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XXVII International Symposium on Lepton Photon Interactions at High Energies
Ljubljana, Slovenia

Lattice QCD at non-zero Temperature and Density

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K computer

Lattice QCD/Lattice Field Theory

adapted from C. Davies, LP2013

Particle physics

QCD parameters

Hadron spectrum

Hadron structure

Glueballs and exotica

CKM elements

QCD at high temperature and density

Nuclear physics

Nuclear masses and properties

Theories beyond the Standard Model

Composite Higgs, Many-flavor theories, near-conformality
Super-symmetric models, etc

Quantum gravity

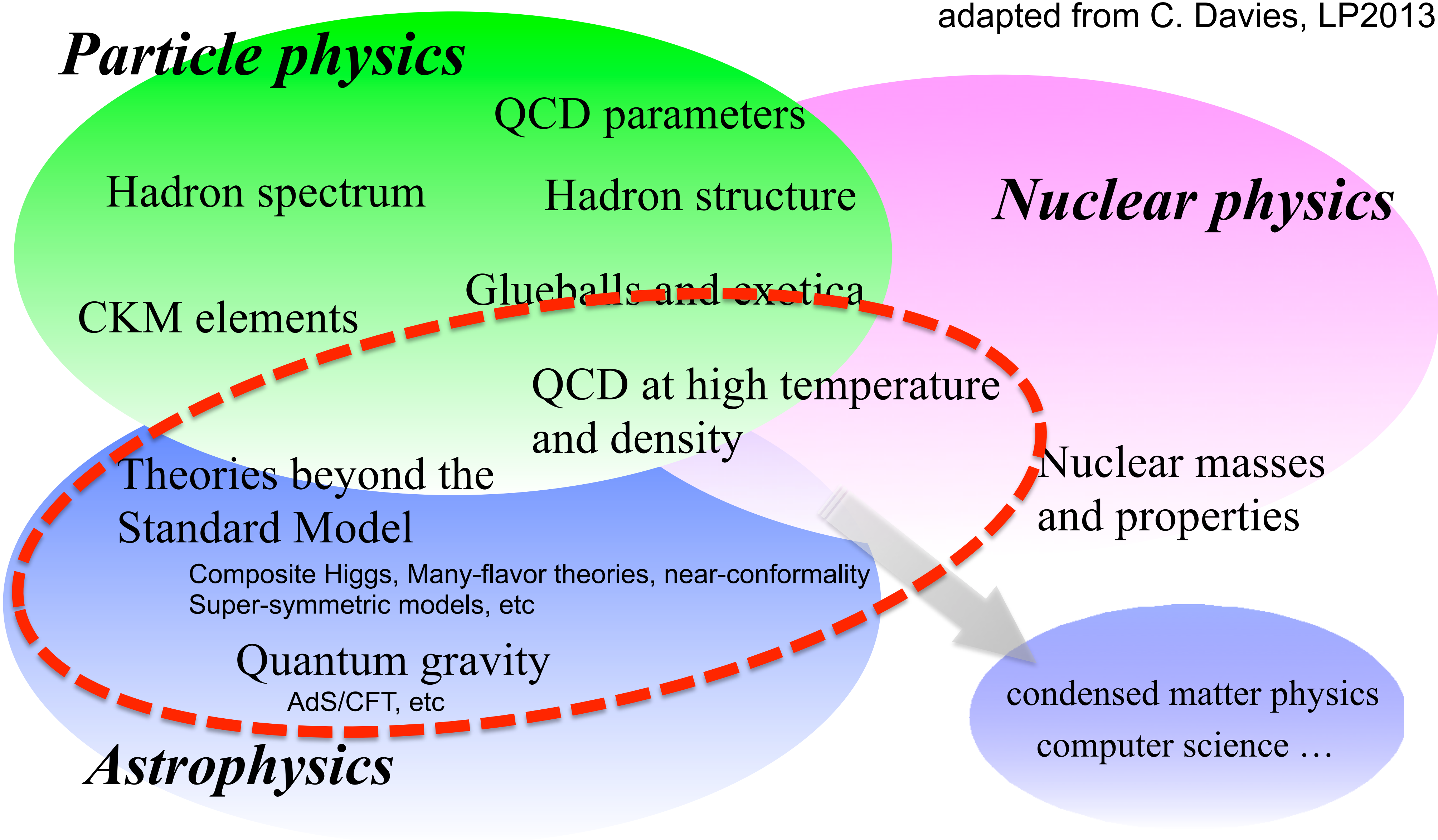
AdS/CFT, etc

Astrophysics

condensed matter physics
computer science …

Lattice QCD/Lattice Field Theory

adapted from C. Davies, LP2013



Plan of talk

- Non-zero temperature but Zero baryon density
  - Phase diagram and equation of state (EoS)
  - Loose ends?
- Non-zero but small baryon density
  - Lattice QCD and comparison with phenomenology
- Conserved charge fluctuations and experiment
- Search for the critical end point?
- Summary

Development since 80's

- Deeper theoretical understandings
- Better numerical algorithms
- O(10^{10}) increase of FLOPS power

Titan

2015

K computer

Sequoia

VAX 11/780

1980

1st Monte Carlo of lattice gauge theory by M. Creutz, L. Jacobs, C. Rebbi

1st hadron mass calculations by D. Weingarten, H. Hamber&G. Parisi

State of the art of lattice QCD

- includes all light quarks (u, d, s), and even charm

  $N_f = 2(\text{degenerate u \&d}) + 1(\text{s})$ or $N_f = 2(\text{u \&d}) + 1(\text{s}) + 1(\text{c})$

- physical values for the quark masses

  $(m_u + m_d)/2 \approx 3.5\text{MeV}, m_s \approx 95\text{MeV}, m_c \approx 1.3\text{GeV} \quad (\overline{MS}\ at\ 2GeV)$

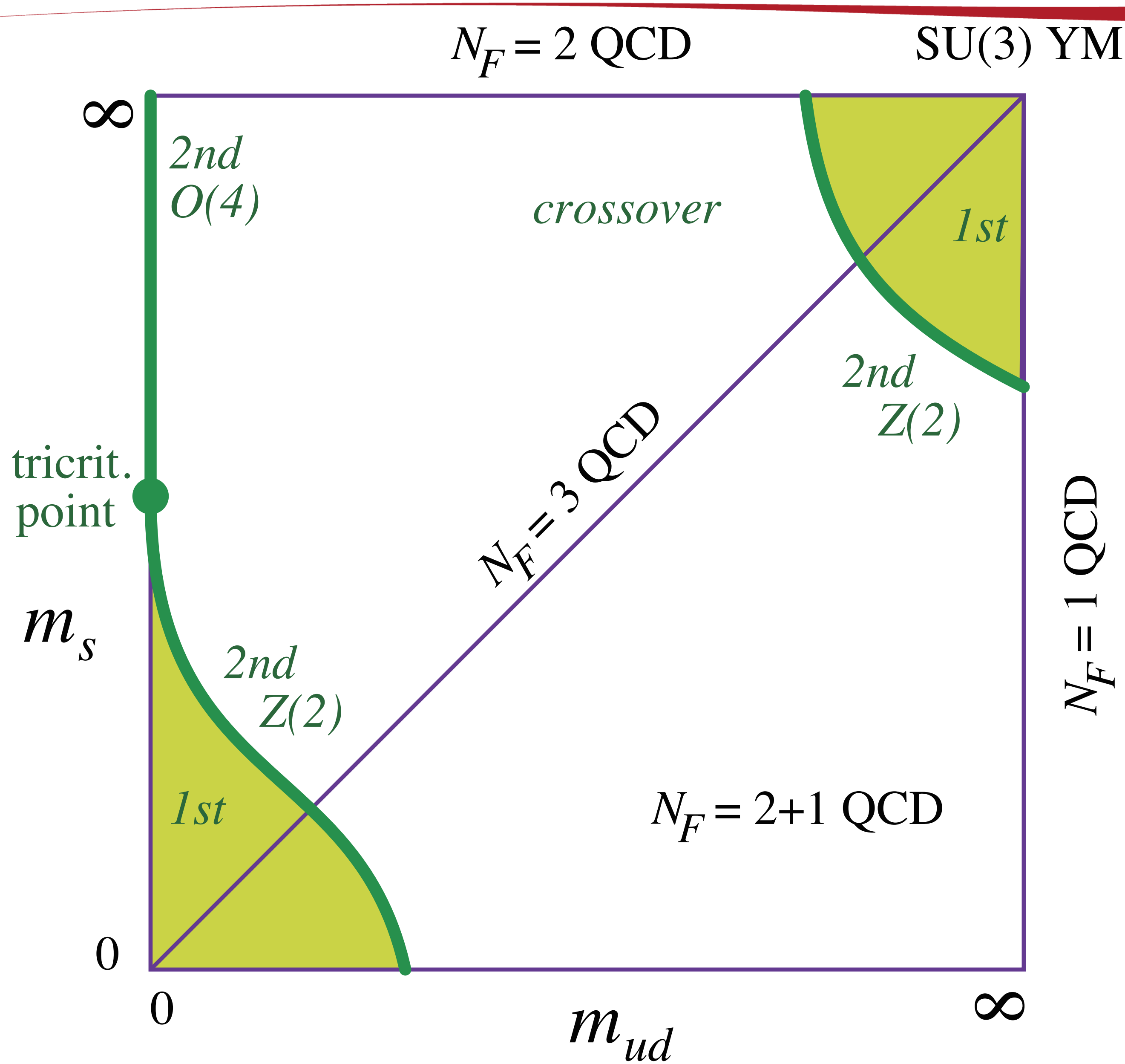
- results are extrapolated to *the continuum limit*

  $a(lattice\ spacing) \approx 0.1\ \text{fm} - 0.05\ \text{fm} \Rightarrow 0(continuum)$

- A word of caution:thermodynamic calculations still use
  - mostly "staggered quark action" with some unwanted properties (continuum limit mandatory)
  - Full-fledged results with chirally symmetric actions (e.g., domain-wall) needs more FLOPS to catch up.

