NLO matched with shower in Monte Carlo scheme for Drell-Yan

Sebastian Sapeta

CERN

in collaboration with:
S. Jadach, W. Płaczek, A. Siódmok and M. Skrzypek

EW precision physics at the LHC, CERN, 30 June 2014

This work is partly supported by the Polish National Science Centre grant DEC-201103BST202632 and UMO-201204MST200240.

Outline and motivation

I will talk about **a new method for NLO+PS matching applied to Drell-Yan process.**

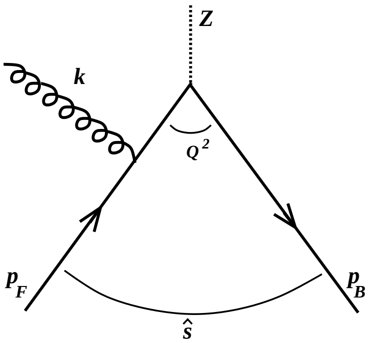
Key ingredients:

- new factorization scheme leading to new MC PDFs
- NLO correction applied to PS via reweighting of MC events

But there are two well established methods... so what is the problem?

- By departing from $\overline{\text{MS}}$, the NLO+PS matching becomes very simple $\rightarrow$ just multiplying by a positive MC weight.
- Possibility of correcting all vertices $\rightarrow$ hence no need for k_T-ordered or truncated showers.
- That alone could have not been worth the effort. But, if it is so simple at NLO, you may hope that pushing it to NNLO+LO PS or NNLO+NLO PS should be possible.

Drell-Yan process ($q\bar{q}$ channel)



$$\hat{s} = (p_F + p_B)^2$$

$$z = \frac{Q^2}{\hat{s}}$$

$$\alpha = \frac{2k \cdot p_B}{\sqrt{\hat{s}}} = \frac{2k^+}{\sqrt{\hat{s}}}$$

$$\beta = \frac{2k \cdot p_F}{\sqrt{\hat{s}}} = \frac{2k^-}{\sqrt{\hat{s}}}$$

$$z = 1 - \alpha - \beta$$

$$k_T^2 = \hat{s}\alpha\beta$$

$$y = \frac{1}{2} \ln \frac{\alpha}{\beta}$$

A generic problem of NLO+PS matching

DY ($q\bar{q}$ channel) cross section at NLO in collinear $\overline{\text{MS}}$ factorization

$$d\sigma^1_{\text{DY}} \quad = \quad \sigma^B_{\text{DY}}\, D_1^{\overline{\text{MS}}}(x_1,\mu^2) \otimes \frac{\alpha_s}{2\pi} C_q^{\overline{\text{MS}}}(z) \otimes D_2^{\overline{\text{MS}}}(x_2,\mu^2)\,,$$

where

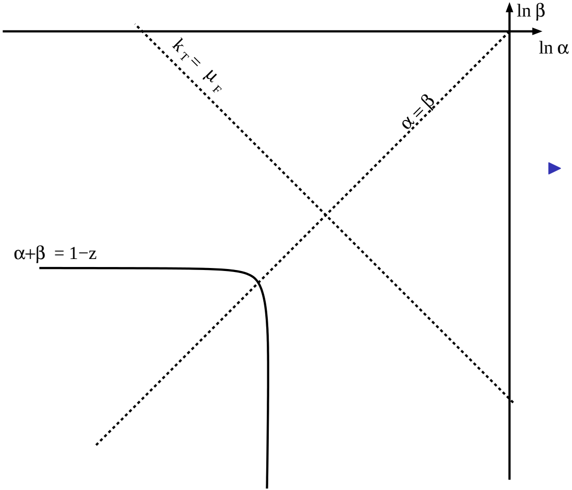
$$C_q^{\overline{\text{MS}}}(z) = C_F\left[4\,(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - 2\frac{1+z^2}{1-z}\ln z + \delta(1-z)\left(\frac{2}{3}\pi^2 - 8\right)\right].$$

We want to reproduce this with Monte Carlo, in a fully exclusive way.

If we decide to use $\overline{\text{MS}}$ PDFs, we need to generate terms of the type $4\,(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+$ that are technical artefacts of $\overline{\text{MS}}$ scheme.

