New Methods in Quantum Field Theory

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- We are used to the idea that a scalar particle can be composite, for example the pion π has constituents which can be resolved if one does experiments at $\sim 1 GeV$.
- We are even used to the idea that a massive spin 1 particle can be composite, e.g. the rho meson ρ_μ.
- The scale of compositeness is the scale when interactions become strong.

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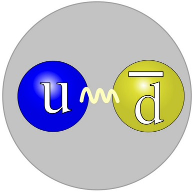
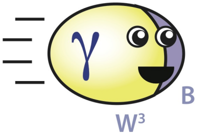


It is clearly an outstanding question to understand whether the W, Z bosons and the Higgs field are fundamental or composite.

But what about the photon?



I am not trying to suggest that this is a phenomenological question that needs an urgent solution. It is just a theoretical question that I will use as an excuse to survey some recent developments in QFT.

Traditionally, one would argue that a massless gauge field A_μ cannot be composite.

- $A_\mu \simeq A_\mu + \partial_\mu \Lambda$. Necessary for unitarity. For example, if two fermions have a massless composite vector bound state

$$A_\mu \sim \bar{\Psi}\gamma_\mu \Psi \qquad (*),$$

where would this gauge symmetry come from?

- Weinberg-Witten theorem: The gauge symmetry cannot be the manifestation of any conserved current in the system:

$$\langle VAC | j_\mu | A_\mu \rangle = 0 \quad \text{for all} \quad \partial^\mu j_\mu = 0 \ .$$

Essentially rules out $(*)$.

Where would the gauge symmetry come from if it did not exist in the fundamental theory?

Take a $U(1)$ Goldstone scalar in $2+1$-dimensional quantum field theory, $\pi \simeq \pi + f$. It can be transformed to a gauge potential

$$\partial_\mu \pi = \epsilon_{\mu\nu\rho}\partial^\nu A^\rho \ .$$

This transformation has the inherent ambiguity

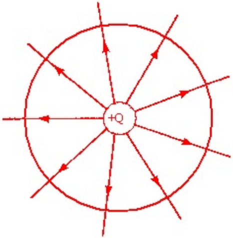
$$A_\mu \to A_\mu + \partial_\mu \Lambda \ .$$

The transformation from the fundamental degrees of freedom to the low energy degrees of freedom could have an inherent ambiguity. So it cannot be like $A_\mu \sim \bar{\Psi}\gamma_\mu\Psi$.

Where does the current to which the gauge field couples come from?

By the Weinberg-Witten theorem, it cannot be one of the conserved currents in the theory.

But the current to which a gauge field couples is not a real current in the theory anyway: If we quantize the theory on $\mathcal{M} \times \mathbb{R}$ with compact $\mathcal{M}$, then the Hilbert space consists of gauge singlets (Gauss' law).



There is thus no theoretical obstruction for the compositeness of massless spin 1 particles. In nature, it could actually pertain to the photon and also the W, Z bosons (the Higgsing would come "later").

We will discuss some phenomenological possibilities at the end.

As we mentioned, in $2+1$ dimensions there are easy examples that demonstrate the emergence of Abelian gauge symmetries. It essentially happens anytime there is spontaneous symmetry breaking.

Nonabelian gauge fields in $2+1$ are harder to realize as composite particles.

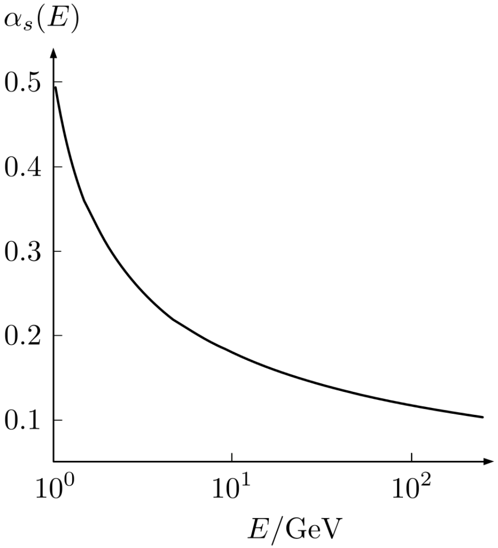
The simplest existence proof in $3+1$ dimensions is provided by $d = 4$, $\mathcal{N} = 1$ theories [Seiberg].

The simplest possible case is that we start with $SU(4)$ gauge theory with 6 fundamental fields Q_i^A and 6 anti-fundamental fields $\tilde{Q}_A^i$. This theory has a negative beta function

$$\beta \sim -3 \times 3 + 4 = -5 \ .$$

It therefore develops strong coupling at some scale Λ_{QCD} and becomes intractable.

What happens in the infrared?


$\alpha_s(E)$
0.5
0.4
0.3
0.2
0.1
10^0
10^1
10^2
E/GeV

Seiberg has made a guess that the infrared theory is actually weakly coupled but in terms of different variables:

It is an $SU(2)$ gauge theory with 12 fundamentals and 36 neutral scalar fields.

The beta function is

$$\beta = -6 + 6 = 0$$

at one-loop, but it is positive at two loops. So the theory is free in the infrared.