Non-zero temperature but Zero baryon density

“Standard” picture of the QCD phases

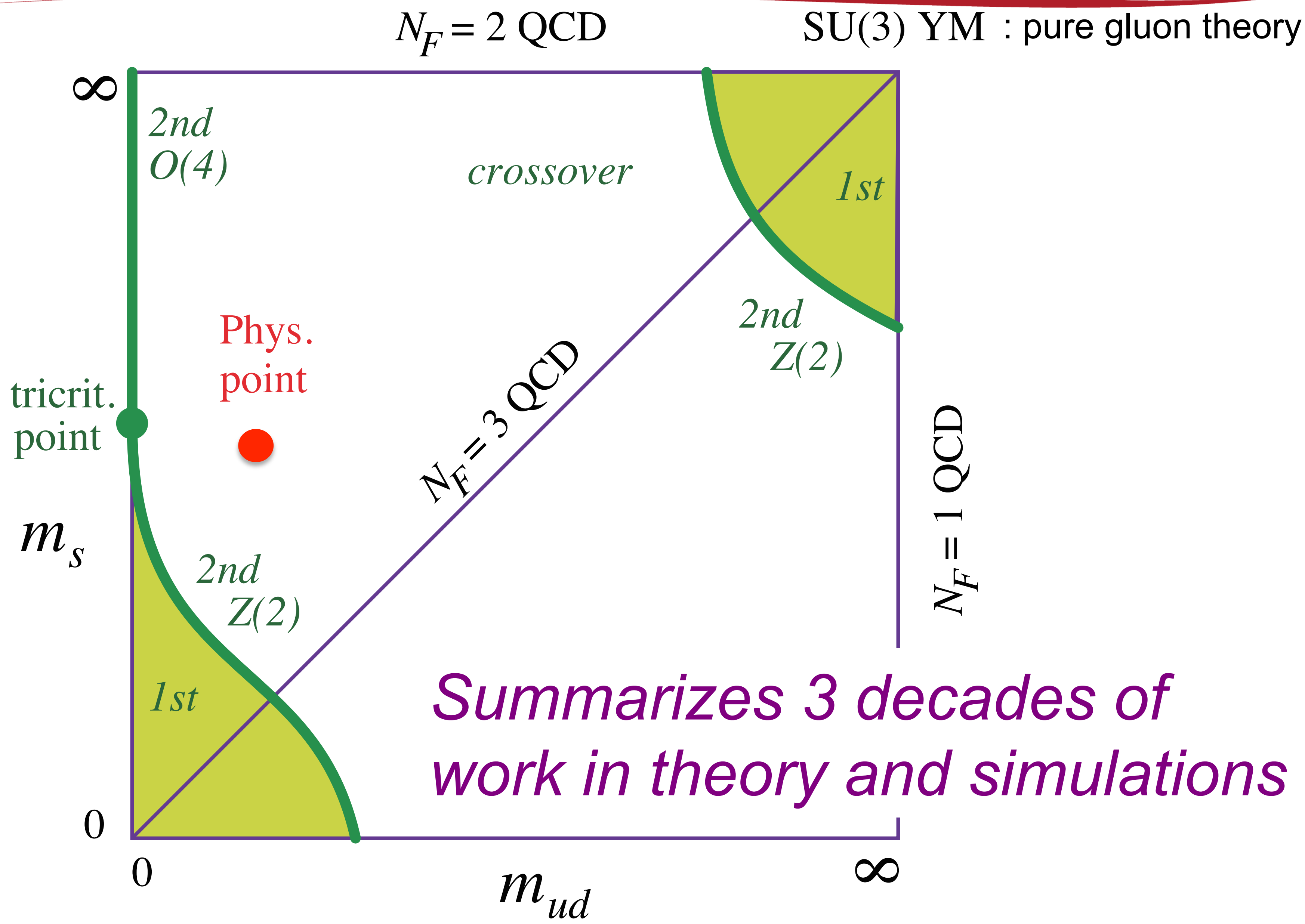


: pure gluon theory

“Columbia Plot”

F. R. Brown et al,
PRL 65, 2491 (1990)

“Standard” picture of the QCD phases

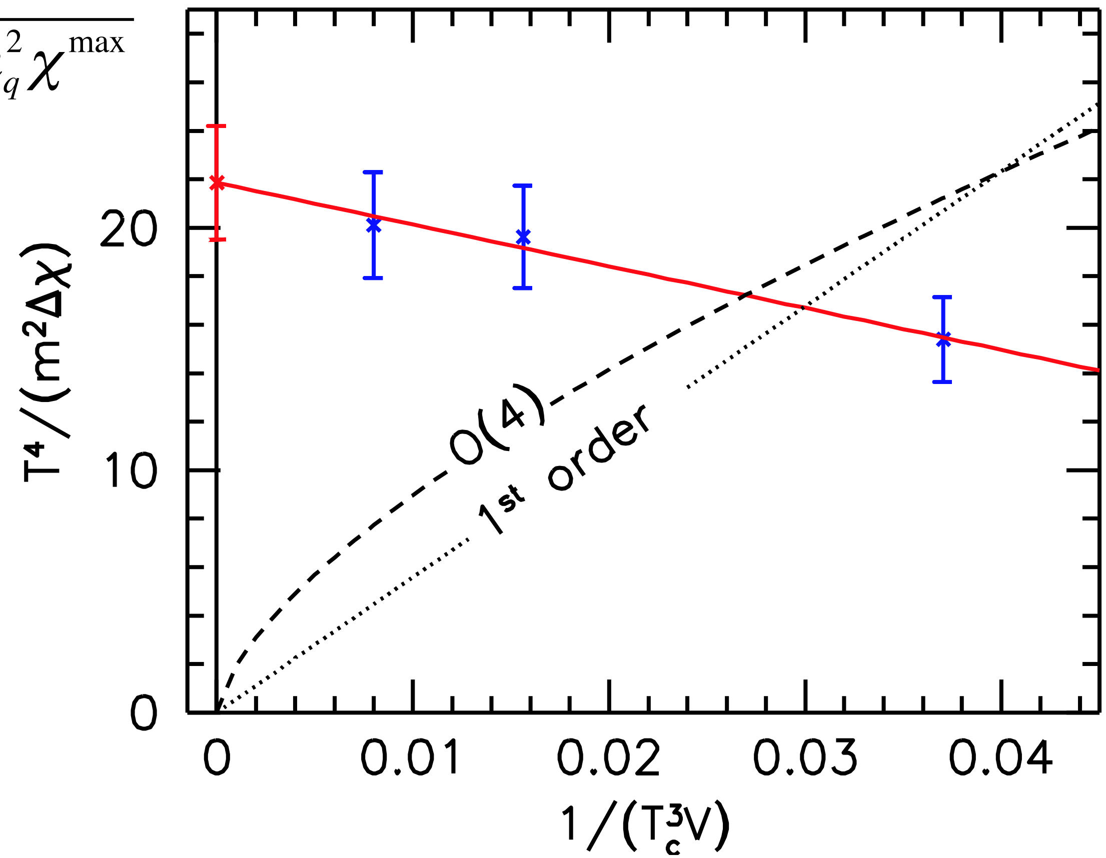


Summarizes 3 decades of work in theory and simulations

Transition at the physical point (I)

- *Continuous crossover* as a function of temperature
  i.e., no jumps or divergences in the observables

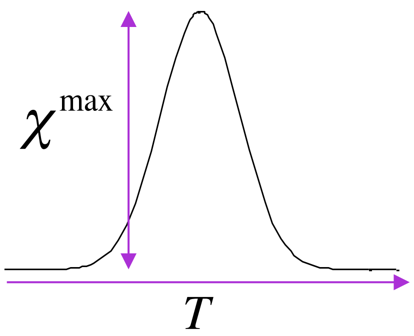
$$\frac{T^4}{m_q^2\chi^{max}}$$



$$\frac{1}{T_c^3 V}$$

Y. Aoki et al,
Nature 443:675 (2006)

$$\chi = \int dx \left(\langle \bar{q}q(x)\bar{q}q(0) \rangle - \langle \bar{q}q(x) \rangle \langle \bar{q}q(0) \rangle \right)$$



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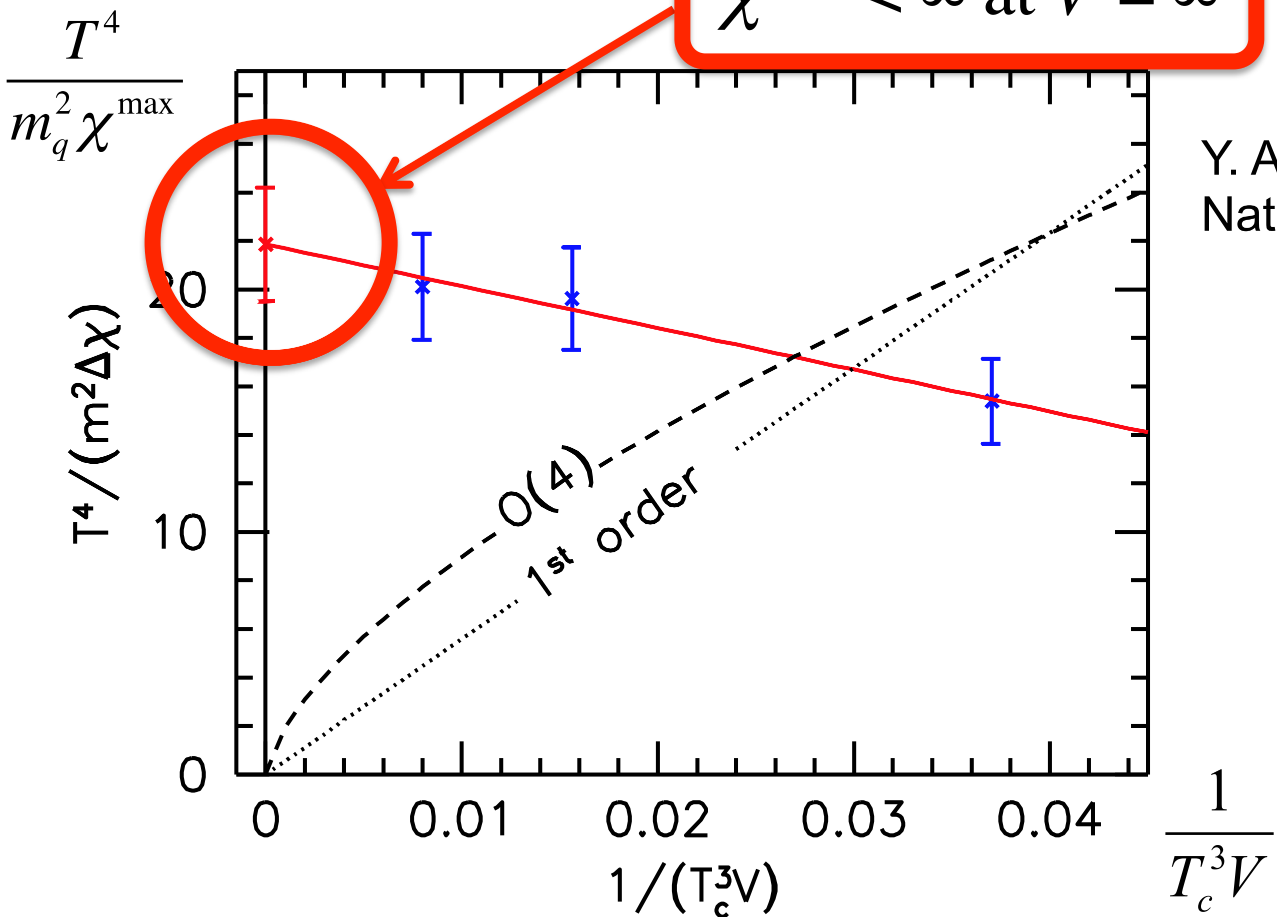
$$\chi^{\max} < \infty \text{ at } V = \infty$$



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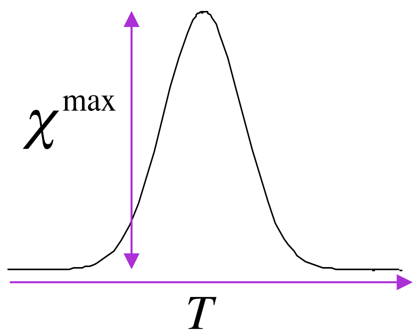
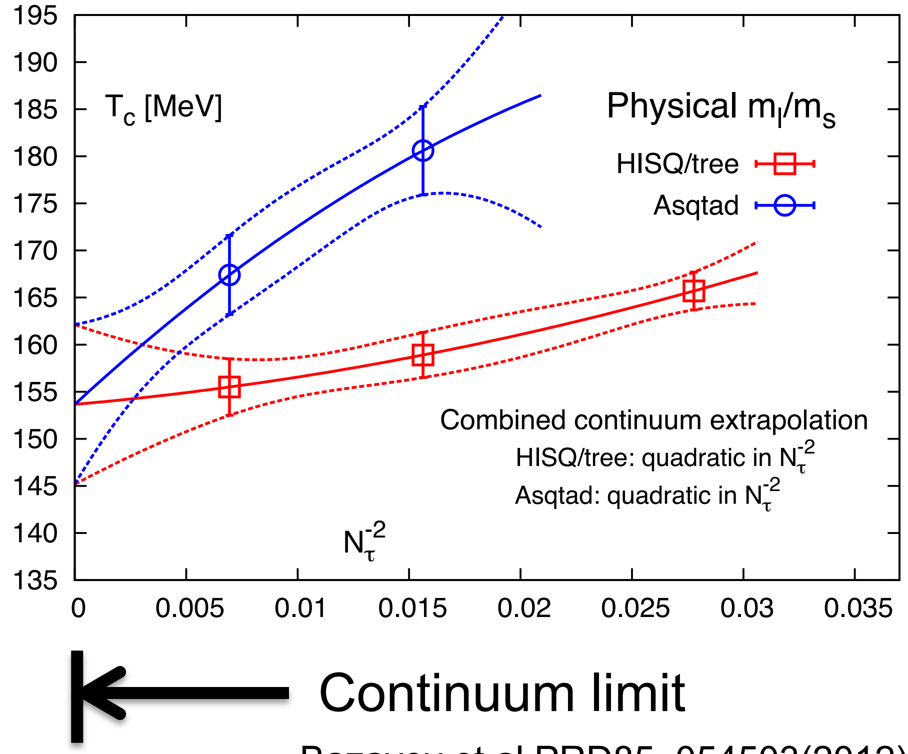


