

Observation of $J/\psi p$ Resonances Consistent With Pentaquark States

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On behalf of the LHCb Collaboration



Tetra- and Penta-quarks conceived at the birth of Quark Model

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PHYSICS LETTERS

1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN

California Institute of Technology, Pasadena, California

Received 4 January 1964

. . .

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u\bar{3}$, $d\bar{3}$, and $s\bar{3}$ of the triplet as "quarks" q and the members of the anti-triplet as anti-quarks \bar{q} . Baryons can now be constructed from quarks by using the combinations (qqq) , $(qqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assumed that the lowest baryon configuration (qqq) gives just the representations 1, 8, and 10 that have been observed, while

8419/TH.412

21 February 1964

 AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

II *)

G. Zweig

CERN---Geneva

*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

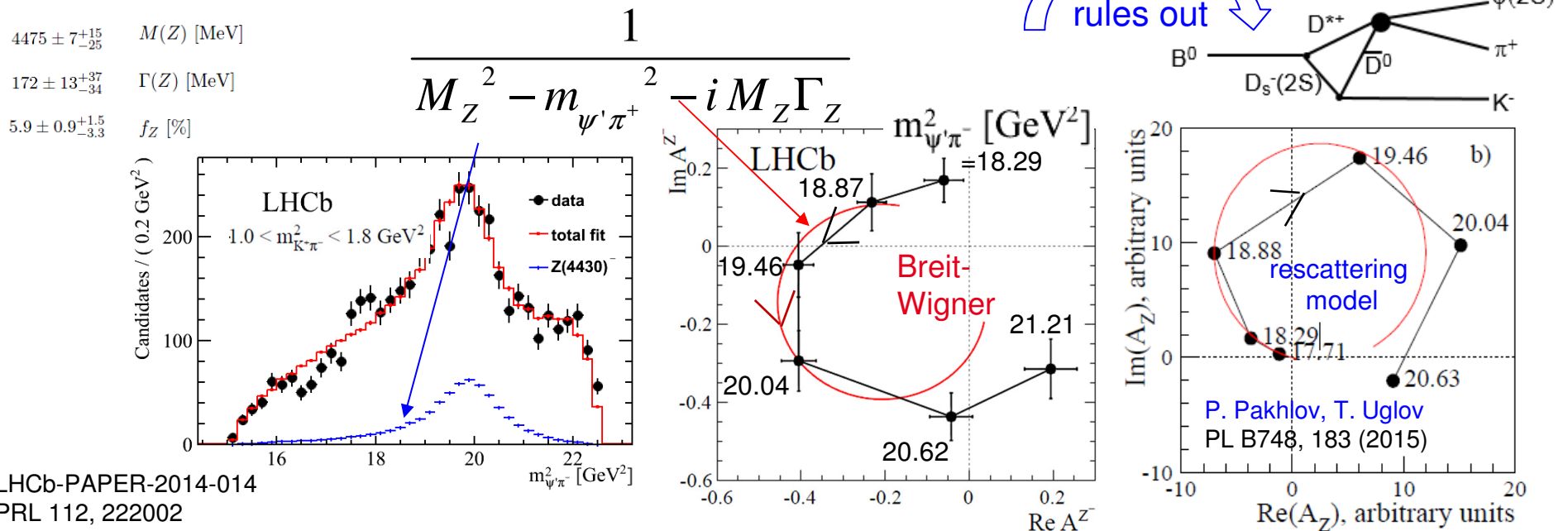
. . .

6) In general, we would expect that baryons are built not only from the product of three aces, AAA , but also from $\bar{A}AAAA$, $\bar{A}AAAAA$, etc., where \bar{A} denotes an anti-ace. Similarly, mesons could be formed from $\bar{A}A$, $\bar{A}AAA$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A}A$ and AAA , that is, "deuces and treys".

- Searches for such states made out of the light quarks (u,d,s) are ~50 years old, but no undisputed experimental evidence have been found for them

Charmonium and Quark Model

- **November revolution 1974:** discovery of J/ψ and other charmonium states convinces remaining skeptics that the quarks are real and mesons are made out of $q\bar{q}$.
- **Discovery of the super narrow ($\Gamma < 1.2$ MeV) $X(3872)$ state ($\rightarrow J/\psi\pi^+\pi^-$)** at the $D^0\bar{D}^{0*}$ threshold in **2003** by Belle renews hopes for establishing **tetraquark** states (molecular or tightly bound), but $\chi_c(2^3P_{1++})$ is likely in the mix.
- Discovery of the **charged $Z(4430)^+$ ($\rightarrow \psi'\pi^+$)** in **2007** by Belle provides “smoking gun” for 4-quark effect. However, not confirmed until the last year.
- 4D amplitude analysis by LHCb of $B^0 \rightarrow \psi'\pi^+K^-$, $\psi' \rightarrow \mu^+\mu^-$ with interfering $K^{0*} \rightarrow \pi^+K^-$ and $Z(4430)^+ \rightarrow \psi'\pi^+$ contributions confirms $Z(4430)^+$ and provides **evidence for its resonant character via Argand diagram**:



LHCb-PAPER-2014-014
 PRL 112, 222002
 See also LHCb-PAPER-2015-038

LHCb has 12.5 times more events than Belle with better purity thanks to...

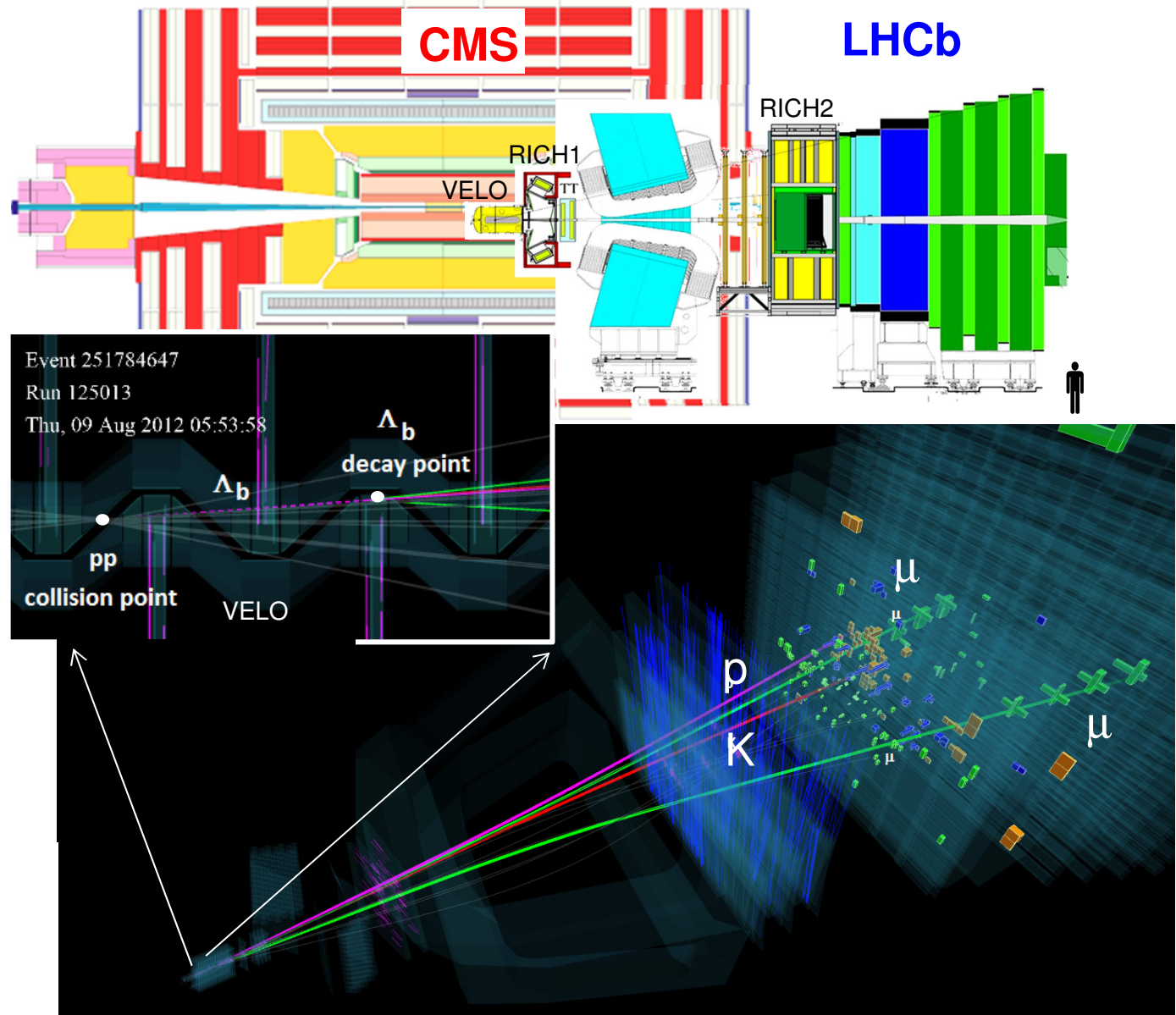
LHCb detector at LHC

- Advantages over e^+e^- B-factories:

- ~1000x larger b production rate
- **produce b-baryons at the same time as B-mesons**
- long visible lifetime of b-hadrons (no backgrounds from the other b-hadron)

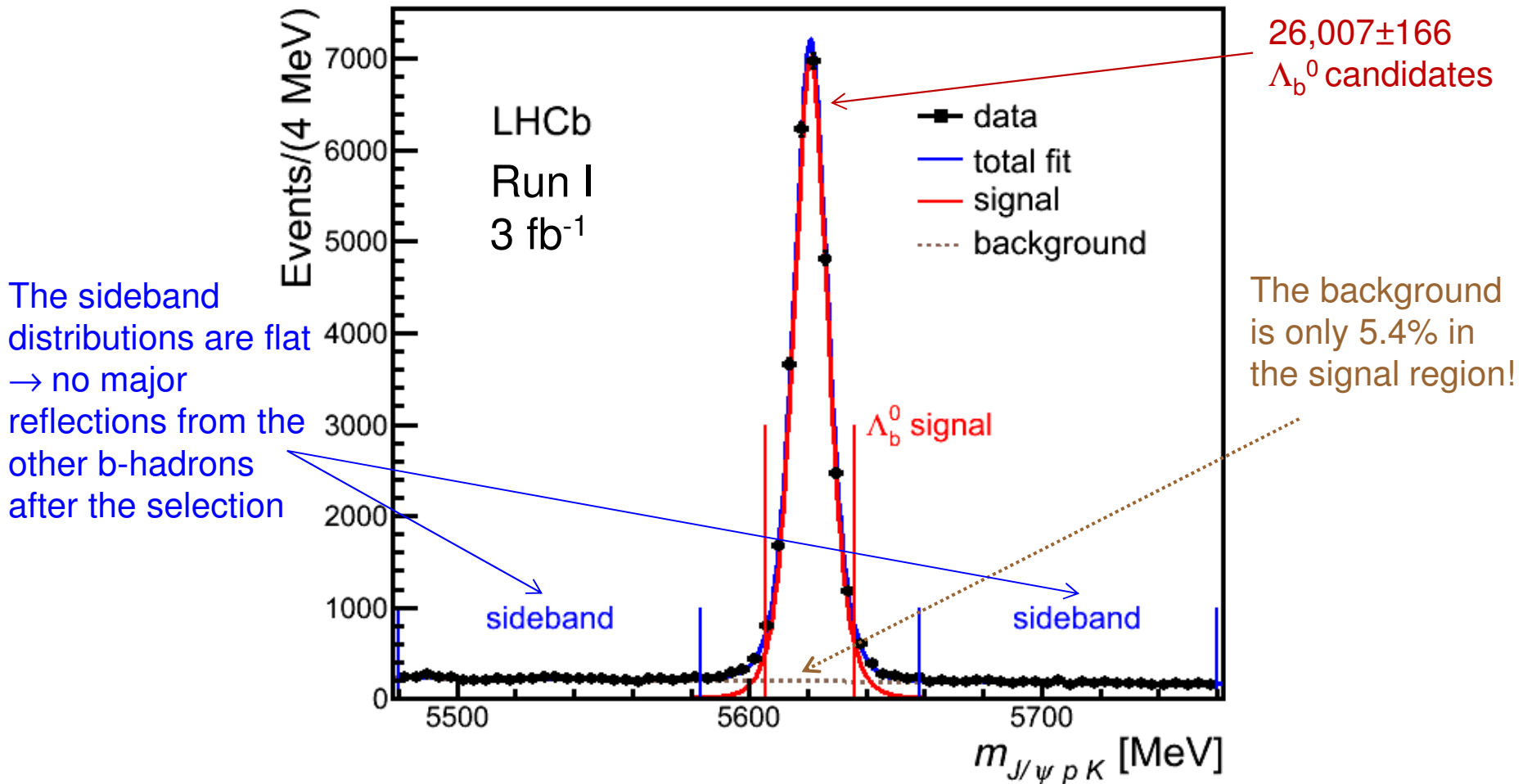
- Advantages over GPDs:

