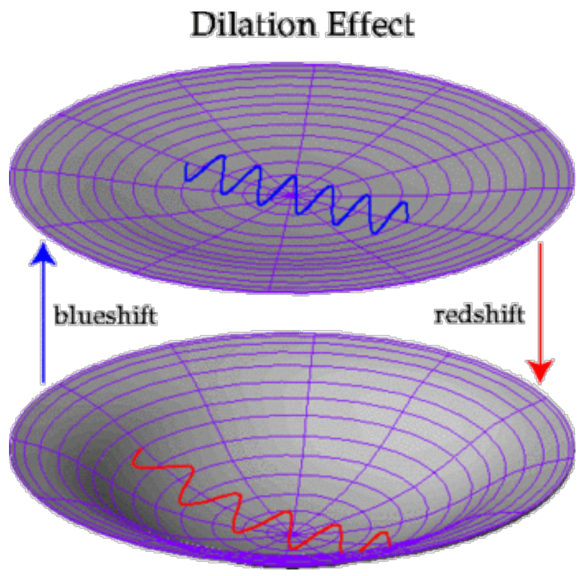
-> measurement of the geometry of the universe

how flat is the world?



Planck 2015 (preliminary)

(integrated) Sachs-Wolfe eff.



Impact of gravitational potential on CMB:

$$\frac{\delta T}{T} \sim (\Phi - \Psi)|_{\rm dec} + \int_{t_{\rm dec}}^{t_0} \left(\dot{\Phi} - \dot{\Psi}\right) dt$$

First term: SW -> ~ constant contribution

Second term: ISW -> depends on evolution of the gravitational potential along photon path!

Poisson eq. in matter dom. $\nabla^2 \Phi = 4\pi G a^2 \rho_m \delta_m$, $\rho_m \sim a^{-3}$, $\delta_m \sim a$

No ISW effect in a pure matter dominated universe.
But when dark energy begins accelerating the expansion: Φ, Ψ decay
-> ISW provides direct test of accelerated expansion
-> cosmic variance: large uncertainties … about 3σ when correlating with large scale structure

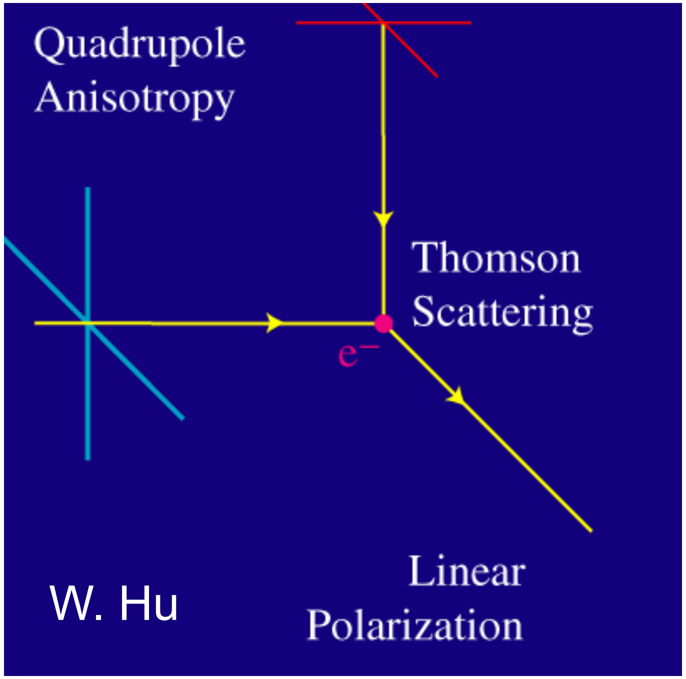
polarization

Scattering of light depends on polarisation angle -> last scattering polarizes light depending on local quadrupole.

-> also reionization probe (scattering again)

Scalar (density) perturbations do not lead to vorticity in polarization pattern ("B-modes")

BUT gravitational waves (tensor perturbations) do! (as does lensing)

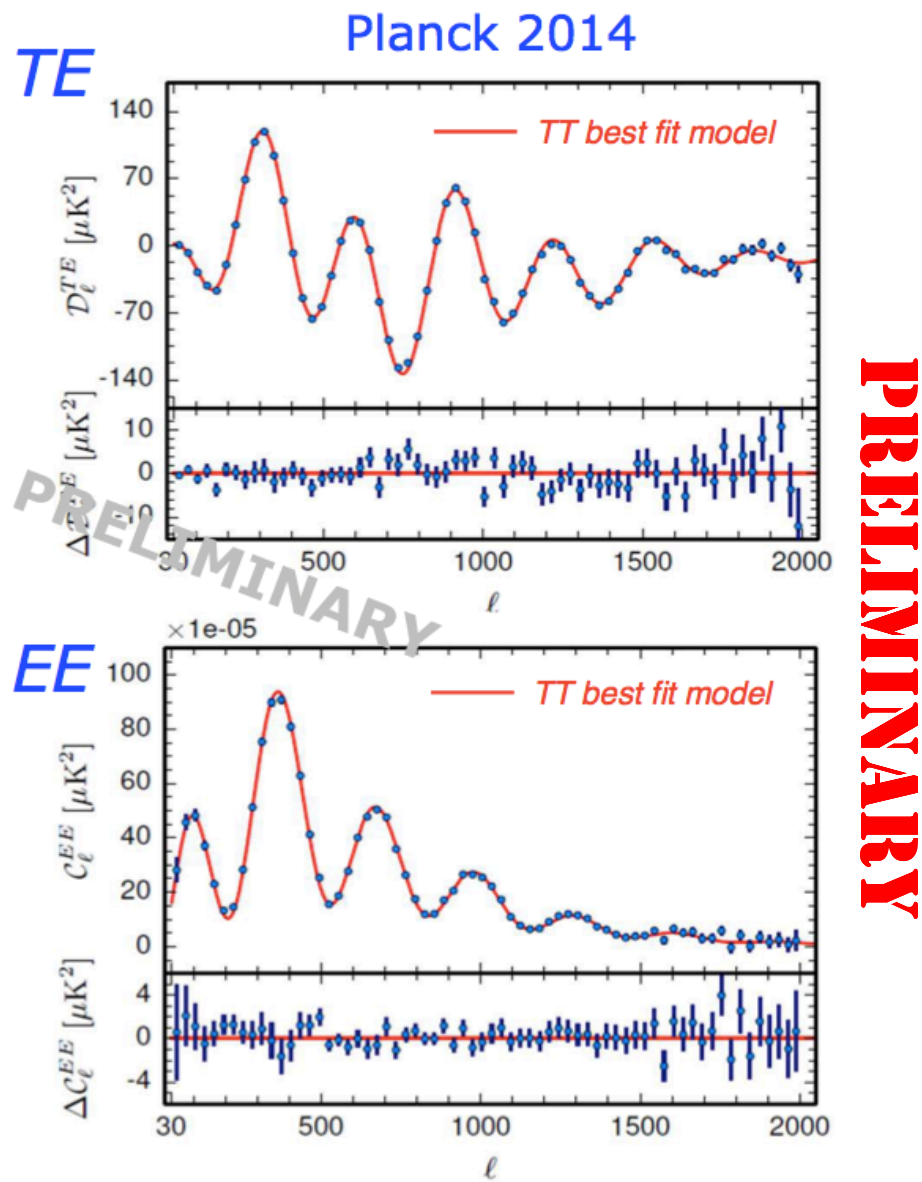


"B-mode" polarization is a probe of exotic (exciting) physics!

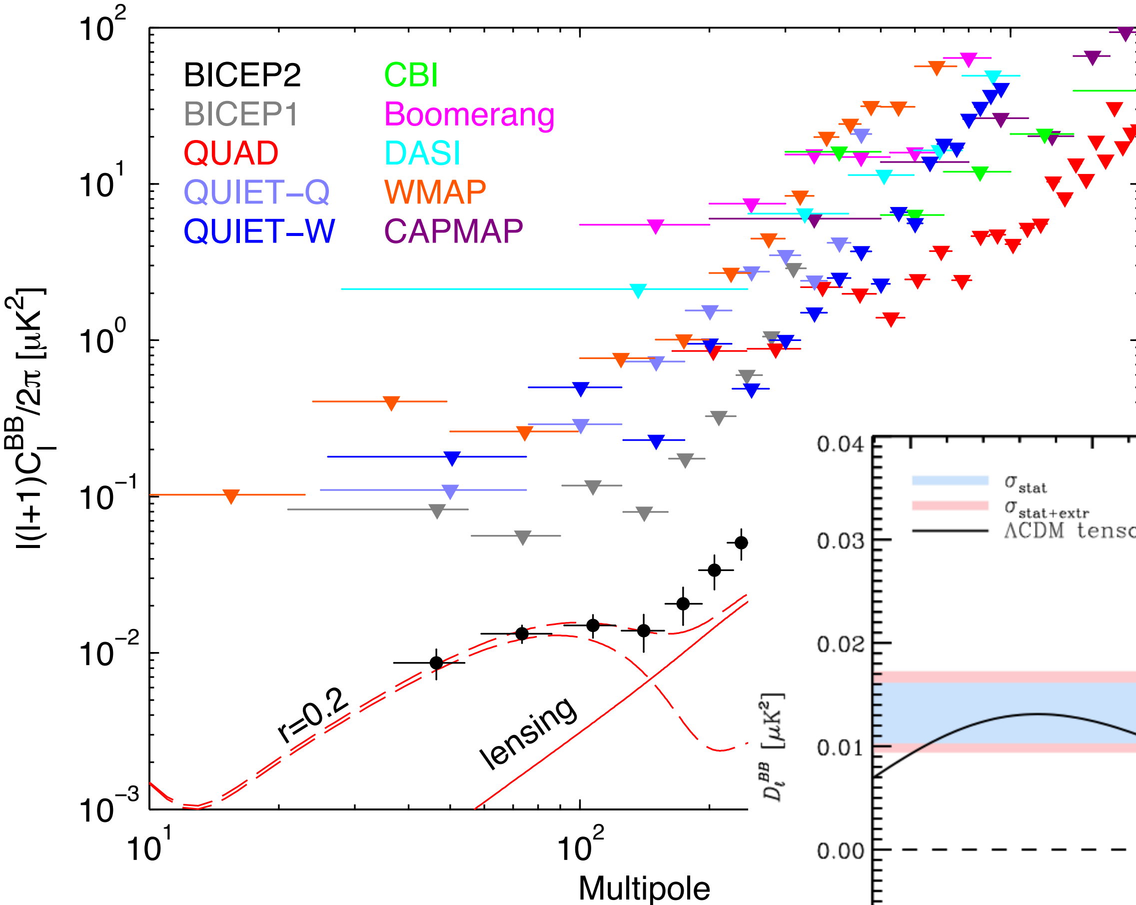
HOT TOPIC

2014 polar power spectrum

- polarisation decomposed into
  - E: gradient type
  - B: vector / rotation type
- for density / scalar perturbations alone, TT predicts TE and EE (and no B-type polarisation)
- CMB lensing and other constituents (e.g. grav. waves) create B-type polarisation
- so do 'foregrounds'

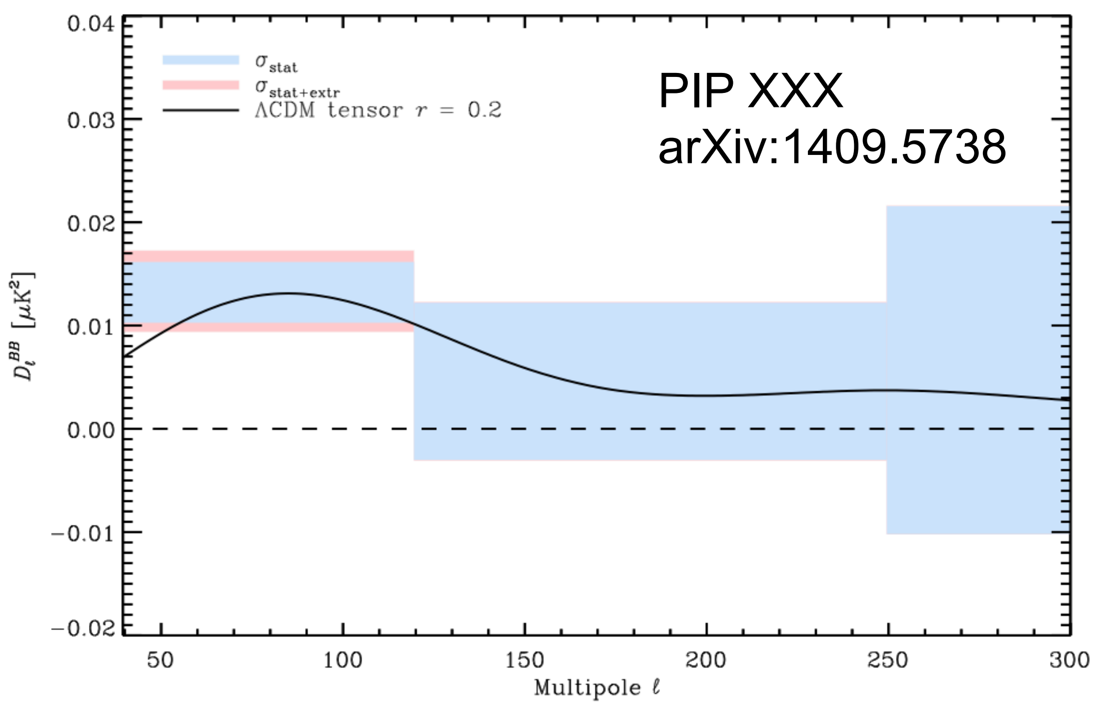


B-modes & BICEP2



BICEP2 B-modes, arXiv:1403.3985

Primordial tensor waves are expected in all 'inflation-like' models (all



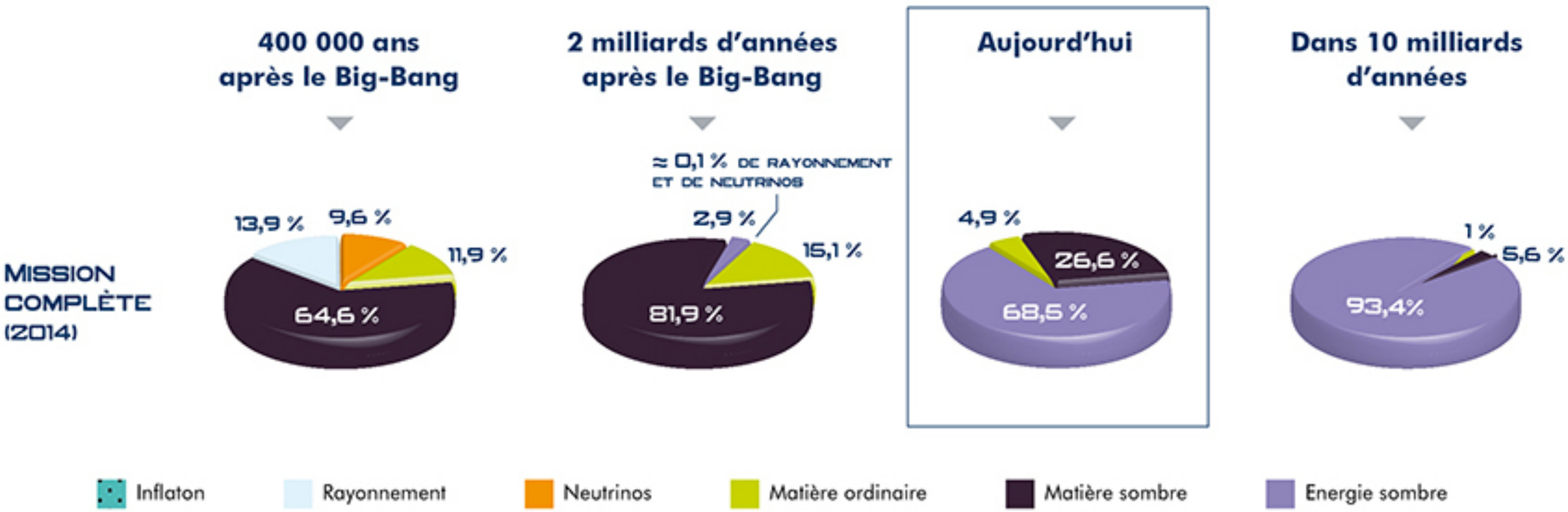
- determines energy scale of
- large field excursion, $\Delta\varphi/M_P$
- could test consistency relation $r = -8n_T$

CMB summary

- CMB: left-over radiation from initial hot state, “photo of the big-bang”
- Basically we are seeing sound-waves from … what? Inflation?
- Key cosmological observable due to theoretical cleanness, measures many parameters directly
- Even more when combined with other observations (or things like lensing, SZ, ...)
- Large-scale polarisation pretty much rules out any “causal” late-time source of perturbations!
- Lots of exciting stuff: Polarisation (grav. waves), non-Gaussianity (origin of perturbations), …

HOT TOPIC

“precision cosmology”



n_s	0.9652±0.0062	0.9639±0.0047	5.6σ / 7.7σ
H_0	67.3±1.0	67.6 ± 0.6 (+BAO)	
Ω_m	0.316±0.014	0.316±0.009	
σ_8	0.830±0.015	0.831±0.013	
z_{re}	9.9±1.9	10.7±1.7	

age: 13.813 ± 0.026 Gyr (Planck TT,TE,EE+lowP)

perturbation evolution

period	scale	CDM	radiation	baryons
$t < t_{eq}$	$k < aH$	grows $\sim a^2$	grows $\sim a^2$	grows $\sim a^2$
$t > t_{eq}$	$k < aH$	grows $\sim a$	grows $\sim a$	grows $\sim a$
$t < t_{eq}$	$k > aH$	$\sim$ constant ($\ln a$)	oscillates	oscillates
$t_{eq} < t < t_{dec}$	$k > aH$	grows $\sim a$	oscillates	oscillates
$t_{dec} < t$	$k > aH$	grows $\sim a$	free-streams	grows $\sim a$

CDM: inside horizon grows only after matter-radiation equality -> scale imprinted in power spectrum where power-law will change!

radiation: oscillates, then free-streams after decoupling -> oscillations remain imprinted in power spectrum -> acoustic oscillations in CMB!

baryons: oscillate with photons until decoupling, then fall into CDM potential wells -> small imprint of acoustic oscillations also in matter power spectrum -> BAO

matter power spectrum P(k)

scales entering before t_{eq}: $\lambda<\lambda_{eq}$
growth delayed until a_{eq}

$$\delta_\lambda(t) \simeq \delta_\lambda(t_{enter})(a/a_{eq})$$

scales entering after t_{eq}: $\lambda>\lambda_{eq}$

$$\delta_\lambda(t) = \delta_\lambda(t_{enter})(a/a_{enter})$$

$$= \delta_\lambda(t_{enter})(a/a_{eq})(a_{eq}/a_{enter})$$

horizon: $t_{enter} = \lambda\, a_{enter} \sim \lambda t_{enter}^{2/3}$

$$\rightarrow (a_{eq}/a_{enter}) = (\lambda_{eq}/\lambda)^2$$

in terms of k:

$P(k,t_{enter})$: $k^3 P(k) \sim k^{n-1} \sim$ const.

scales entering before t_{eq}:

$$|\delta_k(t)|^2 \propto k^{n-4}(a/a_{eq})^2$$

scales entering after t_{eq}:

$$|\delta_k(t)|^2 \propto k^{n}(a/a_{eq})^2$$

(& growth rate, redshift-space distortions, non-linear growth)