Transition at the physical point (II)

- Checked with 3 types of staggered actions (stout, Asqtad, HISQ)
- Significant difference at finite lattice spacings, but consistent results in the continuum limit
- Pseudo critical temperature:

$$T_{pc} = \begin{cases} 155 \pm 6 MeV & \text{WB} \\ 154 \pm 9 MeV & \text{HotQCD} \end{cases}$$

pseudo critical temperature (MeV)



Bazavov et al,PRD85, 054503(2012)

- Wuppertal-Butapest Collaboration
  - Y. Aoki et al, Nature 443: 675 (2006); S. Borsanyi et al, JHEP 2010:73 (2010)
- HotQCD Collaboration
  - A. Bazavov et al, PRD 85:054503 (2012)

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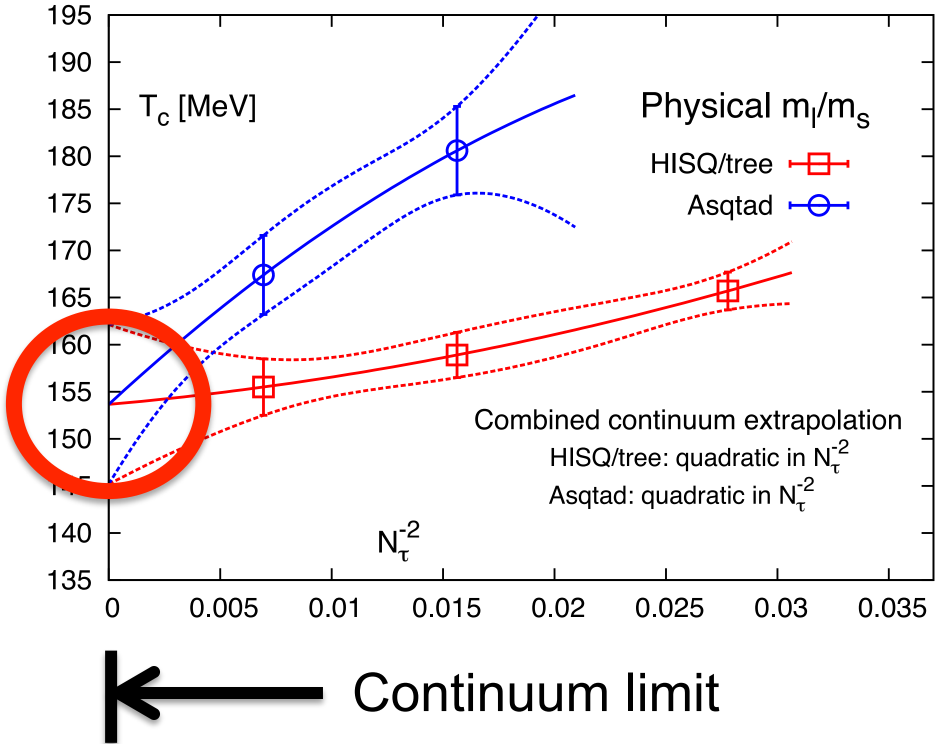

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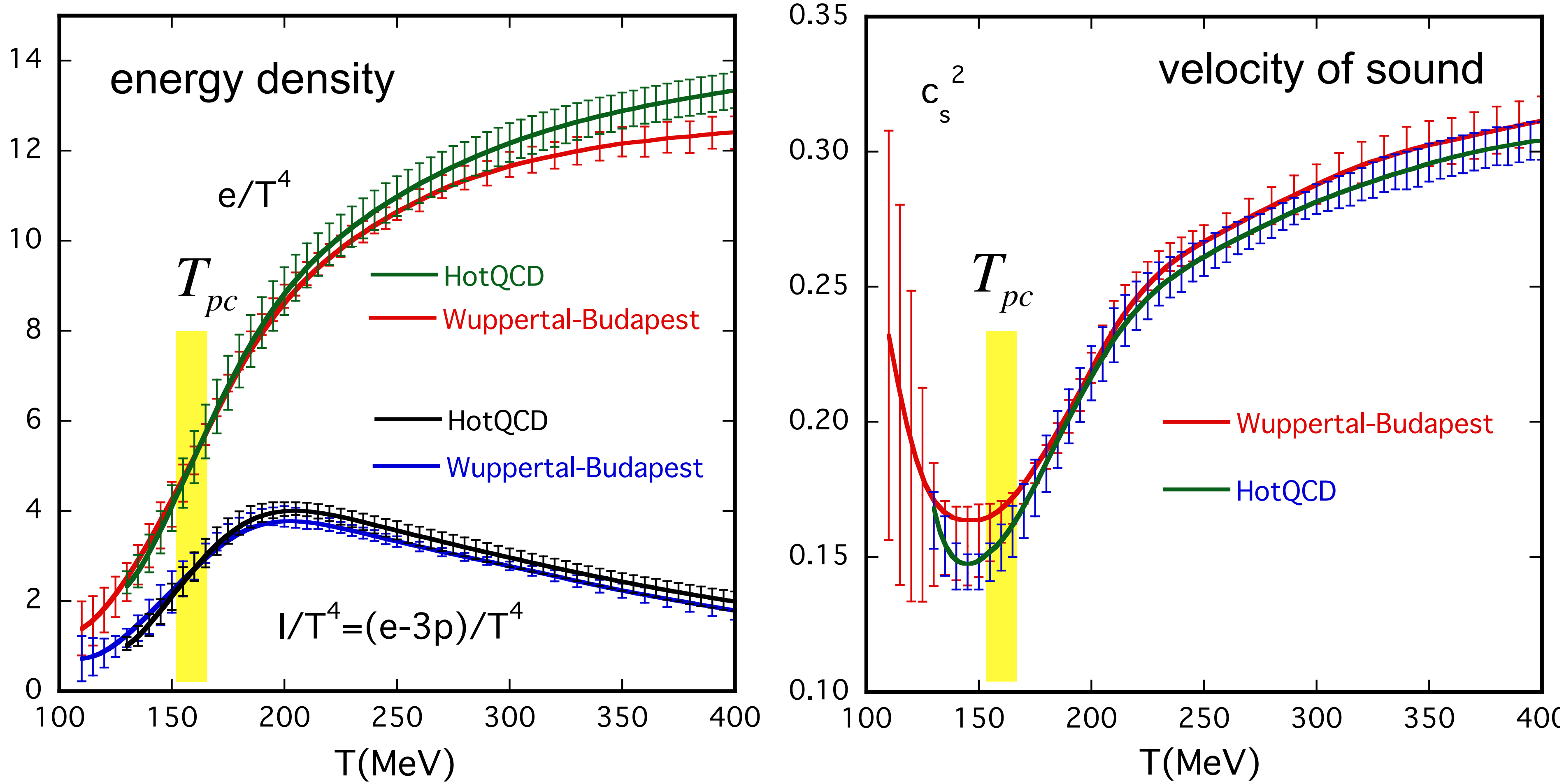
Equation of State



- Wuppertal-Budapest: S. Borsanyi et al, Phys. Lett. B730, 99 (2014)
- HotQCD: A. Bazavov et al, PRD90, 094503(2014)

QGP in heavy ion collisions?

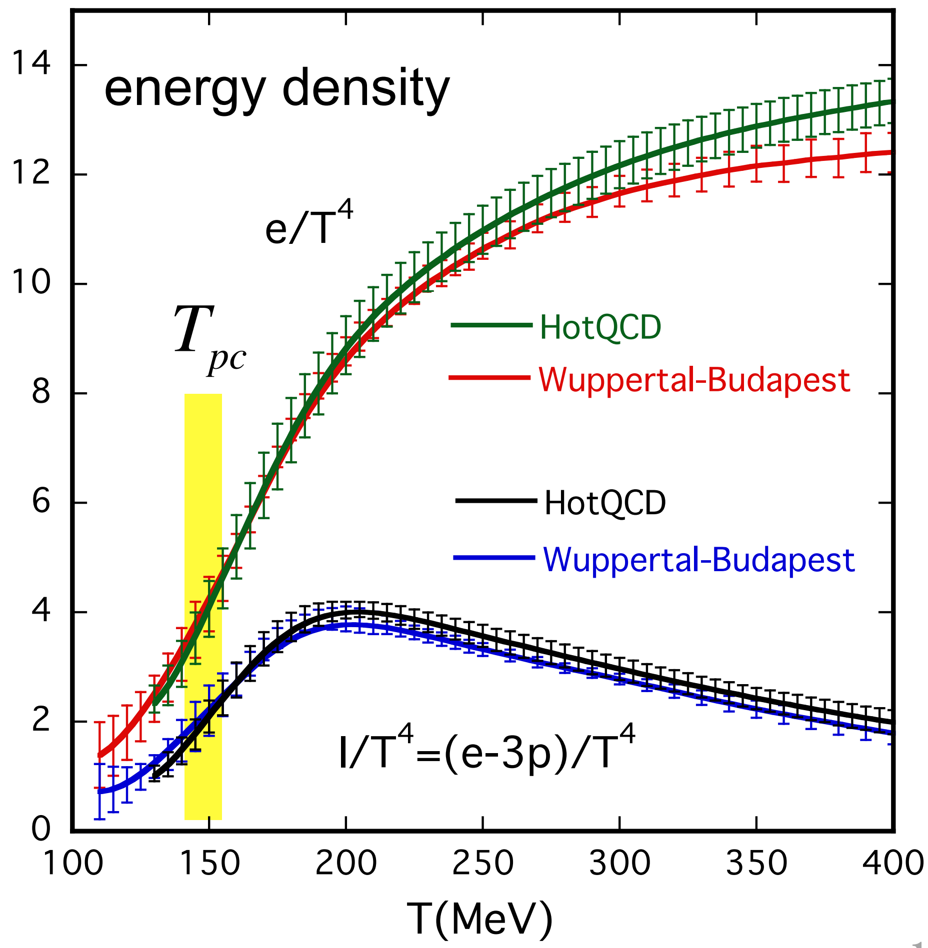
- Direct photon transverse distribution as a probe of temperature in heavy ion collisions

$$n_\gamma \propto \exp\left(-\frac{p_T}{T_\gamma}\right)$$

$$T_\gamma = \begin{cases} 221 \pm 27 MeV & RHIC\ \sqrt{s} = 200 GeV \\ 304 \pm 51 MeV & LHC\ \sqrt{s} = 2.7 TeV \end{cases}$$

PHENIX collaboration , PRL104, 132301(2010)

