Threshold Resummation and Approximate NNLO of W^+W^- production at the LHC

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Sally Dawson, IL, Mao Zeng, PRD88 (2013) 054028

EW Miniworkshop

CERN

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Current Status

- Electroweak corrections known to NLO Bierweiler, Kasprzik, Kuhn, Uccirati, JHEP1211, 093 (2012)
  Baglio, Ninh, Weber, PRD 288 (2013) 113005
  Billoni, Dittmaier, Jager, Speckner, JHEP 1312 (2013) 043
  - Contribute less than $1-2\%$ to total cross section at LHC.
- $pp \to W^+W^-$ known at NLO in QCD Ohnemus, PRD44, 1403 (1991); Frixione, NPB410, 280 (1993)
  Dixon, Kunszt, Signer, NPB531, 3 (1998)
  - Increase LO cross section by $\sim 50\%$
  - $gg \to W^+W^-$ contributes another $\sim 3\%$ at 7 TeV and $\sim 4\%$ at 14 TeV.
    Dicus, Kao, Repko, PRD36, 1570 (1987); Glover, van der Bij, Phys. Lett. B219, 488 (1989)
    Binoth et. al. JHEP0612, 046 (2006); JHEP0503, 065 (2005)
  - $3-4\%$ uncertainty from pdfs and scale variation Campbell, Ellis, Williams, JHEP1107, 190 (2013)
- NLO QCD corrections incorporated in MCFM Campbell, Ellis, Williams, JHEP1107, 190 (2013)
- Interface with parton shower in POWHEG Melia, Nason, Rontsch, Zanderighi, JHEP1111, 078 (2011)
  Hamilton, JHEP1101, 009 (2011)
  Hoche, Krauss, Schonherr, Siegert, JHEP1104, 024 (2011)
- Recent work on N^2LO Henn, Melnikov, Smirnov 1402.7078; Caola, Henn, Melnikov, Smirnov 1404.5590;
  Gehrmann, Tancredi, Weihs JHEP 1308 (2013) 070; Gehrmann, von Manteuffel, Tancredi, Weihs JHEP 1406 (2014) 032

Threshold Logs

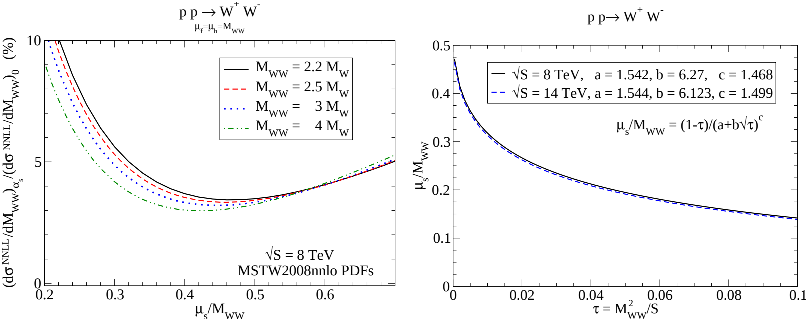
- Will try to improve by resumming threshold logs to NNLL+NLO.
- With these results, will obtain approximate NNLO cross sections fo W pair production.
- Use SCET to perform threshold resummation.
- Two extra ingredients:
  - Hard function consisting of QCD virtual corrections Frixione, NPB410, 280 (1993)
  - Soft function, same as DY known to N^3LL Becher, Neubert, Xu, JHEP0807, 030 (2008)

$$\frac{d\sigma^{Thresh.}}{dMd\cos\theta} = \int_\tau^1 \frac{dz}{z}\mathcal{H}(M,\cos\theta,\mu_f)\mathcal{S}(\sqrt{\hat{s}}(1-z),\cos\theta,\mu_f)\mathcal{L}\left(\frac{\tau}{z},\mu_f\right),$$

- Relevant scale:
  - Evaluate hard function at hard scale: $\mu_h \sim M_{WW}$
  - Evaluate soft function at soft scale: $\mu_s \sim 1 - \frac{M_{WW}^2}{S}$
- Run to factorization scale and resum logs.
- Perform resummation to order NNLL:

$$\alpha_s^n \ln^m(\mu_s/M_{WW}): \quad 2n-3 \leq m \leq 2n$$

Scale Choice



- Soft scale:
  - The soft scale is chosen by minimizing the one-loop contribution to the soft function, allowing perturbative calculation Becher, Neubert, Xu, JHEP 0807, 030 (2008).
  - Enforcing $\mu_s \propto (1-\tau)$ as $\tau = M_{WW}^2/S \to 1$: $\dfrac{\mu_s^{\min}}{M_{WW}} = \dfrac{1-\tau}{(1.542+6.270\sqrt{\tau})^{1.468}}$
  - Little difference between 8 and 14 TeV.
- Hard scale:
  - Hard scale set to scale of hard scattering process: $\mu_h = M_{WW}$.