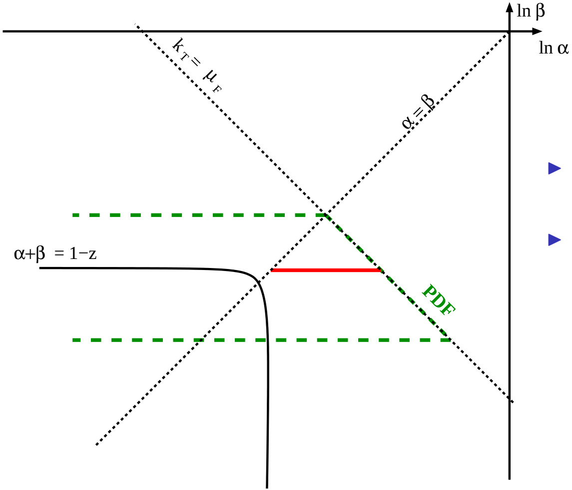
This is problematic since those terms correspond to the collinear limit but Monte Carlo lives in 4 dimensions and not in the phase space restricted by $\delta(k_T^2)$.

Origin of $4\frac{\ln(1-z)}{1-z}$ in $\overline{\text{MS}}$



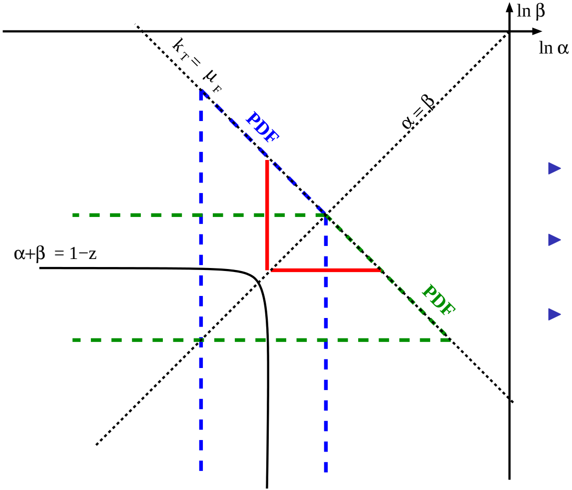
- Integration extends up to a fixed $k_T = \mu_F$.

Origin of $4\frac{\ln(1-z)}{1-z}$ in $\overline{\text{MS}}$



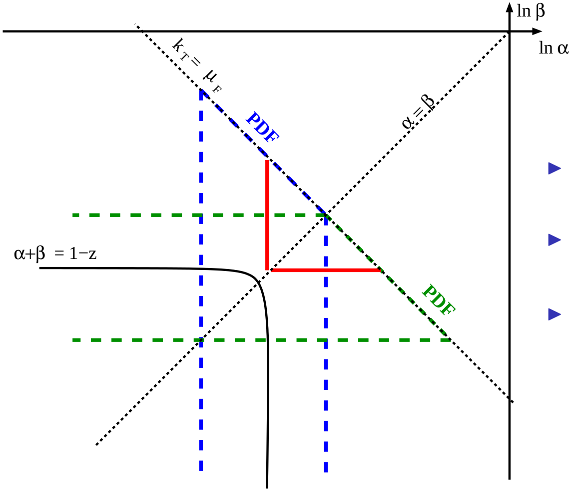
- Integration extends up to a fixed $k_T = \mu_F$.
- For one PDF we get $2\frac{\ln(1-z)}{1-z}$

Origin of $4\frac{\ln(1-z)}{1-z}$ in $\overline{\text{MS}}$



- Integration extends up to a fixed $k_T = \mu_F$.
- For one PDF we get $2\frac{\ln(1-z)}{1-z}$
- Combining two PDFs leads to overcounting by $4\frac{\ln(1-z)}{1-z}$

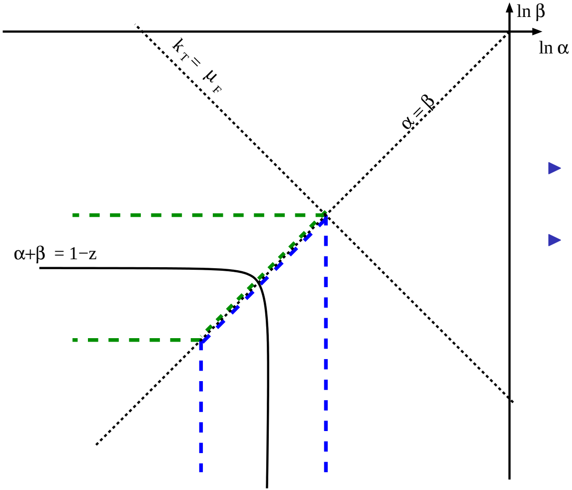
Origin of $4\frac{\ln(1-z)}{1-z}$ in $\overline{\text{MS}}$