M. Wilde for ALICE collaboration , arXic:1210.5958 (2012)



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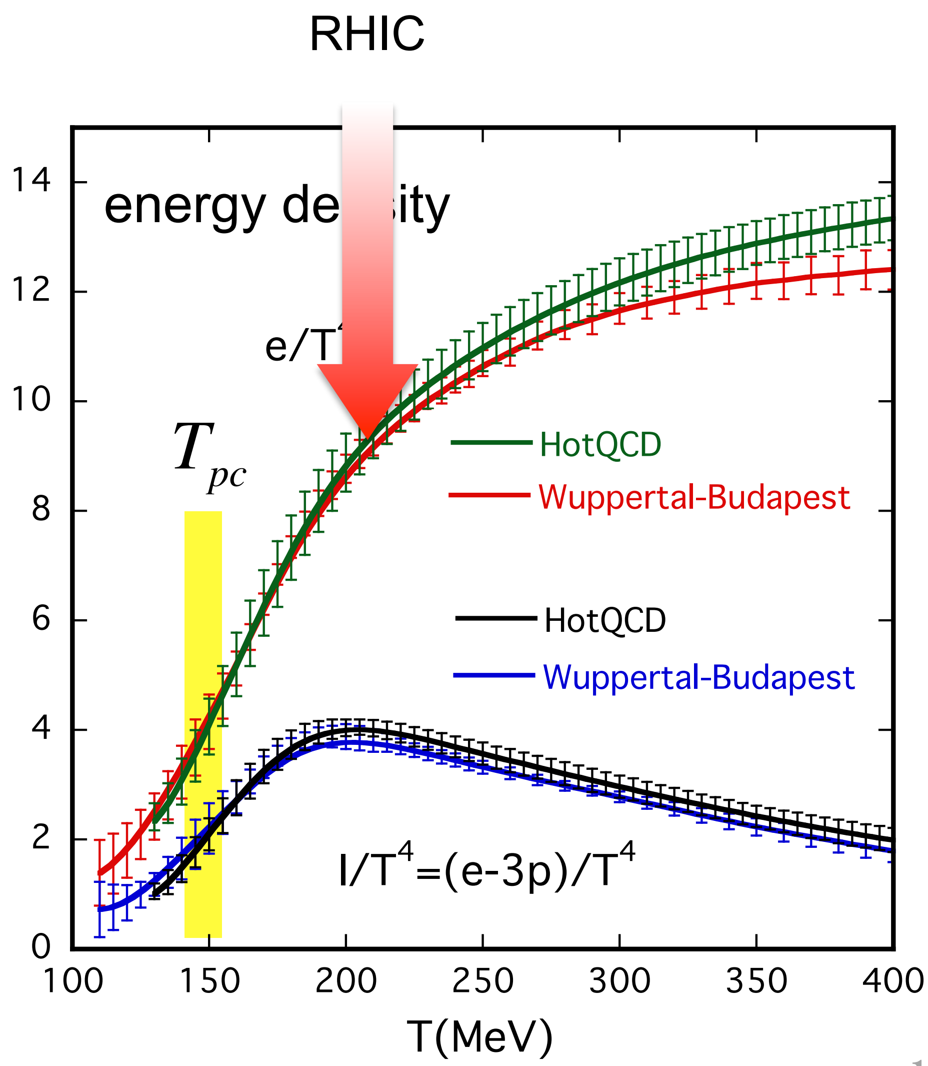

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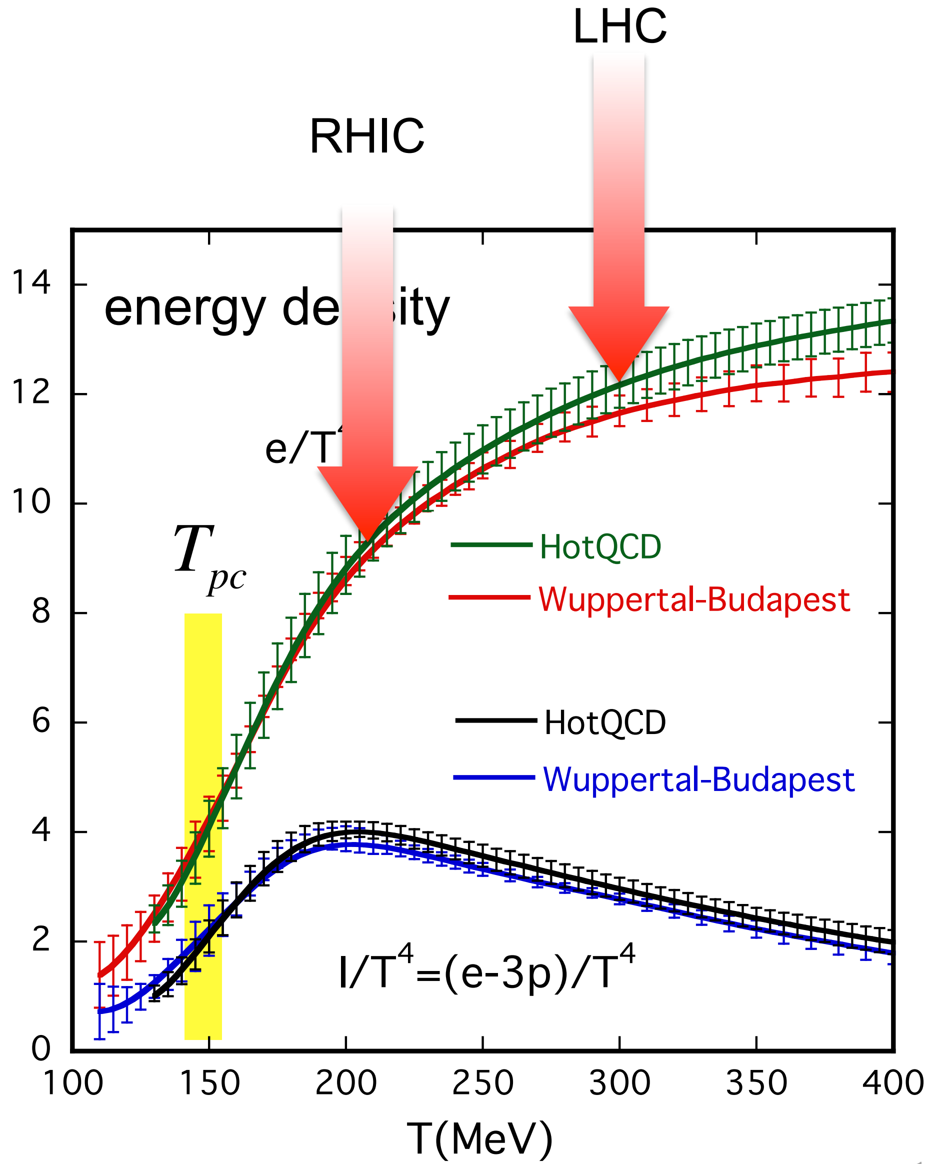

PHENIX collaboration , PRL104, 132301(2010)

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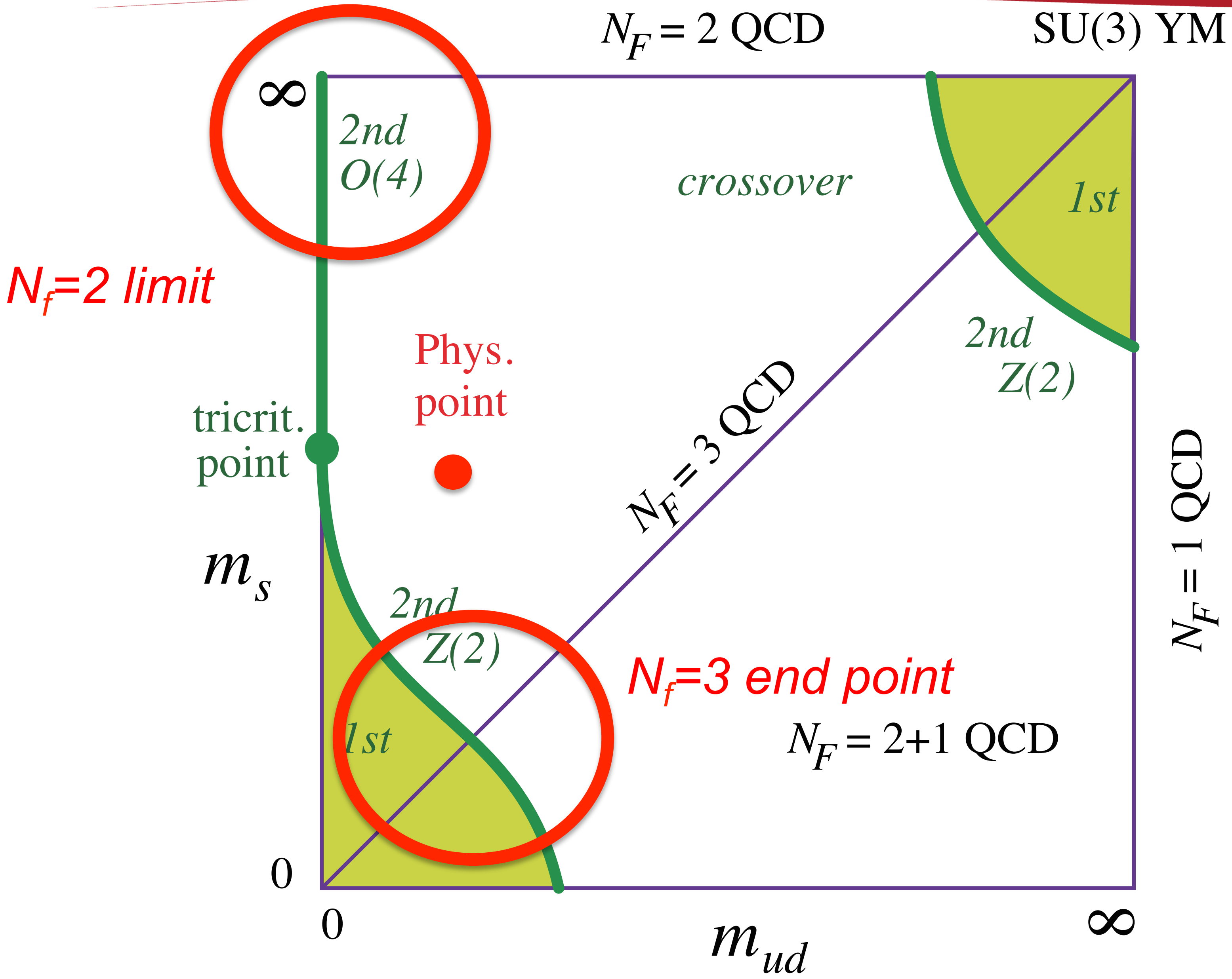
Loose ends?


$N_F = 2$ QCD
SU(3) YM
∞
2nd
O(4)
crossover
1st
N_f=2 limit
Phys.
point
2nd
Z(2)
$N_F = 3$ QCD
tricrit.
point
$N_F = 1$ QCD
m_s
2nd
Z(2)
N_f=3 end point
1st
N_F = 2+1 QCD
0
0
m_{ud}
∞