Matching

- Resummed result depends on factorization that is only valid at threshold.
- Fixed order calculation valid away from threshold, without large logs.
- Need to combine these two results to obtain result valid for all z:

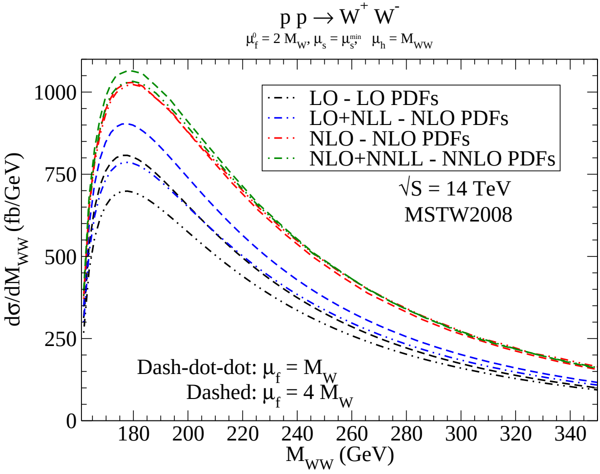
$$d\sigma^{matched} = d\sigma^{Thresh} + d\sigma^{F.O.} - d\sigma^{Leading}$$

- Have introduced the leading singularity term:

$$d\sigma^{Leading} = d\sigma^{Thresh}\big|_{\mu_s=\mu_h=\mu_f}$$

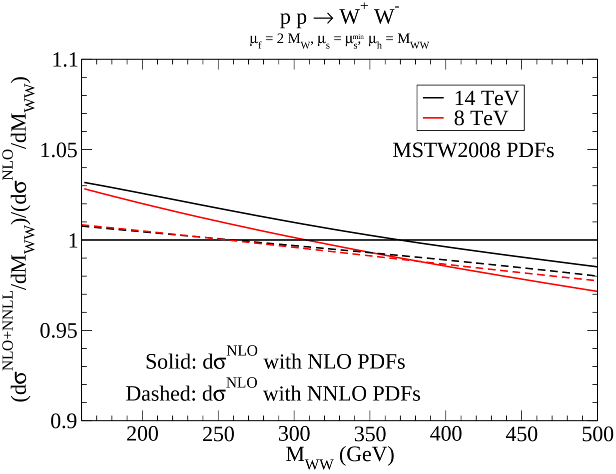
- The leading singularity is subtracted to prevent double counting between the fixed order and resummed results.

Invariant Mass Distribution



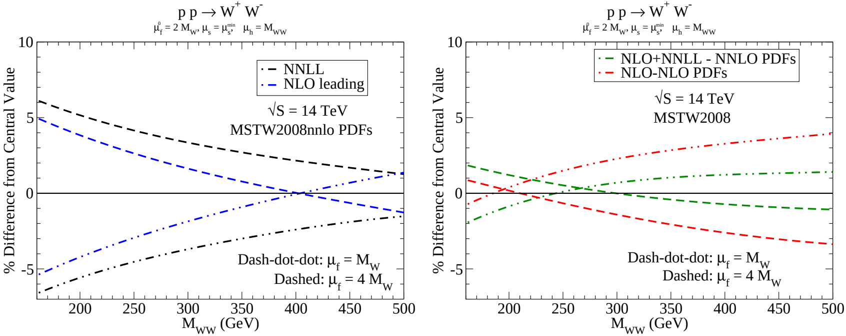
- Increase in cross section at peak.

Invariant Mass Distribution



- Much of the change in shape from NNLO pdfs.