- RICH detectors for $\pi/K/p$ discrimination (smaller backgrounds)
- Small event size allows large trigger bandwidth (up to 5 kHz in Run I); all devoted to flavor physics



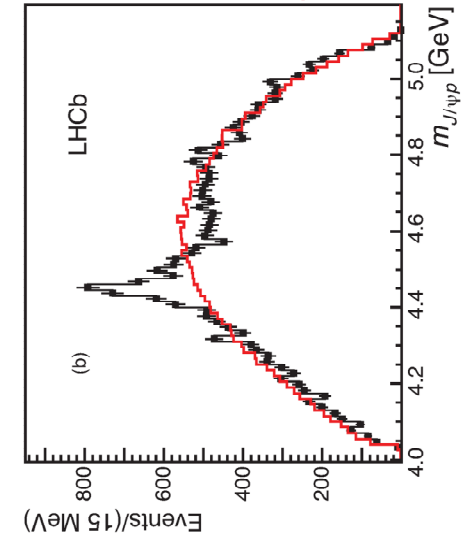
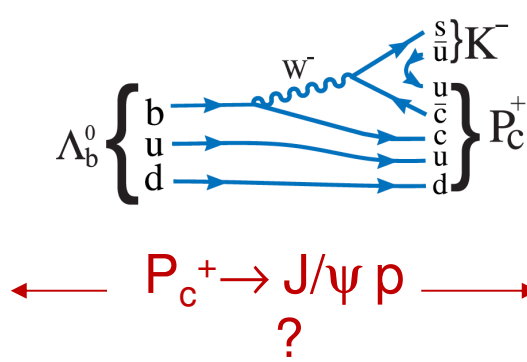
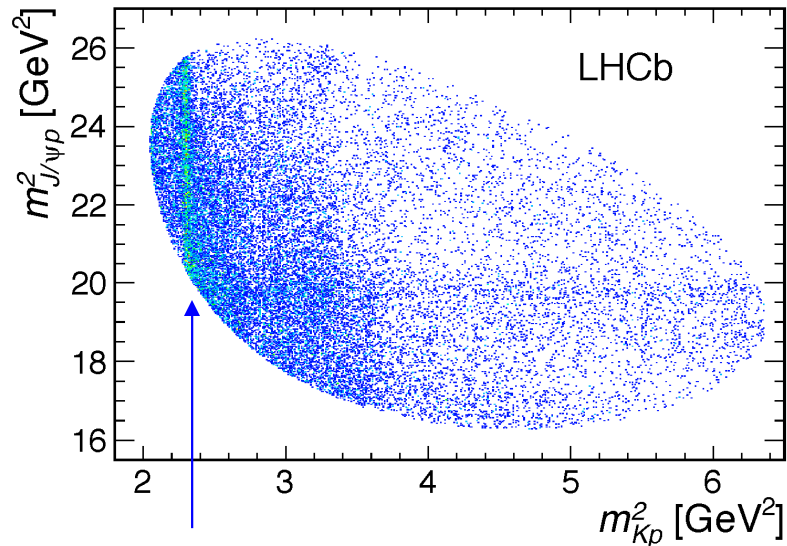
LHCb $\Lambda_b^0 \rightarrow J/\psi p K^-$

LHCb-PAPER-2015-029, arXiv:1507.03414, PRL 115, 07201

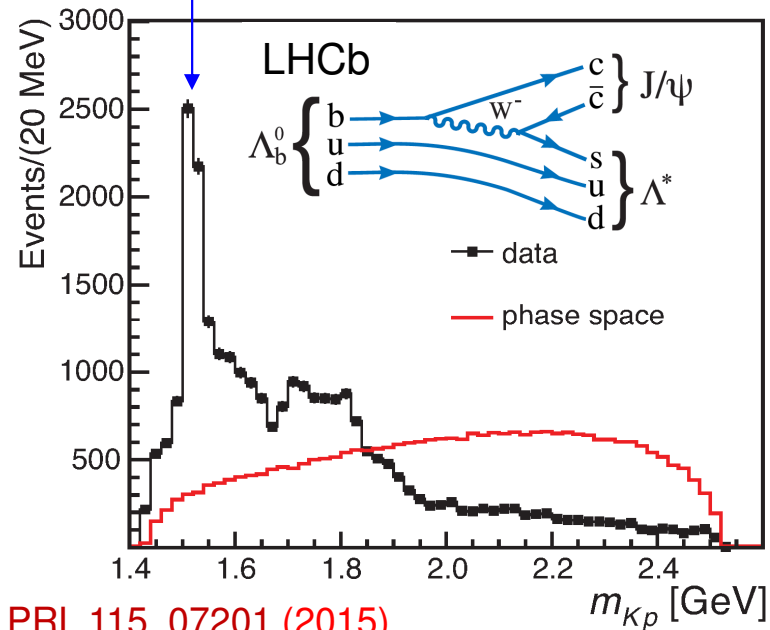


- The decay first observed by LHCb and used to measure Λ_b^0 lifetime:
 - LHCb-PAPER-2013-032 (PRL 111, 102003)

$\Lambda_b^0 \rightarrow J/\psi p K^-$: unexpected structure in $m_{J/\psi p}$



$\Lambda(1520)$ and other Λ^* 's $\rightarrow p K^-$

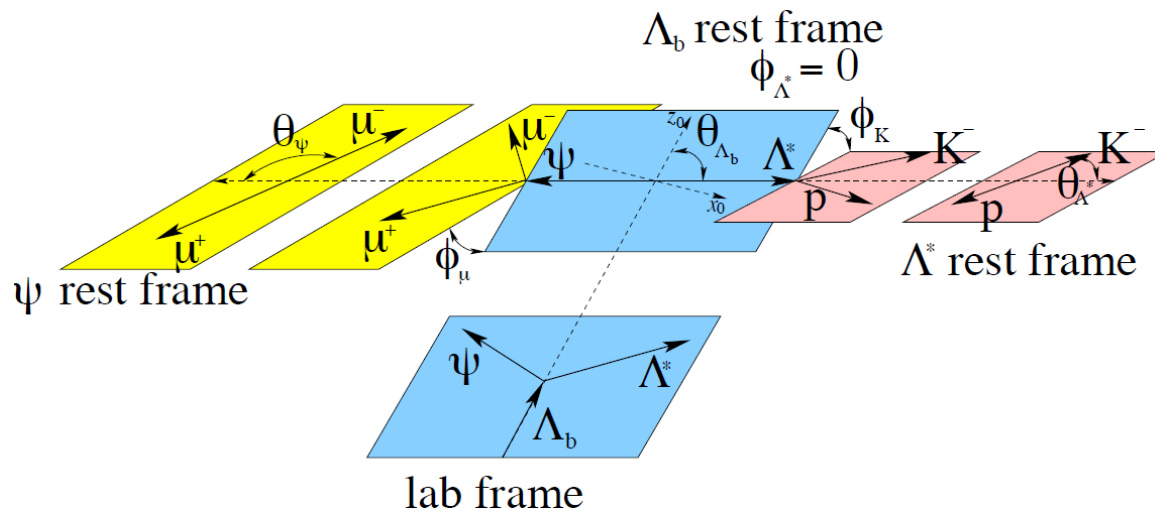


PRL 115, 07201 (2015)

- Unexpected, narrow peak in $m_{J/\psi p}$
- Many checks done to ensure it is not an “artifact” of selection:
 - Veto $B_s \rightarrow J/\psi K^- K^+$ & $B^0 \rightarrow J/\psi K^- \pi^+$ after changing p to K, or K to π
 - Clone and ghost tracks carefully eliminated
 - Exclude Ξ_b decays as a possible source
- Could it be a reflection of interfering Λ^* 's $\rightarrow p K^-$?
 - Proper amplitude analysis absolutely necessary!

Amplitude Analysis of $\Lambda_b^0 \rightarrow J/\psi p K^-$, $J/\psi \rightarrow \mu^+ \mu^-$

- Follows in footsteps of the $Z(4430)^+$ 4D amplitude analysis in $B^0 \rightarrow J/\psi \pi^+ K^-$, $J/\psi \rightarrow \mu^+ \mu^-$
- Analyze all dimensions of the $\Lambda_b^0 \rightarrow J/\psi p K^-$, $J/\psi \rightarrow \mu^+ \mu^-$ decay kinematics:
 - to maximize sensitivity to the decay dynamics
 - to avoid biases due to averaging over some dimensions in presence of the non-uniform detector efficiency
 - two additional dimensions (6D) because Λ_b^0 has a spin



6 independent data variables:
1 mass (m_{Kp}), 5 angles

4-6 independent **complex** helicity couplings per Λ_n^* resonance to fit depending in its J^P

- Use 6D unbinned maximum likelihood fit of the matrix element parameters
- Two different background subtraction methods:
 - parametrized $m_{J/\psi p K}$ sidebands (cFit) or sWeighted log-likelihood (sFit)

Λ^* resonance model

$$\mathcal{H}_{\lambda_B, \lambda_C}^{A \rightarrow BC} = \sum_L \sum_S \sqrt{\frac{2L+1}{2J_A+1}} B_{L,S} \begin{pmatrix} J_B & J_C & S \\ \lambda_B & -\lambda_C & \lambda_B - \lambda_C \end{pmatrix} \times \begin{pmatrix} L & S & J_A \\ 0 & \lambda_B \mp \lambda_C & \lambda_B - \lambda_C \end{pmatrix}$$