The $SU(2)$ gauge fields have nothing to do with the original $SU(4)$ gauge theory. The $SU(2)$ gauge fields are new, emergent, weakly-coupled massless composite spin 1 particles.

We often refer to this guess of the low energy degrees of freedom as "duality." It is a duality in the sense that the original strongly-coupled $SU(4)$ description has a more useful, weakly-coupled, description as an $SU(2)$ gauge theory.

We would like to know

- Can we actually test Seiberg's guess?
- When do such new classical limits in the infrared emerge? When are there composite massless gauge fields at long distances?

On both questions there has been progress lately.

Suppose we have a theory with a conserved fermionic charge Q such that $Q^2 = H$ and $Q|boson\rangle = |fermion\rangle$ and vice versa.

Suppose $H|\Psi\rangle = E|\Psi\rangle$, $E \neq 0$. Then, $Q|\Psi\rangle = |\Psi'\rangle \neq 0$. Thus, $Tr(-1)^F = 0$ for all the states with $E \neq 0$.

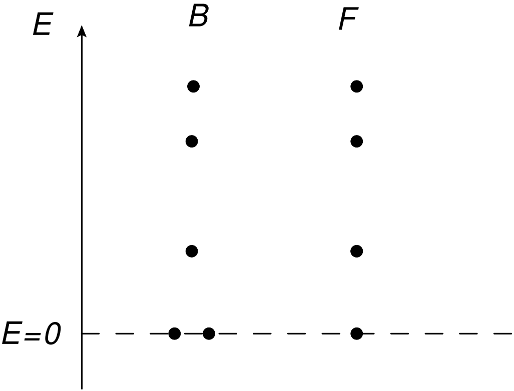
Define

$$I = Tr_{\mathcal{H}}(-1)^F$$

over the whole Hilbert space $\mathcal{H}$. The contributions only come from $H = 0$ states (vacua).

$$I = n_B - n_F$$

This is the Witten Index.



$I = 2 - 1 = 1$

Since the index does no depend on the renormalization group scale, we would like to compare the Index I of our $SU(4)$ and $SU(2)$ gauge theories.

The trouble is that it diverges. This is due to the infinitely many SUSY vacua on both sides (flat directions).

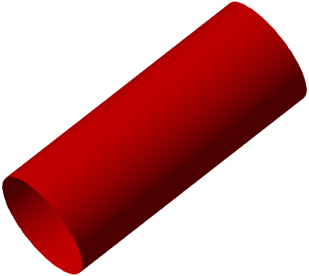
We now know how to overcome this longstanding problem! The first steps were done by Römelsberger. We can study the theory on $\mathcal{M}_3 \times \mathbb{R}$ rather than on $\mathbb{R}^4$ and consider

$$I(\mu_i) = Tr_{\mathcal{H}}((-1)^F e^{\mu_i q_i})$$

for various charges q_i that commute with the supercharge.

This is well defined for interesting theories such as our $SU(4)$ and $SU(2)$ theories.

A particularly nice example is $\mathcal{M}_3 \sim S^3$. So the theory lives on a four-dimensional cylinder $\mathcal{M}_3 \times \mathbb{S}^1$



The gauged charges do not correspond to chemical potentials because the corresponding charges vanish. One finds that if $SU(4)$ indeed flows to $SU(2)$ with massless emergent gauge fields, then the following identity should hold true (schematically):

$$(p,p)^2(q,q)^2 \int \prod_{i=1,..,4} [dr_i] \frac{\prod_{i,j\leq 4} \Gamma(\mu_i r_j, 1/(\tilde{\mu}_i r_j), p, q)}{\prod_{i,j\leq 4} \Gamma(r_i/r_j, r_j/r_i, p, q)}$$

$$= \left[\prod_{i,j\leq 2} \Gamma(\mu_i/\tilde{\mu}_j, p, q)\right] \int \prod_{i=1,2} [dr_i] \frac{\prod_{i,j\leq 2} \Gamma(\mu_i r_j, 1/(\tilde{\mu}_i r_j), p, q)}{\prod_{i,j\leq 2} \Gamma(r_i/r_j, r_j/r_i, p, q)}$$

$\Gamma(\cdot)$ is the elliptic hypergeometric gamma function. $(\cdot,\cdot)$ is the q-Pochhammer symbol.

It appears that mathematicians have independently proved such identities quite recently [Spiridonov, Rains, Rahman, van de Bult...]. In particular, the identity above holds true!

The elliptic hypergeometric function Γ that appeared above is a "higher version" of the Jacobi theta function $\Theta_n(z;q)$. The latter are central in complex analysis in *two dimensions*.
The computations of

$$I(\mu_i) = Tr_{\mathcal{H}}((-1)^F e^{\mu_i q_i})$$

are closely linked with the theory of complex geometry in *four dimensions* [Closset-Dumitrescu-Festuccia-ZK].