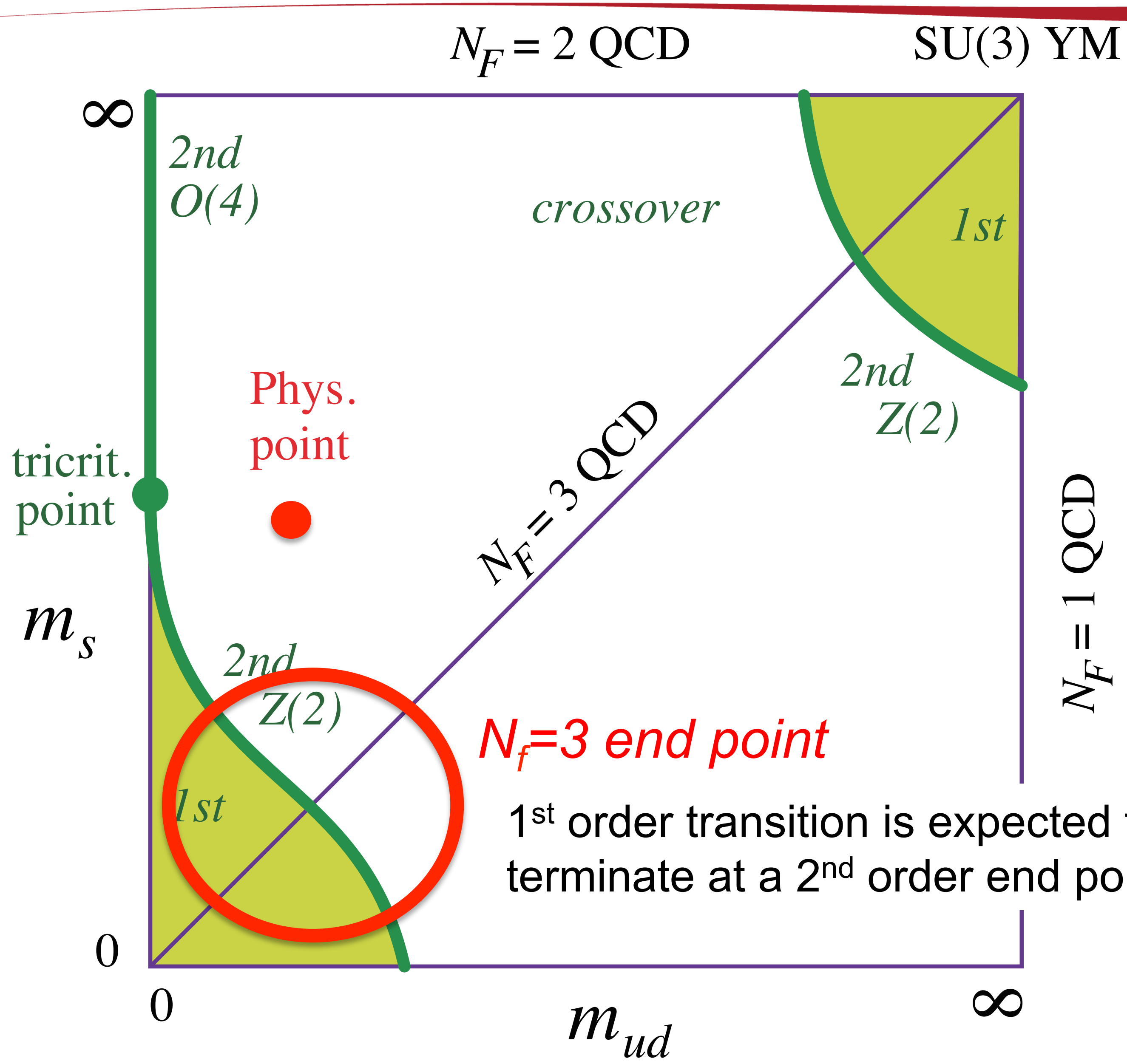
$U_A(1)$ symmetry and N_f=2 limit

- $U_A(1)$ symmetry is broken by anomaly $\partial_\mu J_\mu^{U_A(1)} = \frac{1}{8\pi^2} Tr\left(F_{\mu\nu}\tilde{F}_{\mu\nu}\right)$ but its effect maybe negligible around the transition
- This may affect the *order and universality class* of the N_f=2 transition at m_u=m_d=0
- Long debate, not yet completely settled

	$U_A(1)$ *broken*	$U_A(1)$ *restored*
symmetry	$SU(2)\otimes SU(2)\approx O(4)$	$SU(2)\otimes SU(2)\otimes U(1)\approx O(4)\otimes O(2)$
Pisarski-Wilczek(1984)	*2nd order*	*1st order* (no RG fixed point at 1-loop)
Pellisetto et al(2000)		*2nd order* (RG fixed point at 6-loop)
Aoki et al(2012)	*cannot be* $O(4)$	*not visible in certain correlators*
Nakayama et al(2014)		*2nd order* (conformal bootstrap)

My assessment

- No strong evidence of 1st order; 2nd order likely
- Theoretically important to settle
- But, quantitative effects likely limited at the physical point, since
  - O(4) and O(4)xO(2) critical exponents are similar
    (only about 10% difference)
  - Finite quark mass effects further softens the difference



N_f=3 end point

1st order transition is expected to terminate at a 2nd order end point

N_f=3 end point

- Pion mass marking the end point is very uncertain:
  - For the staggered action, small values, *even consistent with zero?*

    Karsch et al (p4) '03

    Forcrand-Philipsen (unimproved) '07

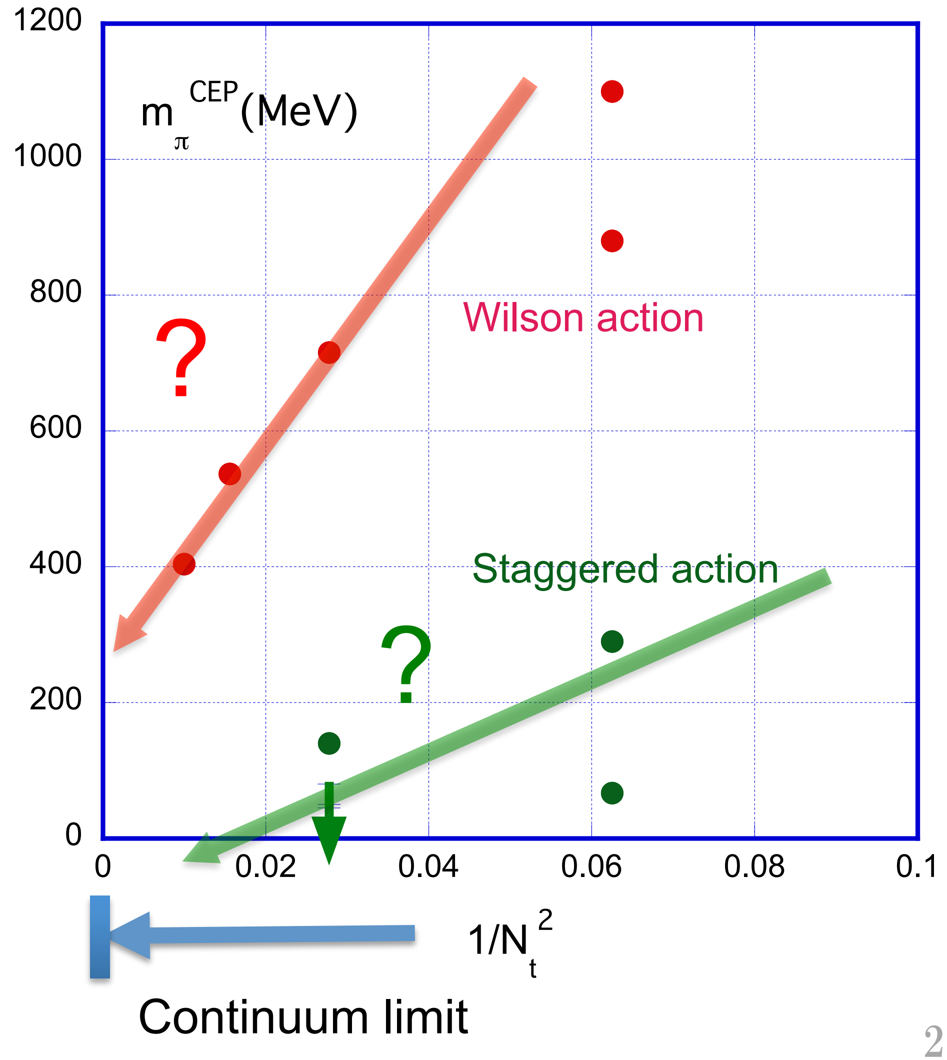
    Endrodi et al (stout) '07

    Ding et al (HISQ) '11
  - For the Wilson action, large scaling violation, and *sizable difference from the staggered values*

    Iwasaki et al '96

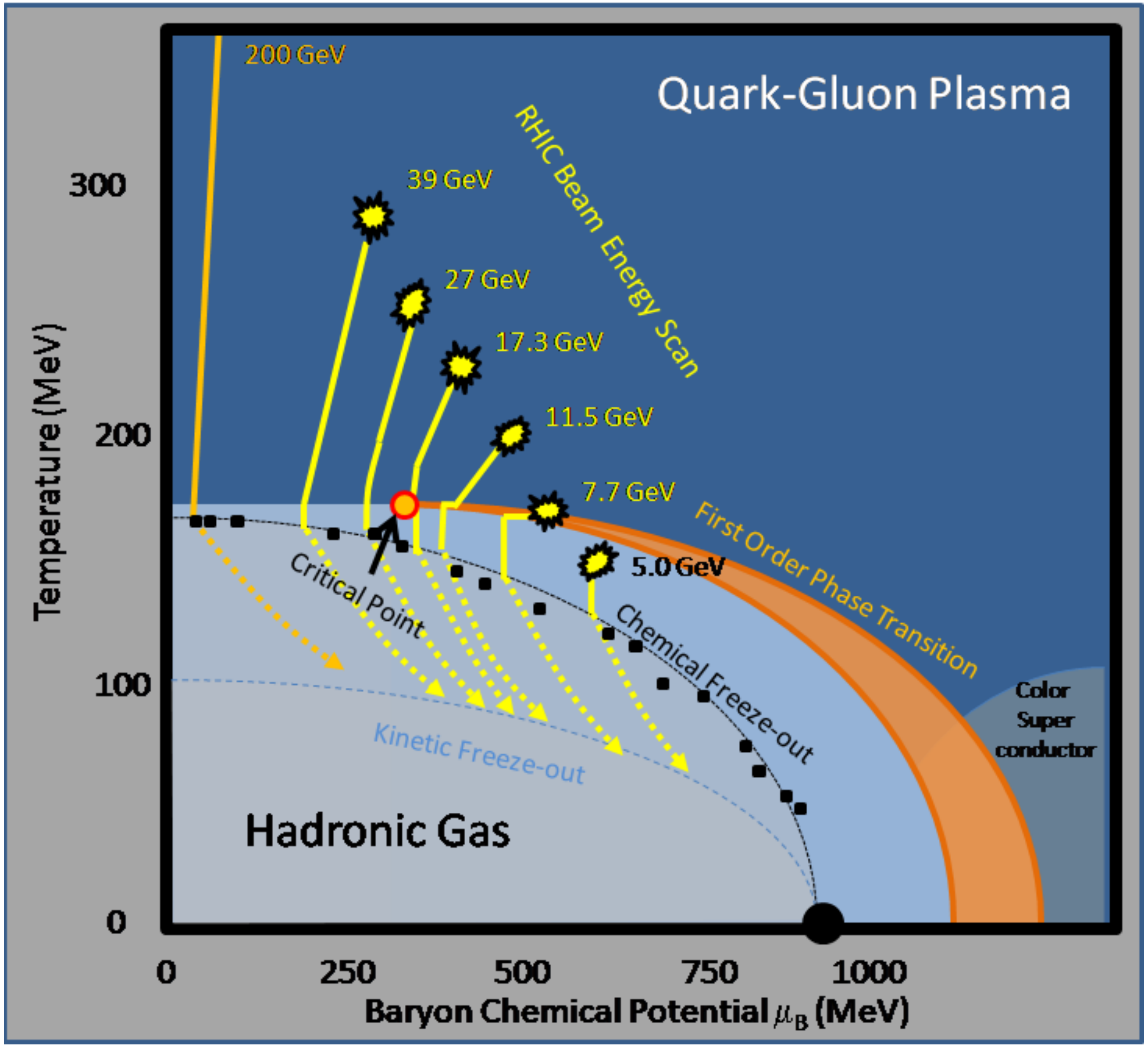
    Nakamura et al '14

pion mass at the end point



Non-zero but small baryon density

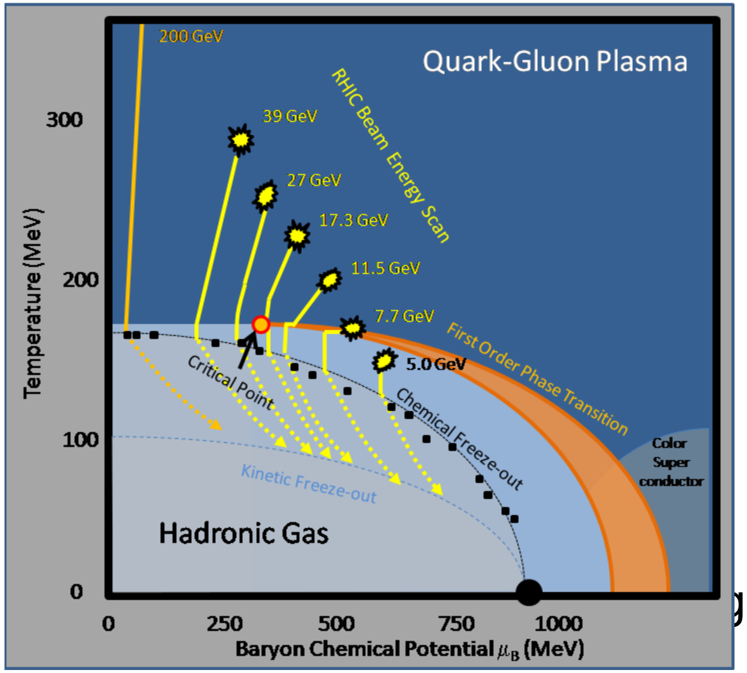
How much do we really know?