- Integration extends up to a fixed $k_T = \mu_F$.
- For one PDF we get $2\frac{\ln(1-z)}{1-z}$
- Combining two PDFs leads to overcounting by $4\frac{\ln(1-z)}{1-z}$

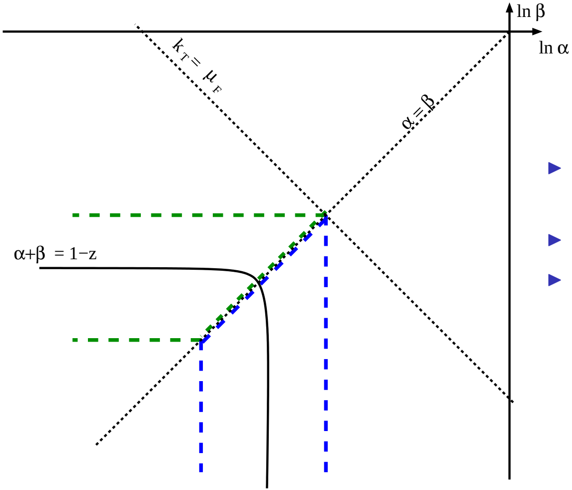
Could we reorganize phase space integration to remove the oversubtraction?

Alternative factorization scheme



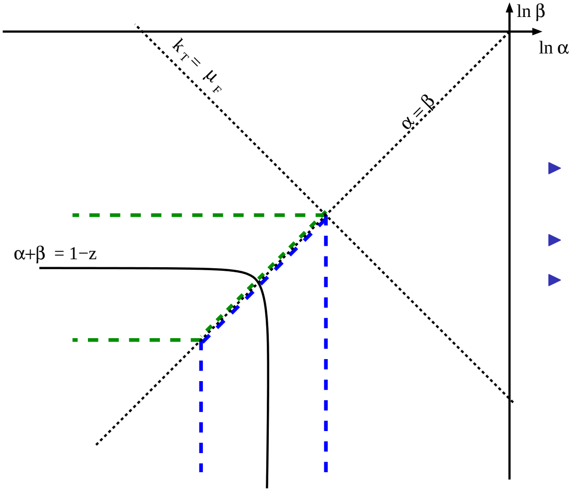
- Integration in angle rather than k_T.
- No overcounting.

Alternative factorization scheme



- Integration in angle rather than k_T.
- No overcounting.
- This is equivalent to saying that the $4\frac{\ln(1-z)}{1-z}$ term gets absorbed into PDFs.

Alternative factorization scheme



- Integration in angle rather than k_T.
- No overcounting.
- This is equivalent to saying that the $4\frac{\ln(1-z)}{1-z}$ term gets absorbed into PDFs.

Could the change of factorization scheme help us to simplify NLO+PS matching?

KRK method [Jadach, Kusina, Płaczek, Skrzypek & Sławińska '13]

- Take a parton shower that covers the (α, β) phase space completely and produces emissions according to approx. matrix element $K \simeq R$.
- Upgrade the real emissions to exact ME by reweighting with R/K.
- Upon integration over transverse d.o.f. this upgraded PS will give an extra term $C_2(z) = \int (R - K)$.
- Redefine PDFs by subtracting the above $C_2(z)$ together with all the z-dependent terms from $\overline{\text{MS}}$ coefficient function. This means transforming PDFs to a new MC factorization scheme.
- Virtual+soft correction, Δ_{S+V}, is just a constant now. Multiply the whole result by $1 + \Delta_{S+V}$ to achieve complete NLO accuracy.

KRK method [Jadach, Kusina, Płaczek, Skrzypek & Sławińska '13]

This has been shown to reproduce exactly the NLO result of fixed order collinear factorization for the case of analytical Parton Shower by Jadach et al.

The MC scheme has been validated by reproducing the scheme-independent relations between DY and DIS.

Could we implement the method in a popular, general purpose MC?

Practical implementation with CS shower

We used Sherpa with the Catani-Seymour (CS) shower.

- The CS shower covers all space of (α, β).
- The evolution variable: $q^2 = (\alpha + \beta)\,\beta\, s$.
- For the $q\bar{q}$ channel:

$$C_2^{\mathrm{CS}}(z) = \int (R - K) = \frac{\alpha_s}{2\pi} C_F \left[-2(1-z)\right] .$$

- Quark and anti-quark PDFs are redefined by:
  - subtracting $-\frac{\alpha_s}{2\pi} C_F\,(1-z)$, coming from $\frac{1}{2} C_2^{\mathrm{CS}}(z)$
  - absorbing $\frac{\alpha_s}{2\pi}\, C_F \left[\frac{1+z^2}{1-z} \ln \frac{(1-z)^2}{z}\right]_+$, coming from $\overline{\mathrm{MS}}$ coeff. function
- Virtual+soft, $\Delta_{V+S} = \frac{\alpha_s}{2\pi} C_F \left(\frac{4}{3}\pi^2 - \frac{5}{2}\right)$, is applied multiplicatively.
- The hardest real emission is upgraded to ME by reweighting.