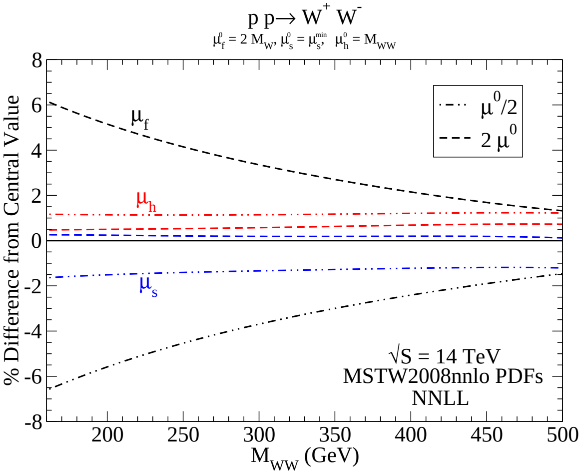
Factorization Scale Dependence



$$d\sigma^{matched} = d\sigma^{Thresh} + d\sigma^{F.O.} - d\sigma^{Leading}$$

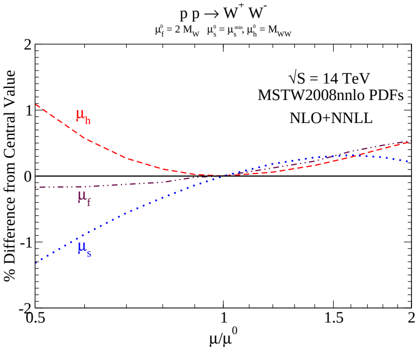
- For $M_{WW} \lesssim 400$ GeV, cancellation between NNLL resummed and leading singularity.
- For $M_{WW} \gtrsim 190$ GeV, cancellation between NNLL resummed and NLO contributions.

Scale Dependence of Threshold Resummed Piece



- Relatively large factorization scale dependence.
- Hard and soft scale dependences smaller and flat with respect to M_{WW}

Full Scale Dependence



$$d\sigma^{matched} = d\sigma^{Thresh} + d\sigma^{F.O.} - d\sigma^{Leading}$$

- In matched contribution, factorization scale dependence cancels among the three pieces.
- Factorization scale dependence less than hard and soft scale dependencies.
- NLO scale dependence: $117.9^{+1.4\%}_{-0.9\%}$ pb
- Factorization scale dependence decreases significantly.

Approximate NNLO

- Scattering kernel known fully up to NLO.
- NNLO piece can be approximated by expanding the scattering kernel in the threshold limit at the factorization scale $\mu_h = \mu_s = \mu_f$ (similar to leading singularity):

$$C(z, M_{WW}, \cos\theta, \mu_f) = \mathcal{H}(M_{WW}, \cos\theta, \mu_f)\mathcal{S}(\sqrt{\hat{s}}(1-z), \mu_f)$$

- Take power expansions of hard and soft functions, then NNLO piece is

$$C_2 = \mathcal{H}_0\mathcal{S}_2 + \mathcal{H}_1\mathcal{S}_1 + \mathcal{H}_2\mathcal{S}_0$$

- Soft function evaluated at $\mu_f = \mu_s$, no running:
  - Soft function same as Drell-Yan, known to NNLO Becher, Neubert, Xu, JHEP 0807, 030 (2008).
  - Captures behavior that is singular as $z \to 1$, that is $\delta(1-z)$ and "plus-functions."

Hard Piece

- Hard piece known fully to NLO, can approximate NNLO piece from RG Running:
  - Expand in powers of logs: $\mathcal{H}_2 = \sum_{n=0}^{4} h^{(2,n)} L_{WW}^n, \quad L_{WW} = \ln \frac{M_{WW}^2}{\mu_f^2}$
  - Insert into RGE get scale dependent pieces, now scale independent up to α_s^3
  - Introduce new scale $Q_h \sim M_{WW}$ and make replacement

$$L_{WW} = \ln \frac{M_{WW}^2}{\mu_f^2} \to \ln \frac{Q_h^2}{\mu_f^2}$$

  - Variations in Q_h correspond to variations in α_s^2 scale independent piece.

Total Cross Section

$\sigma(\text{pb})$	$\sqrt{S}=7$ TeV	$\sqrt{S}=8$ TeV	$\sqrt{S}=13$ TeV	$\sqrt{S}$=14 TeV
σ^{NLO}	$45.7^{+1.5}_{-1.1}$	$55.7^{+1.7}_{-1.2}$	$110.6^{+2.5}_{-1.6}$	$122.2^{+2.5}_{-1.8}$
σ^{gg}	$1.0^{+0.3}_{-0.2}$	$1.3^{+0.4}_{-0.3}$	$3.5^{+0.9}_{-0.7}$	$4.1^{+0.9}_{-0.7}$
$\sigma^{NLO+NNLL}$	$44.9^{+0.6}_{-0.6}$	$54.8^{+0.7}_{-0.8}$	$108.2^{+1.3}_{-1.5}$	$119.5^{+1.5}_{-1.6}$
σ^{NNLO}_{approx}	$45.0^{+0.4}_{-0.1}$	$54.9^{+0.5}_{-0.05}$	$108.3^{+1.0}_{-0.4}$	$119.6^{+1.2}_{-0.5}$
$\sigma'^{NLO+NNLL}$	$45.9^{+0.5}_{-0.6}$	$56.1^{+0.7}_{-0.8}$	$111.7^{+1.8}_{-1.6}$	$123.6^{+2.0}_{-1.8}$
σ'^{NNLO}_{approx}	$46.0^{+0.4}_{-0.047}$	$56.2^{+0.6}_{-0.1}$	$111.8^{+1.7}_{-1.1}$	$123.7^{+1.8}_{-1.2}$