Helicity couplings

LS couplings

No high- J^P high-mass states

In Λ^* decay: $P_A = P_B P_C (-1)^L$

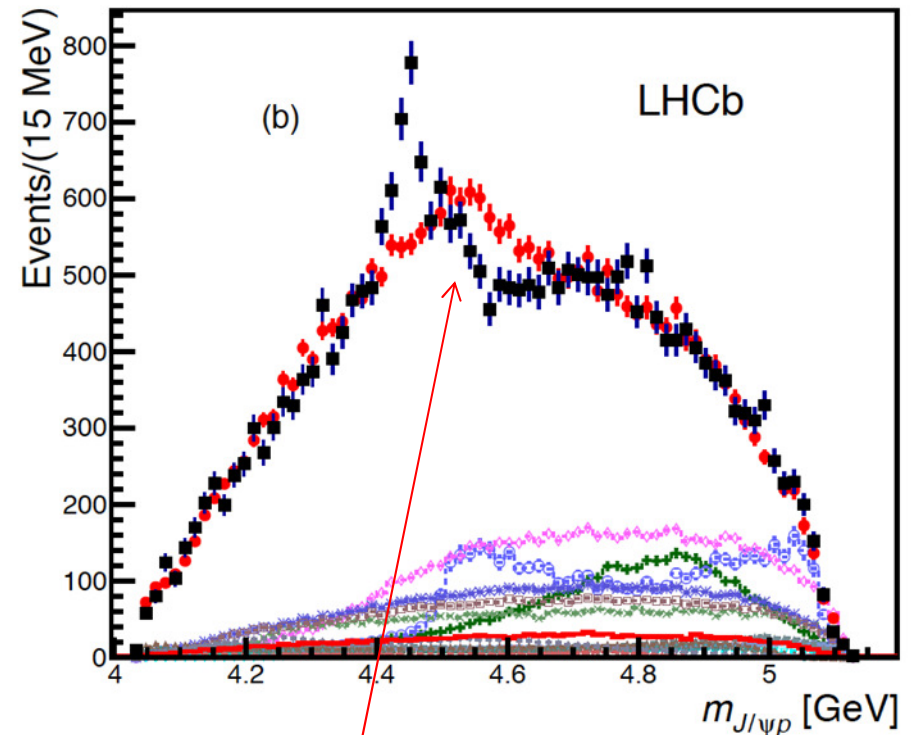
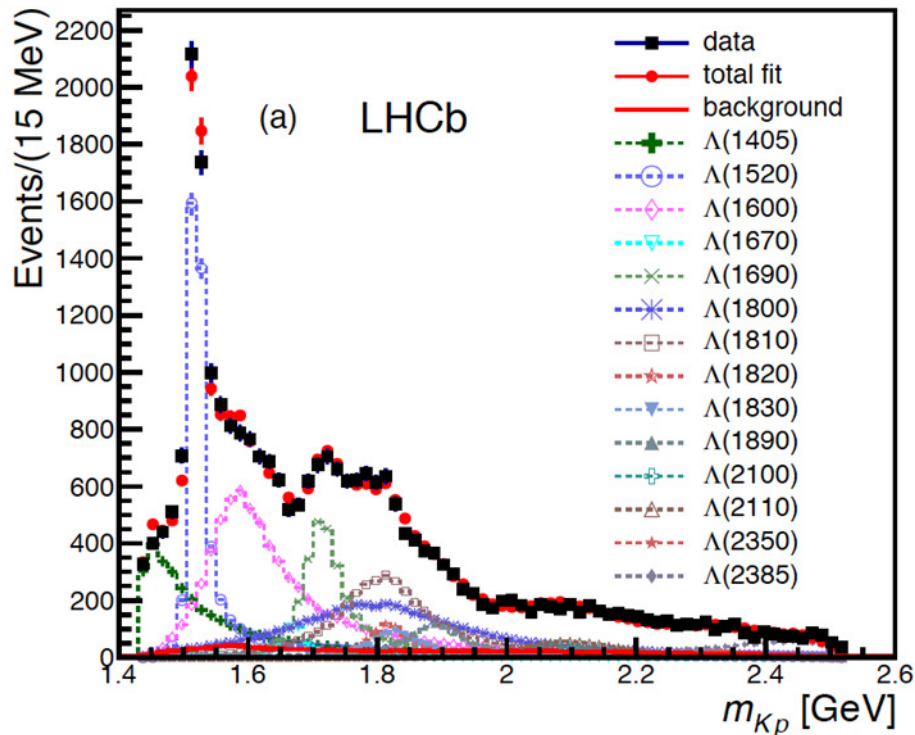
limit L

All states, all L

State	J^P	M_0 (MeV)	Γ_0 (MeV)	# Reduced	# Extended
$\Lambda(1405)$	$1/2^-$	$1405.1_{-1.0}^{+1.3}$	50.5 ± 2.0	3	4
$\Lambda(1520)$	$3/2^-$	1519.5 ± 1.0	15.6 ± 1.0	5	6
$\Lambda(1600)$	$1/2^+$	1600	150	3	4
$\Lambda(1670)$	$1/2^-$	1670	35	3	4
$\Lambda(1690)$	$3/2^-$	1690	60	5	6
$\Lambda(1800)$	$1/2^-$	1800	300	4	4
$\Lambda(1810)$	$1/2^+$	1810	150	3	4
$\Lambda(1820)$	$5/2^+$	1820	80	1	6
$\Lambda(1830)$	$5/2^-$	1830	95	1	6
$\Lambda(1890)$	$3/2^+$	1890	100	3	6
$\Lambda(2100)$	$7/2^-$	2100	200	1	6
$\Lambda(2110)$	$5/2^+$	2110	200	1	6
$\Lambda(2350)$	$9/2^+$	2350	150	0	6
$\Lambda(2585)$	$5/2^-?$	≈ 2585	200	0	6

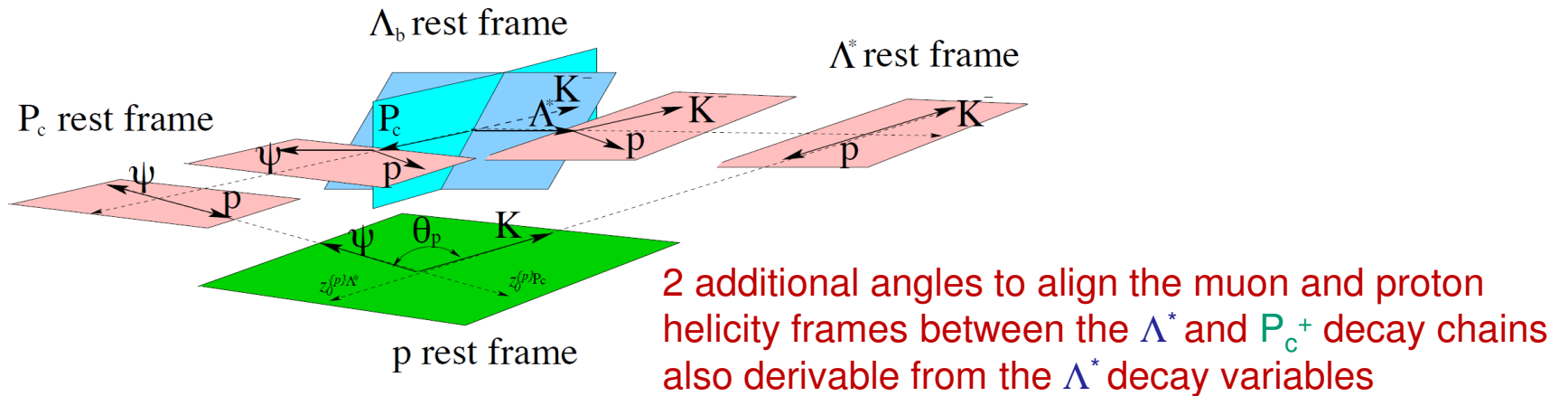
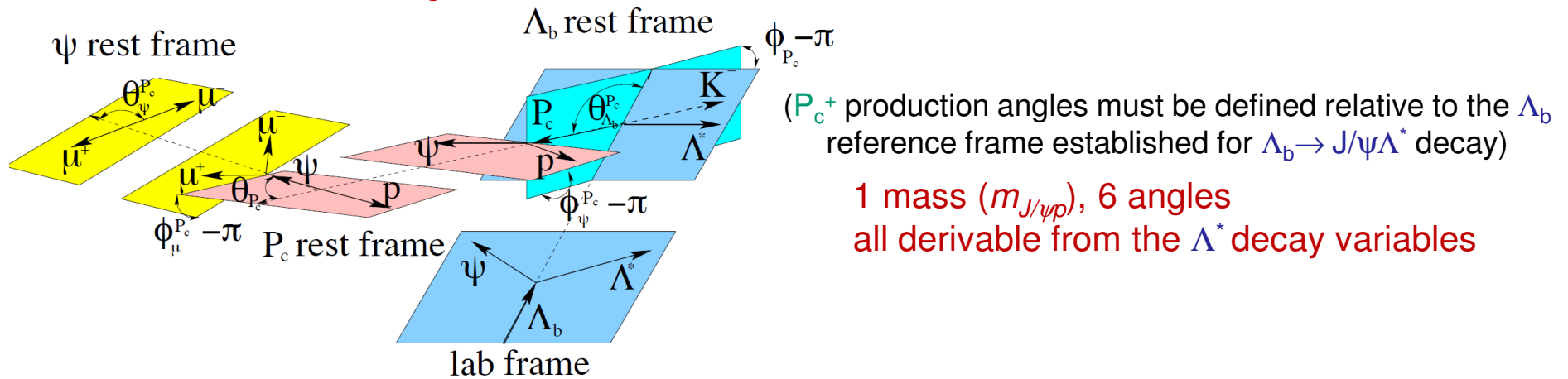
All known Λ^* states

Fit with $\Lambda^* \rightarrow pK^-$ contributions only



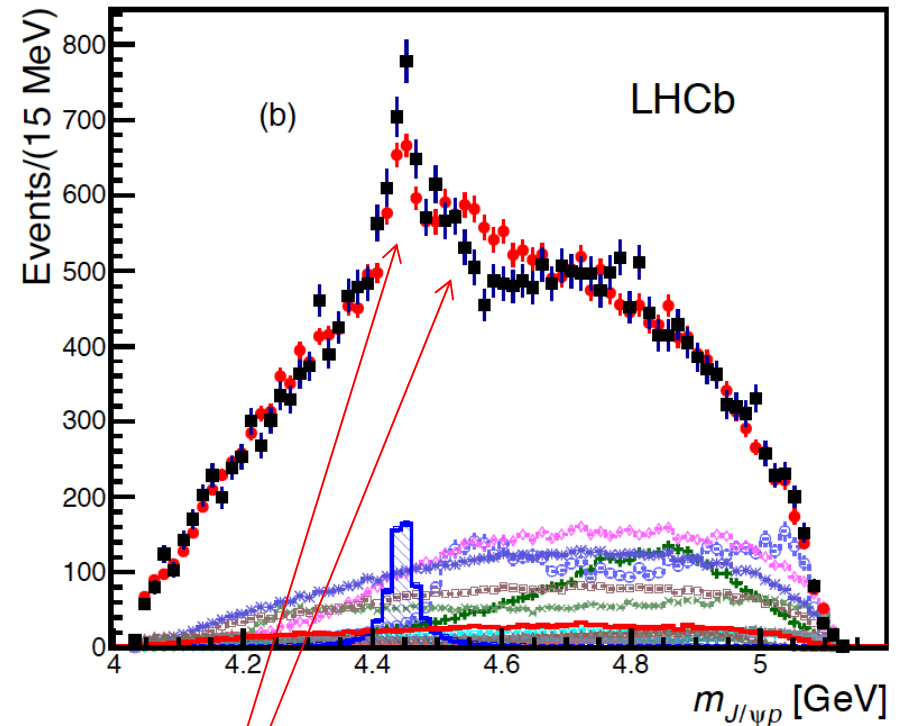
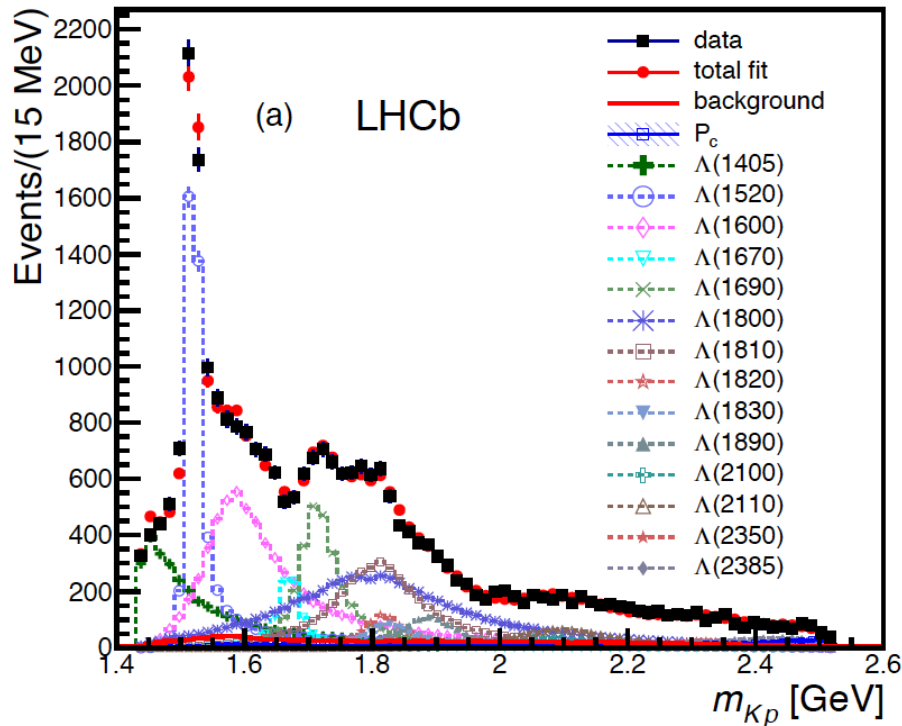
- Use extended model, so all possible known Λ^* amplitudes: m_{Kp} looks fine, but not $m_{J/\psi p}$
- Additions of non-resonant term, Σ^* 's or extra Λ^* 's doesn't help