Two essential steps

- **change the factorization scheme from $\overline{\text{MS}}$ to MC**
  (that means producing new MC PDFs)

- **reweight parton shower**

MC PDFs

Transformation of q and $\bar{q}$ PDFs from $\overline{\mathrm{MS}}$ to MC scheme

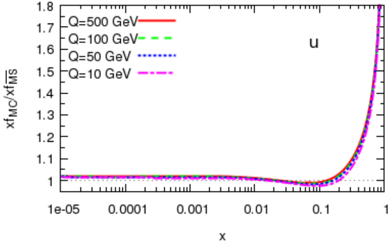
$$q_{\mathrm{MC}}(x, Q^2) = q_{\overline{\mathrm{MS}}}(x, Q^2) + \int_x^1 \frac{dz}{z}\, q_{\overline{\mathrm{MS}}}\left(\frac{x}{z}, Q^2\right) \Delta C_{2q}(z)$$

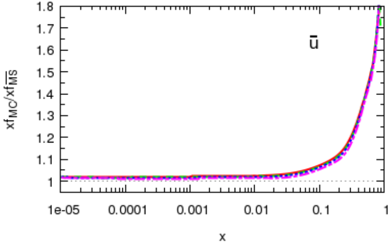
$$\Delta C_{2q}(z) = \frac{\alpha_s}{2\pi}\, C_F \left[\frac{1+z^2}{1-z} \ln \frac{(1-z)^2}{z} + 1 - z\right]_+$$

How big is the difference?

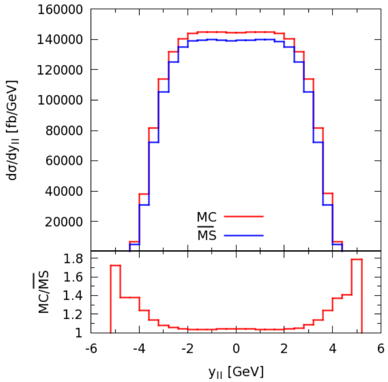
MC PDFs

- Ratios with respect to standard $\overline{\text{MS}}$ PDFs for light quarks.





- Effect on LO cross section



$$x_{1,2} = \frac{m_Z}{\sqrt{s}} e^{\pm y_Z}$$

$\overline{\text{MS}}$ scheme vs MC scheme at NLO

$$
\begin{aligned}
\sigma_{\text{tot}}^{\overline{\text{MS}}} &= f_q \otimes (1 + \alpha_s\, C_q^{\overline{\text{MS}}}) \otimes f_{\bar{q}} \\
\sigma_{\text{tot}}^{\text{MC}} &= (f_q + \alpha_s \Delta f_q) \otimes (1 + \alpha_s\, C_q^{\text{MC}}) \otimes (f_{\bar{q}} + \alpha_s \Delta f_{\bar{q}}) \\
&= f_q f_{\bar{q}} + \alpha_s \left(\Delta f_q f_{\bar{q}} + \Delta f_{\bar{q}} f_q + C_q^{\text{MC}} f_q f_{\bar{q}} \right) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\alpha_s^3)
\end{aligned}
$$

At $\mathcal{O}(\alpha_s)$:

$$
C_q^{\overline{\text{MS}}} f_q f_{\bar{q}} = \Delta f_q f_{\bar{q}} + \Delta f_{\bar{q}} f_q + C_q^{\text{MC}} f_q f_{\bar{q}}
$$

Drell-Yan, $q\bar{q}$ channel, $\alpha_s = \alpha_s(m_Z)$, MCFM, MSTW2008LO

$$
(336.36 \pm 0.09)\,\text{pb} = \underbrace{25.79\,\text{pb} + 25.79\,\text{pb} + 284.77\,\text{pb}}_{(336.35 \pm 0.09)\,\text{pb}}
$$

- Final result is scheme independent up to $\mathcal{O}(\alpha_s)$.
- A nontrivial test as $\Delta f_{q,\bar{q}}$ and C_q come from totally different places. In particular, the former includes convolutions.
- Terms $\mathcal{O}(\alpha_s^2) \simeq 16\,\text{pb}$, for this example; $\mathcal{O}(\alpha_s^3) \simeq 0.2\,\text{pb}$.