$$\mu_f^0 = 2M_W\,,\quad \mu_h^0 = M_{WW}\,,\quad \mu_s^0 = \mu_s^{\min},\quad Q_h^0 = M_{WW}$$

- *gg* piece included in NLO cross section and primed cross sections.
- Factor of two scale variation around central values.
- Varied independently, added in quadrature.
- *gg* contributes $\sim 3-4\%$ to cross section.
- Increases NLO cross section by $1-3\%$
- Threshold resummed and approximate NNLO cross sections very close, indicating stability of perturbative series with respect to soft gluon emissions.

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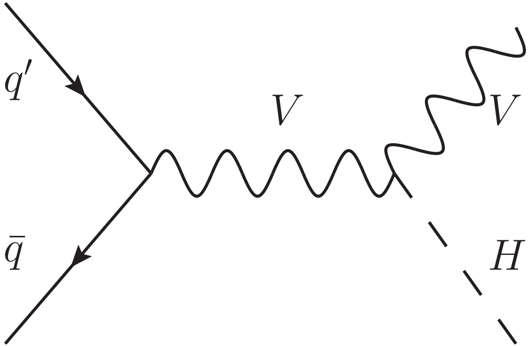
$$\mu_f^0 = M_{WW}\,, \quad \mu_h^0 = M_{WW}\,, \quad \mu_s^0 = \mu_s^{\min} \quad Q_h = M_{WW}$$

- Compare different choices of factorization scale.
- Again, stable against scale variation.

Higgs Associated Production



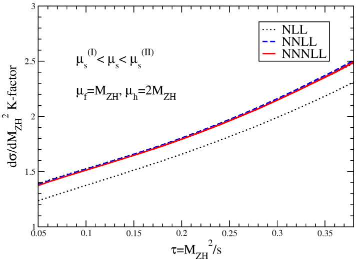
Sally Dawson, Tao Han, Wai-Kin Lai, Adam Leibovich, IL PRD86, 074007 (2012)

Current Status

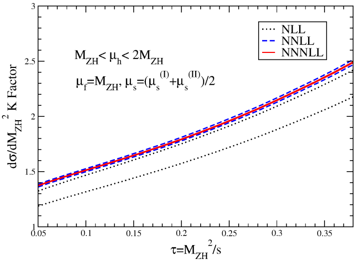
- Known to NNLO in QCD. Brein, Djouadi, Harlander, PLB579 149 (2004); Brein et al, EPJ C72 1868 (2012)
  - NLO increase cross sections by $\sim 20\%$ Han, Willenbrock, PLB2737, 167 (1991)
    Baer, Bailey, Owens, PRD47, 2730 (1993)
    Ohnemus, Stirling, PRD47, 2722 (1993)
  - NNLO increases WH by another $1-2\%$
  - NNLO increases ZH by another $7-8\%$
  - gg initial state makes $\sim 5\%$ contribution to ZH
- NLO electroweak corrections also known Ciccolini, Dittmaier, Kramer, PRD68, 073003 (2003)
  - Decrease cross section by $5-10\%$
- As with W^+W^- and Drell-Yan, has $q\bar{q}$ initial state.
  - Again, use known soft and hard functions.
- Soft scale choices:
  - $\mu_s^I = \dfrac{M_{VH}(1-\tau)}{2\sqrt{1+100\tau}}$ chosen to minimize 1-loop correction to soft piece.
  - $\mu_s^{II} = \dfrac{M_{VH}(1-\tau)}{0.9+12\tau}$ chosen when 1-loop correction drops below 10%
- Hard scale: $\mu_h = 2M_{VH}$

Scale dependence

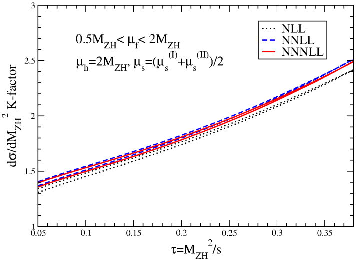
Soft Scale:



