Charm-loop and charmonium effects in $B \to K\ell\ell$

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based on [AK, Th.Mannel, Yuming Wang, arXiv:1211.0234[hep-ph]]

□ $B \to K\,\ell^+\ell^-$, the effective Hamiltonian

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu)\Big|_{\mu\sim m_b}$$

- hierarchy of operators and Wilson coeffs:

$O_{9(10)} = \frac{\alpha_{em}}{4\pi}[\bar{s}_L\gamma_\mu b_L]\ell\gamma^\mu(\gamma_5)\ell$ $O_{7\gamma} = -\frac{em_b}{8\pi^2}[\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b]F^{\mu\nu}$	$C_9(m_b) \simeq 4.2$, $C_{10}(m_b) \simeq= -4.4$ $C_7(m_b) \simeq -0.32$
$O_1^{(c)} = [\bar{s}_L\gamma_\rho c_L][\bar{c}_L\gamma^\rho b_L]$ $O_2^{(c)} = [\bar{c}_L\gamma_\rho c_L][\bar{s}_L\gamma^\rho b_L]$	$C_1(m_b) \simeq 1.1$ $C_2(m_b) \simeq -0.27$
$O_{8g} = -\frac{m_b}{8\pi^2}\bar{s}\sigma_{\mu\nu}(1+\gamma_5)bG^{\mu\nu}$ O_{3-6} - quark-penguin operators	$C_8(m_b) \simeq -0.17$ $\lvert C_{3,4,5,6}\rvert < 0.03$

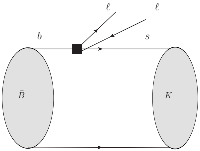
$\sim V_{ub}V_{us}^*$ part neglected

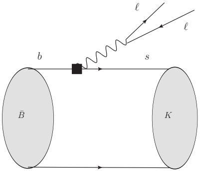
□ The $B \to K\ell^+\ell^-$ decay amplitude

$$A^{(B\to K\ell^+\ell^-)}(q^2) = \frac{G_F}{\sqrt{2}}\frac{\alpha_{em}}{\pi}V_{tb}V_{ts}^*\Bigg[\bar{\ell}\gamma_\mu\ell\, p^\mu\Bigg(C_9 f^+_{BK}(q^2)$$

$$+\frac{2(m_b+m_s)}{m_B+m_K}C_7^{eff} f^T_{BK}(q^2) + \sum_{i=1,2,\dots,6,8} C_i\,\mathcal{H}_i^{(BK)}(q^2)\Bigg) + \bar{\ell}\gamma_\mu\gamma_5\ell\, p^\mu C_{10} f^+_{BK}(q^2)\Bigg]$$

- the main part $\sim C_{9,10}$ and C_7,
  hadronic input: $B \to K$ form factors
- $f^+_{BK}(q^2)$ and $f^T_{BK}(q^2)$,
  QCD on the lattice (large q^2), LCSR (small q^2),
  current accuracy $\sim 10-15\%$





☐ Nonlocal hadronic effects in $B \to K\ell^+\ell^-$

- nonlocal hadronic matrix element for each $O_i = O_{1,2}^{(c)}, O_{3,\ldots,6}, O_{8g}$:

$$\mathcal{H}_i^{(BK)}(q^2) \sim \langle K(p)| i \int d^4x\, e^{iqx}\, T\{j_\mu^{em}(x), O_i(0)\}|B(p+q)\rangle$$

$j_\mu^{em} = \sum\limits_{q=u,d,s,c,b} e_q \bar{q}\gamma_\mu q$ - quark e.m. current,

- separate flavour contributions/topologies,

  to be estimated one by one

- parametrization: $\Delta C_9^{(BK)}(q^2) \equiv \sum\limits_{i=1,2,\ldots,6,8} C_i\, \mathcal{H}_i^{(BK)}(q^2)/f_{BK}^+(q^2)$,

$$\boxed{C_9 \to C_9 + \Delta C_9^{(BK)}(q^2)}$$

process- and q^2-dependent

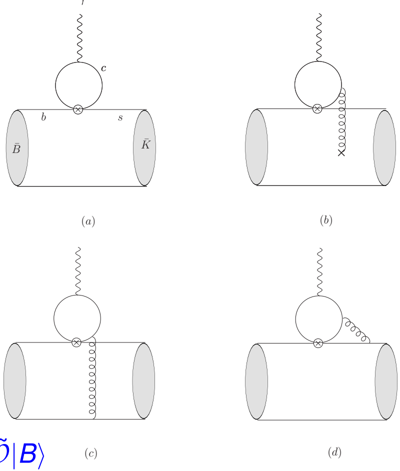
- dominated by c-quark e.m. current and $O_{1,2}^{(c)}$