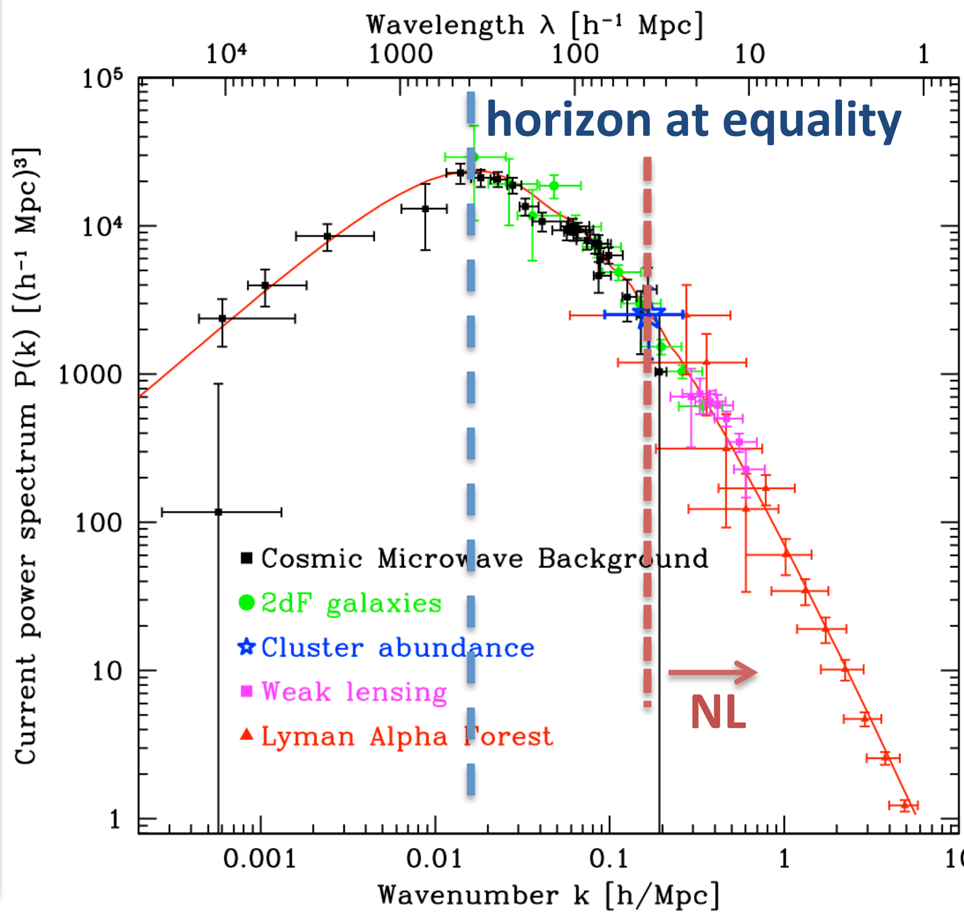
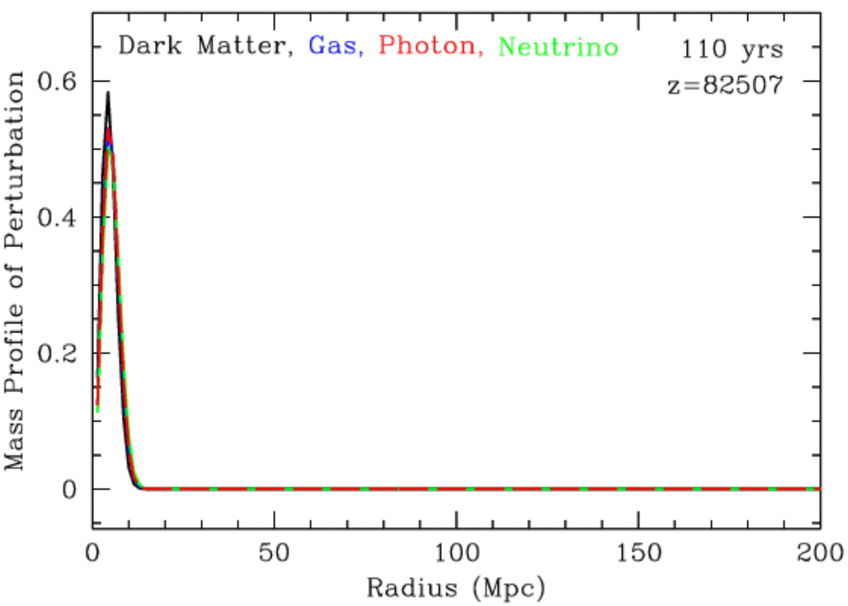


figure from Tegmark & Zaldarriaga

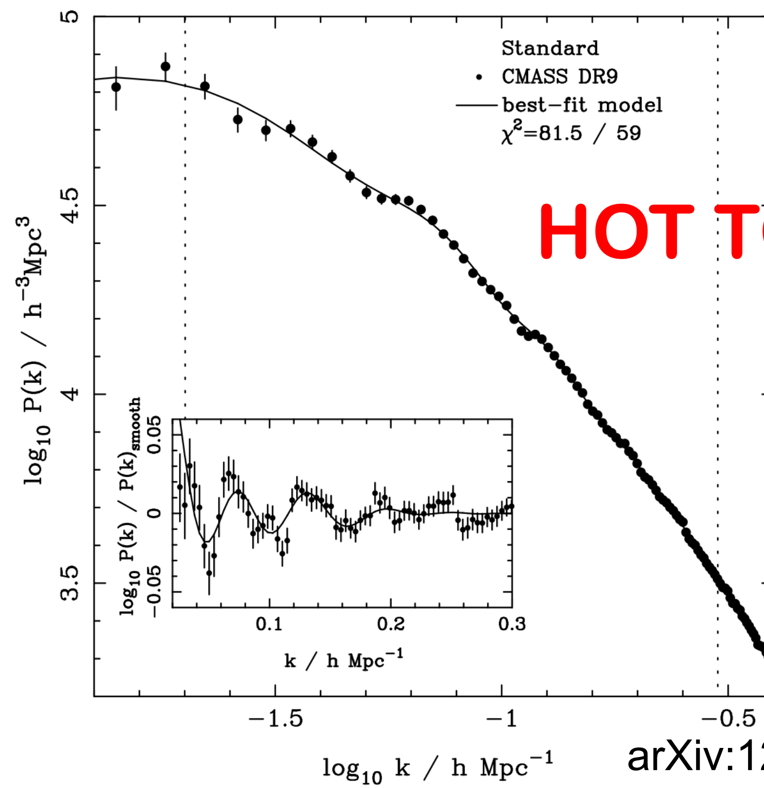
BAO

- On sub-horizon scales, the baryon-photon fluid oscillates until t_{dec}
- After t_{dec}, the photons free-stream away, and the baryons fall into the potential wells of the cold dark matter
- But the CDM also falls a bit into the baryon potential wells
- This imprints the oscillations also into the matter power spectrum

-> Baryonic Acoustic Oscillations feature -> **standard ruler!**



animation by Eisenstein, uses cmbfast

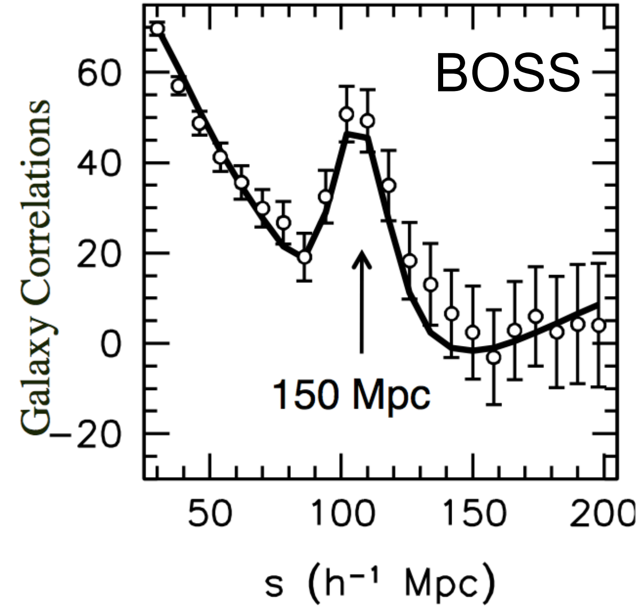


HOT TOPIC

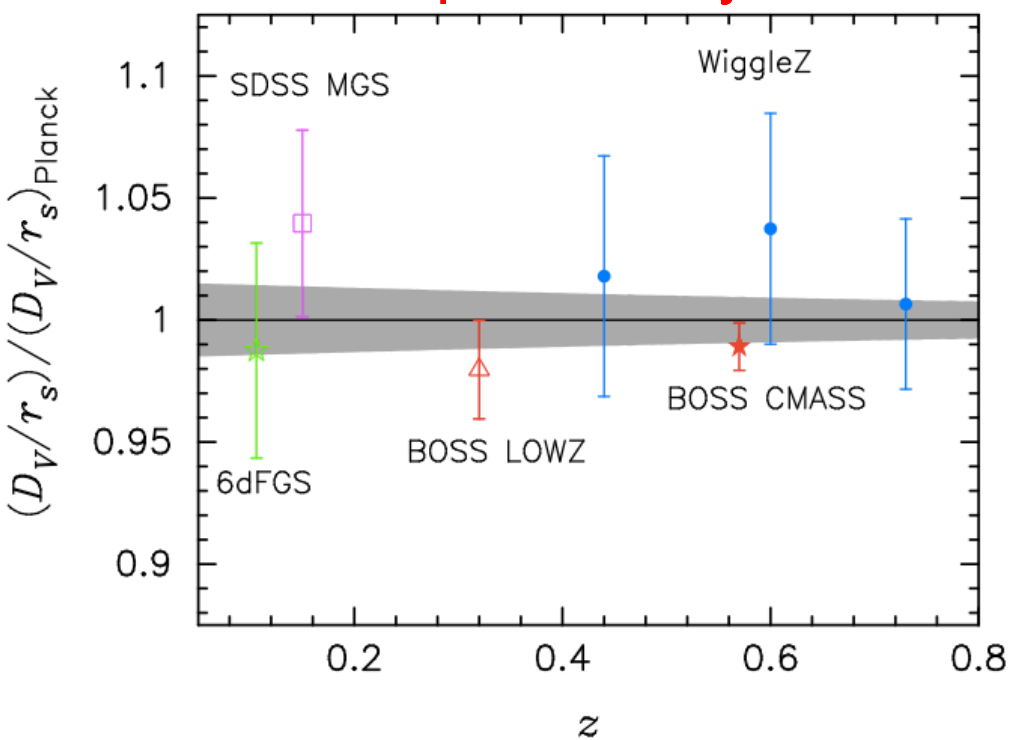
arXiv:1203.6594

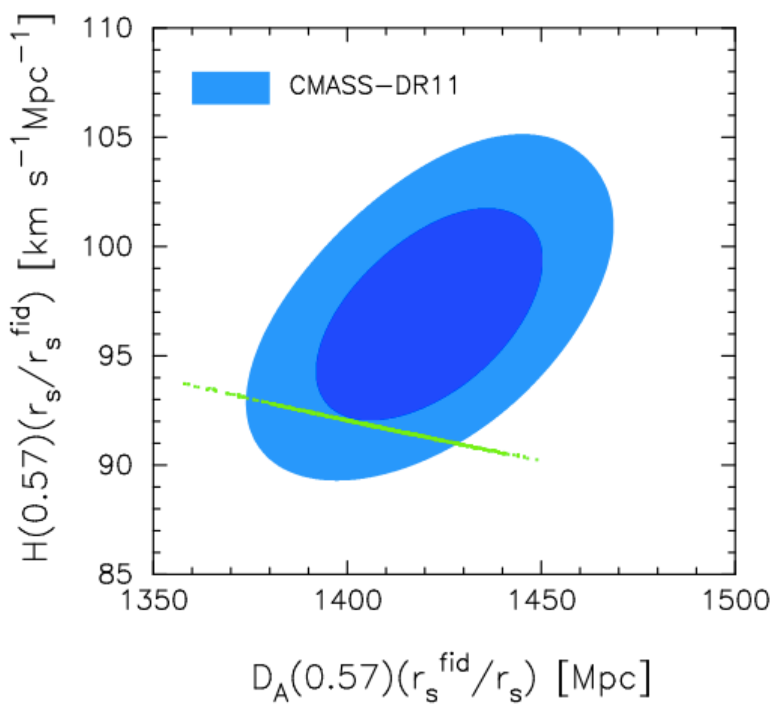
BAO distances

a standard ruler of ~150 comoving Mpc gives us an angular diameter distance (linked to same scale as CMB peak position!)



Planck 2015 preliminary

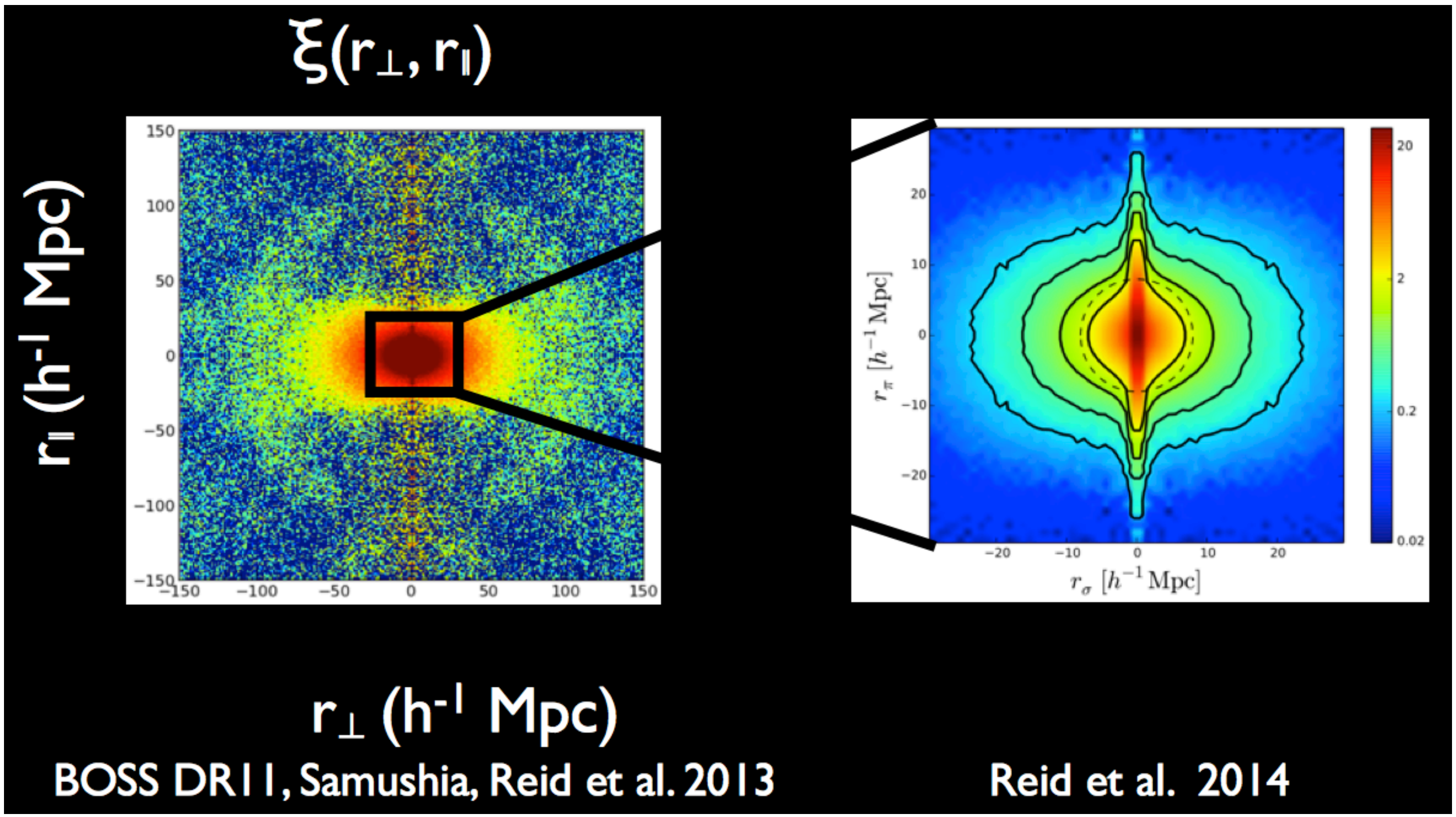




redshift space distortions

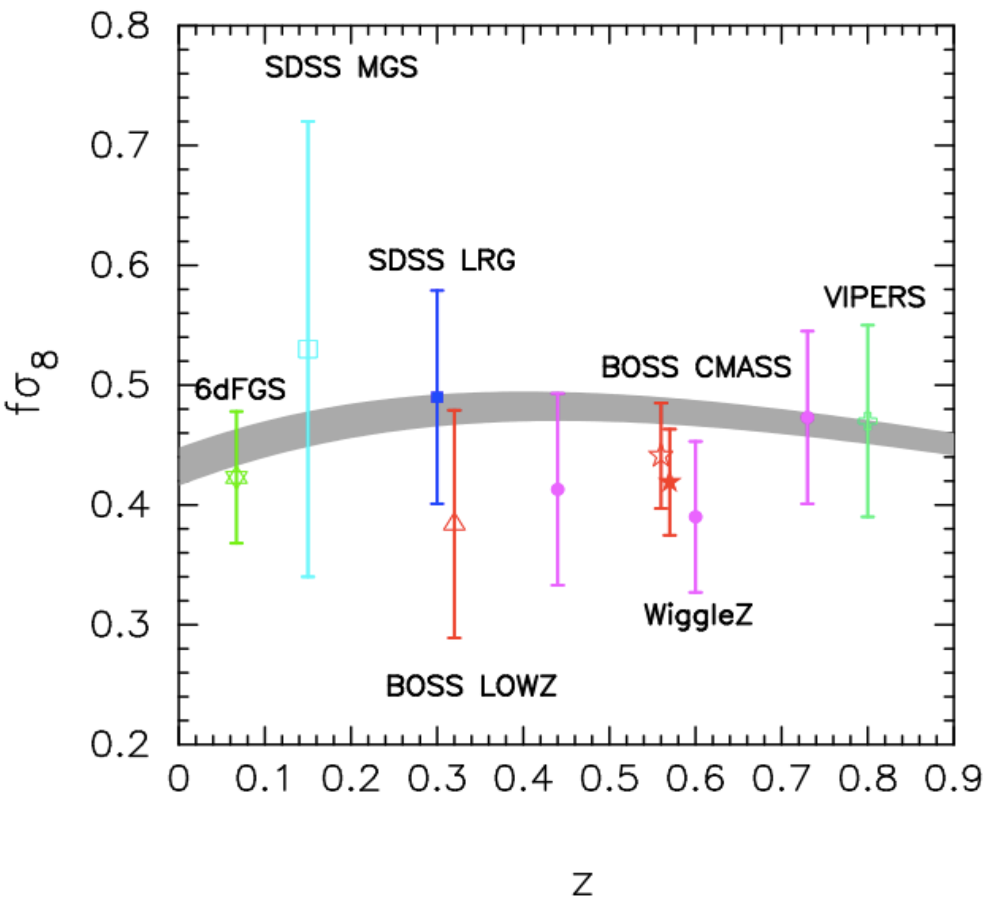
We observe galaxies in redshift space, not real space

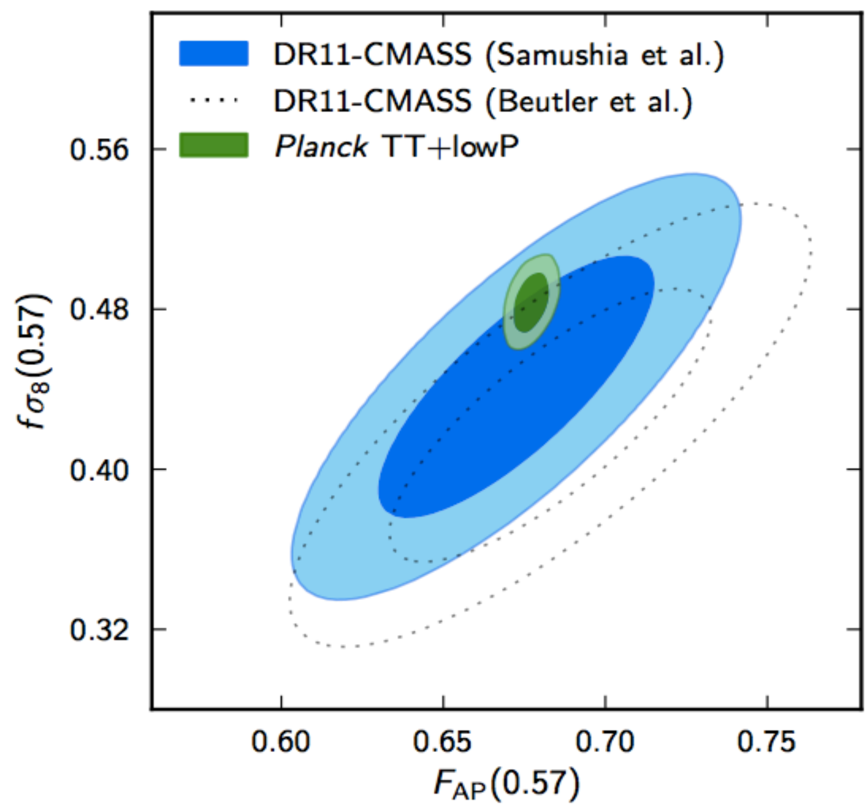
- large scales: coherent infall → squashing
- small scales random motion → elongation (`finger of god')



redshift space distortions

- particle conservation: velocities → growth
  → RSD measure combination $f\sigma_8$, $f = d\ln D/d\ln a$
- particle acceleration ~ grad Ψ