P_c^+ Matrix Element



3-4 independent complex helicity couplings per P_{cj}^+ resonance depending on its J^P plus its mass and width to fit

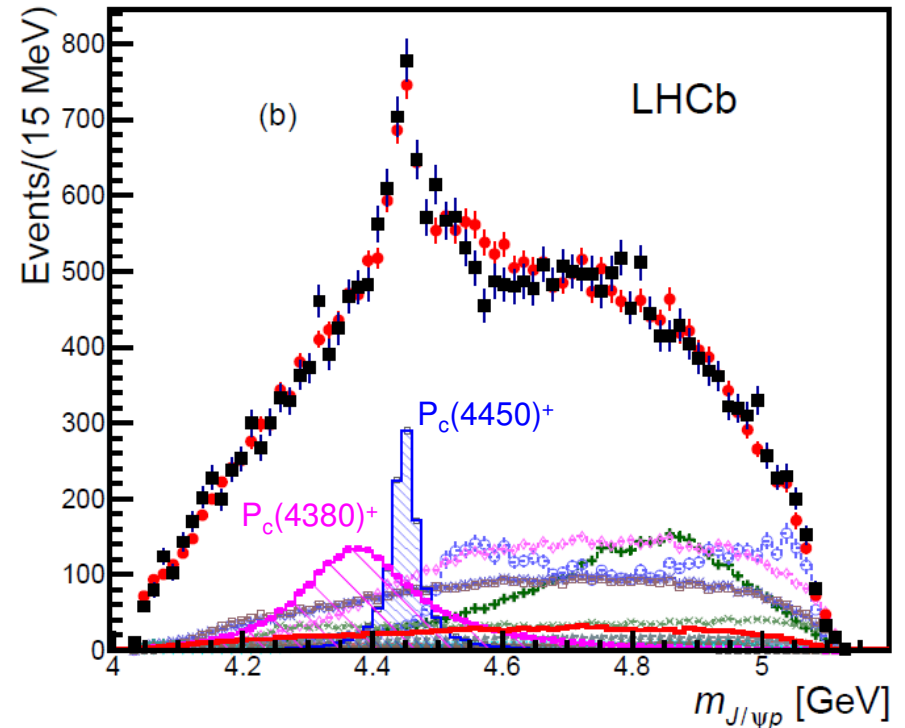
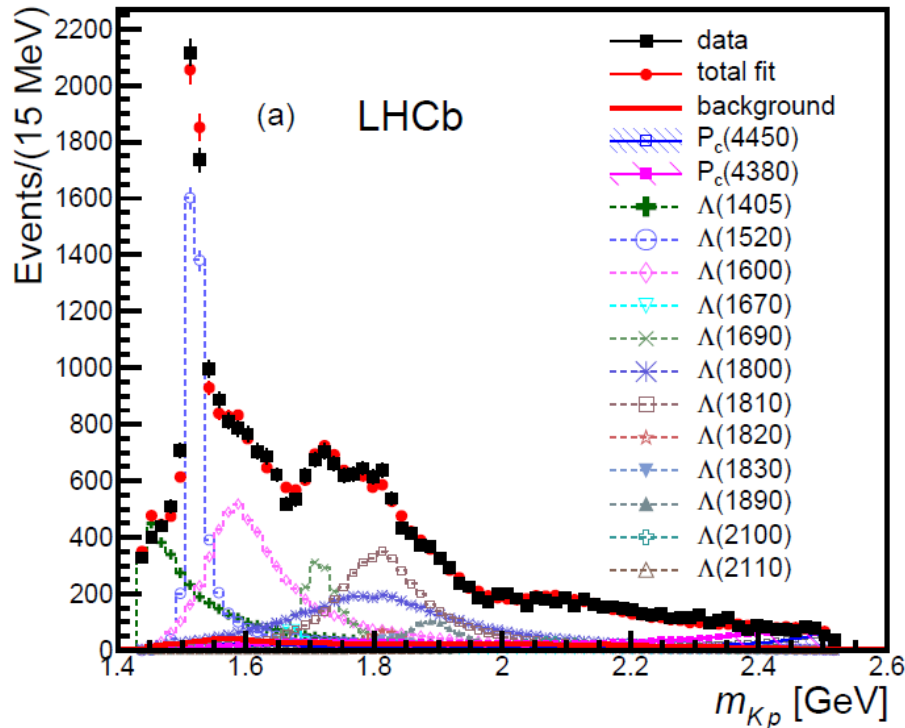
Fit with Λ^* 's and one $P_c^+ \rightarrow J/\psi p$ state



(extended Λ^* model)

- Try all J^P of P_c^+ up to $7/2^\pm$
- Best fit has $J^P = 5/2^\pm$. Still not a good fit

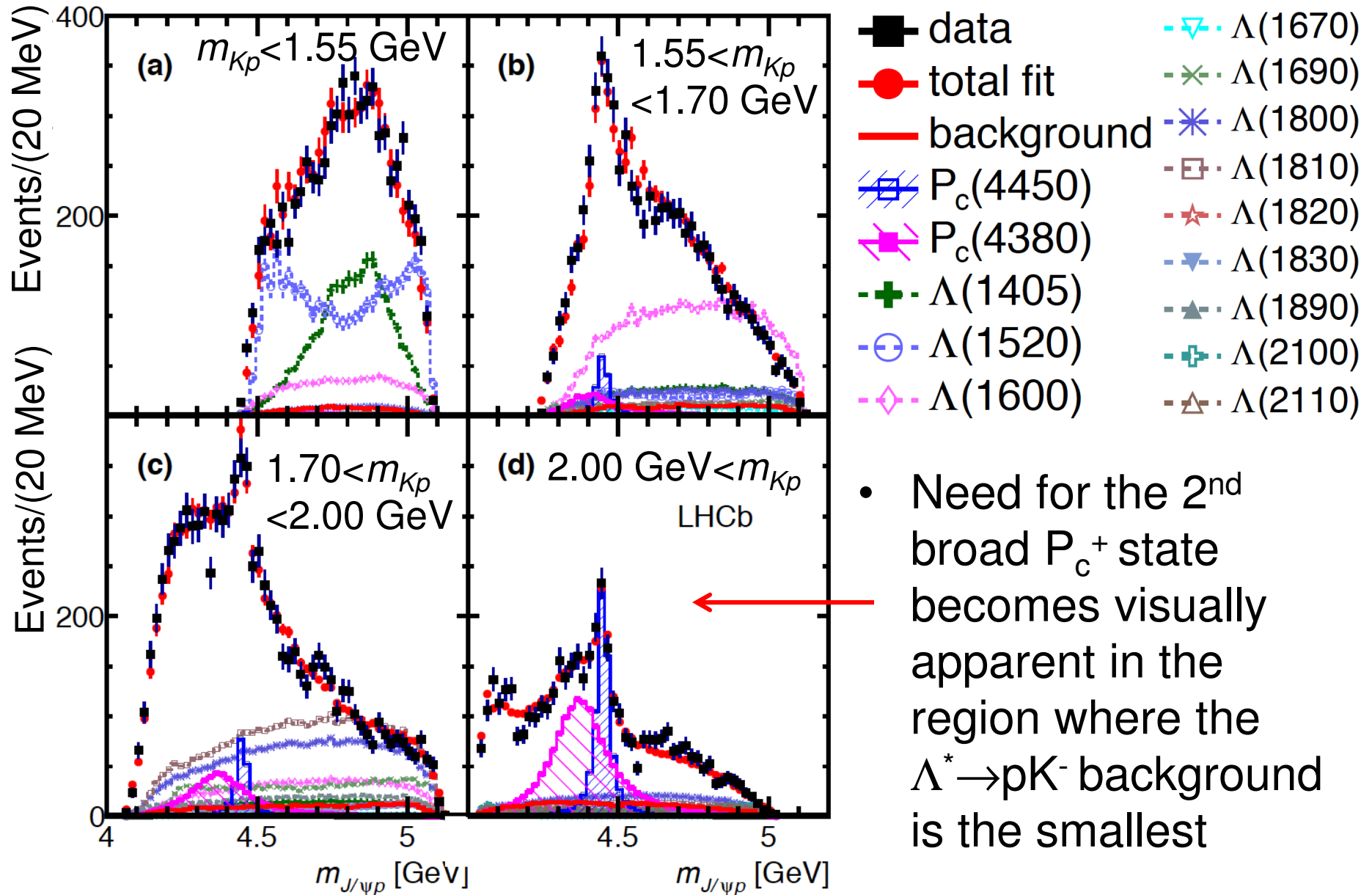
Fit with Λ^* 's and two $P_c^+ \rightarrow J/\psi p$ states



(reduced Λ^* model)