CS parton shower

The "Sudakov" form factor

$$S(Q^2, \Lambda^2, x) = \int_{\Lambda^2}^{Q^2} \frac{dq^2}{q^2} \int_{z_{\min}(q^2)}^{z_{\max}(q^2)} dz \;\; K(q^2, z, x)\,,$$

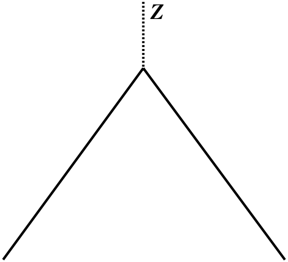
where

$$K(q^2, z, x) = \frac{C_F \alpha_s}{2\pi} \frac{1+z^2}{1-z} \; \frac{D(q^2, x/z)/z}{D(q^2, x)}\,.$$

- z, q^2 - internal variables of the shower
- $D(q^2, x)$ - parton distribution functions

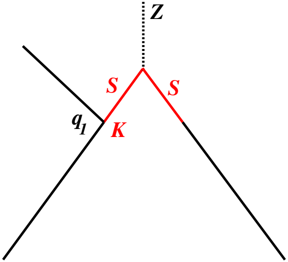
The kernel K is just a CS dipole written in terms of shower's internal variables multiplied by the ratio of PDFs due to backward evolution.

Upgrading to NLO: the hardest emission



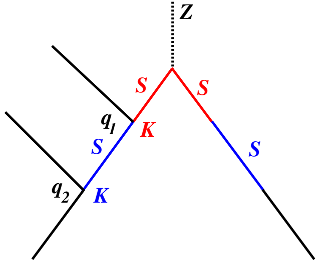
$$\sigma^{\mathrm{LO}} = \sigma_B \otimes D_{\oplus}(Q^2, x_{\oplus}) \otimes D_{\ominus}(Q^2, x_{\ominus})$$

Upgrading to NLO: the hardest emission



$$\sigma_{1+}^{\mathrm{PS}} = \sigma_B \otimes D_{\oplus}(Q^2, x_{\oplus}) \otimes D_{\ominus}(Q^2, x_{\ominus})$$
$$\otimes \Big\{ S_{\oplus}(Q^2, q_1^2) K_{\oplus}(q_1^2, z_1) S_{\ominus}(Q^2, q_1^2) + S_{\ominus}(Q^2, q_1^2) K_{\ominus}(q_1^2, z_1) S_{\oplus}(Q^2, q_1^2) \Big\}$$

Upgrading to NLO: the hardest emission



$$\sigma_{2+}^{\mathrm{PS}} = \sigma_B \otimes D_\oplus(Q^2, x_\oplus) \otimes D_\ominus(Q^2, x_\ominus)$$

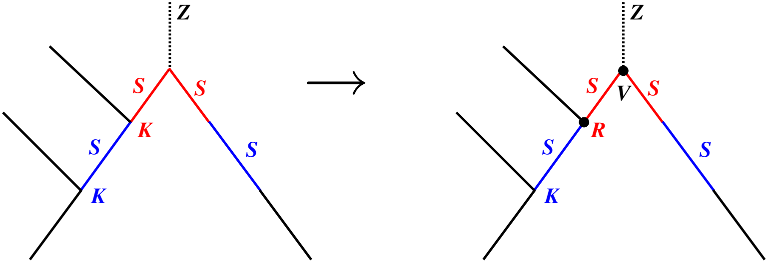
$$\otimes\Big\{ S_\oplus(Q^2, q_1^2) K_\oplus(q_1^2, z_1) S_\ominus(Q^2, q_1^2)$$

$$\otimes\Big\{ S_\oplus(q_2^2, q_1^2) K_\oplus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) + S_\oplus(q_2^2, q_1^2) K_\ominus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) \Big\}$$

$$+\ S_\ominus(Q^2, q_1^2) \otimes K_\ominus(q_1^2, z_1) \otimes S_\oplus(Q^2, q_1^2)$$

$$\otimes\Big\{ S_\oplus(q_2^2, q_1^2) K_\oplus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) + S_\oplus(q_2^2, q_1^2) K_\ominus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) \Big\}\Big\}$$