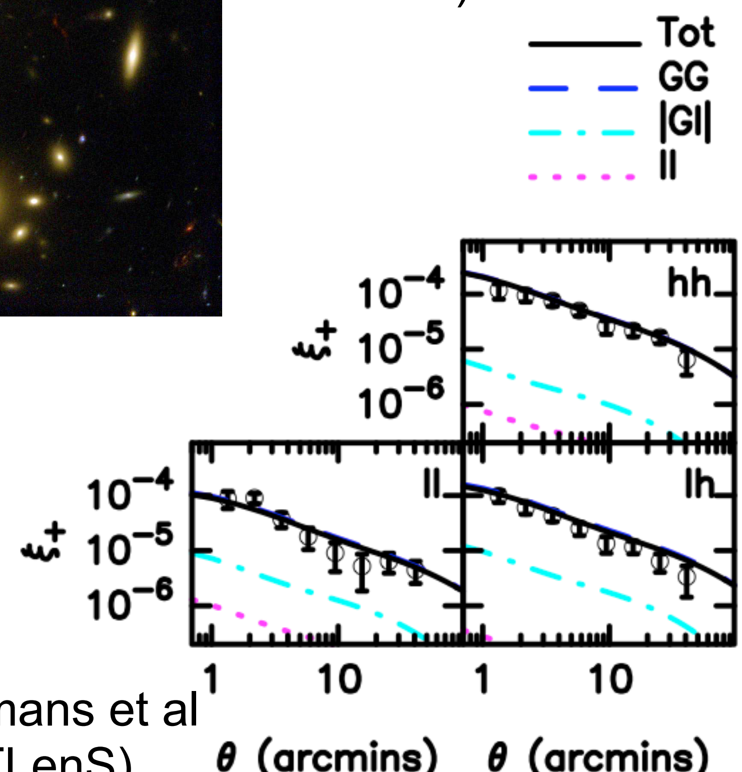




weak lensing

mass deflects light
this distorts galaxy
shapes a tiny bit

(lensing potential
~ Φ+Ψ)

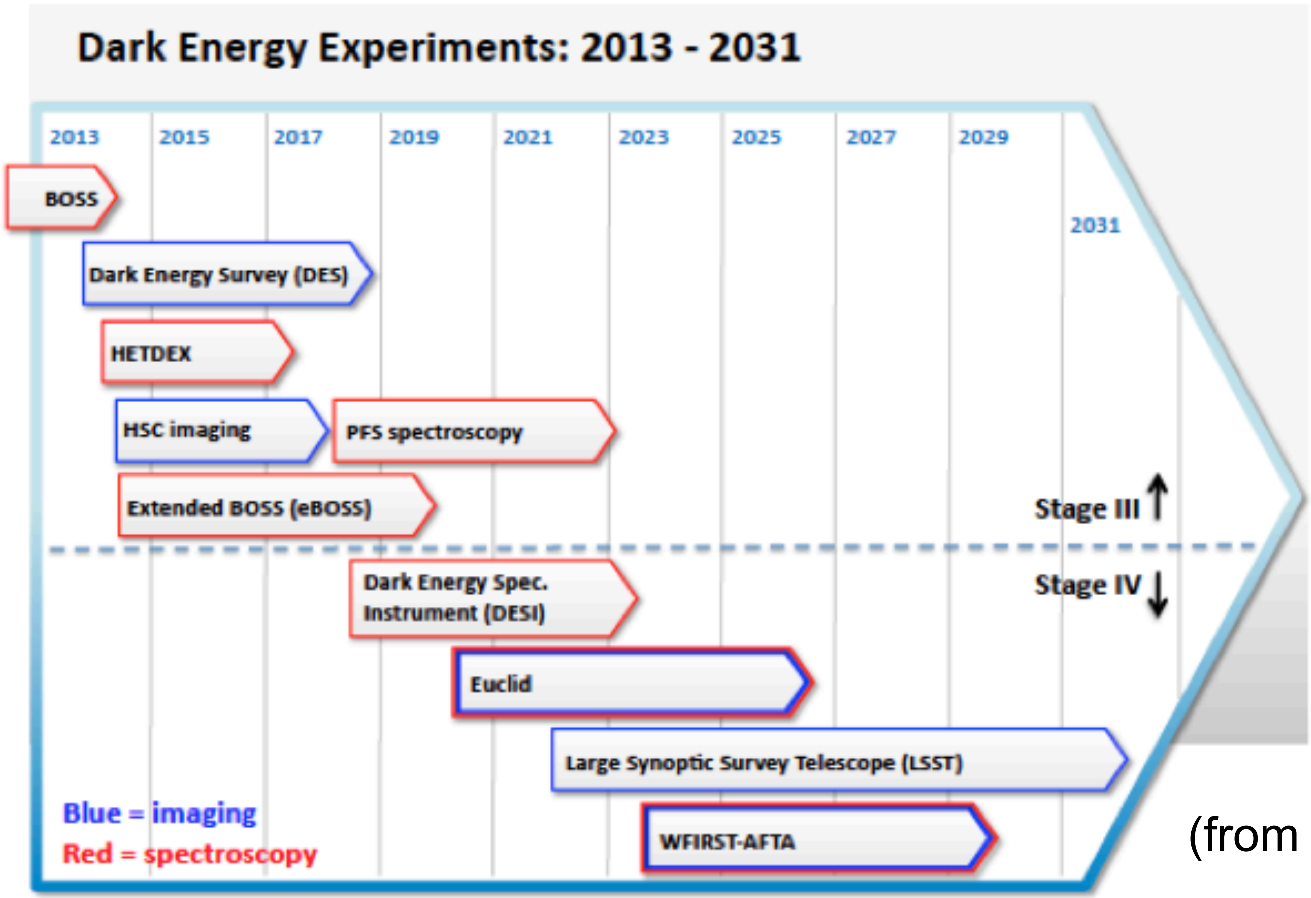


(Heymans et al
CFHTLenS)

seen as a future key probe,
but difficult:

- **non-linear scales**
- **baryons**
- **intrinsic alignments**

overview of future surveys



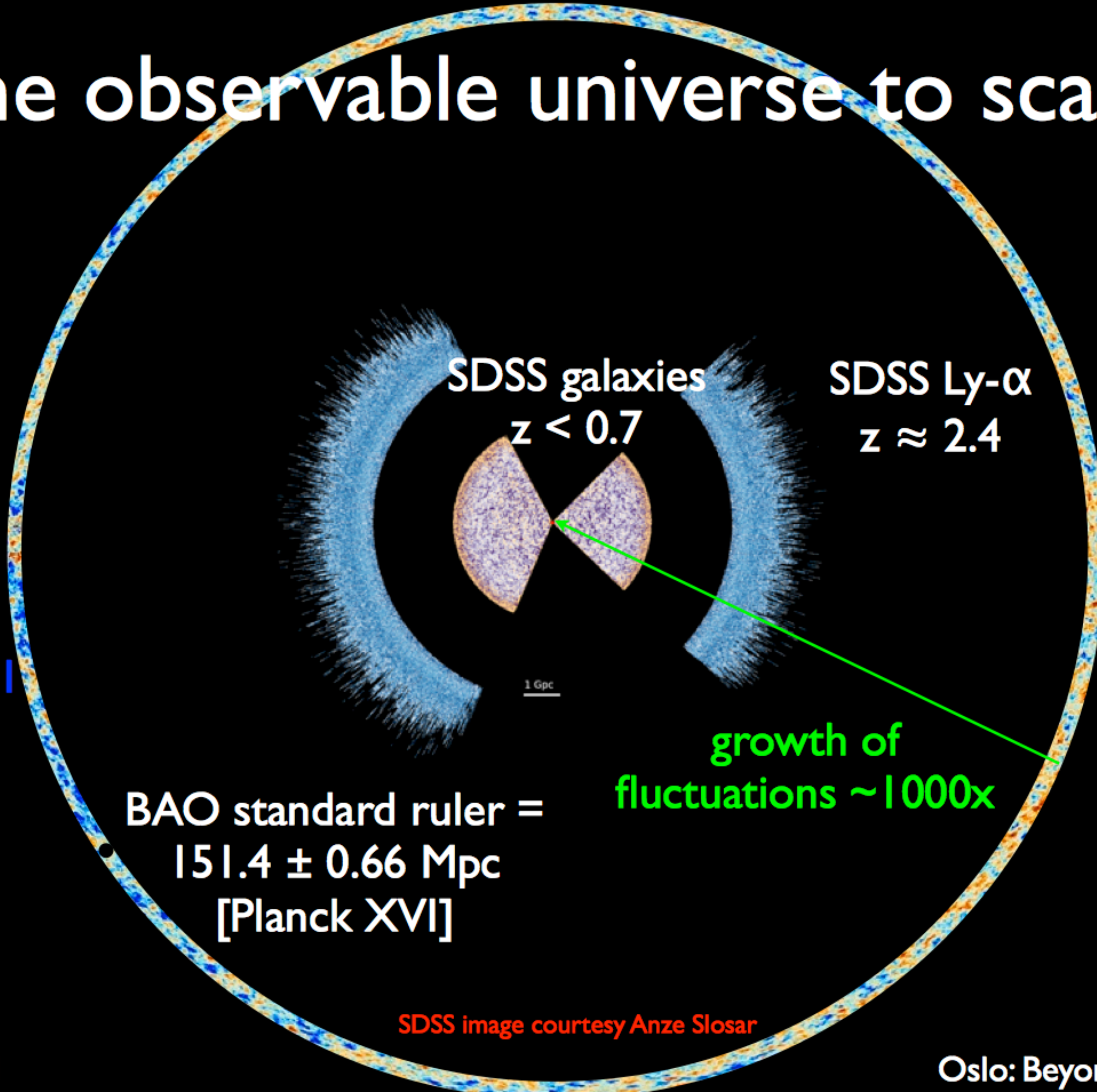
(from Ofer Lahav)

+ SKA (radio telescope)
+ CMB experiments (ground, space)
+ gravitational waves + lots more (neutrinos, cosmic rays, X-rays, …)!

overview of cosmological data

- distances ('pure background')
  - CMB peak locations: ~ angular diameter distance
  - supernovae: luminosity distance
  - Baryonic Acoustic Oscillations: angular diameter distance, H
  - change in redshift of distant objects: H
- perturbations:
  - full CMB spectrum (temperature, polarisation, ISW)
  - full shape galaxy power spectrum P(k) [but: bias]
  - redshift space distortions & peculiar velocities
  - growth rate of matter perturbations [P(k,z)]
  - gravitational lensing: CMB, weak, strong
  - galaxy clusters
  - "relativistic effects"

The observable universe to scale*

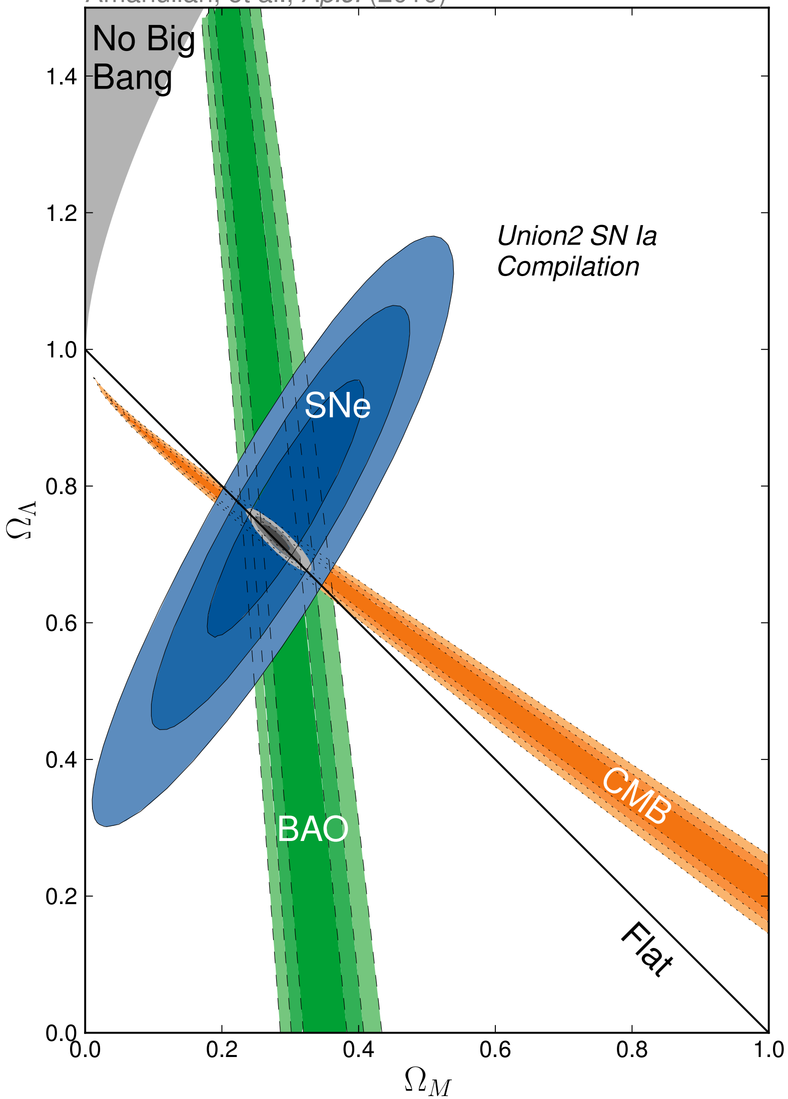


status report

- we have a full ‘model chain’ that explains cosmological observations
- **the GR + FLRW + LCDM + inflation model is consistent with current data, no significant deviations are observed**
- (some issues with isotropy of the CMB, the structure of galaxies and possibly the growth of perturbations notwithstanding)
- **main problems are theoretical:**
  - we don’t understand 95% of the contents: DE and DM
  - especially the cosmological constant is highly problematic
  - (the model also does not explain how inflation started)
  - (and we can’t explain the baryon asymmetry)

Dark Energy

Supernova Cosmology Project
Amanullah, et al., *Ap.J.* (2010)



Physics Nobel prize 2011:
"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"

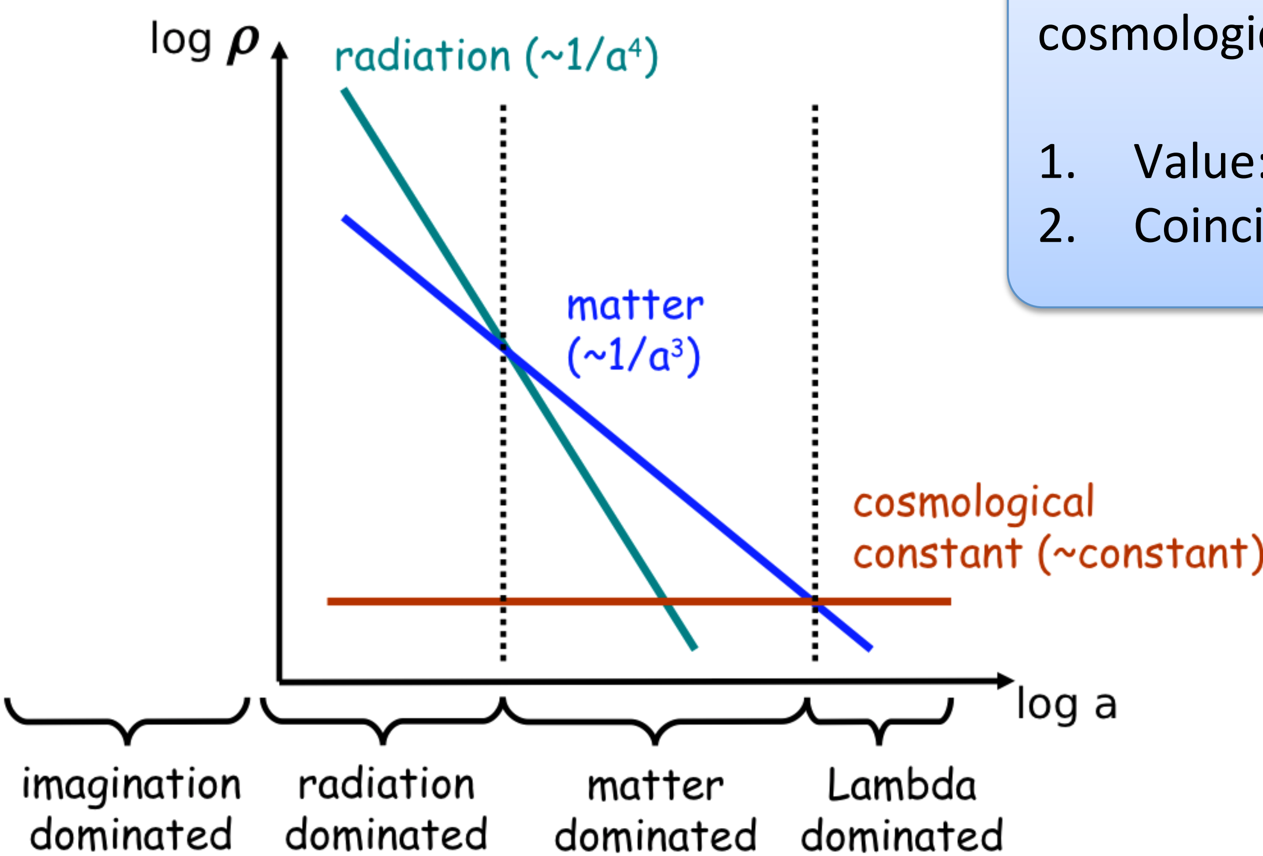
accelerating expansion: w < -1/3

- we know that for Λ: w = -1
- data is consistent with Λ

why look elsewhere?

What's the problem with Λ?