From BNL@gov

How much do we really know?

- How does the pseudo critical temperature extend to non-zero density?
- How does it compare with phenomenological "freeze-out" temperatures?
- Is there a critical end point, and if yes, where?



Simulation for $\mu_B \neq 0$

- Standard Monte Carlo simulation methods not applicable

- Two alternative methods in use for small μ_B / T
  - Taylor expansion in μ_B / T

$$O(T, \mu_B) = O_0(T) + O_1(T)\frac{\mu_B}{T} + O_2(T)\left(\frac{\mu_B}{T}\right)^2 + \cdots$$

expansion coefficients $O_n(T)$ are calculable with simulations at $\mu_B = 0$

  - Analytic continuation from imaginary potential $\mu_B / T = i\rho / T$

$$O(T, \mu_B = i\rho)$$

calculable by Monte Carlo, hence analytically continued to real potential

Pseudo critical temperature

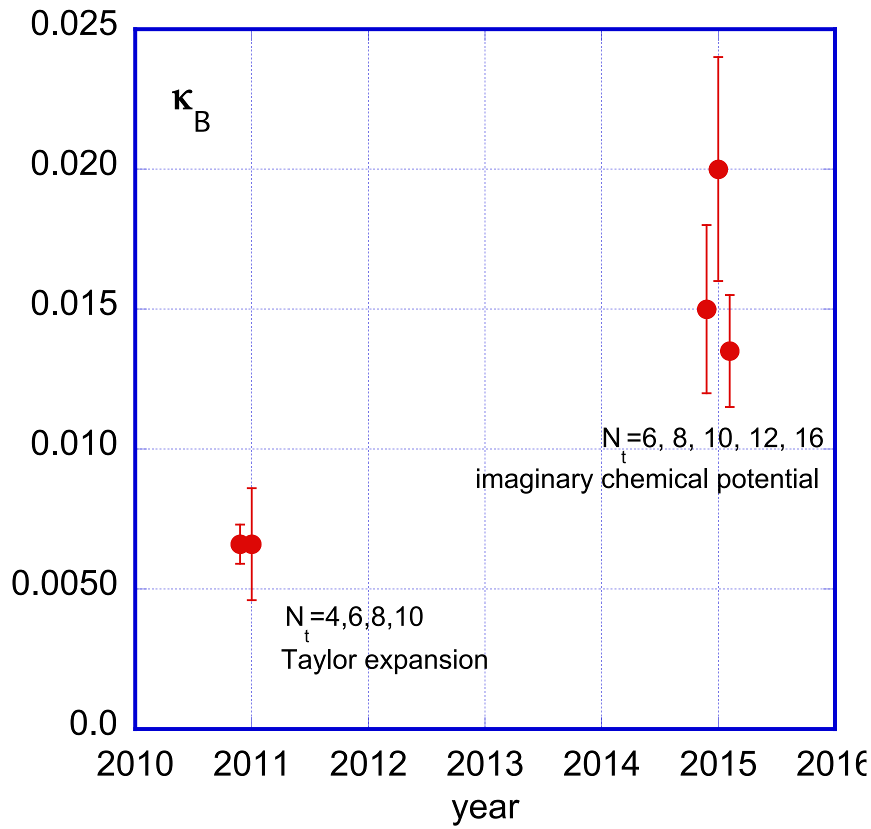
- Numerical methods for small chemical potential:
  - Taylor expansion in μ_B / T
  - Imaginary chemical potential $\mu_B / T = i\rho / T$

O. Kacmarek et al, Phys.Rev.D83:014504('11)
G. Endrodi et al, JHEP04, 001('11)

$$\frac{T_{pc}(\mu_B)}{T_{pc}(0)} = 1 - \kappa_B \left(\frac{\mu_B}{T_{pc}(\mu_B)} \right)^2 + \cdots$$

- Recent continuum results sizably larger than the pioneering numbers

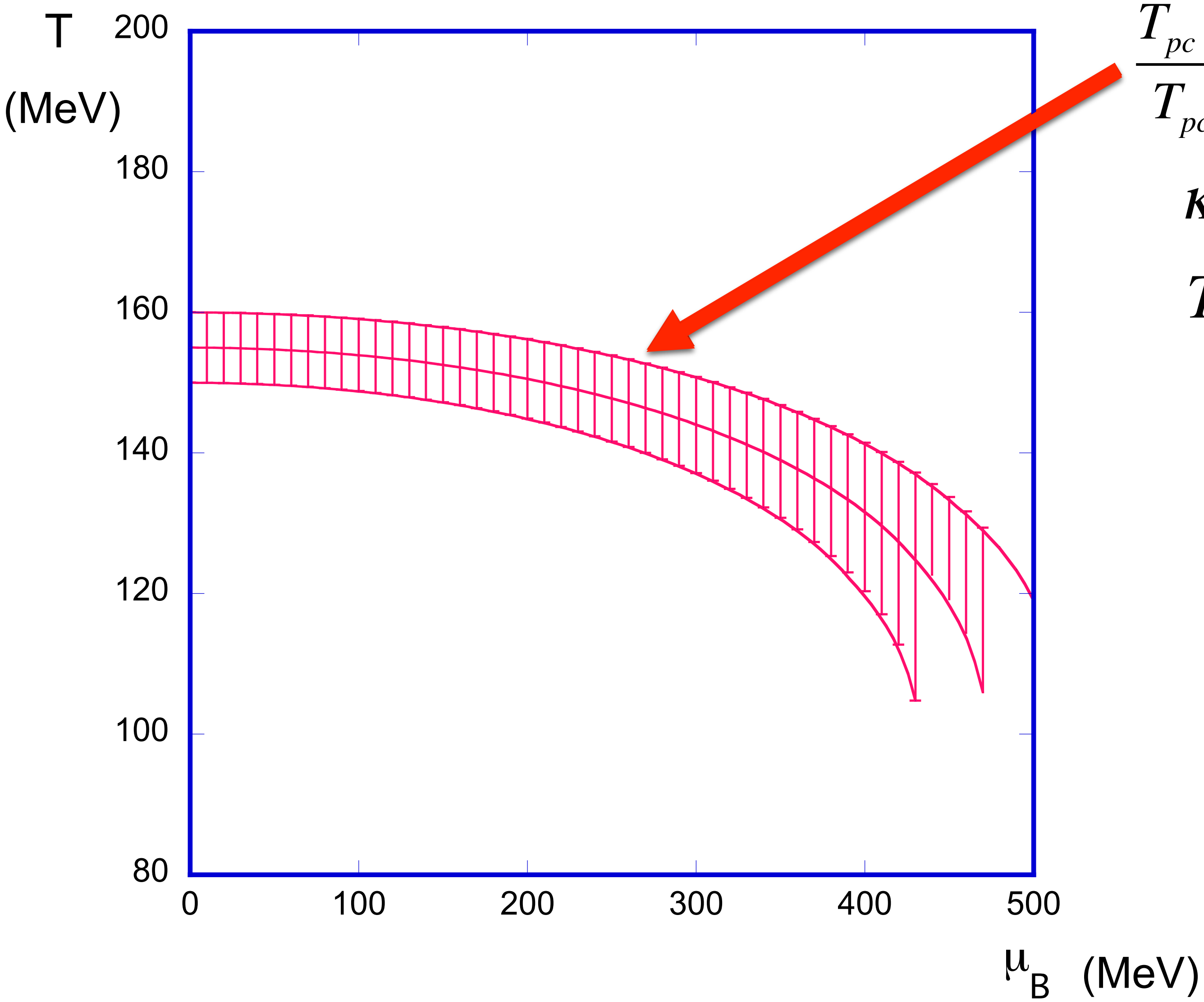
L. Cosmai et al, Lattice 2015
R. Bellwied et al, Lattice 2015
C. Bonatti et al, Lattice 2015



$$\kappa_B \approx 0.0162 \pm 0.0017$$

My eyeball value

Lattice QCD



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$$\kappa_B \approx 0.0162 \pm 0.0017$$

$$T_{pc} \approx 155 \pm 5\ MeV$$

Heavy ion phenomenology

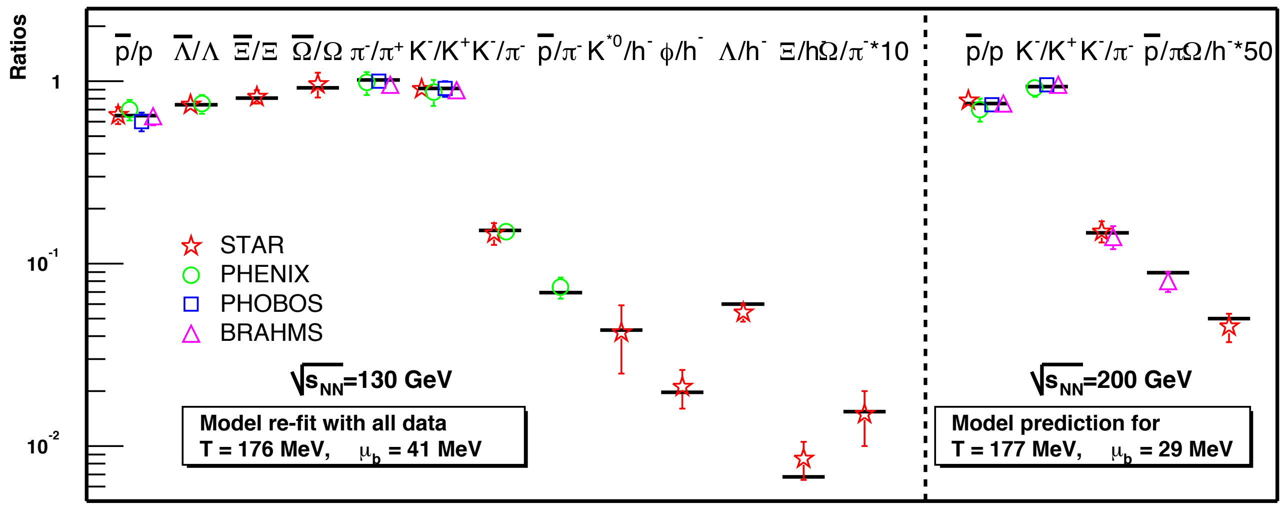
- Statistical hadronization model

  e.g., review by P. Braun-Munzinger et al, Nucl-th/0304013(2003)