Upgrading to NLO: the hardest emission



$$\sigma_{2+}^{\mathrm{NLO+PS}} = \sigma_B (1+V) \otimes D_\oplus(Q^2, x_\oplus) \otimes D_\ominus(Q^2, x_\ominus)$$
$$\otimes \Big\{ S_\oplus(Q^2, q_1^2) K_\oplus(q_1^2, z_1) S_\ominus(Q^2, q_1^2)\, R_\oplus(q_1^2, z_1)/K_\oplus(q_1^2, z_1)$$
$$\otimes \Big\{ S_\oplus(q_2^2, q_1^2) K_\oplus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) + S_\oplus(q_2^2, q_1^2) K_\ominus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) \Big\}$$
$$+ S_\ominus(Q^2, q_1^2) \otimes K_\ominus(q_1^2, z_1) \otimes S_\oplus(Q^2, q_1^2)\, R_\ominus(q_1^2, z_1)/K_\ominus(q_1^2, z_1)$$
$$\otimes \Big\{ S_\oplus(q_2^2, q_1^2) K_\oplus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) + S_\oplus(q_2^2, q_1^2) K_\ominus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) \Big\} \Big\}$$

The MC weight for CS shower

Real part:

$$W_R^{q\bar{q}}(\alpha,\beta) = 1 - \frac{2\alpha\beta}{1+(1-\alpha-\beta)^2},$$

where $\alpha = \frac{2k\cdot p_B}{\sqrt{\hat{s}}}$ and $\beta = \frac{2k\cdot p_F}{\sqrt{\hat{s}}}$.

Virtual + soft:

$$W_{V+S}^{q\bar{q}} = \frac{\alpha_s}{2\pi} C_F \left[\frac{4}{3}\pi^2 - \frac{5}{2}\right].$$

- **Real weight is a simple function of kinematic variables.**
  One can compute it on the fly, inside an MC, or outside, using information from event record.
- **Virtual+soft weight is a constant.**
  No need to generate strictly collinear contributions (like $d\Sigma^{c_-^+}$ terms in MC@NLO).

Matched results: total cross section

Schematically:

$$\sigma_{\rm tot}^{\rm MCFM,\overline{MS}} = f_q^{\overline{\sf MS}} \otimes (1+\alpha_s\, C_2^{\overline{\rm MS}}) \otimes f_{\bar q}^{\overline{\sf MS}}\,,$$

$$\sigma_{\rm tot}^{\rm MCFM,MC} = (f_q^{\overline{\sf MS}} + \alpha_s \Delta f_q) \otimes (1+\alpha_s\, C_2^{\rm MC}) \otimes (f_{\bar q}^{\overline{\sf MS}} + \alpha_s \Delta f_{\bar q})\,,$$

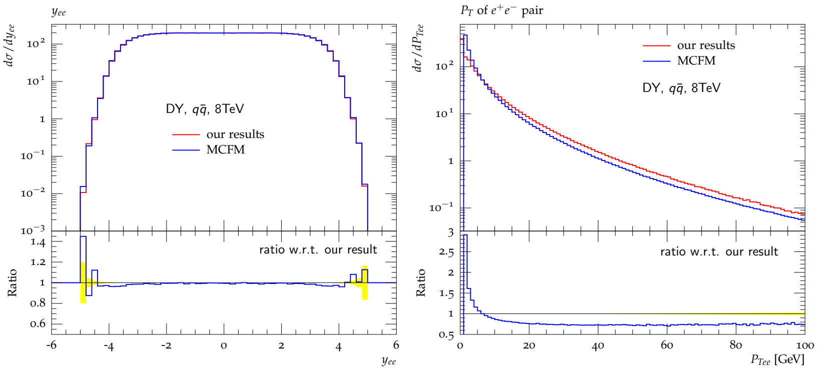
$$\sigma_{\rm tot}^{\rm NLO+PS,MC} = (f_q^{\overline{\sf MS}} + \alpha_s \Delta f_q) \otimes (1+\alpha_s \int K \tfrac{R}{K}) \otimes (1+\alpha_s \Delta_{\rm V+S}) \otimes (f_{\bar q}^{\overline{\sf MS}} + \alpha_s \Delta f_{\bar q}) \otimes e^{-S_q} \otimes e^{-S_{\bar q}}$$

Total cross section for DY, $q\bar{q}$ channel, 8 TeV

	$\sigma_{\rm tot}$ [pb]
MCFM ($\overline{\text{MS}}$ PDFs)	1344.1 $\pm$ 0.1
MCFM (MC PDFs)	1361.6 $\pm$ 0.3
PS (MC PDFs)	1044.1 $\pm$ 0.9
PS+real (MC PDFs)	1031.1 $\pm$ 0.9
PS+full NLO (MC PDFs)	1355.9 $\pm$ 0.8