  $\Rightarrow$ "charm loops" at $q^2 < m_{J/\psi}^2$,

  $\Rightarrow$ $J/\psi, \psi(2S), \ldots$ /open charm pairs at $q^2 \geq m_{J/\psi}^2$

□ Charm loops in $B \to K\ell^+\ell^-$

- at $q^2 \ll 4m_c^2$ virtual $\bar{c}c$-loop, apply OPE
- factorizable part: loop $\otimes f^+_{BK}$
- "soft" gluon emission yields a nonlocal operator $\tilde{\mathcal{O}} \sim \bar{s}Gb$
- each extra gluon $\sim \frac{\Lambda^2_{QCD}}{4m_c^2 - q^2}$



- use LCSR to calculate $\langle K^{(*)}|\tilde{\mathcal{O}}|B\rangle$ *[A.K., Th. Mannel, A. Pivovarov ,Yu-M. Wang, (2010)];*
- perturbative ("hard") gluons: use QCDF *[M. Beneke,T. Feldmann,D. Seidel (2001)]*
- result $\boxed{\Delta C_9^{(BK,\bar{c}c)}(q^2) = \mathcal{H}^{(BK,\bar{c}c)}(q^2)/f^+_{BK}(q^2)}$ valid at $q^2 \ll 4m_c^2$

☐ Nonlocal effects at very large q^2

- at large $\sqrt{q^2} \sim m_b, \gg 2m_c$, local OPE for $\mathcal{H}^{(BK)}(q^2)$
  expansion in $1/\sqrt{q^2} \sim 1/m_b, 1/(2m_c)$,
  [Grinstein, Pirjol (2004)]
- the zero recoil of kaon in $B \to K\ell\ell$ corresponds to $\sqrt{q^2_{max}} = (m_B - m_K) \sim 4.8$ GeV, is that large enough?
- a detailed analysis of low recoil region nonlocal effects, modeling the pattern of quark-hadron duality for charmonium resonances
  [Beylich,Buchalla,Feldmann(2011)],

□ Accessing the $q^2 \gtrsim 4m_c^2$ region from below

- analyticity of the hadronic amplitude in $q^2 \oplus$ unitarity

hadronic dispersion relation

$\Downarrow$

$$\mathcal{H}^{(BK,\bar{c}c)}(q^2) - \mathcal{H}^{(BK,\bar{c}c)}(0) = q^2 \Big[\sum_{\psi=J/\psi,\psi(2S)} \frac{f_\psi A_{B\psi K}}{m_\psi^2 (m_\psi^2 - q^2 - i\, m_\psi \Gamma_\psi^{tot})} + \int_{4m_D^2}^{\infty} ds \frac{\rho(s)}{s(s-q^2-i\epsilon)} \Big]$$

$\Uparrow$

calculable at $q^2 \ll 4m_c^2$

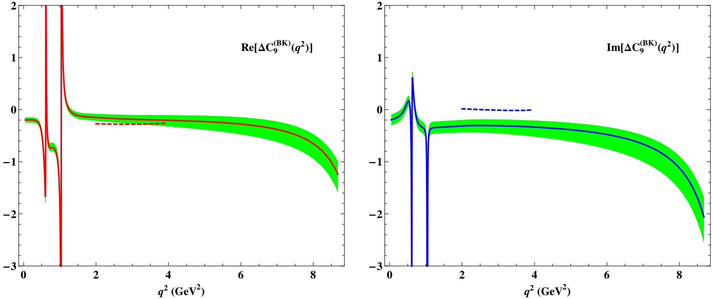
- match l.h.s. and r.h.s. at $q^2 \ll 4m_c^2$
  (previously: double counting $\psi' s \oplus$ loops, neglecting nonfact. contributions, using quark loops at $q^2 \gtrsim 4m_c^2$)