Hard Scale:

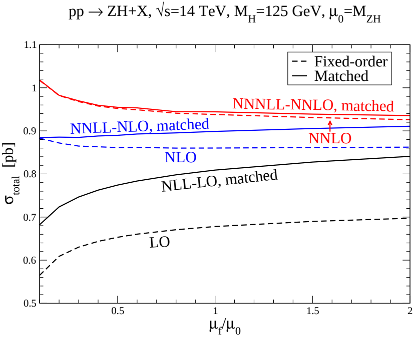


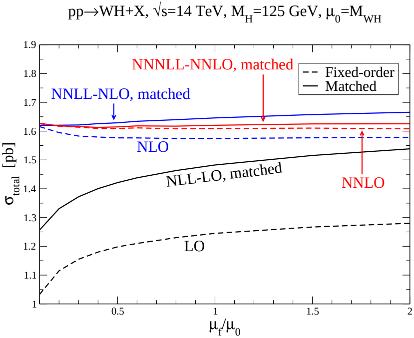
Factorization Scale:



- Scale variation of resummed piece.
- Black dotted: NLL
  Blue: NNLL
  Red: NNNLL
- Area between curves indicates scale variation.
- All cross sections evaluated using MSTW2008NNLO pdfs

14 TeV Cross Sections

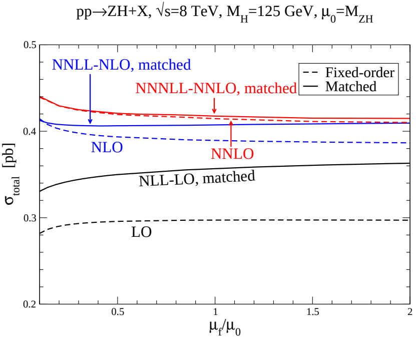


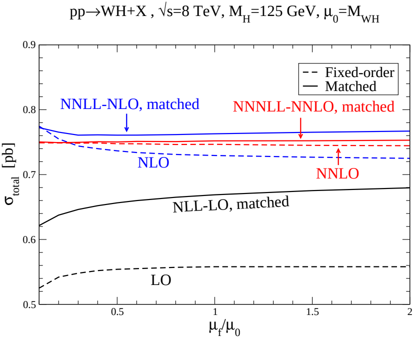


- NNNLL has little effect.
- NNLL increases cross section $\sim 7\%$ for *ZH* and $\sim 3\%$ for *WH*
- Including threshold logs does not introduce added uncertainty.

- $\mu_s = \frac{1}{2}(\mu_s^I + \mu_s^{II})$
- $\mu_h = 2M_{VH}$
- MSTW2008 68% CL
- Use VH@NNLO for fixed order NNLO result
  Brein, Djouadi, Harlander, PLB579, 149 (2004)

8 TeV Cross Sections





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- Use VH@NNLO for fixed order NNLO result
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Conclusions

- Calculated threshold resummed and approximate NNLO cross sections for SM W^+W^- production.
  - Threshold resummation calculated to NNLL
  - Both resummation and approximate NNLO increase NLO cross section by $1-2\%$.
  - Invariant mass distribution slightly increased near peak.
  - Factorization scale uncertainty strongly suppressed.
  - Uncertainties from new scales still less than NLO.
- Calculated threshold resummed cross section for VH production.
  - NNLL resummation increase NLO ZH cross section $\sim 7\%$ and WH by $\sim 3\%$.
  - NNNLL resummation makes little difference to NNLO VH production.
  - Including resummation did not introduce increase scale uncertainty.

BACKUP SLIDES

Threshold Resummation

- QCD factorization allows us to factorize the collinear and hard physics:

$$\frac{d\sigma}{dMd\cos\theta} = \int_{\tau}^{1} \frac{dz}{z} C(z, M, \cos\theta, \mu_f) \mathcal{L}\left(\frac{\tau}{z}, \mu_f\right),$$

  - Hard scattering kernel C
  - Parton luminosity $\mathcal{L}$
  - $z = M^2/\hat{s}$, $\tau = M^2/s$.
- Near partonic threshold have a new scale, the energy of soft emissions $\sqrt{\hat{s}}(1-z)$.