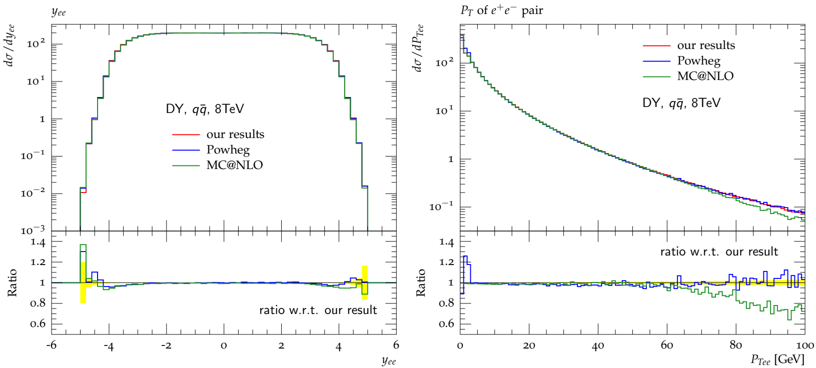
- The difference between fully corrected PS+NLO is at the level of 0.8% w.r.t. MCFM in $\overline{\text{MS}}$ scheme and 0.4% w.r.t. to MCFM in MC scheme.

Matched results: distributions (vs fixed order)



- Excellent reproduction of y_Z distribution at NLO.
- As expected, p_T distribution suppressed at low p_T due to Sudakov.
- Virtual correction spread over a range of p_T.

Matched results: distributions (vs matched results)

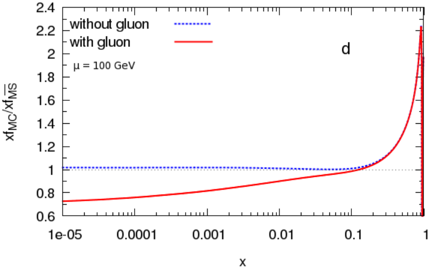


- y_Z and p_T distributions very close to POWHEG
  (difference at low p_T due to slightly different evolution variable)
- y_Z very close to MC@NLO, same for low and intermediate p_T
  (differences for the tail of p_T distributions come from mixed $\mathcal{O}(\alpha_s^2)$ terms)

What about the qg channel?

- Second component in PDFs transformation

$$f_q^{\mathrm{MC}} = f_q^{\overline{\mathrm{MS}}} + f_q^{\overline{\mathrm{MS}}} \otimes \alpha_s \Delta C_q + f_g^{\overline{\mathrm{MS}}} \otimes \alpha_s \Delta C_g$$



- Additional weights

$$W_R^{qg} = 1 + \frac{\alpha(2-\alpha-2\beta)}{1+2(1-\alpha-\beta)(\alpha+\beta)}$$

$$W_{V+S}^{qg} = 0$$

We are almost there. Preliminary results for the total cross section:

	$\sigma_{\mathrm{tot}}^{q\bar{q}+qg}$ [pb]
MCFM	1146.8 ± 0.1
MC@NLO	1144.7 ± 0.6
Powheg	1108.7 ± 0.8
our method	1093.2 ± 1.2

Conclusions

- I have discussed a method of NLO+PS matching:
  - Real emissions are corrected by simple reweighting.
  - Collinear terms are dealt with by putting them to PDFs. This amounts to change of factorization scheme from $\overline{\text{MS}}$ to MC.
  - Virtual correction is just a constant and does not depend on Born kinematics.
- The method has been implemented on top of Catani-Seymour shower in Sherpa event generator.
- It has been validated against fixed order NLO for Drell-Yan process in $q\bar{q}$ channel.
- First comparisons to MC@NLO and POWHEG.

Near future: qg channel (hence full DY), correction of n emissions, public code.

BACKUP SLIDES

MC@NLO [Frixione & Webber '02]

- Naive addition of PS on top of NLO event leads to double counting since PS will generate contributions already present at NLO.
- These affects both resolvable and non-resolvable emissions.
- MC@NLO fixes that by modifying NLO subtraction procedure.

The first emission is generated according to:

$$d\sigma = \mathbb{S}\, d\phi_B + \mathbb{H}\, d\phi_B d\phi_1 \,,$$

where

$$\mathbb{S} = B + V + C + \int K\, d\phi_1 \,, \qquad \mathbb{H} = R - K \,.$$

This is then followed by the emissions from parton shower.

- NLO accuracy of the above is manifest.