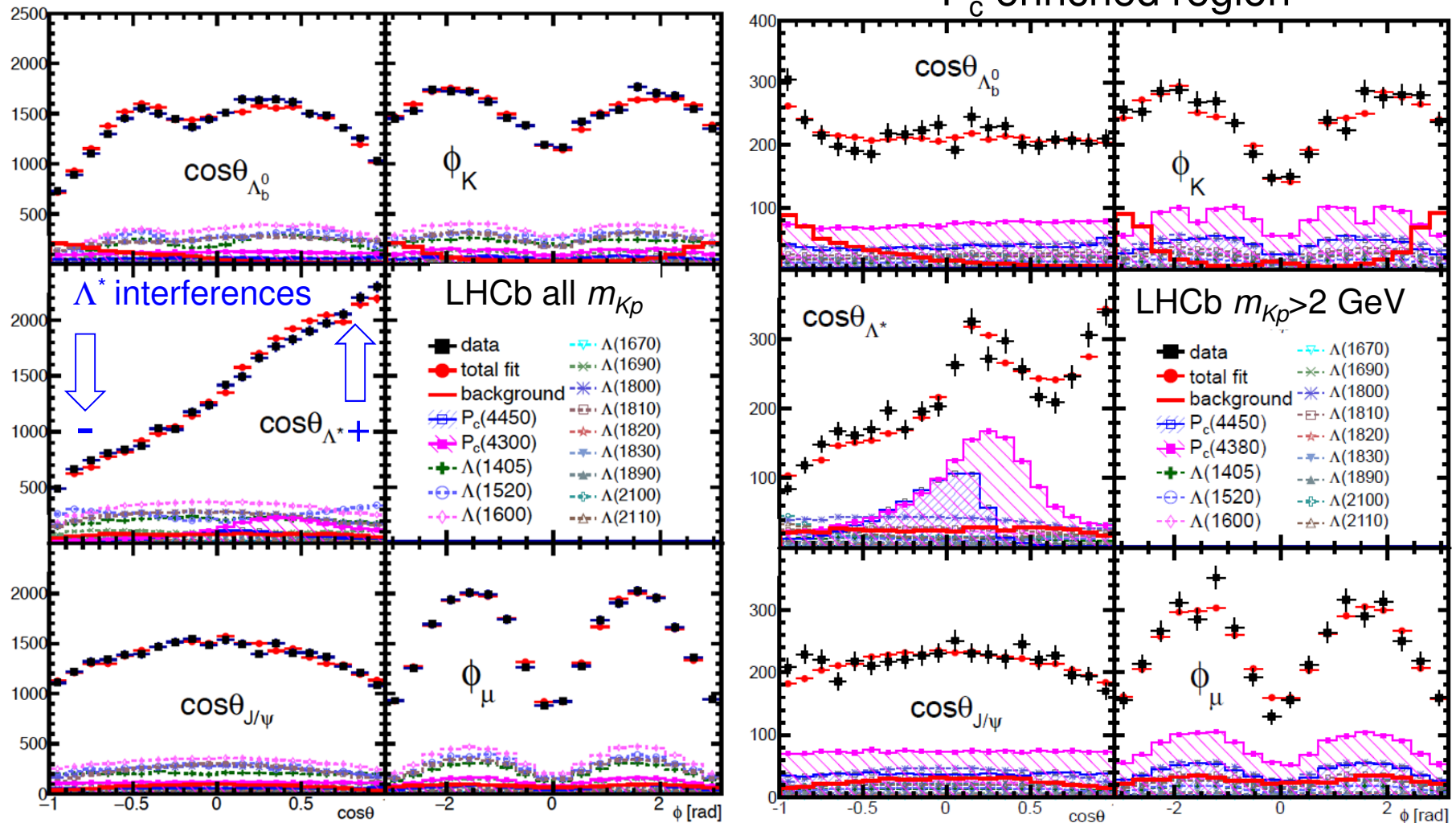
- Obtain good fits even with the reduced Λ^* model
- Best fit has $J^P = (3/2^-, 5/2^+)$, also $(3/2^+, 5/2^-)$ & $(5/2^+, 3/2^-)$ are preferred

Fit with Λ^* 's and two $P_c^+ \rightarrow J/\psi p$ states



Angular distributions

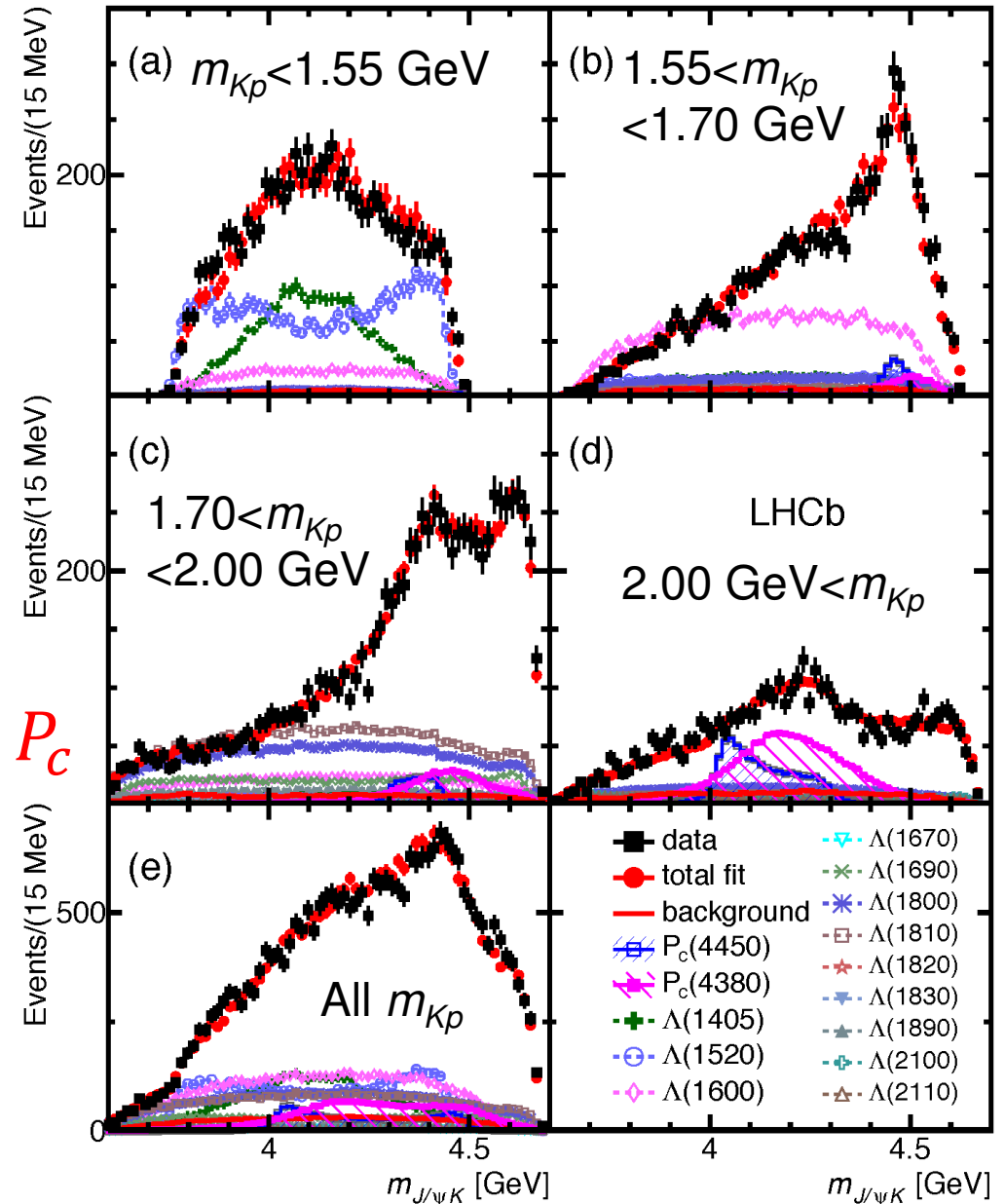
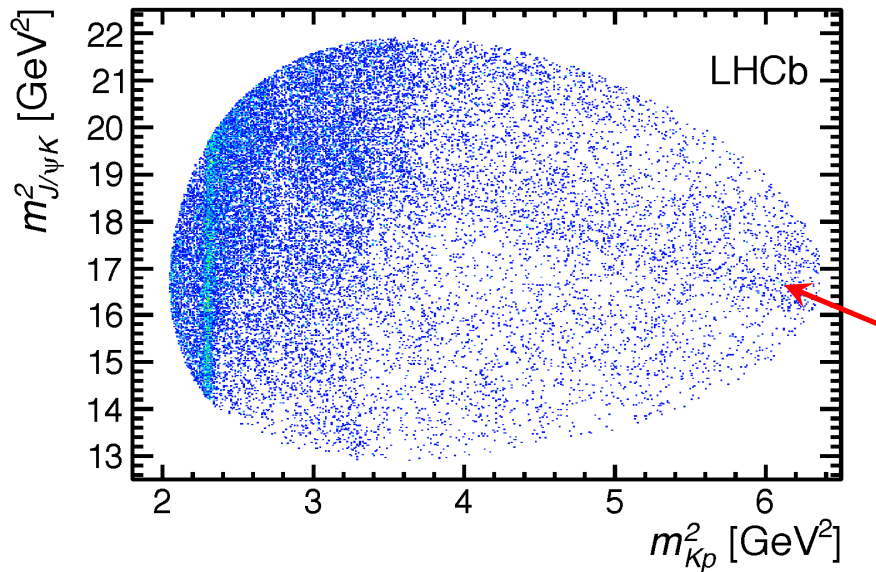
All data

 P_c enriched region


- Good description of the data in all 6 dimensions!

No need for exotic $J/\psi K^-$ contributions

- $J/\psi K^-$ system is well described by the Λ^* and P_c^+ reflections.



Significances and the results

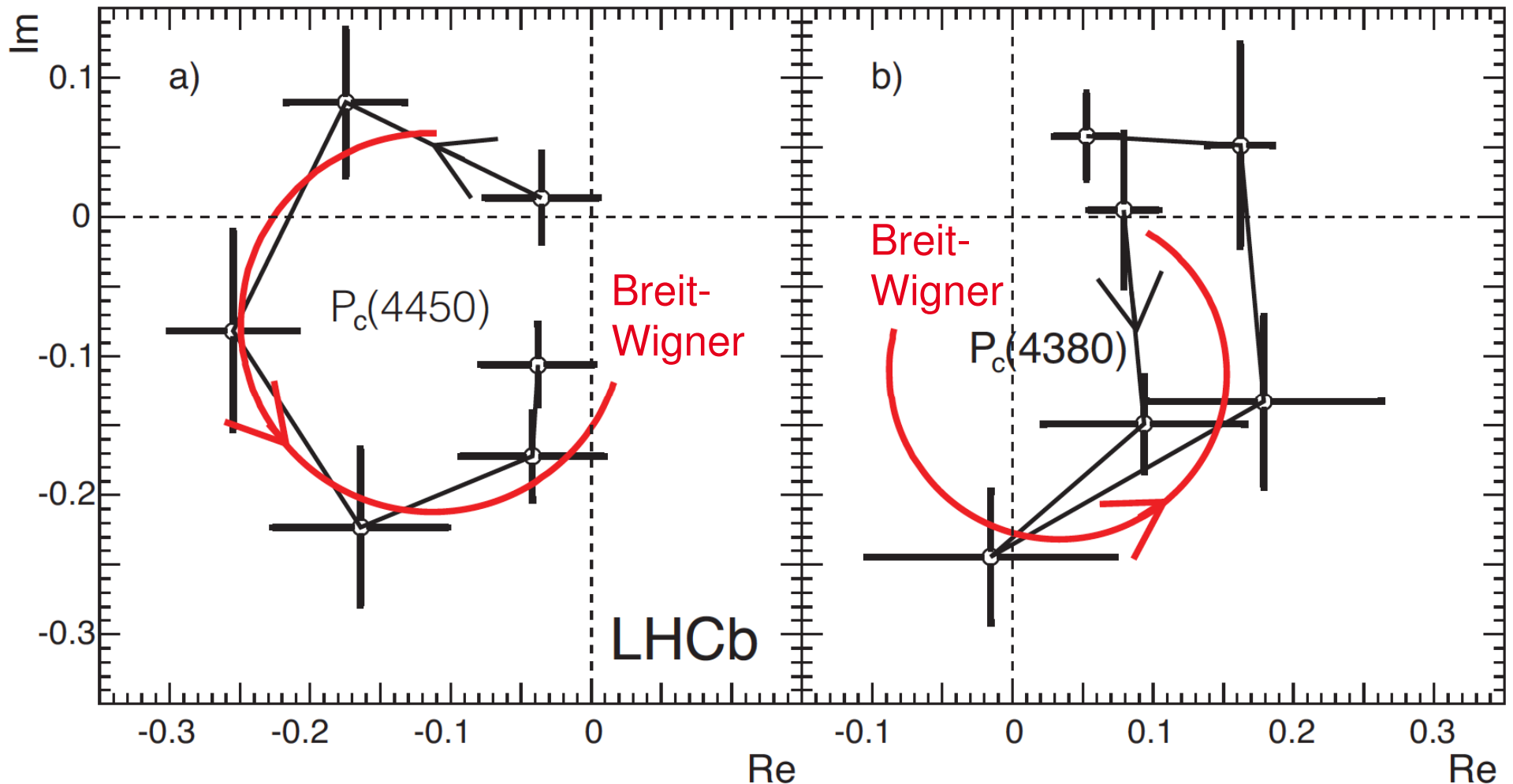
- Fit improves greatly, for 1 P_c $\Delta(-2\ln\mathcal{L})=14.7^2$, adding the 2nd P_c improves by 11.6², for adding both together $\Delta(-2\ln\mathcal{L})=18.7^2$
- Simulations of pseudoexperiments are used to turn the $\Delta(-2\ln\mathcal{L})$ values to significances:
 - significance of $P_c(4450)^+$ state is 12σ
 - significance of $P_c(4380)^+$ state is 9σ
 - combined significance of the two P_c^+ states is 15σ
- This includes the dominant systematic uncertainties, coming from difference between extended and reduced Λ^* model results.
- Parameters of the P_c^+ states (and F.F. of well isolated Λ^* 's)