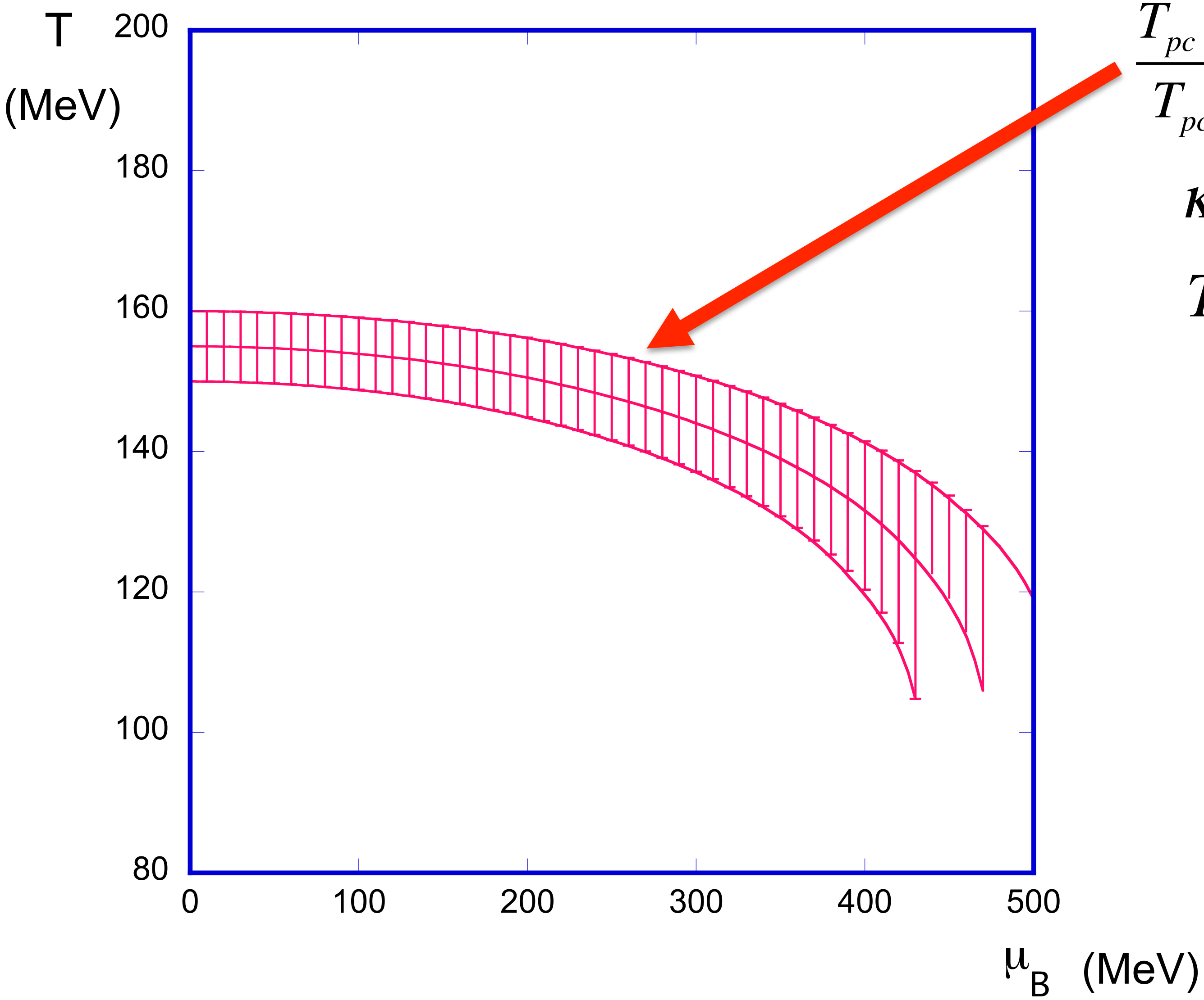
  - Statistical equilibrium at the time of hadronization (freeze out)
  - Boltzman distribution for hadron yields

$$N_h(T,\mu_B,V) = V\int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp\left(\frac{\varepsilon_h(p)-B_h\mu_B}{T}\right)\pm 1}$$

  - Fits to experimental yields provides an estimate of T, μ_B



Lattice QCD



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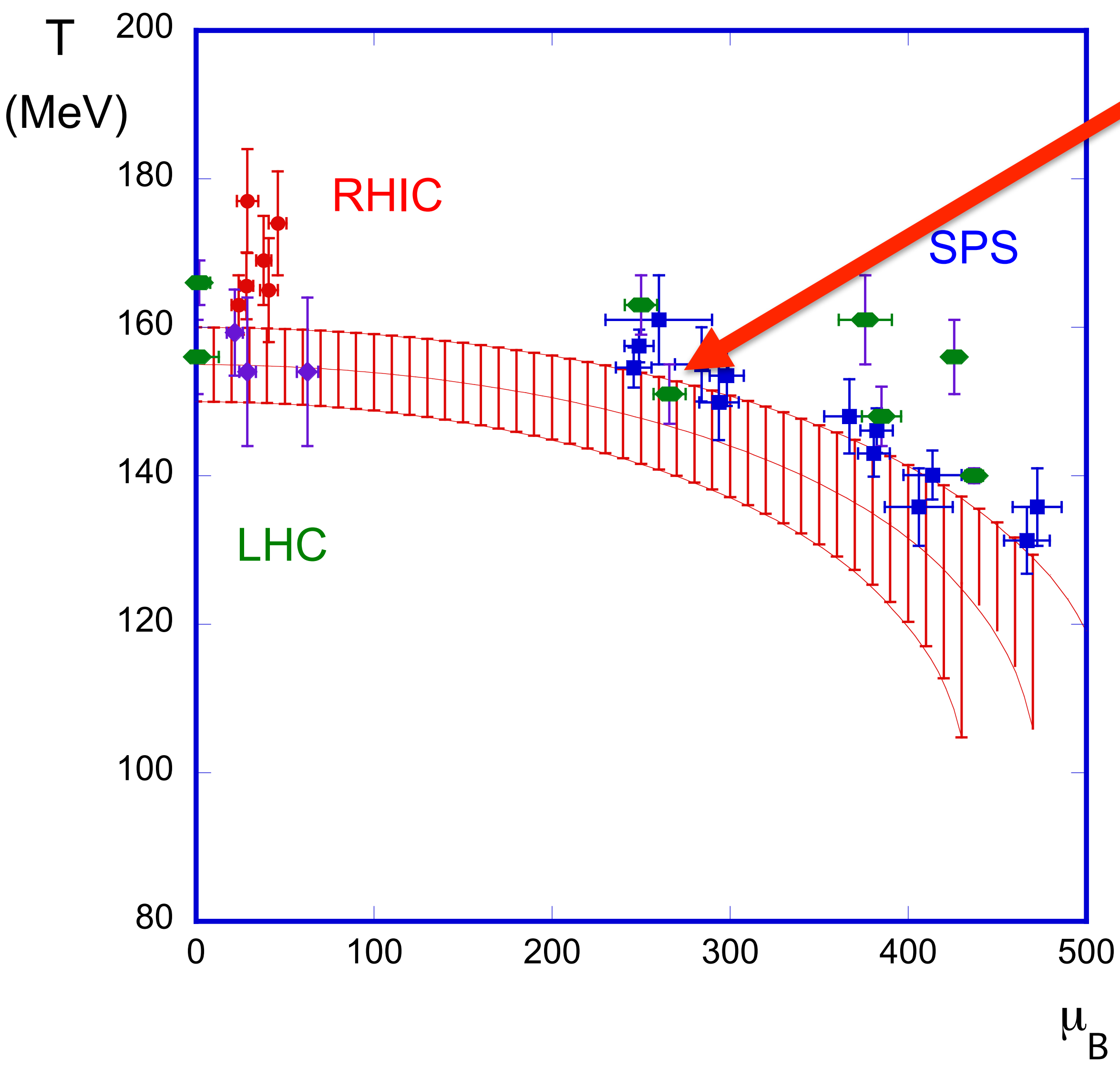
$$T_{pc} \approx 155 \pm 5 \; MeV$$

Lattice QCD vs phenomenology



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RICH, SPS
Cleymans et al, PRC.73.034905 (2006)

RICH Star collaboration
Abelev et al, PRC79, 034909 (2009)

LHC, SPS
Bocattini etal, PRL 111, 082302 (2012)

- Qualitative agreement but higher value from phenomenology?
- "Precise" comparison rather difficult since
  - Lattice QCD side
    - Pseudo critical temperature has inherent ambiguity, e.g., definition, choice of observable etc
    - Statistical and systematic errors of Monte Carlo calculation
  - Heavy ion phenomenology side
    - Inherent ambiguity in the extraction of T, μ_B i.e., at what stage do hadrons materialize?

Conserved charge fluctuations and experiment

Moments as thermometers

- Moments of conserved quantities, e.g., electric charge Q

$$\chi_1^Q(T,\mu_B)=\frac{1}{VT^3}\langle N_Q\rangle$$

$$\chi_2^Q(T,\mu_B)=\frac{1}{VT^3}\left\langle(\delta N_Q)^2\right\rangle,\quad \delta N_Q=N_Q-\langle N_Q\rangle$$

$$\chi_3^Q(T,\mu_B)=\frac{1}{VT^3}\left\langle(\delta N_Q)^3\right\rangle$$

$$\chi_4^Q(T,\mu_B)=\frac{1}{VT^3}\left[\left\langle(\delta N_Q)^4\right\rangle-\left\langle(\delta N_Q)^2\right\rangle^2\right]$$

or

$$M_Q=\langle N_Q\rangle,\quad \sigma_Q^2=\left\langle(\delta N_Q)^2\right\rangle$$

$$S_Q=\frac{\left\langle(\delta N_Q)^3\right\rangle}{\sigma_Q^3},\quad \kappa_Q=\frac{\left\langle(\delta N_Q)^4\right\rangle}{\sigma_Q^4}-3$$

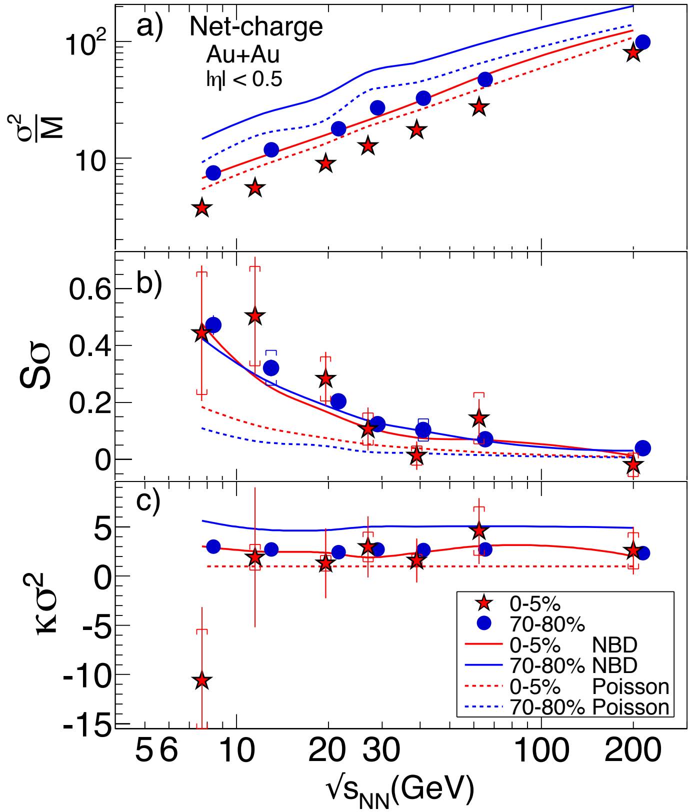
$$\frac{M_Q}{\sigma_Q^2}=\frac{\chi_1^Q}{\chi_2^Q}\equiv R_{12}^Q,\quad \frac{S_Q\sigma_Q^3}{M_Q}=\frac{\chi_3^Q}{\chi_1^Q}\equiv R_{31}^Q$$

  - Measurable in experiments for given beam energy $\sqrt{s}$
  - Measurable in lattice QCD as functions of T,μ_B

- Hence, provide a *direct thermometer* of QGP, e.g.,

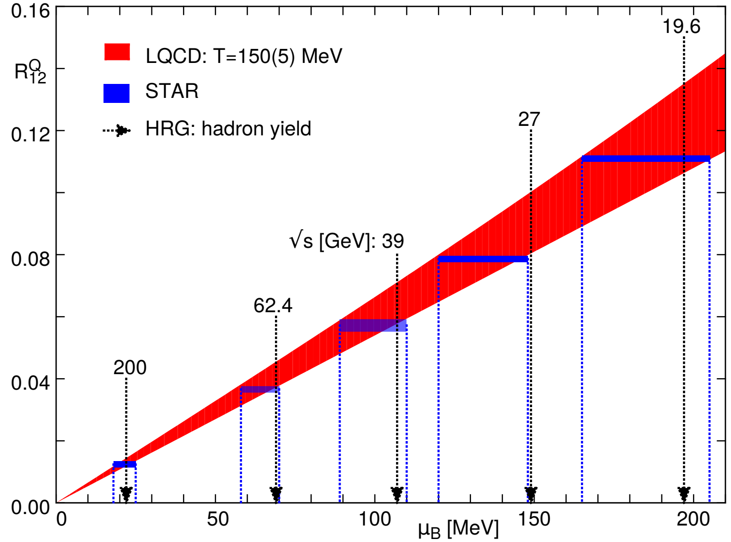
$$\frac{M_Q}{\sigma_Q^2}\left(\sqrt{s}\right)=R_{12}^Q(T,\mu_B)$$