Evolution of the Universe:

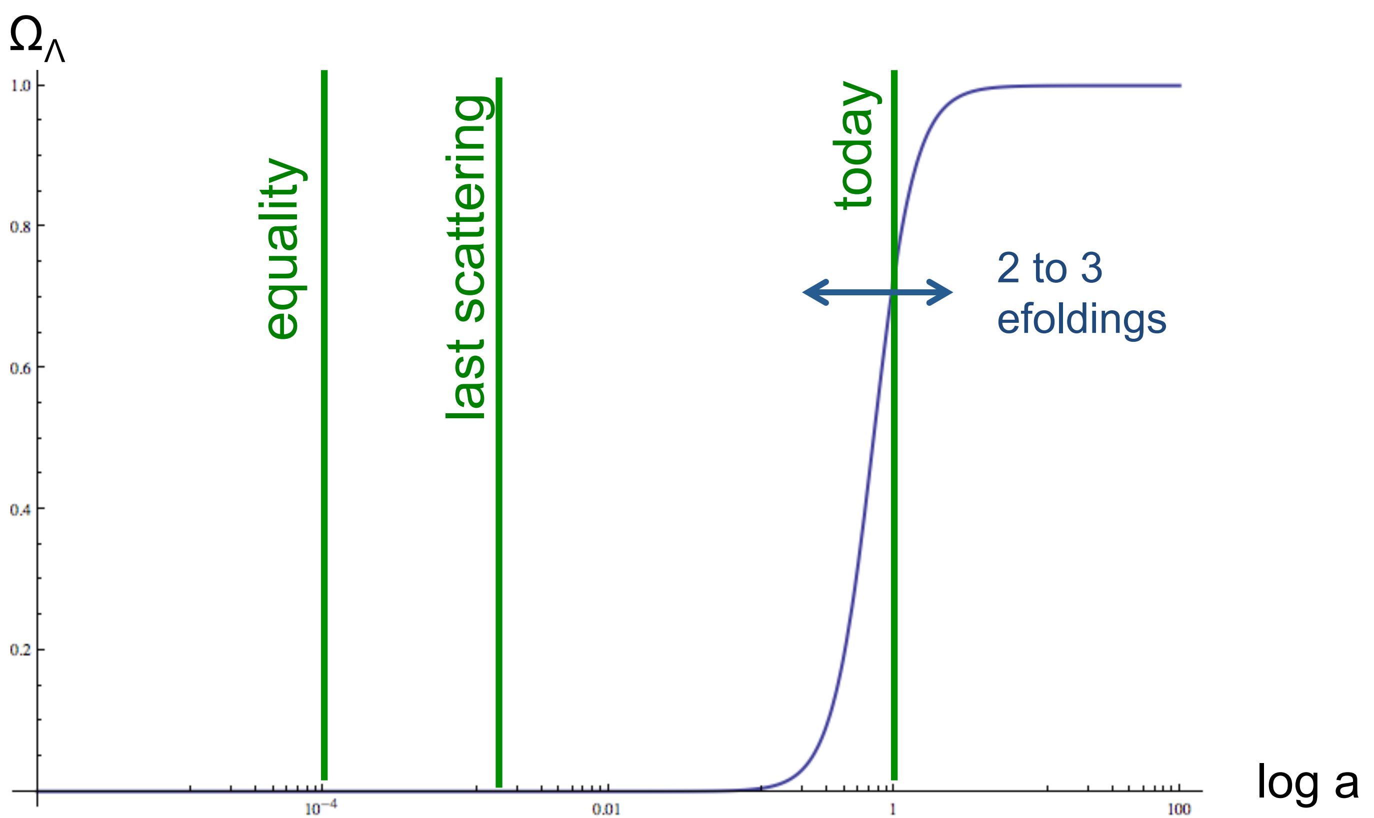


Classical problems of the cosmological constant:

1. Value: why so small? Natural?
2. Coincidence: Why now?

the coincidence problem

- why are we just now observing $\Omega_\Lambda \approx \Omega_m$?
- past: $\Omega_m \approx 1$, future: $\Omega_\Lambda \approx 1$



the naturalness problem

energy scale of observed Λ is ~ $2\text{x}10^{-3}$ eV
zero point fluctuations of a heavier particle of mass m:

cutoff scale

$$\int_0^{\Lambda} \frac{1}{2}\frac{d^3p}{(2\pi)^3}\sqrt{p^2+m^2} \simeq \frac{1}{16\pi^2}\left[\underbrace{\Lambda^4 + m^2\Lambda^2}_{\text{can in principle be absorbed into renormalization of observables}} - \underbrace{\frac{1}{4}m^4\log\left(\frac{\Lambda^2}{m^2}\right)}_{\text{“running” term: this term is measureable for masses and couplings! Why not for cosmological constant?!}}\right]$$

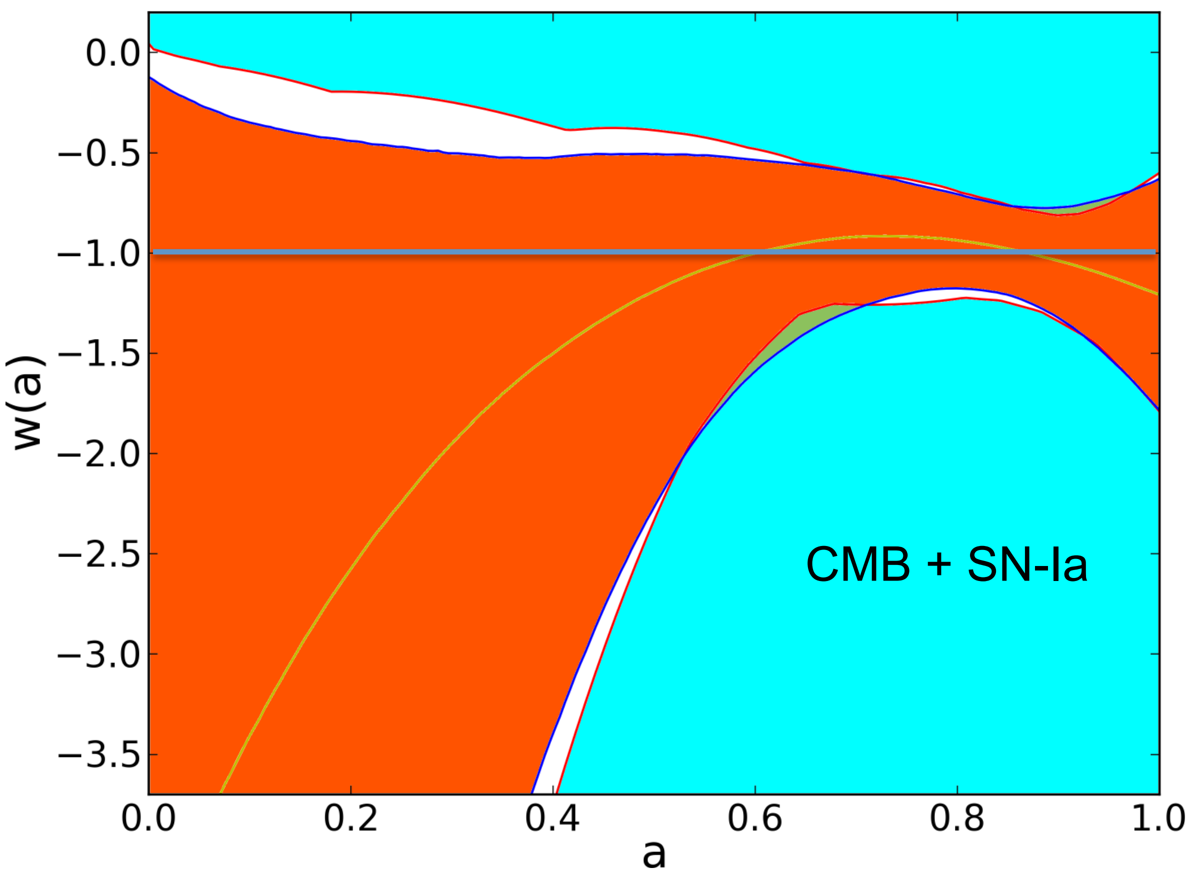
already the electron should contribute at m_e >> eV
(and the muon, and all other known particles!)

Possible explanations

1. It is a cosmological constant, and there is no problem ('anthropic principle', 'string landscape')
2. The (supernova) data is wrong
3. We are making a mistake with GR (aka 'backreaction') or the Copernican principle is violated ('LTB')
4. It is something evolving, e.g. a scalar field ('dark energy')
5. GR is wrong and needs to be modified ('modified gravity')

w(z) of scalar field model

- "standard" DE: minimally coupled scalar field with canonical kinetic term
- has sound speed $c_s^2=1$ and anisotropic stress $\sigma=0$
- general perturbations describe the next class …



cubic expansion of w:
- 95% limits
- w=-1 is a good fit
- best constraints at low z
- ca 10%-15% error on w at 'best' redshift
- not very strong dependence on parametrisation

Is it just Λ?
- remember the problems
- also: inflation

modified gravity models

4D generalisation of GR:

⇨ Scalar/(V)/Tensor : natural generalisation, strong limits from solar system, effects can be screened

⇨ f(R) : modify action: R + f(R) (e.g. R-μ^4/R), related to non-minimally coupled scalar field models

⇨ **EFT / Horndeski** → most general scalar-tensor theories w/ 2^{nd}-order e.o.m., some generalizations **HOT TOPIC**

⇨ massive gravity / bigravity theories / galileons

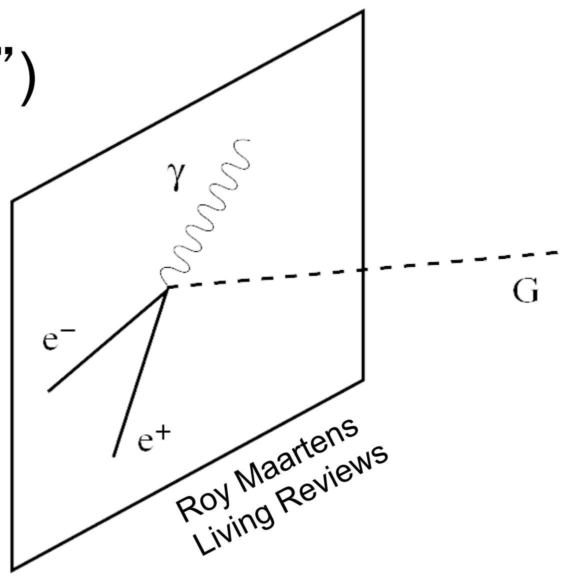
⇨ non-local models

Higher-dimensional gravity (aka "braneworlds")

gravity (closed strings) propagates freely, standard model (open strings) fixed to branes

⇨ DGP : sum of 5D and 4D gravity action

- instabilities, ghosts, finetuning
- solar-system tests
- dependence on background



non-cosmological probes

- fifth force (weak, long-range) from couplings of standard model to new fields

HOT TOPIC

-> screening mechanisms (Chameleon, Vainshtein, …)

- new particles with strange couplings and/or mass hierarchies (KK)
- varying "fundamental constants" and other violations of the equivalence principle
- perihelion shifts / solar system constraints (including double pulsar timings, etc)
- modifications to stellar structure models
- short-distance gravity modified (now well below 0.1mm)

cosmological DE/MG probes

What can we actually measure?

two kinds of equations:

$$\boxed{G_{\mu\nu} = -8\pi G T_{\mu\nu} \qquad T^{\nu}_{\mu;\nu} = 0}$$

$$G_{\mu\nu} = -8\pi G T_{\mu\nu}$$

determine metric coeffs from $T_{\mu\nu}$

$$g_{\mu\nu}$$

$$\nabla_{\nu} T^{\nu}_{\mu} = 0$$

determine evolution of $T_{\mu\nu}$ from metric and "physics"

"modified gravity"

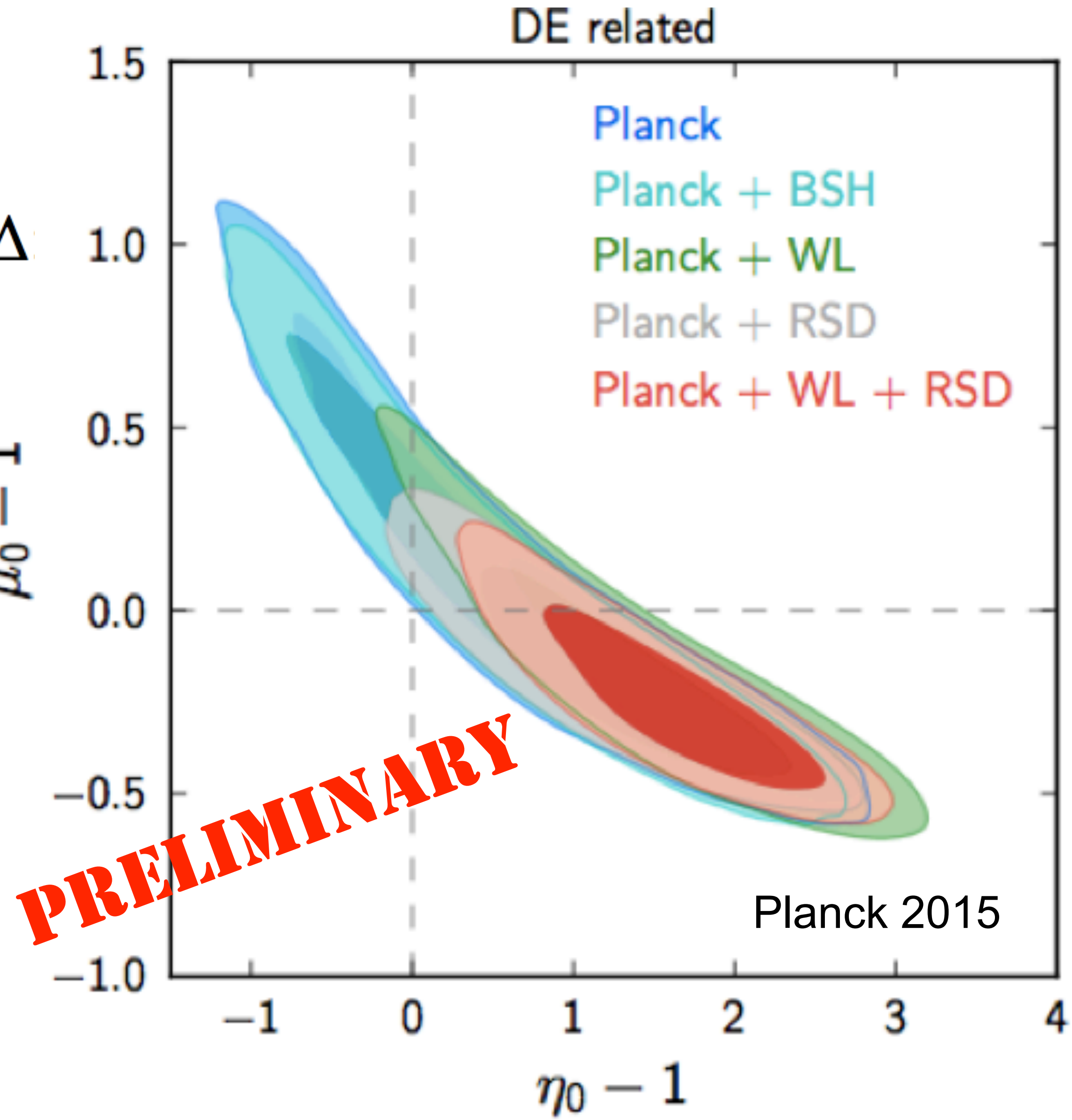
parameterisation of
late-time perturbations:

$$-k^2\Psi \equiv 4\pi G a^2 \mu(a, \boldsymbol{k})\rho\Delta$$

$$\eta(a, \boldsymbol{k}) \equiv \Phi/\Psi$$

functions ~ $\Omega_{DE}(a)$
ΛCDM background

- no scale dependence detected
- deviation mostly driven by WL
- not very significant for two extra d.o.f.

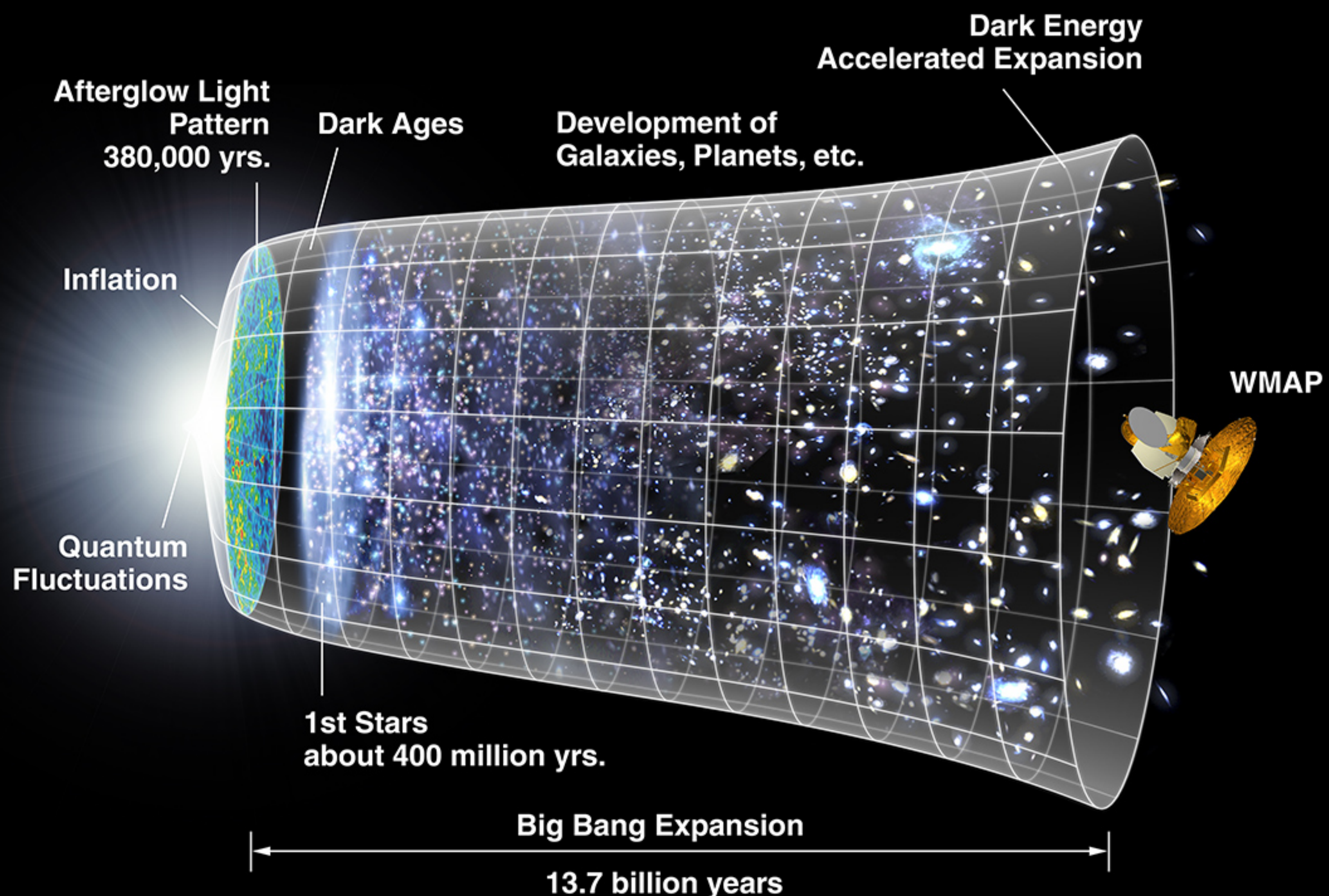


DE/MG summary

- The data clearly sees something incompatible with standard cosmology w/o DE.
- We have no model that we really like.
- Might still be due to mis-understanding of GR.
- Dark energy models need fine-tuning.
- Modified gravity models need screening.
- New d.o.f. necessary, usually look like scalars anyway! (-> difficult to distinguish MG – DE)
- The perturbation evolution contains much more information than w(a).
- But the data is in good agreement with Λ

particle cosmology

- actual particles, not just ‘fluid’
- physics known, can make predictions



Equilibrium distributions

Short-range interactions maintaining thermodynamic equilibrium:

$$f(k,t)d^3k = \frac{g}{(2\pi)^3}\big(\exp[(E-\mu)/T] \pm 1\big)^{-1} d^3k$$

$E = \sqrt{k^2 + m^2}$, T temperature, μ chem. pot.

$$n = \int f(k) d^3k$$ number density

$$\rho = \int E(k) f(k) d^3k$$ energy density

$$p = \int \frac{|k|^2}{3E(k)} f(k) d^3k$$ pressure

Relativistic species, m << T

Crank handle, using m = μ = 0 -> E ~ k
(use x=E/T as integration variable)

$$n_B = T^3 \frac{g\zeta(3)}{\pi^2} \qquad n_F = \frac{3}{4} n_B$$

$$\rho_B = T^4 \frac{g}{30} \pi^2 \qquad \rho_F = \frac{7}{8} \rho_B$$

with $\rho_\gamma \sim a^{-4}$ => $T_\gamma \sim 1/a$
-> ***expanding universe cools down***

-> Stefan-Boltzmann law
$\rho_\gamma \sim T^4$

$$p = \frac{\rho}{3}$$

-> $w_{rad} = p_{rad}/\rho_{rad} = 1/3$

Massive species, m >> T

Expand $E = \sqrt{k^2 + m^2} = m\sqrt{1 + k^2/m^2} \approx m + k^2/(2m)$
and neglect +/-1 wrt exp(m/T)

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-(m-\mu)/T}$$

$$\rho = mn + \frac{3}{2}nT$$ -> E_{kin} / particle: $$E_{\mathrm{kin}} = \frac{3}{2}k_B T$$

$$p = nT \ll \rho$$

Massive particles are suppressed by Boltzmann factor exp(-m/T), so they will quickly drop out of thermal equilibrium when T < m -> 'freeze out' -> effective μ

Multiple relativistic species

If we have several species at different temperatures:

$$\rho_R = \frac{T_\gamma^4}{30}\pi^2 g_* \qquad g_* = \sum_{i\in B} g_i \left(\frac{T_i}{T_\gamma}\right)^4 + \frac{7}{8}\sum_{j\in F} g_j \left(\frac{T_j}{T_\gamma}\right)^4$$

Entropy density: $s = \dfrac{\rho + p}{T} \propto T^3 \longrightarrow d(sa^3)/dt = 0$

(use f and $\dot{\rho} + 3H(\rho + p) = 0$)

$$s = \frac{2\pi^2}{45} g_{*S} T_\gamma^3 \qquad g_{*S} = \sum_{i\in B} g_i \left(\frac{T_i}{T_\gamma}\right)^3 + \frac{7}{8}\sum_{j\in F} g_j \left(\frac{T_j}{T_\gamma}\right)^3$$

- $T_\gamma \propto g_{*S}^{-1/3} a^{-1}$

Now we are ready to study particle evolution in the early universe!