State	Mass (MeV)	Width (MeV)	Fit fraction (%)
$P_c(4380)^+$	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$	$8.4 \pm 0.7 \pm 4.2$
$P_c(4450)^+$	$4449.8 \pm 1.7 \pm 2.5$	$39 \pm 5 \pm 19$	$4.1 \pm 0.5 \pm 1.1$
$\Lambda(1405)$	LHCb-PAPER-2015-029, arXiv:1507.03414, PRL 115, 07201		$15 \pm 1 \pm 6$
$\Lambda(1520)$			$19 \pm 1 \pm 4$

Argand diagrams

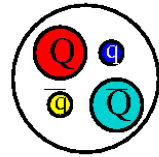
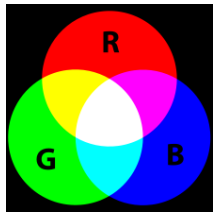
PRL 115, 07201 (2015)

P_c^+ amplitudes for 6 $m_{J/\psi p}$ bins between $+\Gamma$ & $-\Gamma$ around the resonance mass

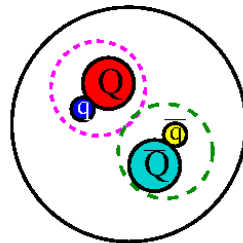
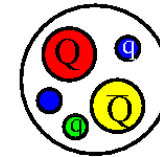


- Good evidence for the resonant character of $P_c(4450)^+$
- The errors for $P_c(4380)^+$ are too large to be conclusive

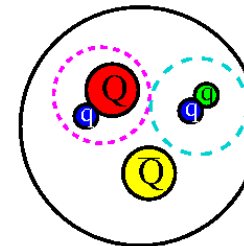
Different types of tetra- and penta-quarks



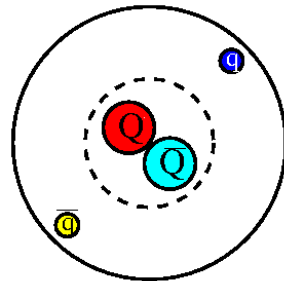
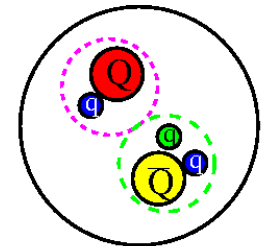
“plain”



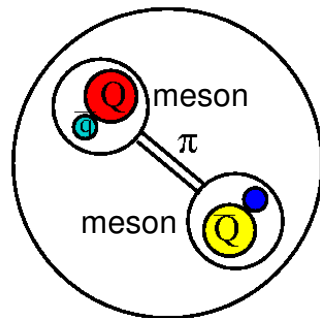
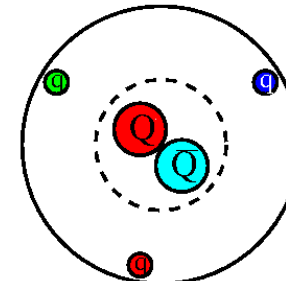
diquark model



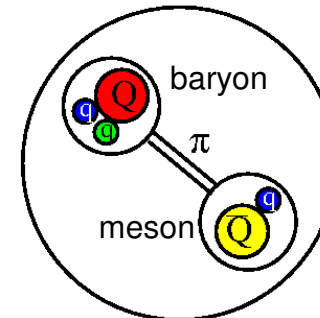
triquark model



hydro-charmonium model



molecular model



Conclusion

- Two pentaquark candidates decaying to $J/\psi p$ observed by LHCb with overwhelming significance in a state of the art amplitude analysis: they will not go away!

Frank Wilczek's tweet on 7/14/15: "Pentaquarks rise from the ashes: a phoenix pair"



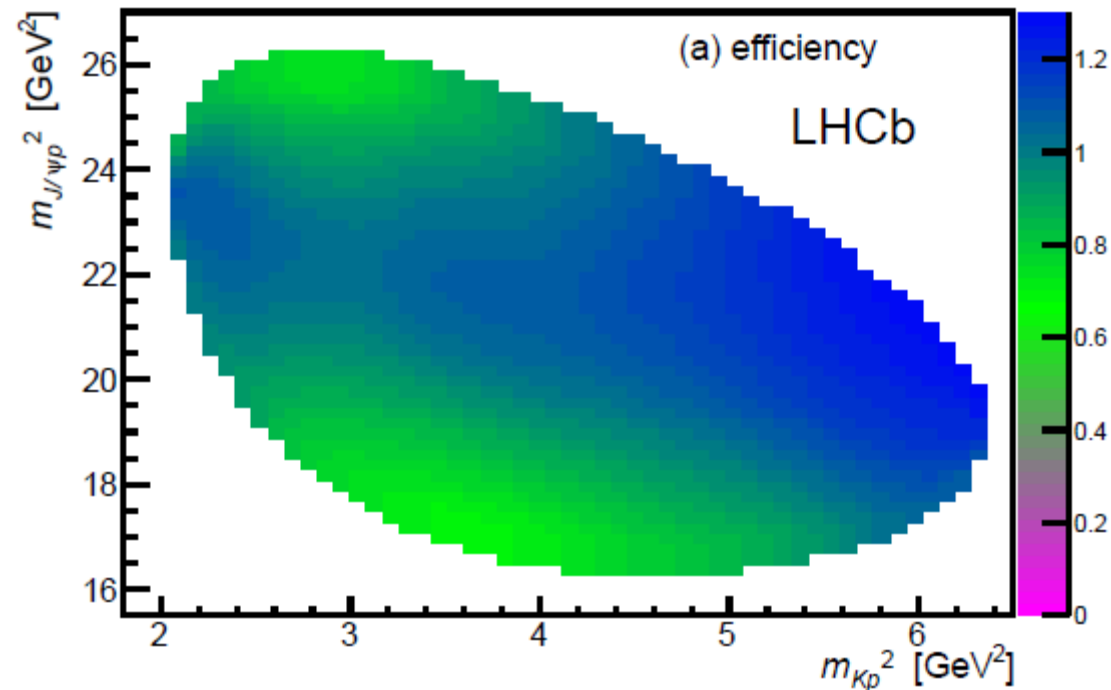
Pentaquark candidates rise from the ashes for the 2nd time.

- LHC resurrects them: should not be a surprise given baryon cross-sections!
 $c\bar{c}$ pair inside:
- Given the history of Quark Model should not be a surprise either.
- Hopefully true July 2015 revolution!

- However, what kind of 5-quark effects are they? 24 paper published in 1 month. Loosely bound meson-baryon molecules? Tightly bound (diquarks, triquarks,...)? Can we decisively rule out rescattering effects?
- Need more statistics for more sensitive tests. Need to identify the other elements of the new periodic table.
 - LHCb expects 8 fb^{-1} in Run 2 (-2018) followed by the detector/luminosity upgrade which will bring $\sim 50 \text{ fb}^{-1}$ by 2028.
 - Other experiments/colliders should be able to contribute (photoproduction?)

BACKUP SLIDES

Efficiency is smooth



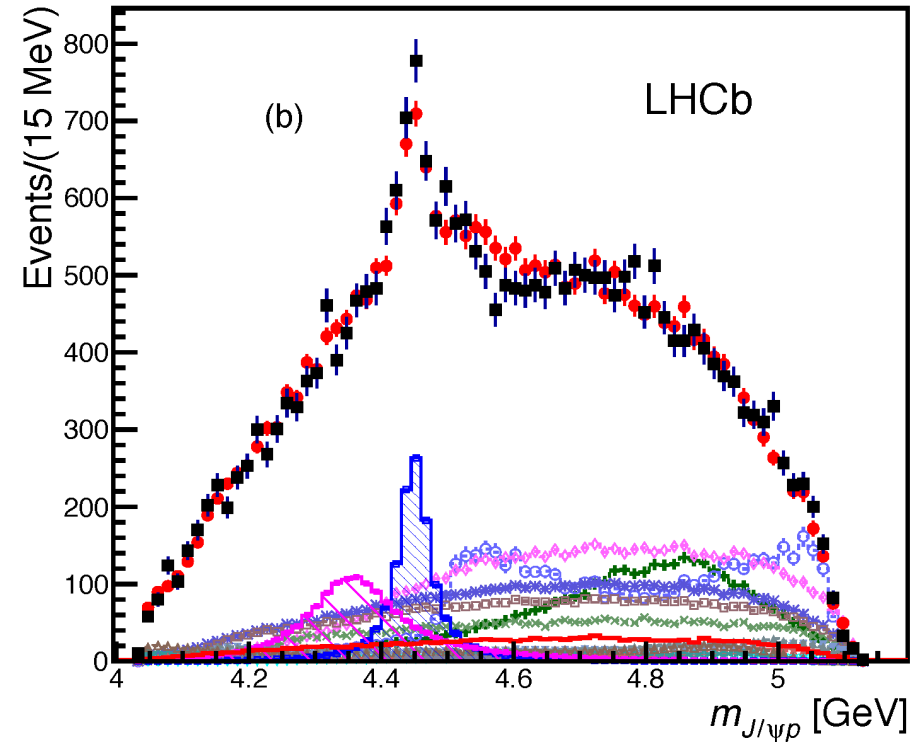
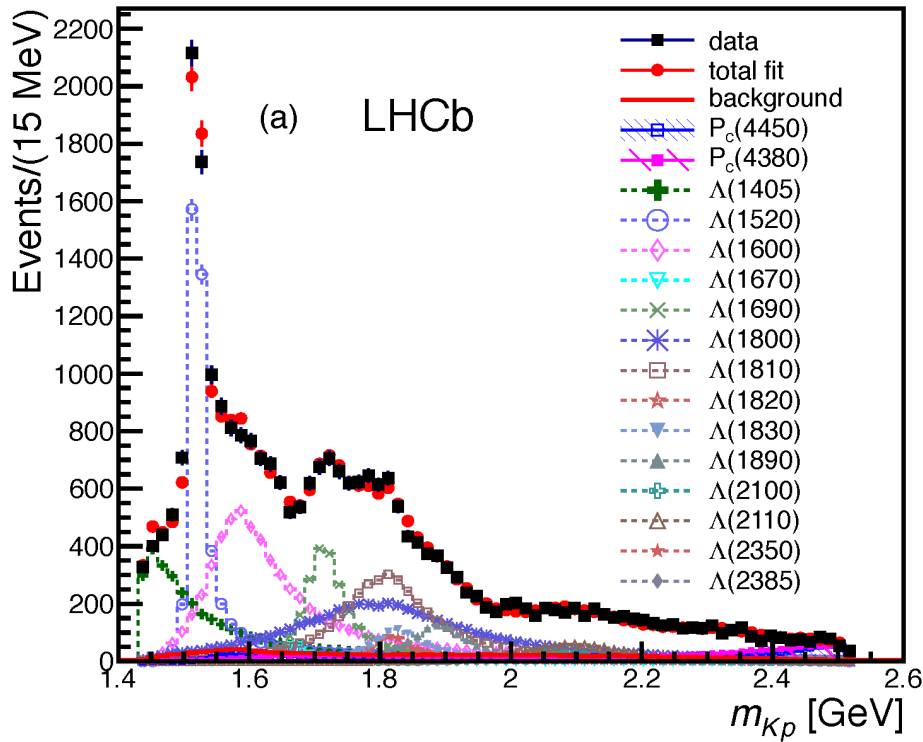
- The Dalitz plane contains the dominant efficiency variations
- Relative changes in efficiency smaller for the other fit variables
- Smooth, cannot be responsible for peaking structures.