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  - Hard scattering kernel C
  - Parton luminosity $\mathcal{L}$
  - $z = M^2/\hat{s}$, $\tau = M^2/s$.
- Near partonic threshold have a new scale, the energy of soft emissions $\sqrt{\hat{s}}(1-z)$.
- Have additional factorization between soft and hard scales near threshold:

$$C(z, M, \cos\theta, \mu_f) = \mathcal{H}(M, \cos\theta, \mu_f)\mathcal{S}(\sqrt{\hat{s}}(1-z), \cos\theta, \mu_f) + \mathcal{O}(1-z)$$

- Two portions:
  - Hard function $\mathcal{H}$: depends on scale of hard process M
  - Soft function $\mathcal{S}$: depends on energy of soft emitted gluons $\sqrt{\hat{s}}(1-z)$
- Separation of scales suggests EFT approach $\Rightarrow$ Soft Collinear Effective Theory (SCET)

Bauer, Fleming, Luke, PRD63, 014006 (2000)
Bauer, Fleming, Pirjol, Stewart, PRD63, 114020 (2001)
Bauer, Pirjol, Stewart, PRD65, 054022 (2002)
Beneke, Chapovsky, Diehl, Feldmann, NPB643, 431 (2002)

SCET

- Near threshold:

$$\frac{d\sigma}{dMd\cos\theta} = \int_{\tau}^{1} \frac{dz}{z} \mathcal{H}(M, \cos\theta, \mu_f) \mathcal{S}(\sqrt{\hat{s}}(1-z), \cos\theta, \mu_f) \mathcal{L}\left(\frac{\tau}{z}, \mu_f\right),$$

- The appropriate EFT is Soft Collinear Effective Theory (SCET)

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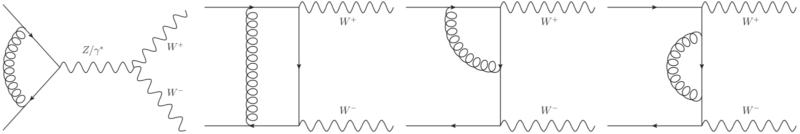
  - EFT consisting of soft and collinear degrees of freedom.
  - Hard virtual modes “integrated out."
- Each component evaluated at their relevant scales:
  - Hard function is a Wilson coefficient evaluated at a hard scale μ_h
  - Soft function evaluated at a soft scale μ_s
- Run components to common scale μ_f via renormalization group equations.
- By choosing $\mu_s \sim \sqrt{\hat{s}}(1-z)$, this running exponentiates and resums large logs.

Hard And Soft Pieces for W^+W^-



- Hard function calculated via Wilson coefficient of SCET operators, C_{WW}:

$$\mathcal{H}(M_{WW}, \cos\theta, \mu_h) = |C_{WW}(M_{WW}, \cos\theta, \mu_h)\mathcal{O}_{WW}|^2$$

- C_{WW} calculated by matching SCET onto full QCD at a hard scale $\mu_h \sim M_{WW}$.
- Known to NLO for a long time Frixione, NPB410, 280 (1993)

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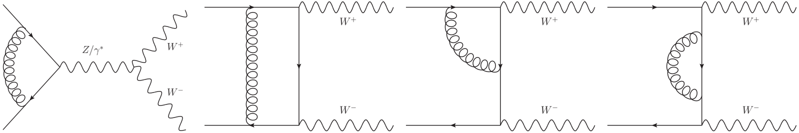

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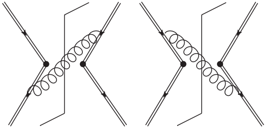
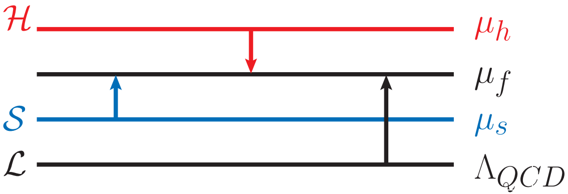


- Soft function describes soft gluon emission.
- Introduce the soft scale μ_s where assume we can do this calculation perturbatively.
- The structure is $q\bar{q} \to$ color singlet, same as Drell-Yan, use known results Becher, Neubert, Xu, JHEP0807, 030 (2008)

RG Running



- Know hard function RGE:

$$\frac{d}{d\ln\mu}\mathcal{H}(M_{WW},\cos\theta,\mu) = 2\left[\Gamma_{\rm Cusp}(\alpha_s)\ln\frac{M_{WW}^2}{\mu^2}+\gamma^V(\alpha_s)\right]\mathcal{H}(M_{WW},\cos\theta,\mu)$$

- In limit $x\to 1$, PDF evolution is known:

$$\frac{d}{d\ln\mu}f_{q/N}(x,\mu) \quad = \quad \int_z^1 P_{q\leftarrow q}(z)f_{q/N}(x/z,\mu)$$

- Total cross section scale-invariant $\Rightarrow$ solve for soft function running in terms of PDFs and hard function.
- Evaluate each piece at appropriate scale then RG evolve to common scale.