- problem: r.h.s. contains too many hadronic degrees of freedom:
  - $A_{B\psi K} = |A_{B\psi K}| exp(i\delta_{B\psi K})$
    strong final-state interaction, largely nonfactorizable, not calculable in QCD
  - $\rho(s) \sim \sum_{[\bar{c}c]=\psi(3770),...,\bar{D}D,...} f_{[\bar{c}c]} A(B \to [\bar{c}c]K) \delta(s - m^2_{[\bar{c}c]})$
    no model-independent ansatz for this spectral function

- $|A_{B\psi K}|$ and f_ψ taken from $BR(B \to \psi K)$ and $BR(\psi \to \ell^+\ell^-)$, model $\rho(s)$, fit disp. relation to OPE calculation, continue to large q^2

□ $\Delta C^{(BK)}(q^2)$ below J/ψ region

(including all operators and flavours)



the red (blue) solid curve corresponds to the Re (Im) part obtained from the hadronic dispersion relation, fitted to the QCD calculation at q2 < 0 (central input, default parametrization). The shaded areas indicate the uncertainties. The dashed curves correspond to the prediction of QCDF.

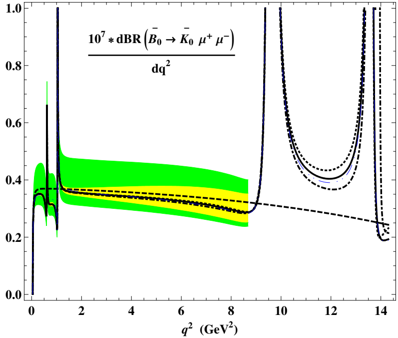
□ $d\mathrm{BR}(B \to K\mu^+\mu^-)/dq^2$ and bins

solid (dotted) lines - central input, default (alternative) parametrization for the dispersion integrals.

long-dashed line -the width calculated without nonlocal hadronic effects.

The green (yellow) shaded area indicates the uncertainties including (excluding) the one from the $B \to K$ FF normalization.

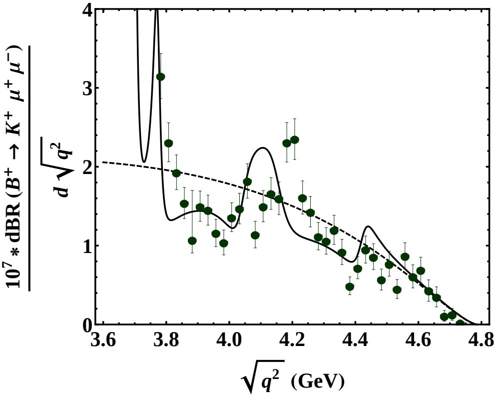
dotted, ... alternative parametrizations of the dispersion integrals



weak dependence on the model of $\rho(s)$ in the low q^2 region

□ Including more ψ resonances (very preliminary !)

- points: recent LHCb data [R. Aaij et al. LHCb Collab. PRL 111, 112003 (2013)]
- dashed line: all nonlocal effects switched off. The observed broad charmonia effects: $\gg$ than our prediction for small q^2 and $\ll$ than for $J/\psi, \psi(2S)$



- solid curve: attempt to fit the LHCb results to the dispersion relation:
  input: low q^2 OPE $\oplus$ |residues| of J/ψ, $\psi(2S)$, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, $\psi(4415)$, model for $\rho(s > m^2_{\psi(4415)})$ fitted/adjusted are the relative phases ; and the parameters of $\rho(s)$
  not a prediction !

☐ Concluding

- nonlocal effects (mainly charm loops) in $B \to K\ell\ell$ are accessible in theory either at $q^2 \ll 4m_c^2$ (light-cone OPE, QCDF, LCSR) or at $q^2 \gg 4m_c^2$ (local OPE, QCDF)
- dispersion relation combined with low q^2 calculation allows to estimate nonlocal effects in $B \to K\ell\ell$ up to $q^2 \sim m^2_{J/\psi}$
- broad charmonia region is hopelessly complicated from the point of view of hadronic degrees of freedom
- contrary to the conclusions drawn in [J.Lyon,R.Zwicky, 1406.0566], (see also the talk by Roman Zwicky at this workshop) our estimates of nonlocal effects $\oplus$ hadronic dispersion relation in gross features do not contradict the LHCb measurements in charmonia region.
- in future both OPE methods, with more accurate data in the charmonia region have the potential to improve the predictions for nonlocal effects at all q^2.