A very partial summary of applications:

- Many checks of dualities, new dualities...
- Wilson loops expectation values (non-perturbative) [Pestun...]
- Exact computations of the metric in theory space and other previously inaccessible observables [Benini-Cremonesi, Doroud-Gomis-le Floch-Lee, Gerchkovit-Gomis-ZK....]
- Relations between field theories in different dimensions [Alday-Gaiotto-Tachikawa,...]
- Novel tests of AdS/CFT [see e.g. Martelli-Sparks, Cassani-Martelli,...]
- Monotonicity of Renormalization Group Flows in $d = 3$ [Jafferis-Klebanov-Pufu-Safdi, Closset-Dumitrescu-Festuccia-ZK-Seiberg]
- Many new relations to mathematics

One can also discuss the phenomenon of emergent gauge symmetry in a wider context, without supersymmetry.

Suppose we have an asymptotically free theory with (N_c, N_f, N_s) gauge fields, Weyl fermions, and scalars, respectively. Suppose it enjoys duality, and it has a new classical limit in the infrared with (N_c', N_f', N_s') gauge fields, Weyl fermions, and scalars, respectively.

It turns out that such a situation must always obey a mysterious-looking inequality [Cardy, ZK-Schwimmer]

$$62N_c + \frac{11}{2}N_f + N_s > 62N_c' + \frac{11}{2}N_f' + N_s'$$

This is a special case of a more general constraint (the a-theorem) on renormalization group flows in four dimensions.

The supersymmetric example we discussed satisfies this inequality because scalars contribute to the inequality much less than gauge fields.

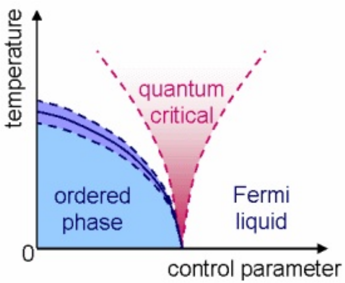
Given that massless (or very light) spin 1 particles can be composite, we can ask whether this has any relevance to nature.

As we have seen in our $SU(4) \to SU(2)$ example, this is a strong/weak relation. It should be viewed as a generalization of Maxwell's Electric-Magnetic duality

$$E \to B \ , \qquad B \to -E \ ,$$

under which $e \to 1/e$.

Three-dimensional versions of this idea may well be realizable in quantum phase transitions. But we are more interested in 3+1 dimensions.



In QCD, there are the ρ_μ^i mesons with $i = 1, 2, 3$ and mass ~ 750MeV. One can describe them in the following way: imagine the unitary matrix of pions

$$U = e^{i\pi^a T^a} \ , \qquad U \to G_L U G_R \ , \qquad G_{L,R} \in SU(2)_{L,R}$$

and factorize it as

$$U = MN^\dagger$$

Now there is a gauge symmetry (i.e. redundancy) that appears [e.g. Georgi and refs]:

$$M \to Mh \ , \qquad N \to Nh \ , \qquad h \in SU(2)_{HLS}$$

We identify the gauge fields of this local symmetry as the rho mesons

$$\rho_\mu^i T^i \to h\rho_\mu^i T^i h^{-1} + ih^{-1}\partial_\mu h$$

Suppose we limit ourselves to 2-derivative Lagrangians that are invariant under the global $G_{L,R}$ symmetry and the local h symmetry. There are three couplings:

$$\mathcal{L} = -\frac{1}{g^2}F^2 + f_\pi^2(\partial_\mu\pi)^2 + af_\pi^2(\epsilon^{abc}\pi^a\partial_\mu\pi^b + \rho_\mu^c)^2 + \cdots ~,$$

$F = \partial\rho + [\rho, \rho]$ the usual field strength. The mass of the rho meson fixes g and the decay constant of the pion fixes f_π. So there is one unknown parameter: a.

QCD phenomenology is best matched if one takes

$$a = 2~.$$

$a = 2$ reproduces the so-called "vector dominance."

If one could continuously deform QCD such that the rho mesons would become lighter and lighter, it is entirely conceivable that this $SU(2)_{LHS}$ would be our emergent massless gauge symmetry. Then, the two-derivative hypothesis would be rigorously justified for light rho mesons.

One could even justify $a = 2$ by appealing to a Weinberg-like sum rule.

In fact, in the SUSY duality we started from, there is an analog of a which is precisely equal to 2 [ZK].

To summarize, some crude aspects of QCD phenomenology are well described by treating the ρ mesons as if they are the light gauge bosons of some emergent "magnetic" $SU(2)_{HLS}$ gauge symmetry.

The challenge here is to make this precise: i.e. find a deformation of QCD for which the rho mesons become parametrically light.

We don't know if such a deformation exists.

Conclusions

- Massless spin-1 particles can be composite.
- One has precise realizations with $\mathcal{N} = 1$ SUSY.
- One can test these ideas using very recent developments in $\mathcal{N} = 1$ SUSY dynamics. Intimate relations to modern mathematics.
- This idea can be consistent with rigorous constraints on the properties of renormalization group flows and the emergence of new classical limits.
- Remains to be seen if one can connect to nature in $3 + 1$ dimensions. Some encouraging hints exist.

Thank You For Your Attention