Resummed Cross Section

- Since renormalization of SCET operator the same as Drell-Yan, can use previous results to finish calculation Becher, Neubert, Xu, JHEP 0807, 030 (2008):

$$\frac{d\sigma^{Thresh}}{dM_{WW}\,d\cos\theta} = \int_\tau^1 \frac{dz}{z} C(z, M_{WW}, \cos\theta, \mu_f)\mathcal{L}\left(\frac{\tau}{z}, \mu_f\right)$$

$$C(z, M_{WW}, \cos\theta\mu_f) = \mathcal{H}(M_{WW}, \mu_h) U(M_{WW}, \mu_h, \mu_s, \mu_f) \frac{z^{-\eta}}{(1-z)^{1-2\eta}}$$

$$\times \quad \tilde{s}\left(\ln\frac{M_{WW}^2(1-z)^2}{\mu_s^2 z} + \partial_\eta, \mu_s\right)\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}$$

- U arises from RGE running and contains exponentiated logs:

$$\ln U(M_{WW}, \mu_h, \mu_s, \mu_f) = 4S(\mu_h, \mu_s) - 2a_{\gamma^V}(\mu_h, \mu_s) + 4a_{\gamma^\phi}(\mu_s, \mu_f) - 2a_\Gamma(\mu_h, \mu_s)\ln\frac{M_{WW}^2}{\mu_h{}^2}$$

  - S is the Sudakov exponent, and a $(\eta = 2a_\Gamma)$ are subleading logs.

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- Since renormalization of SCET operator the same as Drell-Yan, can use previous results to finish calculation Becher, Neubert, Xu, JHEP 0807, 030 (2008):

$$\frac{d\sigma^{Thresh}}{dM_{WW}\, d\cos\theta} = \int_\tau^1 \frac{dz}{z} C(z, M_{WW}, \cos\theta, \mu_f)\mathcal{L}\left(\frac{\tau}{z}, \mu_f\right)$$

$$C(z, M_{WW}, \cos\theta\mu_f) = \mathcal{H}(M_{WW}, \mu_h) U(M_{WW}, \mu_h, \mu_s, \mu_f) \frac{z^{-\eta}}{(1-z)^{1-2\eta}} \times \tilde{s}\left(\ln\frac{M_{WW}^2(1-z)^2}{\mu_s^2 z} + \partial_\eta, \mu_s\right)\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}$$

- U arises from RGE running and contains exponentiated logs:

$$\ln U(M_{WW}, \mu_h, \mu_s, \mu_f) = 4S(\mu_h, \mu_s) - 2a_{\gamma^V}(\mu_h, \mu_s) + 4a_{\gamma^\phi}(\mu_s, \mu_f) - 2a_\Gamma(\mu_h, \mu_s)\ln\frac{M_{WW}^2}{\mu_h{}^2}$$

  - S is the Sudakov exponent, and a $(\eta = 2a_\Gamma)$ are subleading logs.
- Resums logs of form $\alpha_s^n \ln^m \mu_s/M_{WW}$

Order	Accuracy: $\alpha_s^n \ln^m(\mu_s/M_{WW})$	Γ_{cusp}	γ^h, γ^ϕ	$H, \tilde{s}$
NLL	$2n-1 \leq m \leq 2n$	2-loop	1-loop	tree
NNLL	$2n-3 \leq m \leq 2n$	3-loop	2-loop	1-loop

Approximate NNLO

- From knowledge of hard and soft functions, can construct an approximate NNLO result.
- Expand the scattering kernel in a power series:

$$C(z, M_{WW}, \cos\theta, \mu_f) = C_0(z, M_{WW}, \cos\theta, \mu_f) + \frac{\alpha_s}{4\pi} C_1(z, M_{WW}, \cos\theta, \mu_f) + \left(\frac{\alpha_s}{4\pi}\right)^2 C_2(z, M_{WW}, \cos\theta, \mu_f)$$

- Scattering kernel known fully up to NLO.
- NNLO piece can be approximated by expanding the scattering kernel in the threshold limit at the factorization scale $\mu_h = \mu_s = \mu_f$:

$$C(z, M_{WW}, \cos\theta, \mu_f) = \mathcal{H}(M_{WW}, \cos\theta, \mu_f)\mathcal{S}(\sqrt{\hat{s}}(1-z), \mu_f)$$