A. Bazavov et al, PRL 109, 192302 (2012)
S. Borsanyi et al, PRL 111, 062005 (2013)



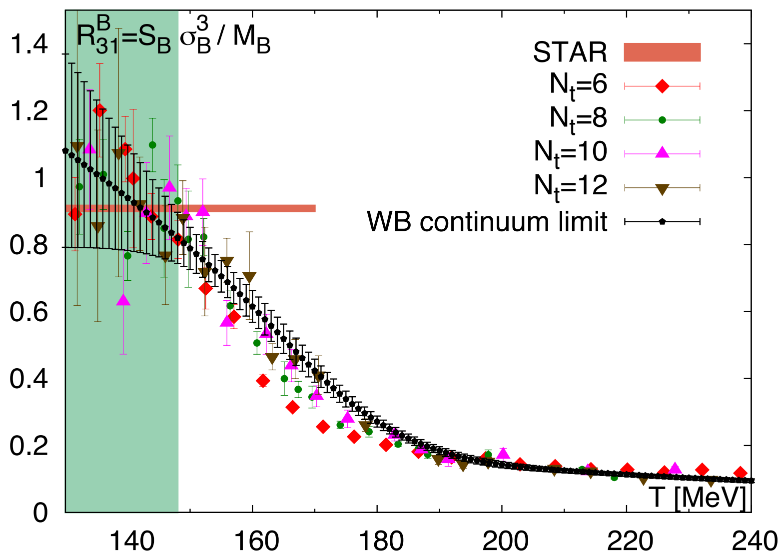
RHIC STAR measurement

L. Adamczyk et al, PRL 113, 092301(2014)



μ_B for given T

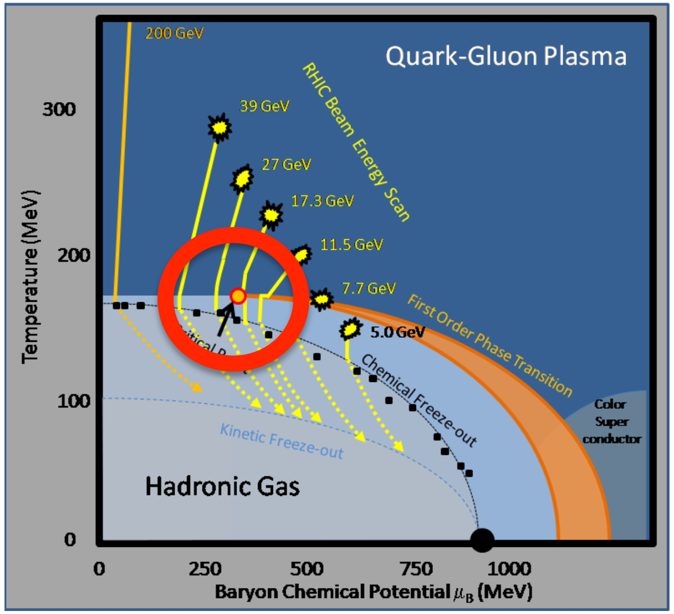
S. Mukherjee,
PoS CPOD2014, 005

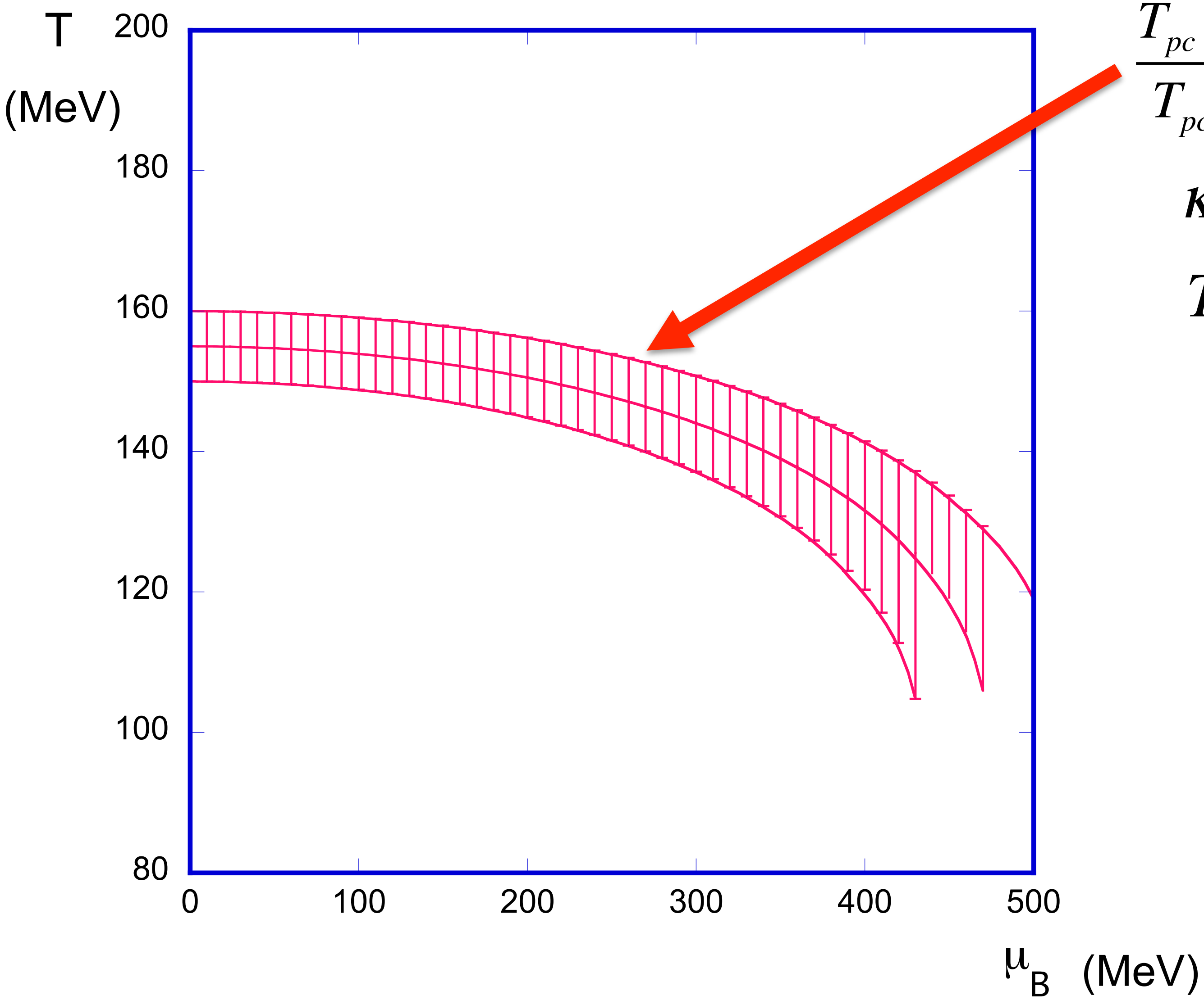


T for $\mu_B \approx 0$

S. Borsanyi et al,
PRL113, 052301(2014)

Search for the critical end point?





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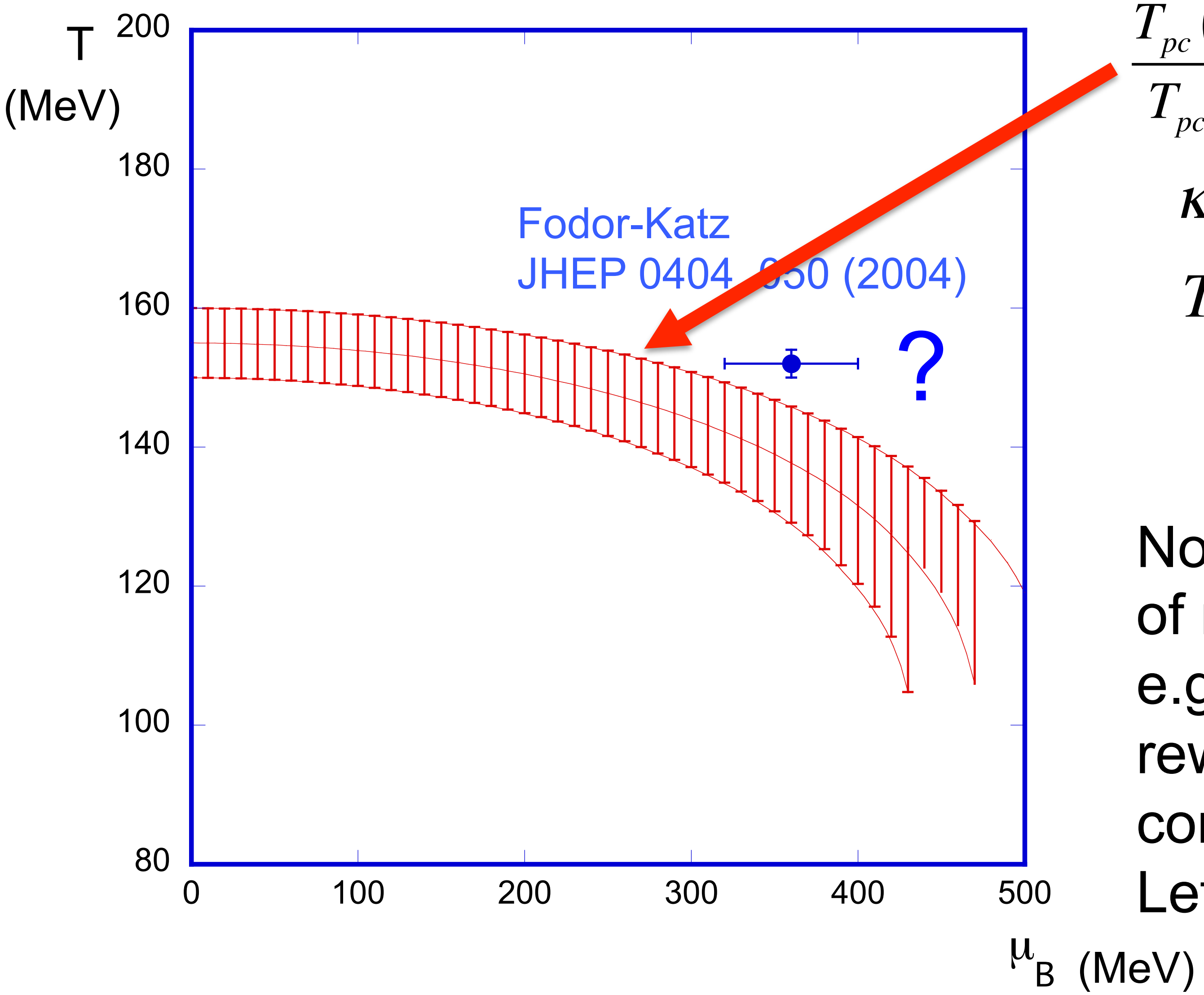
Critical end point?



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No new results in spite of many attempts,
e.g.,
reweighting,
complex Langevin,
Lefschez thimble, …

Summary

- *1st review of lattice QCD thermodynamics at LP/ ICHEP conference series in 10 years!*
- *Significant progress in both lattice QCD and experiment (RHIC, LHC) in the mean time.*
- *Direct comparison of theory and experiment is now beginning to be possible.*
- *The location of critical end point remains elusive; hope for development in near future.*