Neutrino decoupling

Interaction rate: Γ
Expansion rate: H

species in equil.: Γ >> H
species decoupled: Γ << H

$$\Gamma(T) = n(T)\langle\sigma v\rangle_T \qquad \sigma_F \simeq G_F^2 E^2 \simeq G_F^2 T^2 \qquad \Gamma_F \sim G_F^2 T^5$$

$$H(T) = \sqrt{\frac{8\pi G}{3}}\sqrt{\rho_R} \simeq \frac{5.44}{m_P}T^2 \qquad g_* = 2 + \frac{7}{8}(3\times 2 + 2\times 2)$$

$$\Rightarrow \boxed{\frac{\Gamma_F}{H(T)} \simeq 0.24 T^3 G_F^2 m_P \simeq \left(\frac{T}{1\mathrm{MeV}}\right)^3}$$

Neutrinos decouple when temperature drops below ~ 1 MeV because their interactions become too weak.

Temperature of ν background

Shortly after the neutrinos decouple, we reach T=0.5MeV=m_e and the entropy in electron-positron pairs is transferred to photons but not to the neutrinos. Photon + electron entropy $g_{*S}(Ta)^3$ is separately conserved:

$$g_*(T_{\nu\mathrm{dec}} > T > m_e) = 2 + \frac{7}{8} \times 4 = \frac{11}{2}, \quad g_*(T < m_e) = 2$$

How much are the photons heated by the electron-positron annihilation?

Temperature of ν background

Shortly after the neutrinos decouple, we reach $T=0.5\text{MeV}=m_e$ and the entropy in electron-positron pairs is transferred to photons but not to the neutrinos. Photon + electron entropy $g_{*S}(Ta)^3$ is separately conserved:

$$g_*(T_{\nu\mathrm{dec}} > T > m_e) = 2 + \frac{7}{8} \times 4 = \frac{11}{2}, \quad g_*(T < m_e) = 2$$

$$\frac{(aT_\gamma)^3_{\mathrm{after}}}{(aT_\gamma)^3_{\mathrm{before}}} = \frac{(g_*)_{\mathrm{before}}}{(g_*)_{\mathrm{after}}} = \frac{11}{4}$$

Since $(aT_\nu) = (aT_\gamma)_{\mathrm{before}}$ we now have $T_\gamma = (11/4)^{1/3}\, T_\nu$

-> for $T<0.5m_e$: $g_* \sim 3.36$ and $g_{*S} \sim 3.91$ for radiation ($\gamma+\nu$)

relativistic degrees of freedom

relativistic particles change expansion rate during radiation dominated evolution

Preliminary

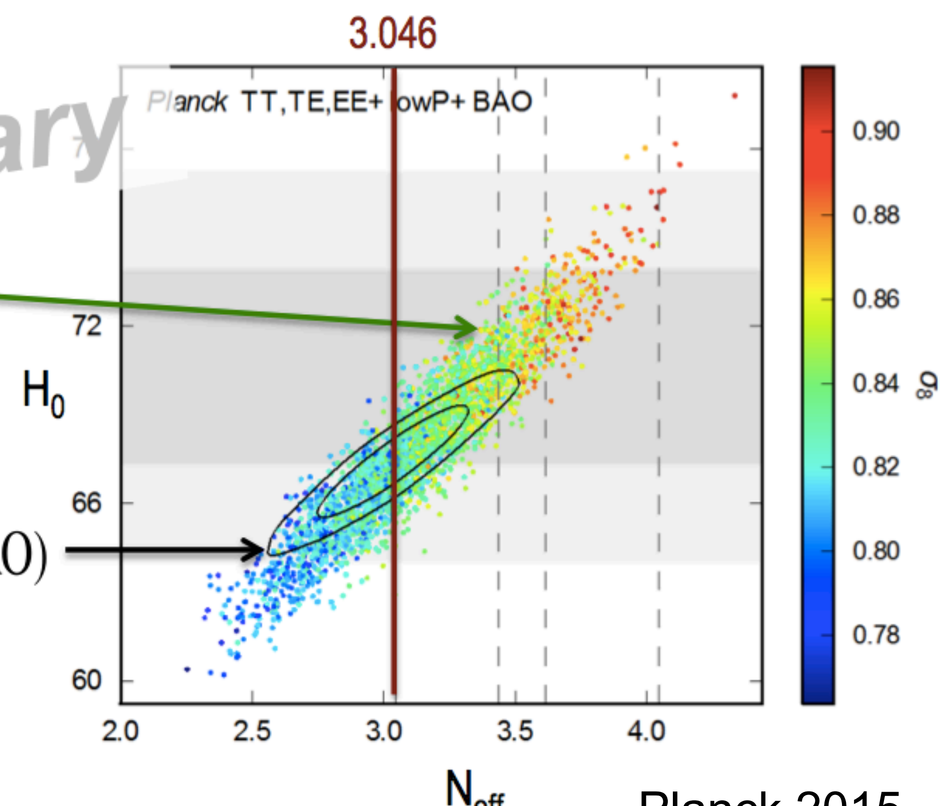
$N_{\rm eff} = 3.13 \pm 0.32$ (*Planck* TT+lowP)

$N_{\rm eff} = 3.15 \pm 0.23$ (*Planck* TT+lowP+ BAO)

$N_{\rm eff} = 2.98 \pm 0.20$ (*Planck* TT,TE,EE+lowP)

$N_{\rm eff} = 3.04 \pm 0.18$ (*Planck* TT,TE,EE+lowP+BAO)

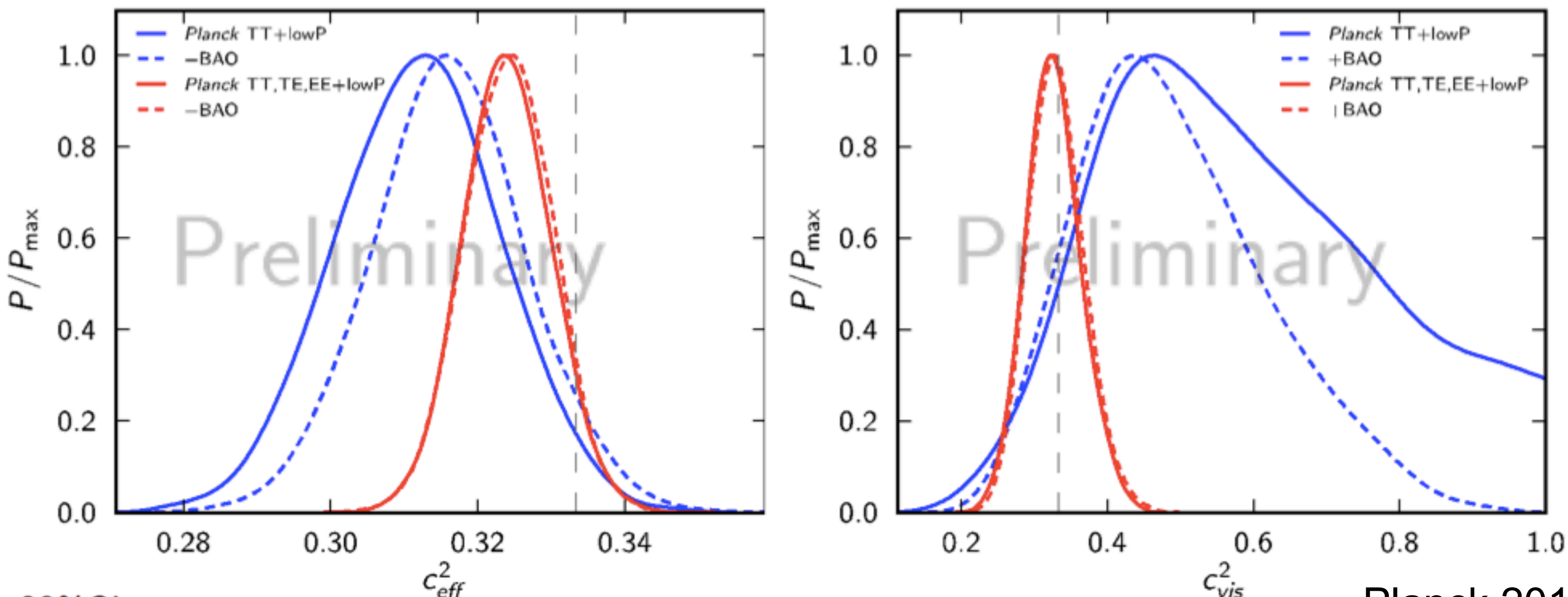
(all at 68% CL, BAO from *6dFGS, SDSS-MGS, BOSS-LOWZ, BOSS-CMASS-DR11*)



Planck 2015

- expected value consistent with data
- zero is not consistent
- $N_{\rm eff}$ = 4 starts to be excluded (but model dependent)
- no sign of any extra light degrees of freedom

neutrino properties



Planck 2015

68%CL

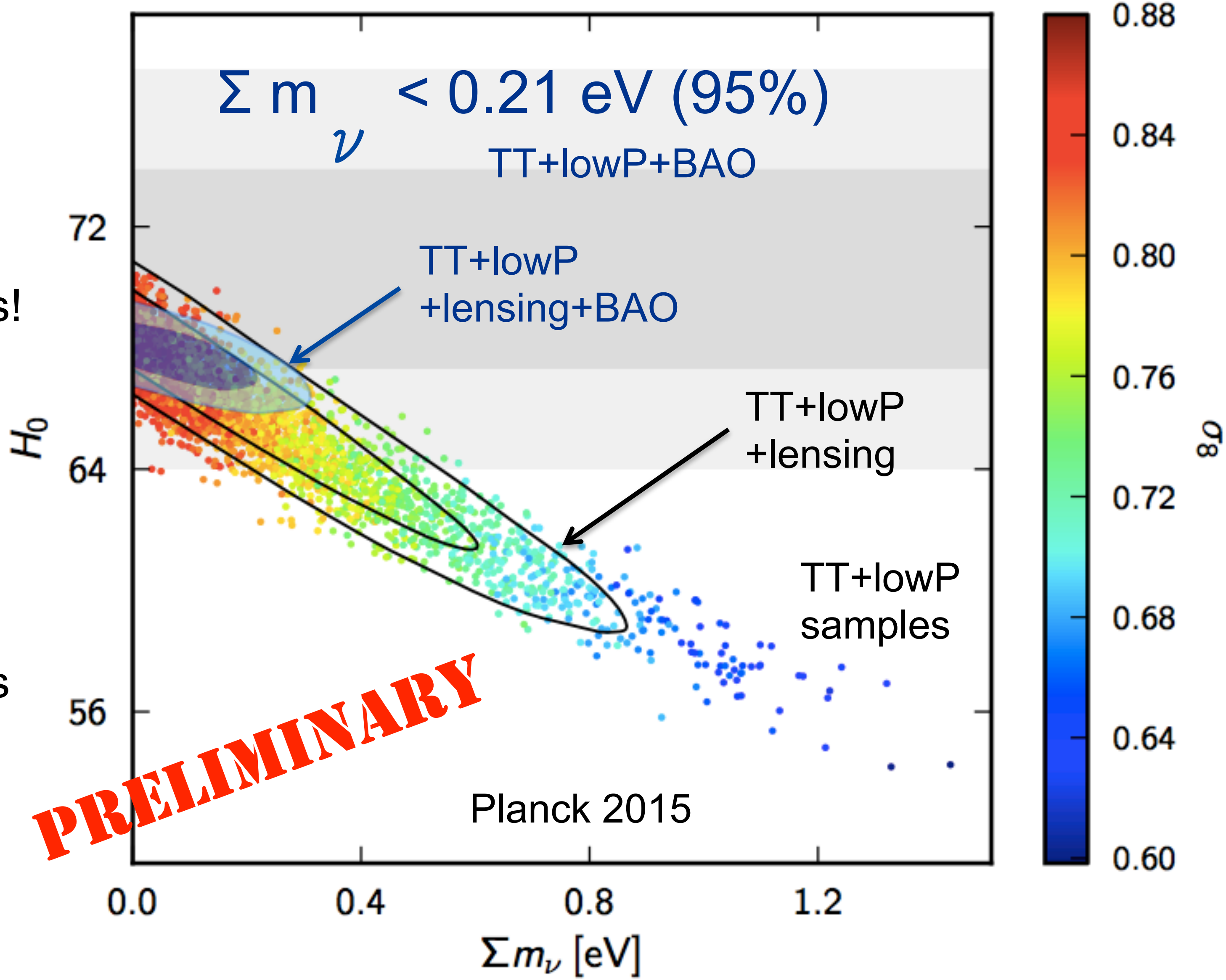
Parameter	TT+lowP	TT+lowP+BAO	TT,TE,EE+lowP	TT,TE,EE+lowP+BAO
c^2_{vis}	$0.47^{+0.26}_{-0.12}$	$0.44^{+0.15}_{-0.10}$	0.327 ± 0.037	0.331 ± 0.037
c^2_{eff}	0.312 ± 0.011	0.316 ± 0.010	0.3240 ± 0.0060	0.3242 ± 0.0059

- significant detection of “neutrino anisotropies”
- compatible with expected values

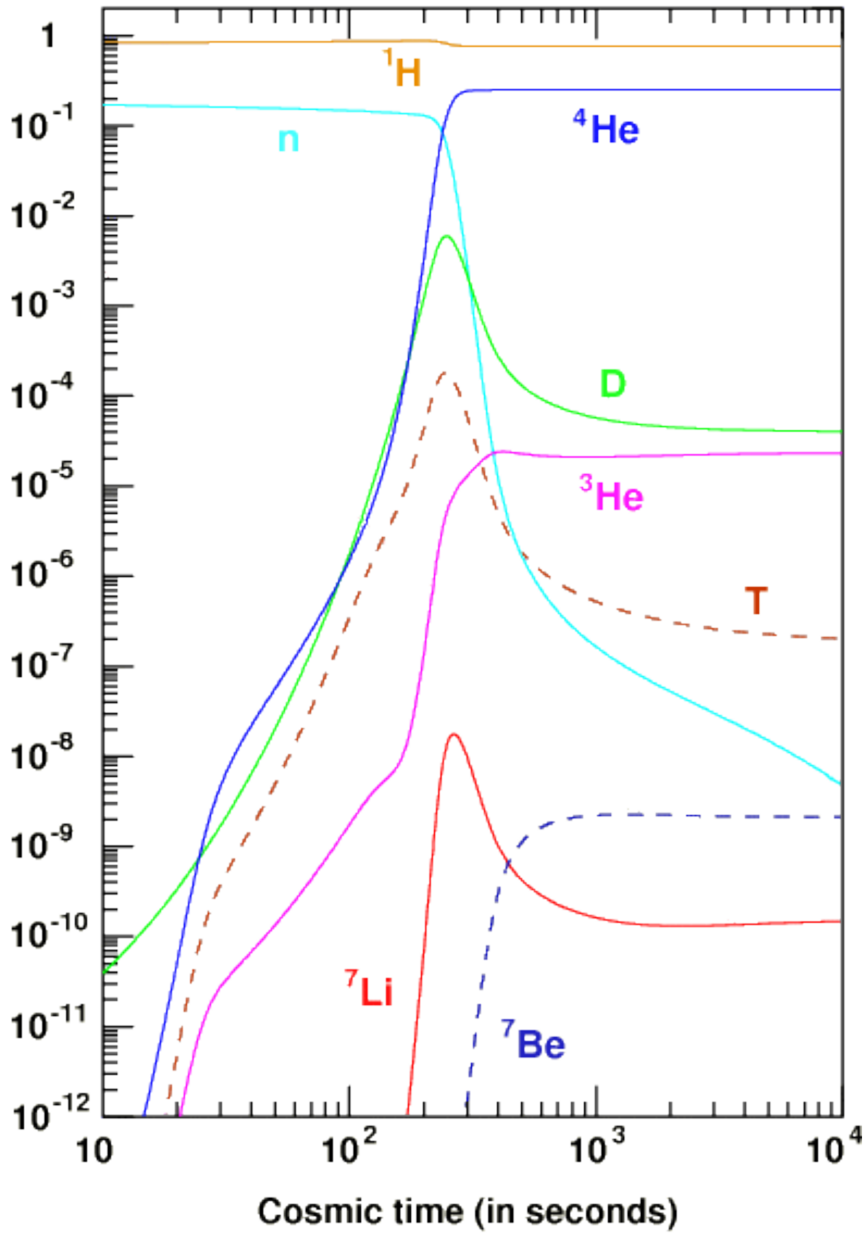
cosmology & neutrino masses

neutrinos affect the evolution of the perturbations!