BACKUP SLIDES

Status of $B \to K$ form factors in QCD

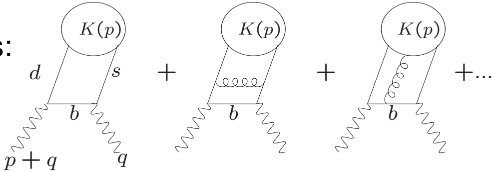
- lattice QCD at $q^2 \geq 15$ GeV^2, recent $n_f = 3$ results:
  [Fermilab-MILC 1211.1390 [hep-lat]; Cambridge-(MILC) 1101.2726[hep-ph]]

- QCD light-cone sum rules (LCSR), at $q^2 \leq 15$ GeV^2:

  correlation function = hadronic sum $\Rightarrow$ $\langle K | j | B \rangle$

  $\Uparrow$ {OPE, light-cone DAs} $\Downarrow$ {quark-hadron duality}

- LCSR with kaon DAs:



- soft-gluon (low virtuality) and hard-gluon effects enter separate terms of OPE
- alternative version of LCSR: B-meson DAs (HQET) and kaon interpolating current (LCSR in SCET)

LCSR results

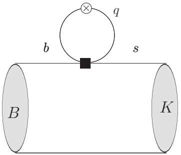
- $q^2 \leq 12 - 15$ GeV^2 accessible,
  complementing the lattice FF's
- LCSR with kaon DAs, the recent update
  [A.K, Th.Mannel, A.Pivovarov, Y-M. Wang (2010)]
- employing z-parameterization:

$$F(q^2) = \frac{F(0)}{1 - q^2/m^2_{B_s(J^P)}} \left\{ 1 + b_1 \left(z(q^2, t_0) - z(0, t_0) + \frac{1}{2}\left[z(q^2, t_0)^2 - z(0, t_0)^2 \right] \right) \right\},$$

$B \to K$ form factor	$F^i_{BK}(0)$	b^i_1	$B_s(J^P)$	input at $q^2 < 12$ GeV^2
f^+_{BK}	$0.34^{+0.05}_{-0.02}$	$-2.1^{+0.9}_{-1.6}$	$B^*_s(1^-)$	
f^0_{BK}	$0.34^{+0.05}_{-0.02}$	$-4.3^{+0.8}_{-0.9}$	no pole	LCSR
f^T_{BK}	$0.39^{+0.05}_{-0.03}$	$-2.2^{+1.0}_{-2.00}$	$B^*_s(1^-)$	with K DA's

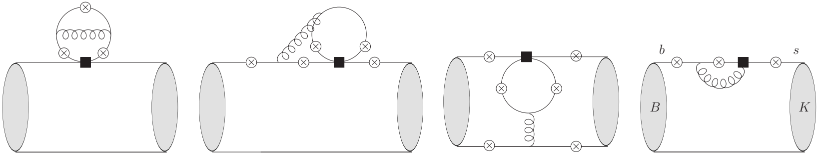
The nonlocal matrix elements

- LO, factorizable and weak annihilation

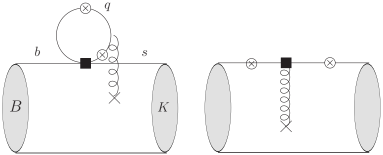


$\otimes$ -virtual photon

- NLO, nonfactorizable ...



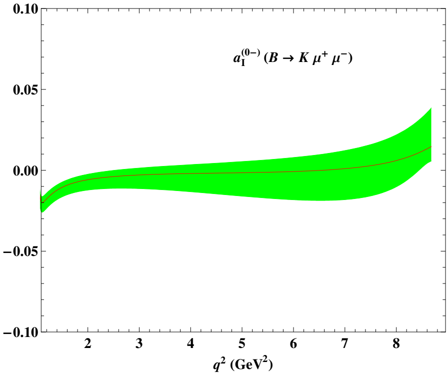
- soft (low virtuality) gluons, nonfactorizable



...

calculated at $q^2 < 0$ and matched to hadronic dispersion relation in the virtual photon channel including $V = \rho, \omega, \phi, ..., J/\psi, \psi(2S)$

Isospin asymmetry: $\bar{B}^0 \to \bar{K}^0 \ell^+\ell^-$ vs $B^- \to K^-\ell^+\ell^-$



- integrated over $1.0 < q^2 < 6.0$ GeV^2.