POWHEG [Nason '04]

- Generate the hardest radiation at NLO accuracy.
- Pass the event, with its positive, NLO weight, to PS for further generation of soft radiation.

The formula for generation of NLO accurate hardest emission:

$$d\sigma = \bar{B}^S \left[\Delta_S(Q_0) + \Delta_S(p_T) \frac{R^S}{B} d\phi_1 \right] d\phi_B + R^F d\phi_R \,,$$

where

$$\bar{B}^S = B + V + \int R^S d\phi_1 \,, \qquad R = R^S + R^F \,,$$

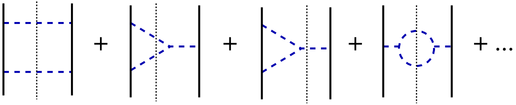
and

$$\Delta_S(p_T) = \exp\left[- \int \frac{R^S}{B} d\phi_1 \Theta(k_T(\phi_1) - p_T) \right] .$$

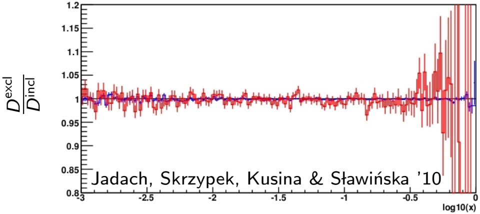
- One can show that the above formula yields NLO accuracy.

Perspectives of including NLO in the ladder

Upgrading kernels to NLO, example diagrams:



- The proof of concept exists! [Jadach, Skrzypek, Kusina & Sławińska '10]
- NLO emissions given above, simulated for the non-singlet case in a toy model MC.
- Perfect agreement with results from standard collinear factorization.



- Take the same path as for NLO+PS matching and try to transfer the toy model solution to a realistic MC generator.

Fixed order calculations in QCD

General structure of NLO cross sections:

$$d\sigma = \Big[B + V(\alpha_s) + C(\alpha_s)\Big] d\phi_B + R(\alpha_s)\, d\phi_B d\phi_1$$

- B, R, V - Born, real and virtual part
- C - collinear subtraction counterterm (for initial state radiation case)

Each part: V, C and $\int R d\phi_1$ is separately divergent (soft and collinear). Divergences cancel in the sum.

Calculation possible e.g. by means of subtraction procedure

$$\begin{aligned} d\sigma &= \Big[B + V(\alpha_s) + \int_1 A(\alpha_s) d\phi_1 + C(\alpha_s)\Big] d\phi_B + \\ & \int_1 \Big[R(\alpha_s) - A(\alpha_s)\Big]\, d\phi_1 d\phi_B\,, \end{aligned}$$

where $A \simeq R$, such that it reproduces collinear and soft singularities.

- Good for inclusive observables or distributions at high-p_T.

Parton shower

In the collinear region, fixed order calculation becomes unreliable because each α_s^n is accompanied by a large, logarithmic coefficient, $\ln^n$, and

$$(\alpha_s \ln)^n \sim 1 \text{ for all } n\,.$$

These terms must be summed to all orders and this is what the Parton Shower (PS) is aiming at. In the collinear limit

$$d\sigma_{n+1} \simeq d\sigma_n \frac{\alpha_s(q^2)}{2\pi}\frac{dq^2}{q^2}P(z)dz\,.$$

This can be iterated and used to resum all leading log contributions. In particular, non-emission probability (Sudakov form factor) is given by

$$\Delta(q_1, q_2) = \exp\left[-\int_{q_1}^{q_2}\frac{\alpha_s(q^2)}{2\pi}\frac{dq^2}{q^2}\int_{z_0}^{1}P(z)\,dz\right].$$

In Monte Carlo event generators, the scale of i$^{\text{th}}$ emission, q_i, is found by solving

$$\Delta(q_{i-1}, q_i) = R_i\,,$$

where $R_i \in [0,1]$ is a random number and q_{i-1} is a scale of previous emission.

Matching fixed order with parton shower

PS gives correct behaviour at low p_T and only approximate at high p_T.

The production of a gluon with p_{Tg} is given by

$$d\sigma_1^{\rm PS} = B \cdot K(p_{Tg})\,\Delta(Q, p_{Tg})\,d\phi_B d\phi_1\,,$$

where $K \simeq R$ and the Sudakov $\Delta(Q, p_{Tg})$ suppresses emissions between scales Q and p_{Tg}.

LO+PS can be achieved by upgrading $B \cdot K$ to the exact R

$$d\sigma_1^{\rm LO+PS} = R(p_{Tg})\Delta(Q, p_{Tg})\,d\phi_B d\phi_1\,.$$