Complete set of fit fractions

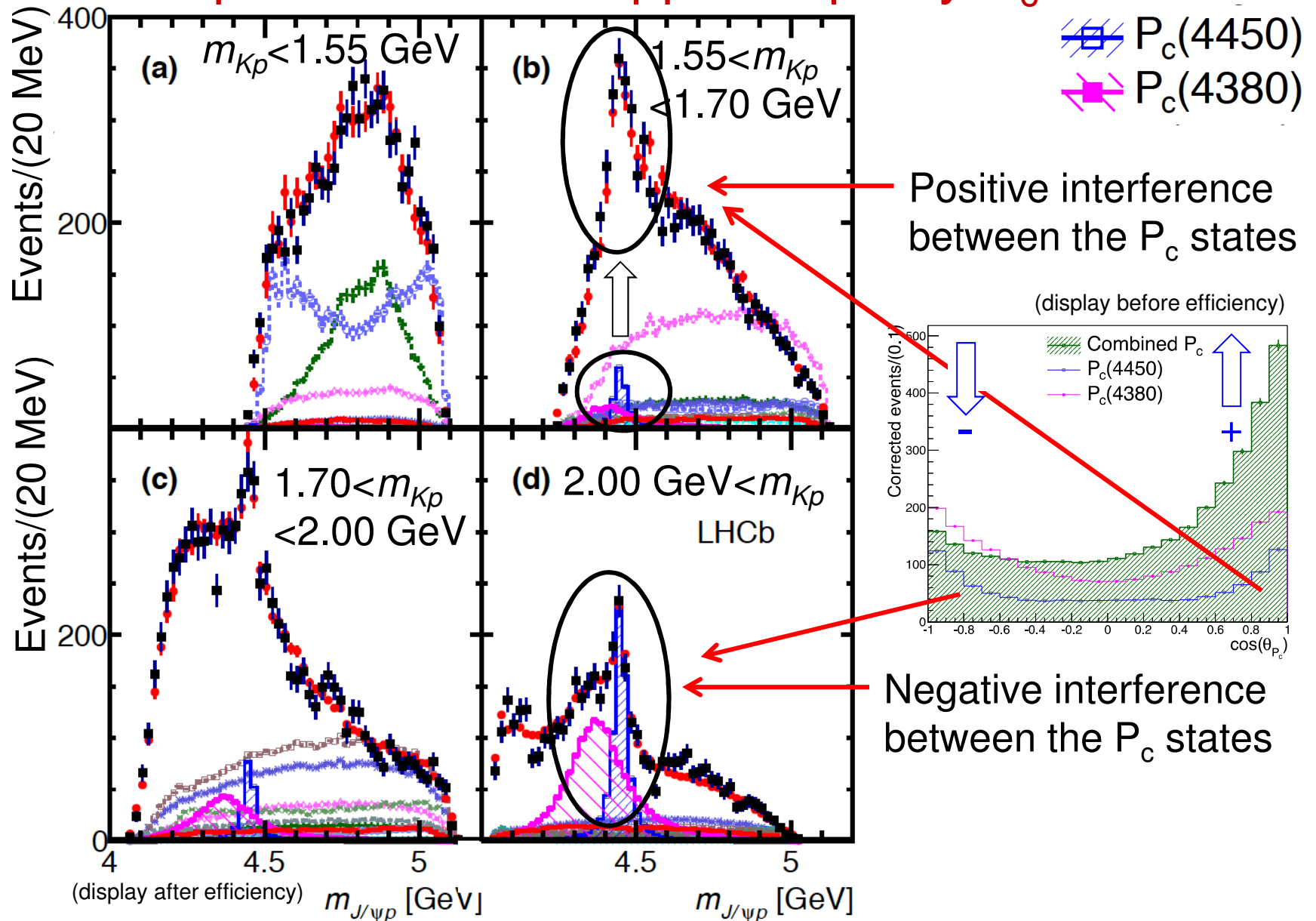
Table 3: Fit fractions of the different components from cFit and sFit for the default ($3/2^-$, $5/2^+$) model. Uncertainties are statistical only.

Particle	Fit fraction (%) cFit	Fit fraction (%) sFit
$P_c(4380)^+$	8.42 ± 0.68	7.96 ± 0.67
$P_c(4450)^+$	4.09 ± 0.48	4.10 ± 0.45
$\Lambda(1405)$	14.64 ± 0.72	14.19 ± 0.67
$\Lambda(1520)$	18.93 ± 0.52	19.06 ± 0.47
$\Lambda(1600)$	23.50 ± 1.48	24.42 ± 1.36
$\Lambda(1670)$	1.47 ± 0.49	1.53 ± 0.50
$\Lambda(1690)$	8.66 ± 0.90	8.60 ± 0.85
$\Lambda(1800)$	18.21 ± 2.27	16.97 ± 2.20
$\Lambda(1810)$	17.88 ± 2.11	17.29 ± 1.85
$\Lambda(1820)$	2.32 ± 0.69	2.32 ± 0.65
$\Lambda(1830)$	1.76 ± 0.58	2.00 ± 0.53
$\Lambda(1890)$	3.96 ± 0.43	3.97 ± 0.38
$\Lambda(2100)$	1.65 ± 0.29	1.94 ± 0.28
$\Lambda(2110)$	1.62 ± 0.32	1.44 ± 0.28

Extended Model with Two P_c Resonances



Data preference for opposite parity P_c^+ states



- This interference pattern only for states with opposite parity

Systematic uncertainties

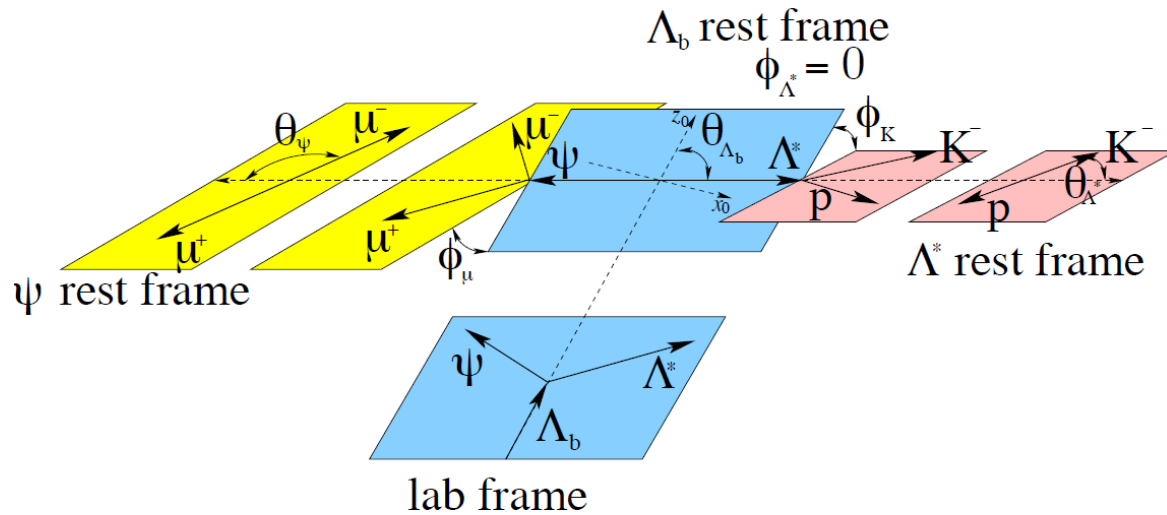
Source	M_0 (MeV)		Γ_0 (MeV)		Fit fractions (%)			
	low	high	low	high	low	high	$\Lambda(1405)$	$\Lambda(1520)$
Extended vs. reduced	21	0.2	54	10	3.14	0.32	1.37	0.15
Λ^* masses & widths	7	0.7	20	4	0.58	0.37	2.49	2.45
Proton ID	2	0.3	1	2	0.27	0.14	0.20	0.05
$10 < p_p < 100$ GeV	0	1.2	1	1	0.09	0.03	0.31	0.01
Nonresonant	3	0.3	34	2	2.35	0.13	3.28	0.39
Separate sidebands	0	0	5	0	0.24	0.14	0.02	0.03
J^P ($3/2^+$, $5/2^-$) or ($5/2^+$, $3/2^-$)	10	1.2	34	10	0.76	0.44		
$d = 1.5 - 4.5$ GeV $^{-1}$	9	0.6	19	3	0.29	0.42	0.36	1.91
$L_{\Lambda_b^0}^{P_c} \Lambda_b^0 \rightarrow P_c^+ (\text{low/high}) K^-$	6	0.7	4	8	0.37	0.16		
$L_{P_c} P_c^+ (\text{low/high}) \rightarrow J/\psi p$	4	0.4	31	7	0.63	0.37		
$L_{\Lambda_b^0}^{\Lambda^*} \Lambda_b^0 \rightarrow J/\psi \Lambda^*$	11	0.3	20	2	0.81	0.53	3.34	2.31
Efficiencies	1	0.4	4	0	0.13	0.02	0.26	0.23
Change $\Lambda(1405)$ coupling	0	0	0	0	0	0	1.90	0
Overall	29	2.5	86	19	4.21	1.05	5.82	3.89
sFit/cFit cross check	5	1.0	11	3	0.46	0.01	0.45	0.13

- Uncertainties in the Λ^* model dominate

Additional cross-checks

- Many additional cross-checks have been done. Some are listed here:
 - The same P_c^+ structure found using very different selections by different LHCb teams
 - Two independently coded fitters using different background subtractions (cFit & sFit)
 - Split data shows consistency: 2011/2012, magnet up/down, $\bar{\Lambda}_b/\Lambda_b$, $\Lambda_b(p_T \text{ low})/\Lambda_b(p_T \text{ high})$
 - Extended model fits tried without P_c states, but with two additional high mass Λ^* resonances allowing masses & widths to vary, or 4 non-resonant terms of J up to 3/2

Λ^* Matrix Element



4-6 independent **complex** helicity couplings per Λ_n^* resonance

6 independent data variables:
1 mass, 5 angles

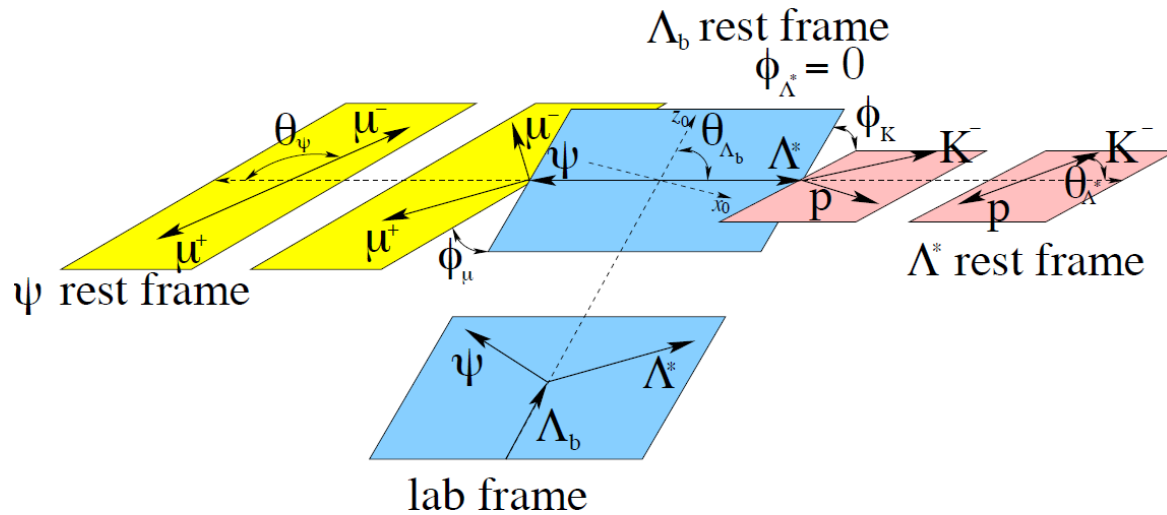
$$\mathcal{M}_{\lambda_{\Lambda_b^0}, \lambda_p, \Delta\lambda_\mu}^{\Lambda^*} \equiv \sum_n \sum_{\lambda_{\Lambda^*}} \sum_{\lambda_\psi} \mathcal{H}_{\lambda_{\Lambda^*}, \lambda_\psi}^{\Lambda_b^0 \rightarrow \Lambda_n^* \psi} D_{\lambda_{\Lambda_b^0}, \lambda_{\Lambda^*} - \lambda_\psi}^{\frac{1}{2}}(0, \theta_{\Lambda_b^0}, 0)^* \mathcal{H}_{\lambda_p, 0}^{\Lambda_n^* \rightarrow Kp} D_{\lambda_{\Lambda^*}, \lambda_p}^{J_{\Lambda_n^*}}(\phi_K, \theta_{\Lambda^*}, 0)^* R_n(m_{Kp}) D_{\lambda_\psi, \Delta\lambda_\mu}^1(\phi_\mu, \theta_\psi, 0)^*$$