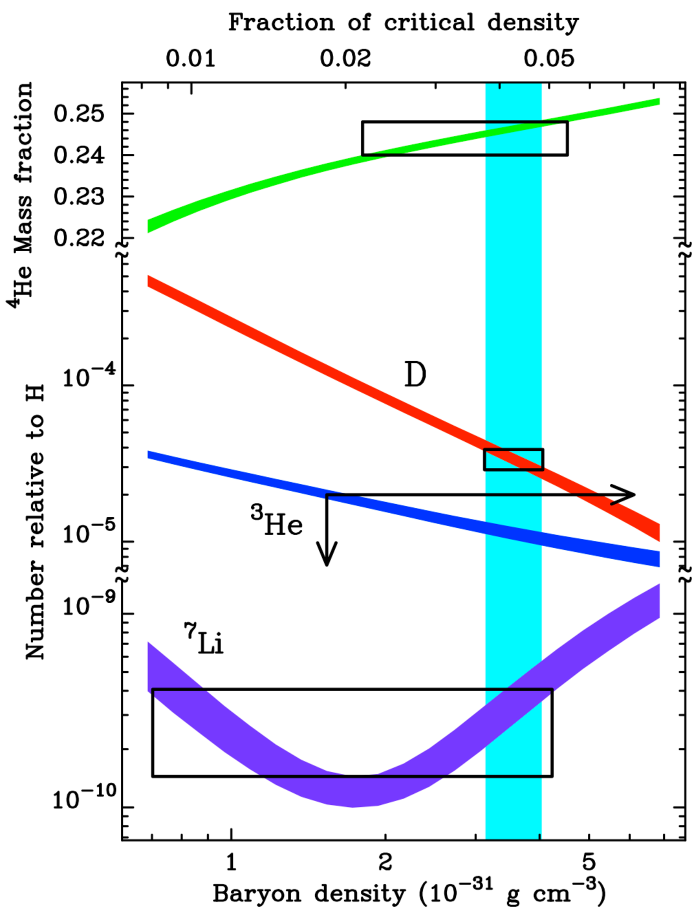
Cosmology has currently the leading absolute mass constraints ☺



formation of light elements



T~10 MeV: $X_n = X_p = ½$
T~1MeV: n <-> p freezout, $X_p \sim 0.85$
T~0.1MeV: ^{4}He stuck because of D
T~65keV: now ^{4}He forms, uses nearly all neutrons that are left



dark matter freeze-out

Consider annihilation processes: $A + \overline{A} \leftrightarrow Y + \overline{Y}$

assume Y in thermal equilibrium

Boltzmann eq (int. momenta): $$\underbrace{\dot{n}_A + 3Hn_A}_{\text{Liouville} \sim d/dt} = \underbrace{\langle \sigma v \rangle \left((n_A^{(eq)})^2 - n_A^2 \right)}_{\text{collision operator}}$$

1) <σv> large -> n -> $n^{(eq)}$
2) <σv> small -> n ~ a^{-3}

Introduce x=m/T and Y=n/T^3 (Y~n/s, constant for passive evol.)

some algebra...

$$\boxed{\frac{x}{Y_A^{(eq)}} Y_A' = -\frac{\Gamma_A}{H(x)} \left[\left(\frac{Y_A}{Y_A^{(eq)}} \right)^2 - 1 \right]}$$

=> freeze-out governed by Γ/H (Γ = $n^{(eq)}$<σv>)

(with $Y^{(eq)}$ = 0.09 g (for fermions) if x<<1 and $Y^{(eq)}$ = 0.16 g $x^{3/2}$ e^{-x} if x>>1)

Hot and cold relics

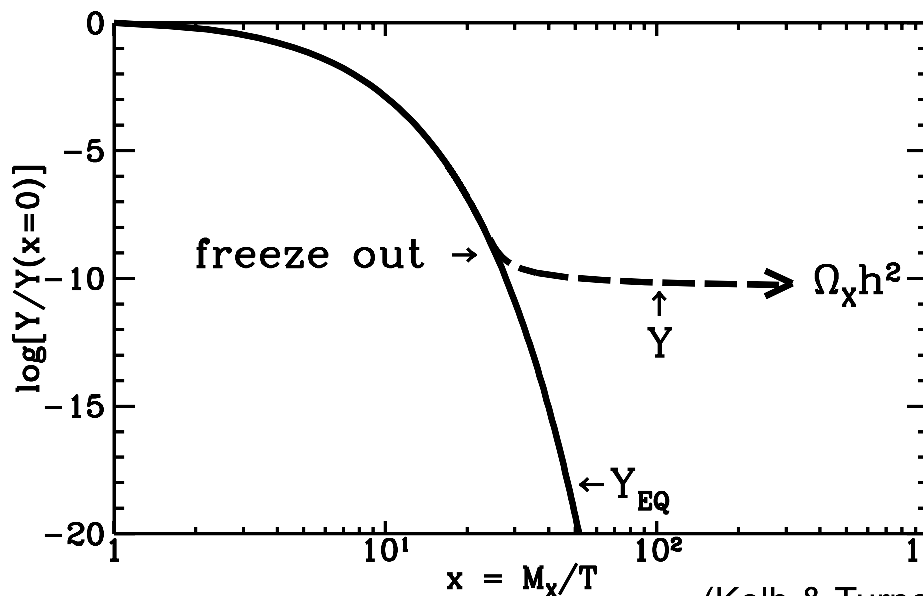
Hot relics: freeze-out when still relativistic (x_f<1)
-> $Y_A(x \to \infty) = Y_A^{(eq)}(x_f) = 0.278 g_A / g_{*S}(x_f)$

Cold relics: freeze out when x_f >> 1 => Y_A suppressed by $e^{-m/T}$
Abundance generically proportional to 1/σ

We can compute
$\rho_{A,0} = m_A\, n_{A,0}$
$\Omega_A = \rho_{A,0}/\rho_{crit}$

numerically, weak cross-sections lead to Ω ~ 1

-> *WIMP miracle*



(Kolb & Turner)

reasons for decoupling / freeze-out

- **neutrinos** decouple from thermal equilibrium because interactions become too weak ($\sim T^5$ scaling of interaction rate)
- **photons** (CMB) decouple from equilibrium because e^- disappear (recombination)
- **baryons** freeze-out from annihilation because of baryon-antibaryon asymmetry (no more antibaryons to annihilate with)
- **WIMPs** (dark matter) freeze out because their density drops due to $e^{-m/T}$ Boltzmann factor (if they are WIMP's)

Timeline summary

Energy (γ)	time	event
1 MeV	7s	neutrino freeze-out
0.5 MeV	10s	e^+/e^- annihilation, $T_\gamma \sim 1.4\ T_\nu$
70 keV	3 minutes	BBN, light elements formed
0.77 eV	70’000 yr	onset of matter domination
0.31 eV	300’000 yr	recombination
0.26 eV	380’000 yr	photon decoupling, origin of CMB
0.2 meV	14 Gyr	today

(WIMPs freeze out a bit below their mass scale)

$$\frac{1\ \mathrm{eV}}{k_\mathrm{B}} = \frac{1.602\,176\,53(14) \times 10^{-19}\ \mathrm{J}}{1.380\,6505(24) \times 10^{-23}\ \mathrm{J/K}} = 11\,604.505(20)\ \mathrm{K}.$$

status report

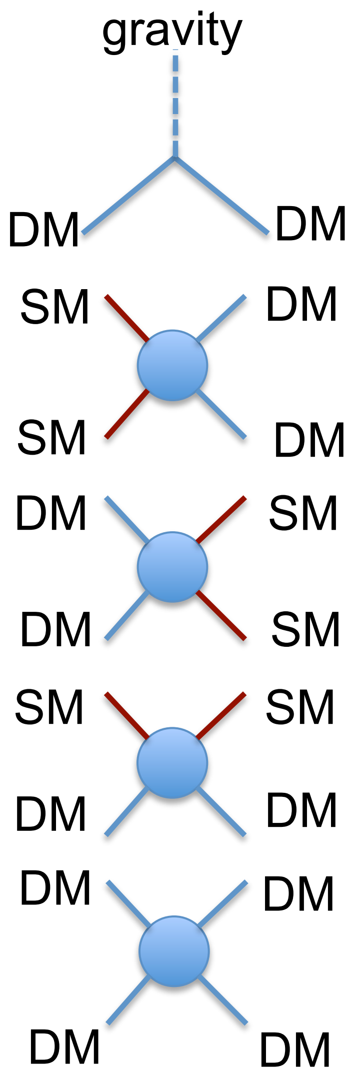
- **we should know physics up to TeV scale**
- **can predict evolution of the universe**
- **Successes:**
  - big-bang nucleosynthesis
  - freeze-out calculations, number of light species
  - constrain neutrino sector (number, mass, …)
- **Issues:**
  - baryogenesis?! why do we exist?
  - where is the dark matter?
- **Next steps:**
  - direct and indirect DM measurements
  - (ultra-) high energy cosmic rays
  - neutrino telescopes
  - multi-messenger astro-particle physics

a cosmologists view of astro-particle physics

- combination of astrophysics and particle physics
- mostly interested in:
  - high-energy photons (gamma rays)
  - cosmic rays
  - neutrinos
- main goals:
  - understand the "extreme universe": blazars, AGN, supernovae, pulsars, …
  - understand the dark matter
- facilities:
  - gamma-ray telescopes (Fermi)
  - X-ray telescopes (Integral, Swift, Chandra, XMM, …)
  - space particle detectors (AMS, Pamela, Fermi)
  - neutrino telescopes (IceCube, Antares, …)
  - Cherenkov telescopes (HESS, Veritas, Magic, …)
  - Air shower arrays (Auger, Telescope Array)

(thank you to Andrii Neronov!)

dark matter



➔ cosmology, cluster dynamics, rotation curves, lensing ✔

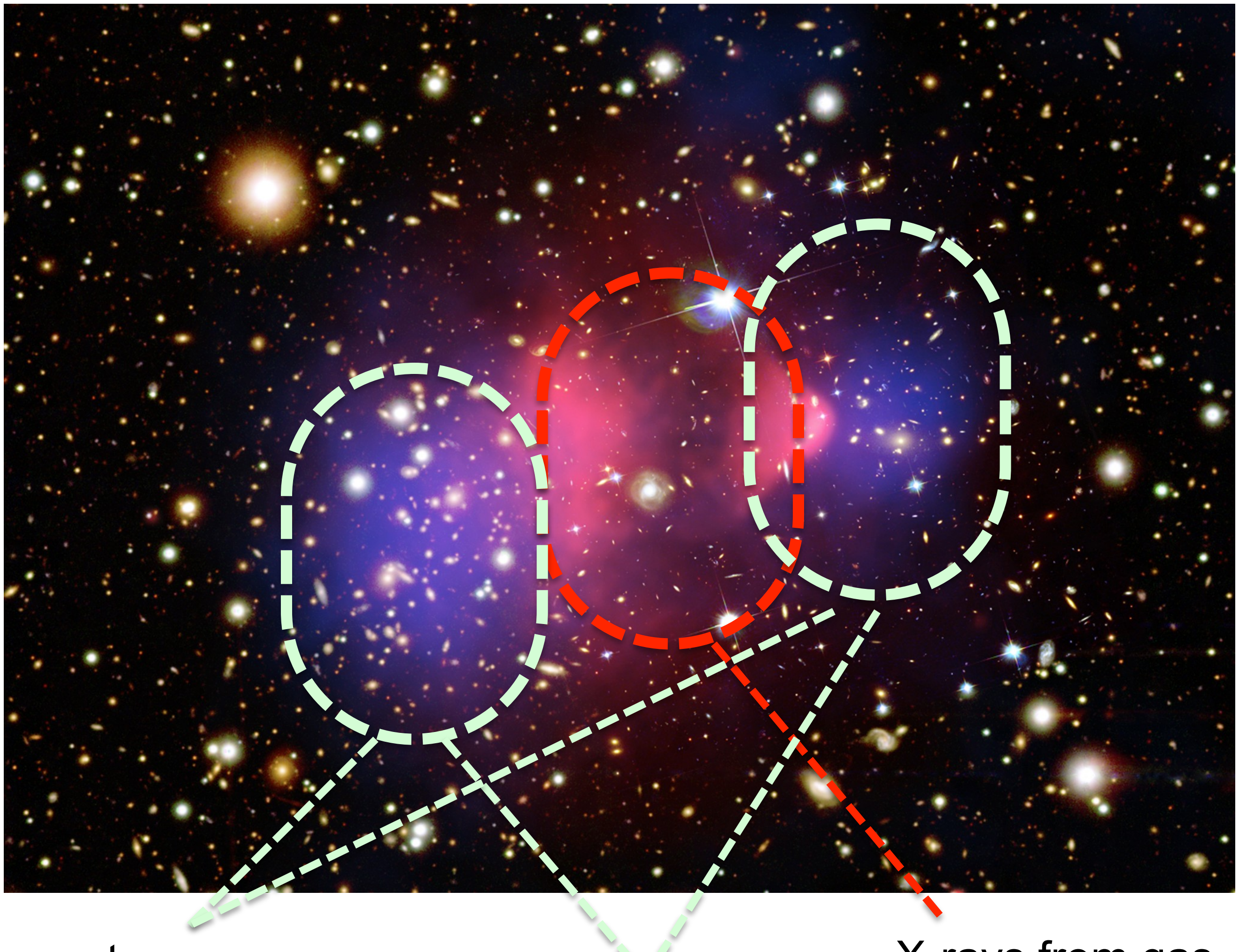
➔ collider searches → LHC, yesterday ✗

➔ indirect detection – CTA, HESS, Pamela, Fermi, … ✗ / ?

➔ direct detection – Xenon, LUX, CoGeNT, DAMA/LIBRA, … ✗ / ?

➔ astrophysics: collisions, structure of galaxy/cluster halos ✗

DM self-interaction



stars
(collisionless)

lensing mass

X-rays from gas
(baryonic mass)

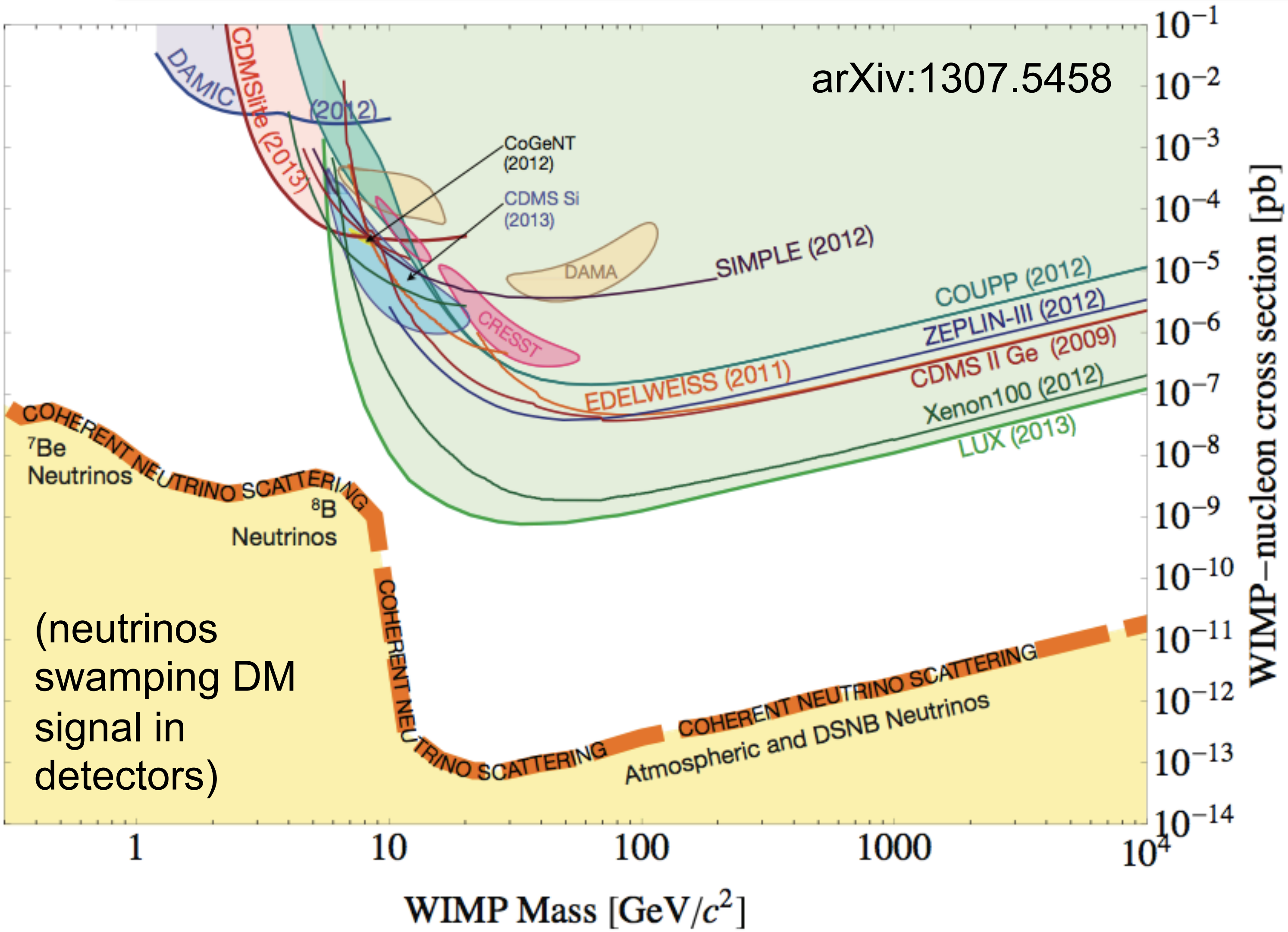
bullet cluster:

cluster collision shows no evidence of DM self-interaction

(but not clear how conclusive)

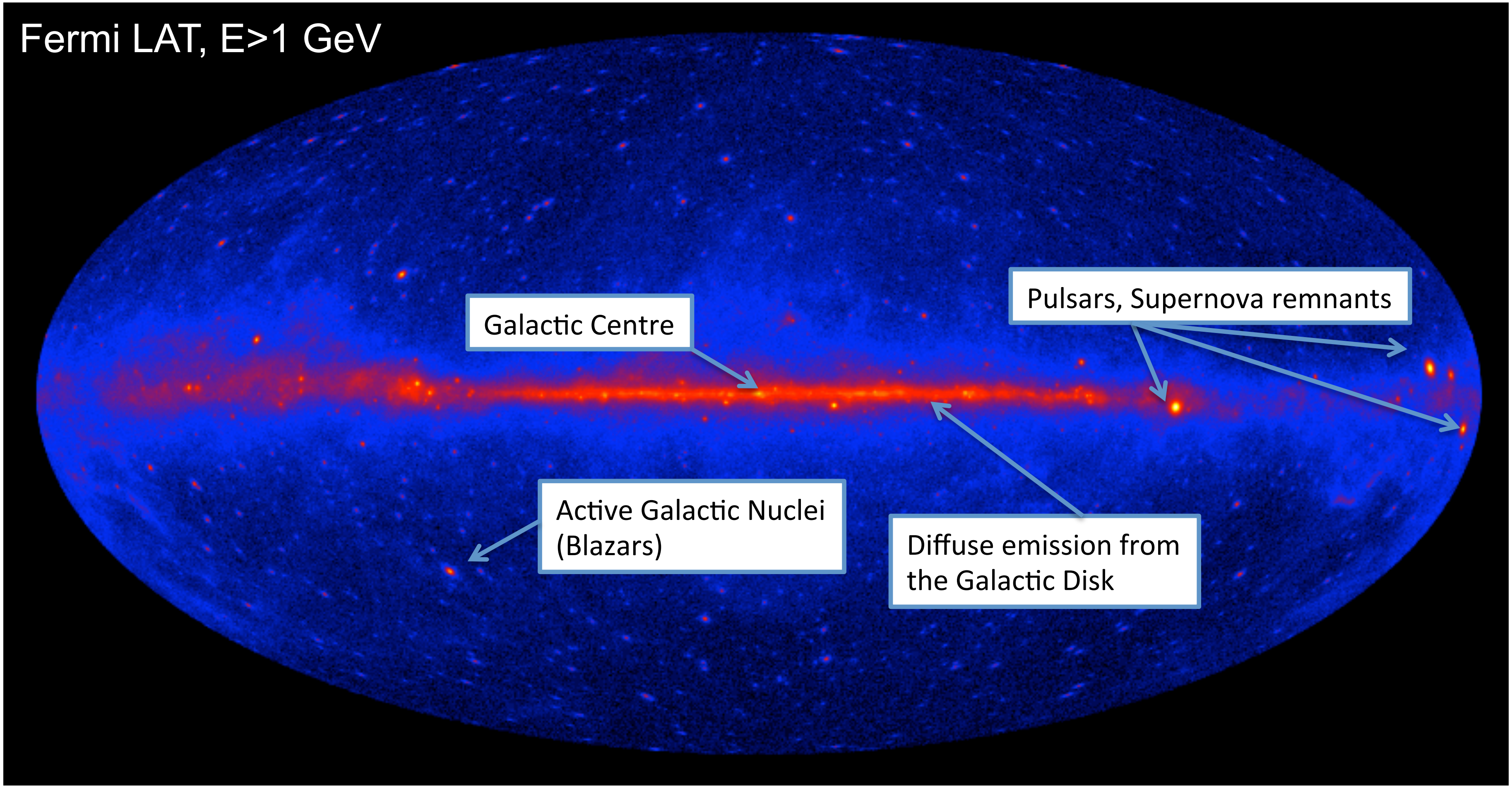
DM direct detection

no clear, consistent signal with convincing explanation



(neutrinos swamping DM signal in detectors)