Belle (2009)	BaBar (2012)	LHCb (2012)	this work
$-0.41^{+0.25}_{-0.20} \pm 0.07$	$-0.41 \pm 0.25 \pm 0.01$	$-0.35^{+0.23}_{-0.27}$	$(-0.4)\% \div (-0.3)\%$

$dBR(B \to K\ell\ell)/dq^2$ in 10^{-7} units integrated over $[q^2_{min} - q^2_{max}]$

$[q^2_{min}, q^2_{max}]$	Belle	CDF	LHCb	LHCb	this work
[0.05, 2.0]	$0.81^{+0.18}_{-0.16} \pm 0.05$	$0.33 \pm 0.10 \pm 0.02$	$0.21^{+0.27}_{-0.23}$	$0.56 \pm 0.05 \pm 0.03$	$0.71^{+0.22}_{-0.08}$
[2.0, 4.3]	$0.46^{+0.14}_{-0.12} \pm 0.03$	$0.77 \pm 0.14 \pm 0.05$	$0.07^{+0.25}_{-0.21}$	$0.57 \pm 0.05 \pm 0.02$	$0.80^{+0.27}_{-0.11}$
[4.3, 8.68]	$1.00^{+0.19}_{-0.08} \pm 0.06$	$1.05 \pm 0.17 \pm 0.07$	1.2 ± 0.3	$1.00 \pm 0.07 \pm 0.04$	$1.39^{+0.53}_{-0.22}$
[1.0, 6.0]	$1.36^{+0.23}_{-0.21} \pm 0.08$	$1.29 \pm 0.18 \pm 0.08$	$0.65^{+0.45}_{-0.35}$	$1.21 \pm 0.09 \pm 0.07$	$1.76^{+0.60}_{-0.23}$

Towards complete analysis of $B \to K^* \ell^+ \ell^-$

- the main challenge: $B \to K^*$ form factors:

only quenched lattice QCD,

LCSR with K^* DAs with $\Gamma^*_K = 0$ (sort of "quenched") [Ball,Zwicky(2004)]

LCSR with B DA's have a large uncertainty;

[AK,Mannel,Pivovarov,Wang(2010)]

- plans to extend the LCSR approach to

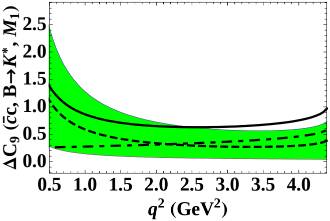
$B \to K\pi\ell\ell$ form factors (embedding K^* and other resonances)

- $\Delta C_9/C_9$ for $B \to K^* \ell^+ \ell^-$ are generally larger at small q^2

(due to $1/q^2$ multiplying $\mathcal{H}_i^{BK^*}$)

- illustration: ΔC_9 in $B \to K^* \ell\ell$

(only charm loop $\oplus$ soft-gluon effect)



a lot of work ahead to assess the hadronic input in $B \to K^* \ell\ell$

☐ Nonfactorizable effects in $B \to [\{\bar{c}c\}]K$ NL decays

- expected to be large, see e.g.: [QCDF: Beneke, Buchalla, Neubert, Sachrajda (2001)]
- are large !

Mode	BR_{exp} [PDG]	factor.approx.
$B^- \to J/\psi K^-$	$(10 \pm 4) \cdot 10^{-4}$	$(1.6 \pm 0.6) \cdot 10^{-4}$
$B^- \to \psi(2S) K^-$	$(6.8 \pm 0.4) \cdot 10^{-4}$	$(0.66 \pm 0.23) \cdot 10^{-4}$
$B^- \to \chi_{c0} K^-$	$(6.0^{+2.4}_{-2.1}) \cdot 10^{-4}$	0
$B^- \to \chi_{c1} K^-$	$(6.8 \pm 1.2) \cdot 10^{-4}$	$(0.61 \pm 0.21) \cdot 10^{-4}$