- Take power expansions of hard and soft functions:

$$\mathcal{H} = \mathcal{H}_0 + \frac{\alpha_s}{4\pi}\mathcal{H}_1 + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathcal{H}_2, \quad \mathcal{S} = \mathcal{S}_0 + \frac{\alpha_s}{4\pi}\mathcal{S}_1 + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathcal{S}_2$$

- Then:

$$C_2 = \mathcal{H}_0\mathcal{S}_2 + \mathcal{H}_1\mathcal{S}_1 + \mathcal{H}_2\mathcal{S}_0$$

Approximate NNLO

$$C_2 = \mathcal{H}_0 \mathcal{S}_2 + \mathcal{H}_1 \mathcal{S}_1 + \mathcal{H}_2 \mathcal{S}_0$$

- Soft function:
  - Soft function same as Drell-Yan, known to NNLO Becher, Neubert, Xu, JHEP 0807, 030 (2008).
  - Captures behavior that is singular as $z \to 1$
  - Expect to be good approximation.

Approximate NNLO

$$C_2 = \mathcal{H}_0\mathcal{S}_2 + \mathcal{H}_1\mathcal{S}_1 + \mathcal{H}_2\mathcal{S}_0$$

- Soft function:
  - Soft function same as Drell-Yan, known to NNLO Becher, Neubert, Xu, JHEP 0807, 030 (2008).
  - Captures behavior that is singular as $z \to 1$
  - Expect to be good approximation.
- Hard function:
  - Hard function known fully to NLO.
  - Can approximate NNLO piece from RG running.
  - Expand NNLO piece of hard function:

$$\mathcal{H}_2 = \sum_{n=0}^{4} h^{(2,n)} L_{WW}^n, \quad L_{WW} = \ln \frac{M_{WW}^2}{\mu_f^2}$$

  - Insert into RGE and can solve for NNLO scale dependent pieces in terms of LO and NLO pieces:

$$\frac{d}{d\ln\mu}\mathcal{H}(M_{WW},\cos\theta,\mu) = 2\left[\Gamma_{\mathrm{Cusp}}(\alpha_s)\ln\frac{M_{WW}^2}{\mu^2} + \gamma^V(\alpha_s)\right]\mathcal{H}(M_{WW},\cos\theta,\mu)$$

  - $h^{(2,0)}$ only determined via full calculation.

Approximate NNLO

- Have approximate NNLO hard piece:

$$\mathcal{H}_2^{approx} = \sum_{n=1}^{4} h^{(2,n)} L_{WW}^n$$

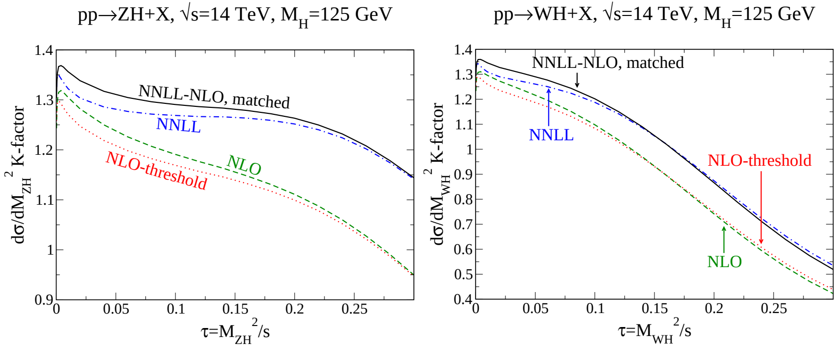
  - Independent of scale up to $\mathcal{O}(\alpha_s^3)$
  - Pieces missing at $\mathcal{O}(\alpha_s^2) \Rightarrow$ simple scale variation underestimates uncertainty.
  - Introduce new scale $Q_h \sim M_{WW}$ and consider replacement

$$L_{WW} = \ln \frac{M_{WW}^2}{\mu_f^2} \rightarrow \ln \frac{Q_h^2}{\mu_f^2}$$

  - $\mathcal{O}(1)$ variations in Q_h corresponds to variations in scale independent piece.
  - Use log expansion and vary Q_h to estimate $\mathcal{O}(\alpha_s^2)$ variation.
- Reproduce all singular pieces up to a scaled independent piece proportional to $\delta(1-z)$:

$$C_2^{approx} = \mathcal{H}_0 \mathcal{S}_2 + \mathcal{H}_1 \mathcal{S}_1 + \mathcal{H}_2^{approx} \mathcal{S}_0$$

Invariant Mass Distribution for *VH*



- K-factor evaluated with LO pdfs for LO distribution and NLO pdfs for all others.