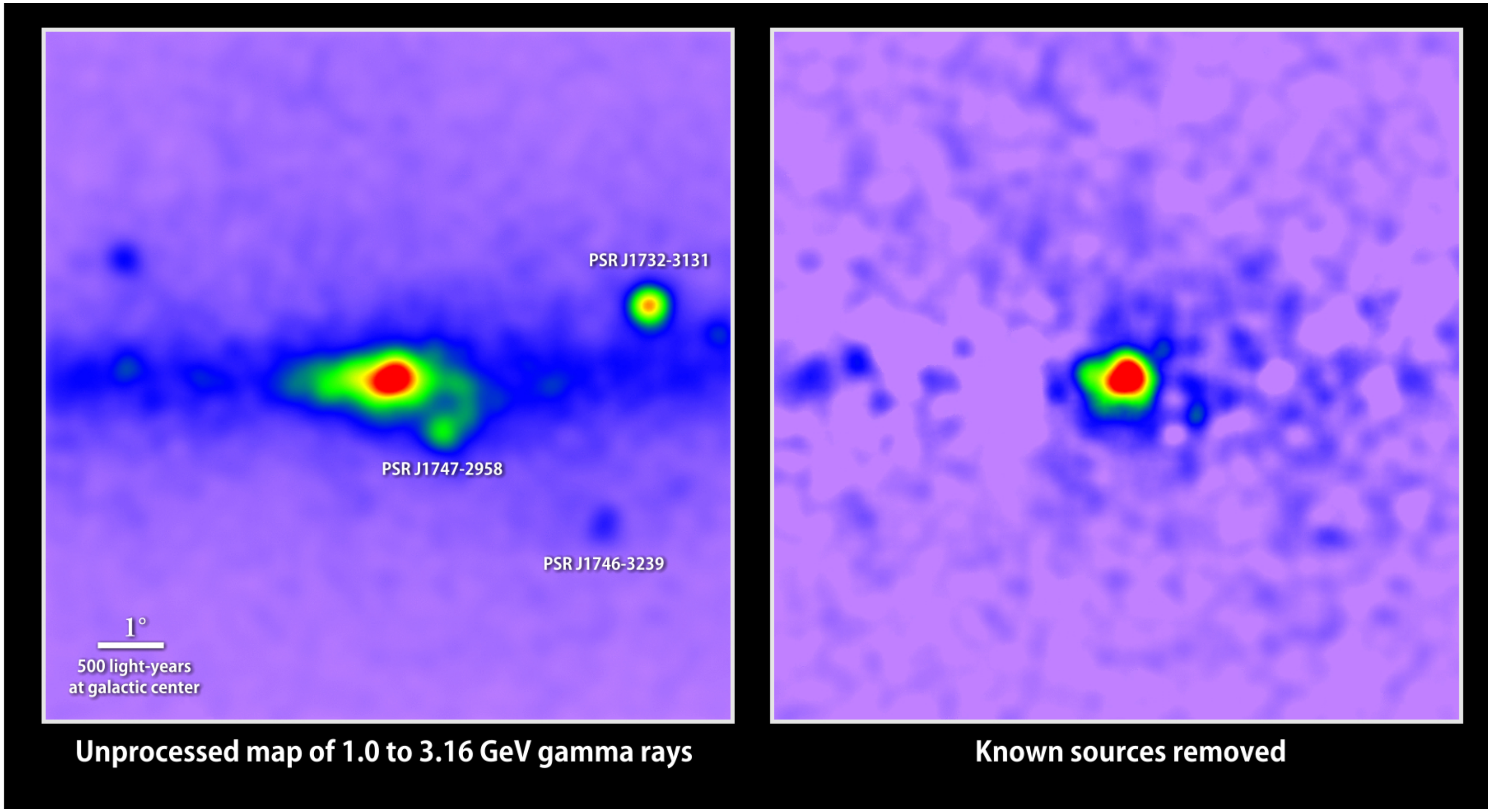
gamma ray astronomy



we observe gamma rays from:

- objects with high energy phenomena (galactic & extragalactic)
- cosmic ray interactions with interstellar medium
- dark matter?

has Fermi seen dark matter?

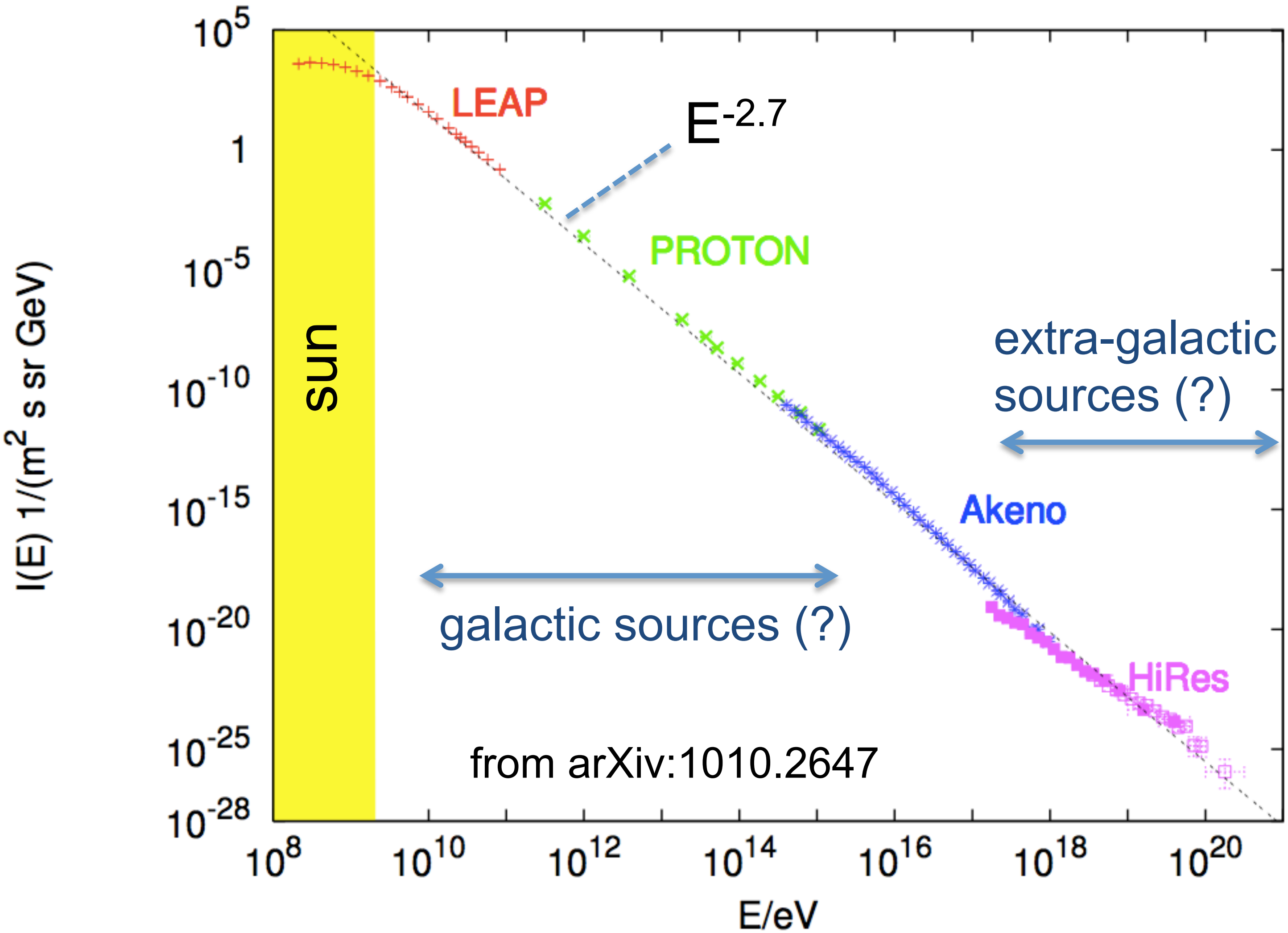


arXiv:1402.6703

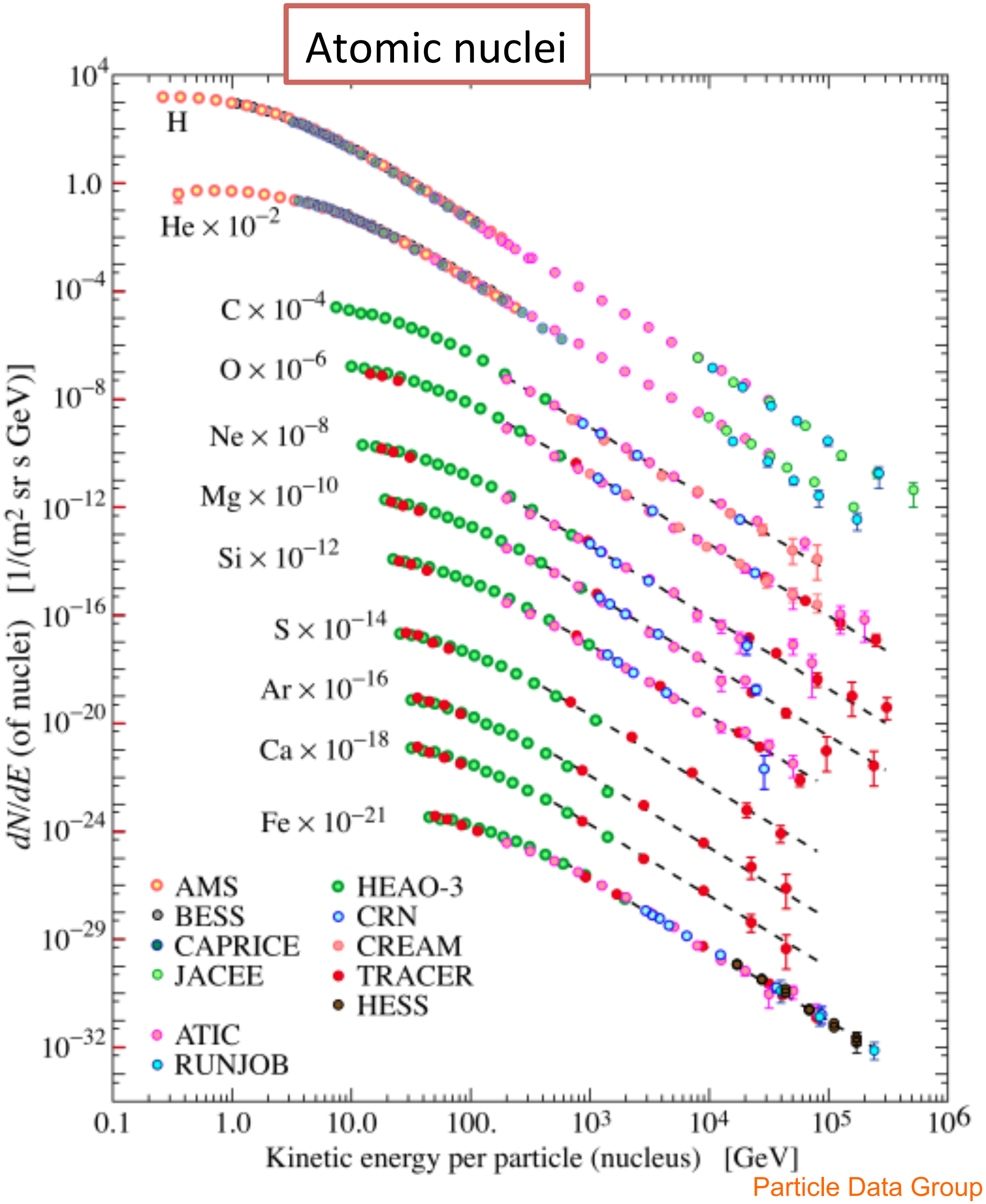
- could be WIMPs with mass of 30-40 GeV
- but purely from ‘elimination argument’ – still very early days…

cosmic rays

- charged particles from space, first seen in 1912 by Victor Hess
- direct detection with space-based detectors (or ballons)
- detection via air-showers & Cherenkov telescopes from ground



composition of cosmic rays

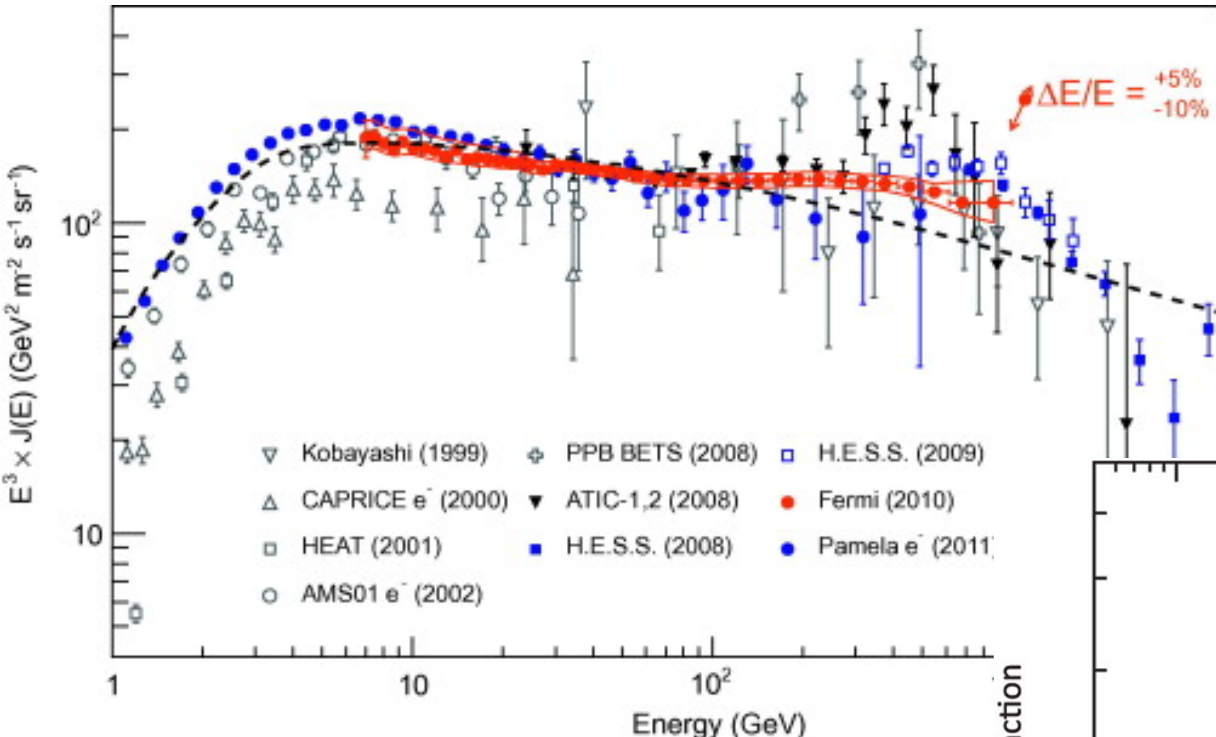


(from Andrii Neronov)

Low energies < 100 TeV: space-based detectors can measure the composition directly

High energies: situation not yet entirely clear – Auger sees a transition from p to heavier elements (Fe) around 10EeV

electrons / positrons



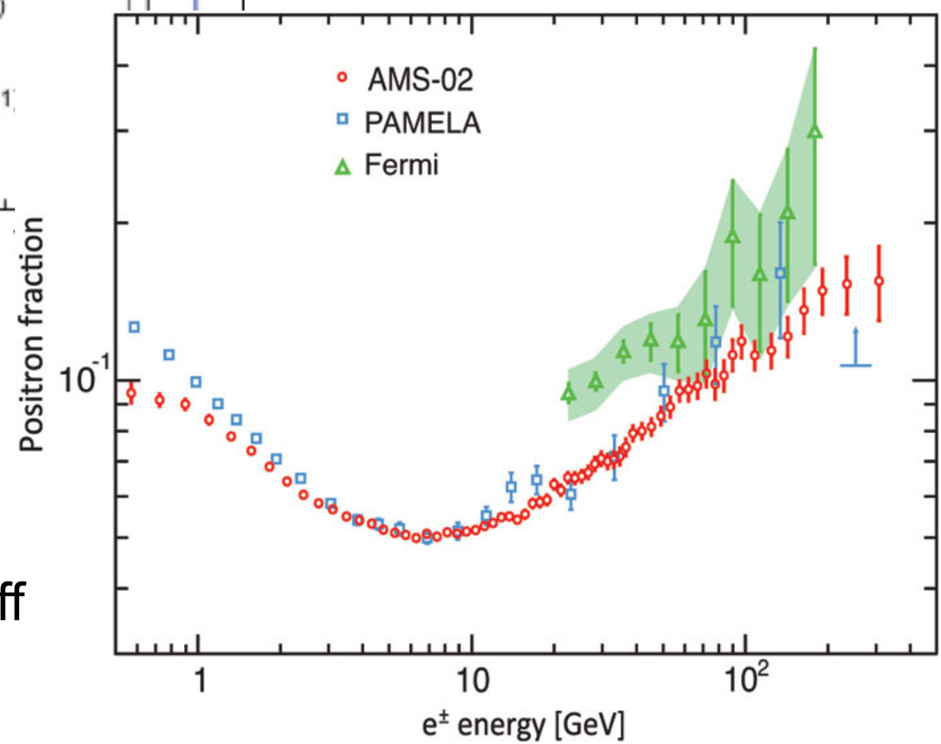
Pamela, Fermi and AMS detect more positrons than expected.

Could be

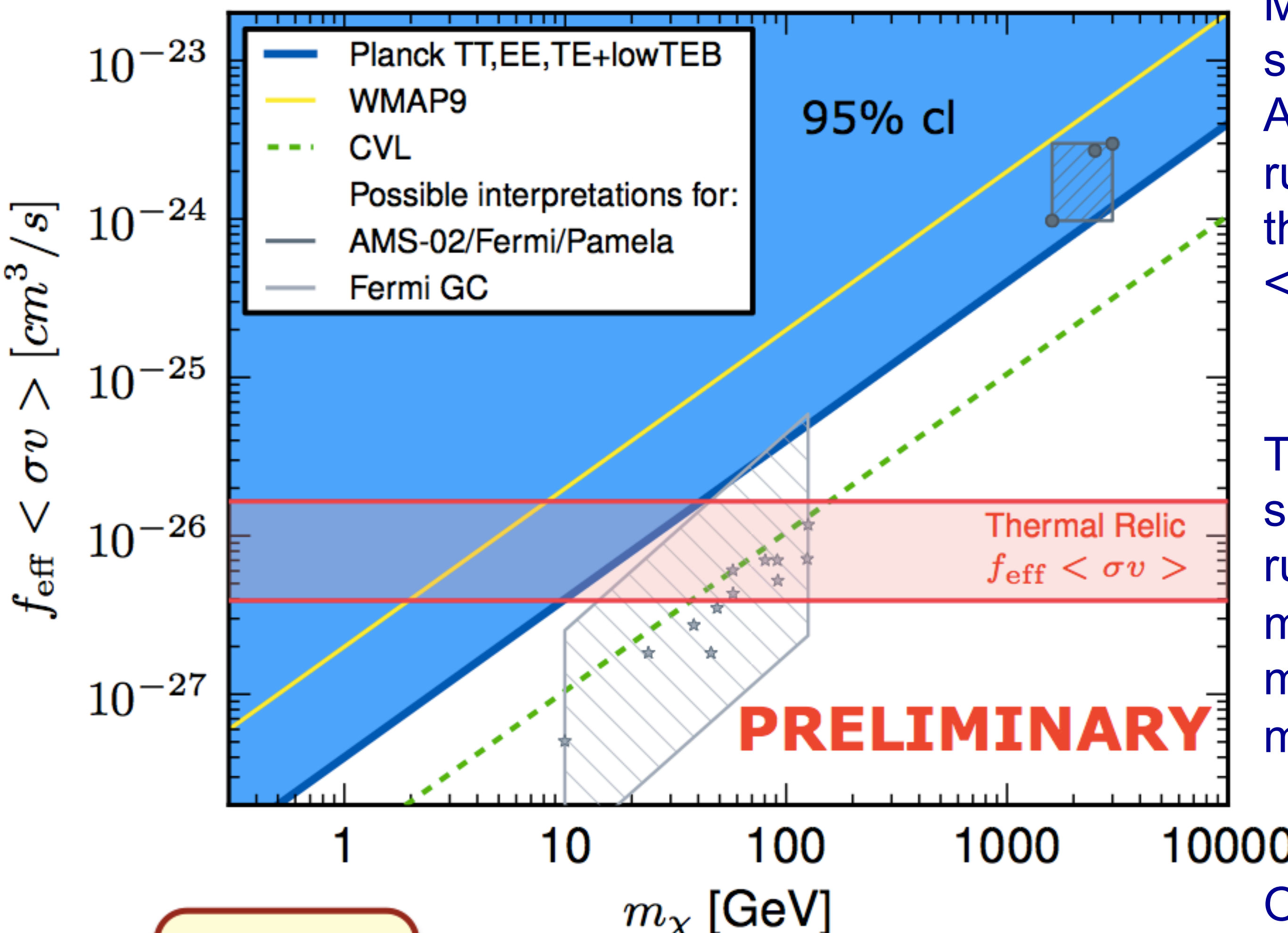
- dark matter annihilation
- pulsars
- something else

→ SUSY WIMPs should lead to cutoff

→ energy injection in CMB?



dark matter annihilation?