$$R_X(m) = B'_{L_{\Lambda_b^0}}(p, p_0, d) \left(\frac{p}{M_{\Lambda_b^0}} \right)^{L_{\Lambda_b^0}^X} \text{BW}(m | M_{0X}, \Gamma_{0X}) B'_{L_X}(q, q_0, d) \left(\frac{q}{M_{0X}} \right)^{L_X} \text{BW}(m | M_{0X}, \Gamma_{0X}) = \frac{1}{M_{0X}^2 - m^2 - iM_{0X}\Gamma(m)}$$

Blatt-Weisskopf functions

Breit-Wigner

Λ^* Matrix Element



4-6 independent **complex** helicity couplings per Λ_n^* resonance

6 independent data variables:
1 mass, 5 angles

$$\mathcal{M}_{\lambda_{\Lambda_b^0}, \lambda_p, \Delta\lambda_{\mu}}^{\Lambda^*} \equiv \sum_n \sum_{\lambda_{\Lambda^*}} \sum_{\lambda_{\psi}} \mathcal{H}_{\lambda_{\Lambda^*}, \lambda_{\psi}}^{\Lambda_b^0 \rightarrow \Lambda_n^* \psi} D_{\lambda_{\Lambda_b^0}, \lambda_{\Lambda^*} - \lambda_{\psi}}^{\frac{1}{2}}(0, \theta_{\Lambda_b^0}, 0)^* \mathcal{H}_{\lambda_p, 0}^{\Lambda_n^* \rightarrow Kp} D_{\lambda_{\Lambda^*}, \lambda_p}^{J_{\Lambda_n^*}}(\phi_K, \theta_{\Lambda^*}, 0)^* R_n(m_{Kp}) D_{\lambda_{\psi}, \Delta\lambda_{\mu}}^1(\phi_{\mu}, \theta_{\psi}, 0)^*$$

$$R_X(m) = B'_{L_{\Lambda_b^0}}(p, p_0, d) \left(\frac{p}{M_{\Lambda_b^0}} \right)^{L_{\Lambda_b^0}^X} \text{BW}(m | M_{0X}, \Gamma_{0X}) B'_{L_X}(q, q_0, d) \left(\frac{q}{M_{0X}} \right)^{L_X} \text{BW}(m | M_{0X}, \Gamma_{0X}) = \frac{1}{M_{0X}^2 - m^2 - iM_{0X}\Gamma(m)}$$

Blatt-Weisskopf functions

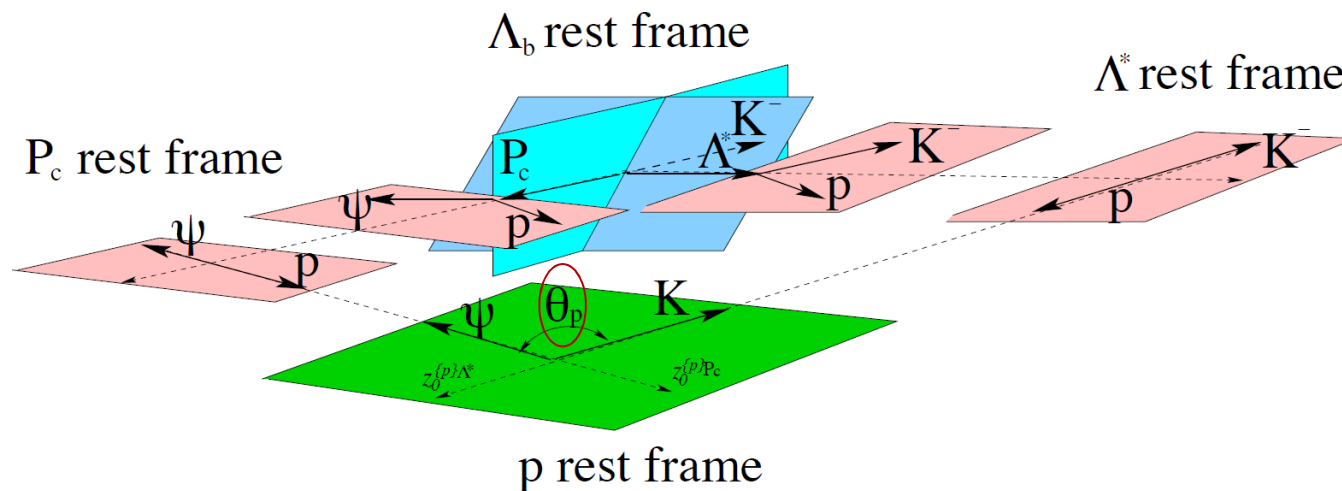
Breit-Wigner

Λ^* Plus P_c^+ Matrix Element

2 additional angles to align the muon and proton helicity frames between the Λ^* and P_c^+ decay chains

also derivable from the Λ^* decay variables

$$|\mathcal{M}|^2 = \sum_{\lambda_{\Lambda_b^0}} \sum_{\lambda_p} \sum_{\Delta\lambda_\mu} \left| \mathcal{M}_{\lambda_{\Lambda_b^0}, \lambda_p, \Delta\lambda_\mu}^{\Lambda^*} + e^{i\Delta\lambda_\mu \alpha_\mu} \sum_{\lambda_p^{P_c}} d_{\lambda_p^{P_c}, \lambda_p}^{\frac{1}{2}}(\theta_p) \mathcal{M}_{\lambda_{\Lambda_b^0}, \lambda_p^{P_c}, \Delta\lambda_\mu}^{P_c} \right|^2$$



- Without this realignment can't describe Λ^* plus P_c^+ interferences properly
- They integrate out to zero in full phase-space but present in the differential 6D fit-PDF

Unbinned 6D maximum likelihood fit

6D $\vec{\omega}$ - Fitted parameters (helicity couplings, M_0, Γ_0)

$$\mathcal{P}_{\text{sig}}(m_{Kp}, \Omega | \vec{\omega}) = \frac{1}{I(\vec{\omega})} |\mathcal{M}(m_{Kp}, \Omega | \vec{\omega})|^2 \Phi(m_{Kp}) \epsilon(m_{Kp}, \Omega), \quad (69)$$

where $\Phi(m_{Kp})$ is the phase space function equal to pq , where p is the momentum of the Kp system (*i.e.* Λ^*) in the Λ_b^0 rest frame, and q is the momentum of K^\mp in the Λ^* rest frame, and $I(\vec{\omega})$ is the normalization integral.

$$\begin{aligned} I(\vec{\omega}) &\equiv \int |\mathcal{M}(m_{Kp}, \Omega | \vec{\omega})|^2 \Phi(m_{Kp}) \epsilon(m_{Kp}, \Omega) dm_{Kp} d\Omega \\ &\propto \sum_j^{N_{\text{MC}}} w_j^{\text{MC}} |\mathcal{M}(m_{Kp_j}, \Omega_j | \vec{\omega})|^2, \end{aligned}$$

Corrections improving MC simulations

$$-2 \ln \mathcal{L}(\vec{\omega}) = -2s_W \sum_i W_i \ln \mathcal{P}(m_{Kp_i}, \Omega_i | \vec{\omega})$$

$$s_W \equiv \sum_i W_i / \sum_i W_i^2$$

Possible data event weights (see next).

sFit

W_i - sWeights based on the fit to $m_{J/\psi p K}$ distribution
 The data in the extended $m_{J/\psi p K}$ range passed to the amplitude fit.

Since the events are weighted in the log-likelihood this is “quasi” maximum-likelihood fit

$$\begin{aligned}
 -2 \ln \mathcal{L}(\vec{\omega}) &= -2s_W \sum_i W_i \ln \mathcal{P}_{\text{sig}}(m_{Kp\ i}, \Omega_i | \vec{\omega}) \\
 &= -2s_W \sum_i W_i \ln |\mathcal{M}(m_{Kp\ i}, \Omega_i | \vec{\omega})|^2 + 2s_W \ln I(\vec{\omega}) \sum_i W_i \\
 &\quad - 2s_W \sum_i W_i \ln [\Phi(m_{Kp\ i}) \epsilon(m_{Kp\ i}, \Omega_i)]. \quad \text{I}
 \end{aligned}$$

No need for parameterization of the signal efficiency

cFit (default)

$W_i=1$ no event weights; true maximum likelihood fit

Data only in the Λ_b^0 peak region passed to the amplitude fit.

Sideband data used to construct 6D model of the background: $\mathcal{P}_{\text{bkg}}^u(m_{Kp\ j}, \Omega_j)$

$$\mathcal{P}(m_{Kp}, \Omega | \vec{\omega}) = (1 - \beta) \mathcal{P}_{\text{sig}}(m_{Kp}, \Omega | \vec{\omega}) + \beta \mathcal{P}_{\text{bkg}}(m_{Kp}, \Omega)$$

$$\beta \pm 5.4\% \quad \text{background fraction}$$

$$-2 \ln \mathcal{L}(\vec{\omega}) =$$

$$\begin{aligned}
 & -2 \sum_i \ln \left[(1 - \beta) \frac{|\mathcal{M}(m_{Kp\ i}, \Omega_i | \vec{\omega})|^2 \Phi(m_{Kp\ i}) \epsilon(m_{Kp\ i}, \Omega_i)}{I(\vec{\omega})} + \beta \frac{\mathcal{P}_{\text{bkg}}^u(m_{Kp\ i}, \Omega_i)}{I_{\text{bkg}}} \right] \\
 & = -2 \sum_i \ln \left[|\mathcal{M}(m_{Kp\ i}, \Omega_i | \vec{\omega})|^2 + \frac{\beta I(\vec{\omega})}{(1 - \beta) I_{\text{bkg}}} \frac{\mathcal{P}_{\text{bkg}}^u(m_{Kp\ i}, \Omega_i)}{\Phi(m_{Kp\ i}) \epsilon(m_{Kp\ i}, \Omega_i)} \right] \\
 & \quad + 2N \ln I(\vec{\omega}) + \text{constant},
 \end{aligned}$$

$$I_{\text{bkg}} \equiv \int \mathcal{P}_{\text{bkg}}^u(m_{Kp}, \Omega) dm_{Kp} d\Omega \propto \sum_j w_j^{\text{MC}} \frac{\mathcal{P}_{\text{bkg}}^u(m_{Kp\ j}, \Omega_j)}{\Phi(m_{Kp\ j}) \epsilon(m_{Kp\ j}, \Omega_j)}.$$

The background term is then efficiency-corrected so it can be added to the efficiency-independent signal probability expressed by $|\mathcal{M}|^2$. This way the efficiency parametrization, $\epsilon(m_{Kp}, \Omega)$, influences only the background component which affects only a tiny part of the total PDF