Most of parameter space preferred by AMS-02/ Pamela/Fermi ruled out at 95%, under the assumption <σv>(z=100)= <σv>(z=0)

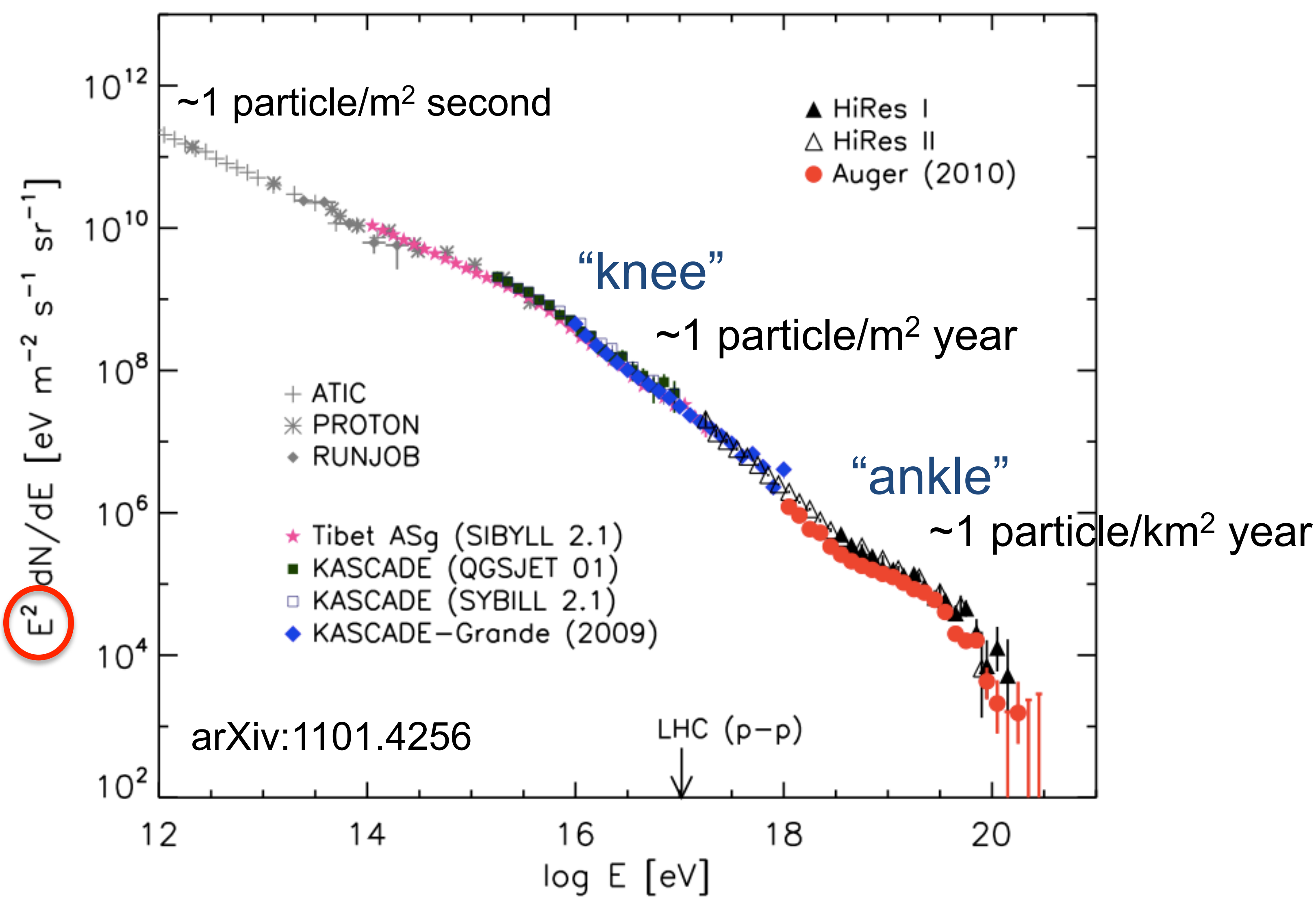
Thermal Relic cross sections at z~1000 ruled out for:
m~<40 GeV (e-e+)
m~<20 GeV (μ+μ-)
m~<10 GeV (τ+τ-).

$$p_{ann} = \boxed{f_{eff}\frac{<\sigma v>}{m_\chi}}$$

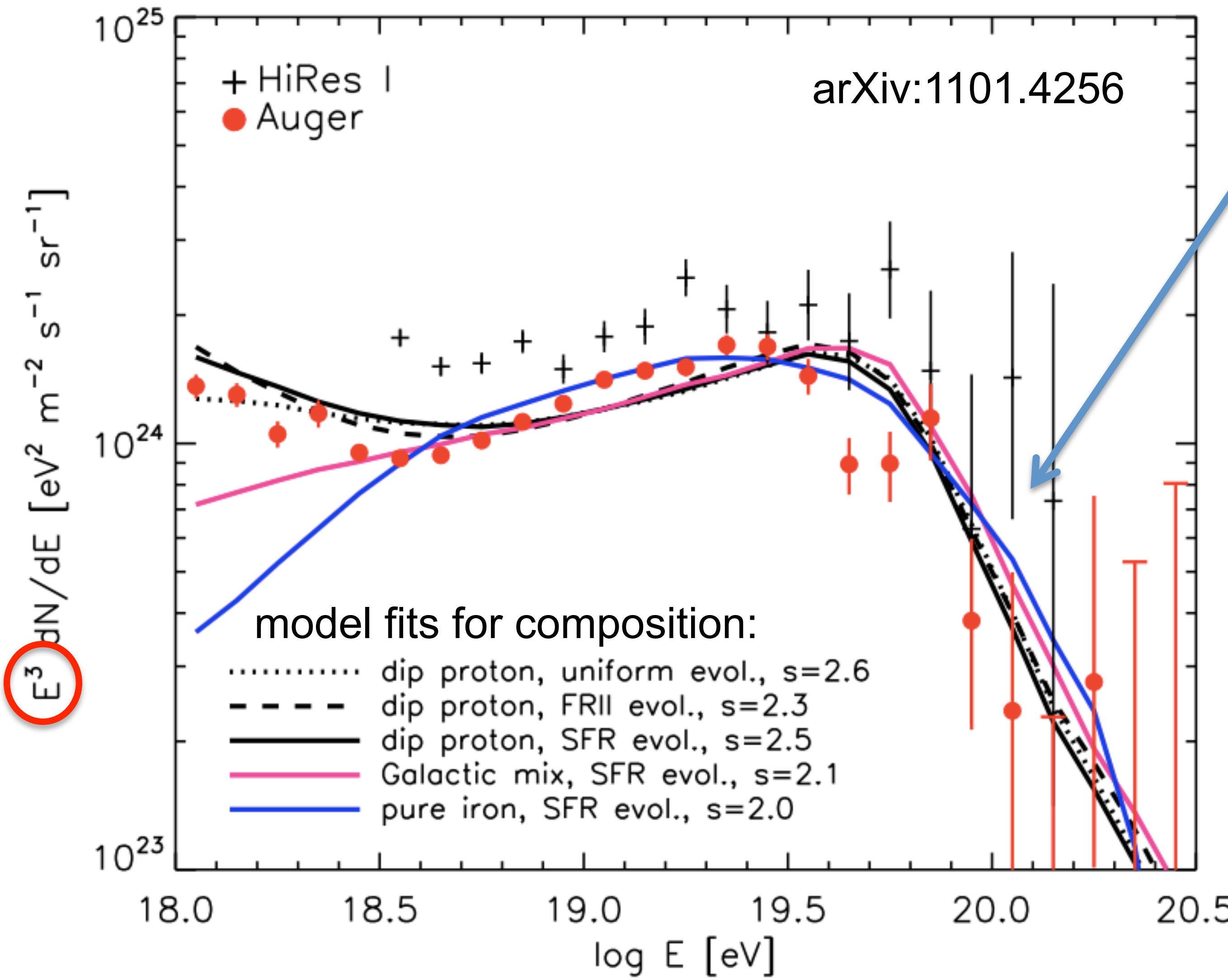
no detection of any energy inject in data

Only a small part of the parameter space preferred by Fermi GC is excluded

ultra high energy CR



ultra high energy CR



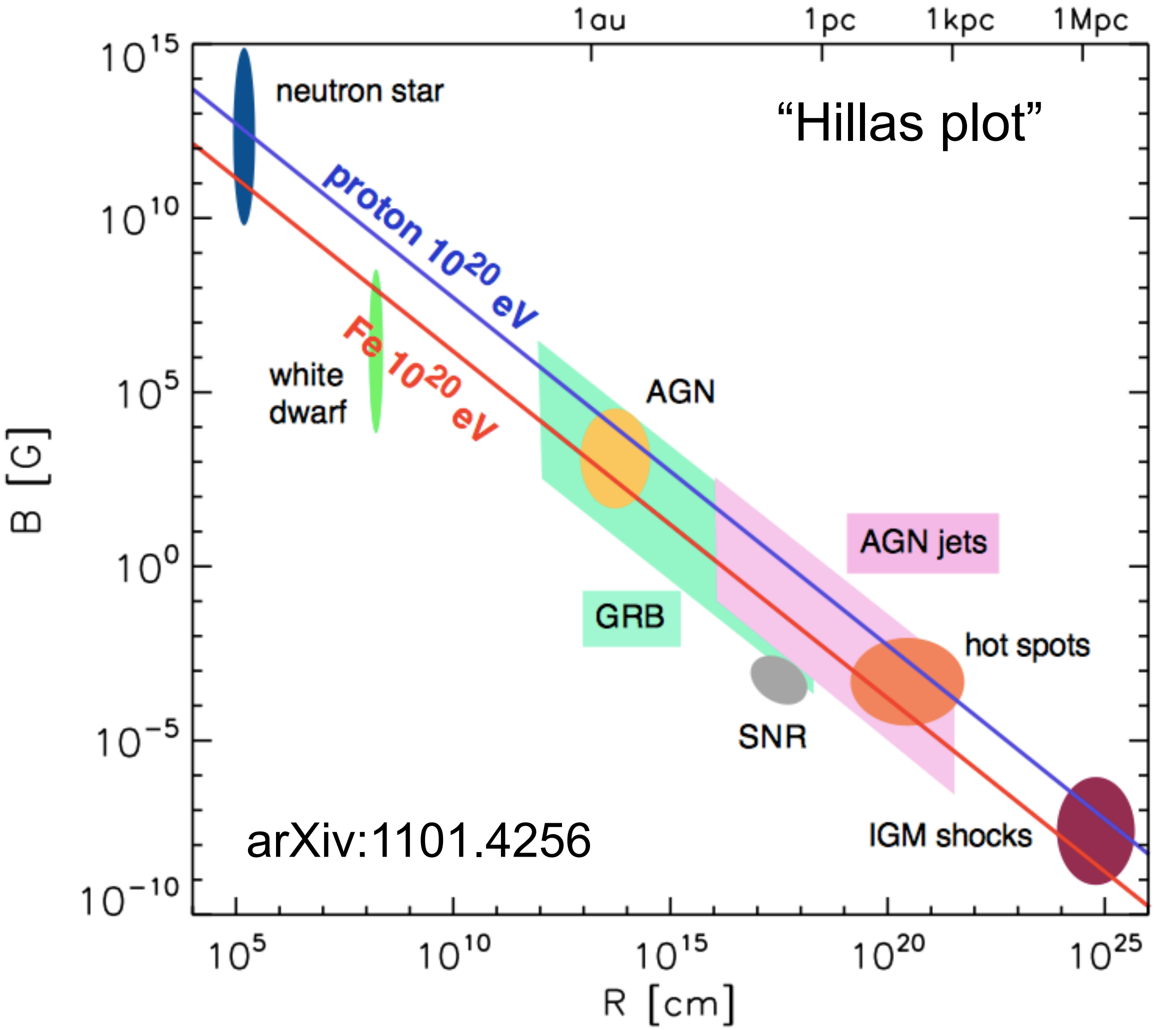
GZK cutoff(?)

$E>10^{15}$ eV:
e^+e^- pair production with CMB: attenuation over ~600 Mpc

$E>10^{20}$ eV:
pion production attenuation over ~100 Mpc

origin of UHECR

- difficult to accelerate particles to such high energies
- some candidates, but mechanism not yet understood



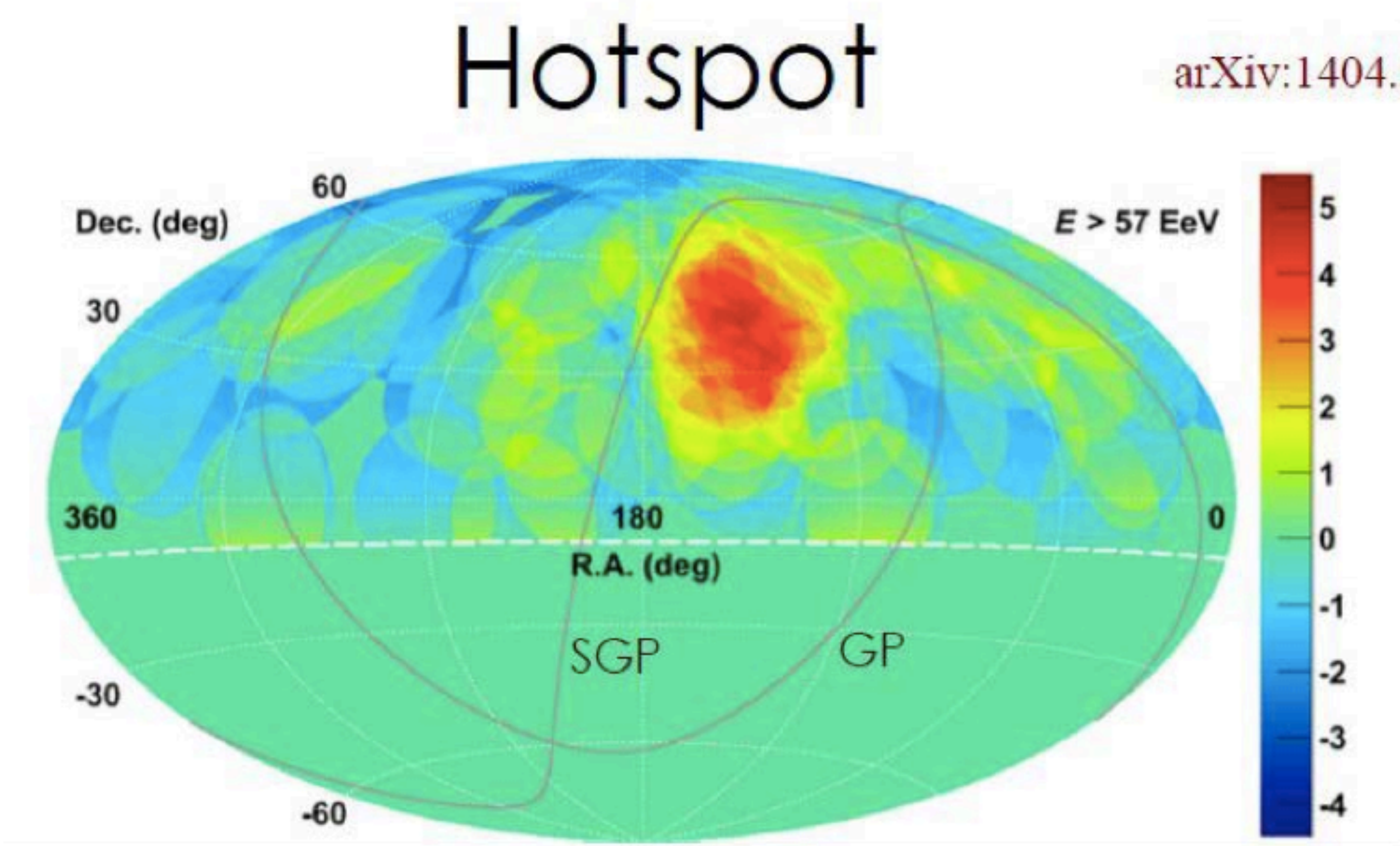
Larmor radius:
$r_L = E/ZeB$

→ UHECR not confined in galactic disk
→ extragalactic origin

confinement:
Hillas condition
$R > r_L$

UHECR astronomy

- searches for sources ongoing since decades, but magnetic field scrambles direction except at highest energies
- situation unclear, detection claims come and go
- 2014: Telescope Array (TA) claims a ~5σ anisotropy – still early days, but keep an eye on this!



direction matches Mrk 421, a BL Lac Blazar

(D ~ 100-150 Mpc)

(thanks, Andrii Neronov!)

neutrino astronomy

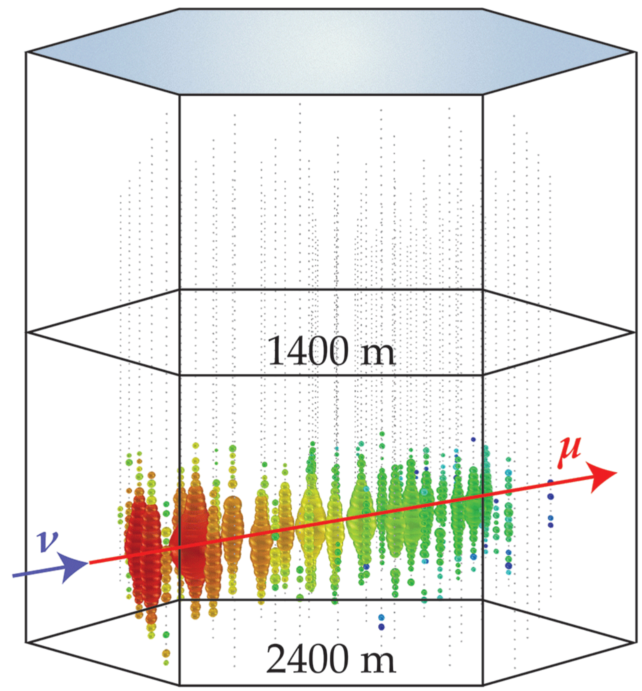
Objects that create high-energy cosmic rays should also create neutrinos; UHECR also create secondary neutrinos in interactions

→ neutrino are not deflected by magnetic fields

→ may point back to the source

IceCube detected 37 events with E ~ 10 TeV to 2 PeV , 6σ in excess of atmospheric background → considered as strong evidence for astrophysical neutrinos

Now waiting for more statistics…



→ has multi-messenger astroparticle physics finally arrived?

final overview

- the universe is a **BIG** physics laboratory
- but only 1 experiment
- need a model to interpret observations
  (→ need *other* models to test 'the' model!)
- standard model of cosmology:
  - **GR with FLRW metric**
  - **particle SM + CDM + Λ**
  - **inflation-like mechanism in early universe**
- fits data well, but 95% of ingredients unexplained – what is wrong?
- data revolution is ongoing

cosmo resources (tiny subset!)

- Books & lecture notes
  - Scott Dodelson, "Modern Cosmology", AP 2003
  - Ruth Durrer, "The Cosmic Microwave Background", CUP 2008
  - Lots of reviews (e.g. Euclid theory group, arXiv:1206.1225)
  - Wayne Hu's webpage, background.uchicago.edu
  - my (old) lecture notes, http://theory.physics.unige.ch/~kunz/lectures/cosmo_II_2005.pdf
- codes
  - Boltzmann codes: CAMB (camb.info), CLASS (class-code.net), etc
  - cosmoMC (with many likelihoods), cosmologist.info/cosmomc/
  - icosmo, icosmos, Fisher4Cast, etc
- lots of cosmological data sets are publicly available!
  - Planck: http://www.sciops.esa.int/index.php?project=planck&page=Planck_Legacy_Archive
  - WMAP (and others): Lambda archive, lambda.gsfc.nasa.gov
  - supernova data (e.g. supernova.lbl.gov/Union/) , BAO, …