Cern 5 November 2014

Theory for Particle Physics

$\Gamma\epsilon\omega\rho\gamma\iota o\varsigma\, \Lambda\epsilon o\nu\tau\alpha\rho\eta\varsigma$
(*George Leontaris*)

Ioannina University
$I\omega\alpha\nu\nu\iota\nu\alpha$
GREECE

Outline of the Talk

- The first steps to unification
- Newton and Maxwell ...
- Standard Model
- roadmap to GUTs
- Strings
- Branes
- Predictions... etc

★ Recent Research in High Energy Physics

★ Physics of Elementary Particles

★ General Relativity and Cosmology

★ Advanced Mathematics

$\mathcal{A}$

Physicists have a ... Dream ...

★ Theory of Elementary Particles: A twofold purpose

▲$\mathcal{A}$: **Attaining Unification of all forces**:

1. Gravity
2. Electro-Magnetism
3. Strong Interactions
4. Weak Interactions

▲$\mathcal{B}$: **Searching for the smallest constituents of matter**

Democritus (c. 400 BC) . . .

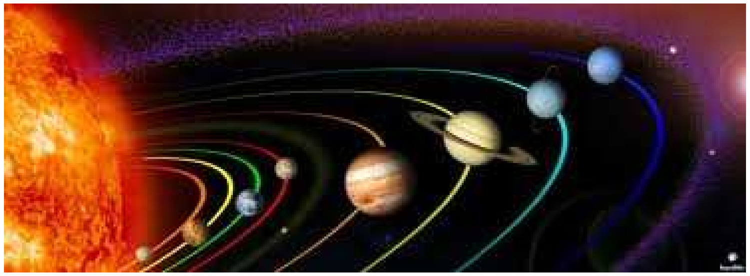
$\mathcal{B}$

The … first … steps

NEWTON

↙ ↘

Falling on Earth | Planetary Motion



↘ ↙

$$F = G\frac{mm'}{r^2}$$

Maxwell

▲ *The four equations unifying Electric and Magnetic Forces*

$$\nabla \cdot \vec{B} = 0 \quad , \quad \nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{1}$$

$$\nabla \cdot \vec{E} = \rho \quad , \quad \nabla \times \vec{B} = \frac{1}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \tag{2}$$

▲ *Unified description with potentials :*

$$\nabla \cdot \vec{B} = 0 \quad \rightarrow \quad \vec{B} = \nabla \times \vec{\mathbf{A}}$$

$$\rightarrow \quad \vec{E} = -\nabla \mathbf{\Phi} - \frac{1}{c}\frac{\partial \vec{\mathbf{A}}}{\partial t}$$

▲ *Remarkable Property*: Duality in Vacuo:

Define $\vec{C} = \vec{E} + i\vec{B}$, then:

$$\nabla \cdot \vec{C} = 0, \; \nabla \times \vec{C} + i\frac{\partial \vec{C}}{\partial t}$$

E/M rotation duality symmetry:

$$\vec{C} \rightarrow e^{i\phi}\vec{C}$$

▲ *Heaviside, Lorenz, Larmor ...* seeking invariance of M.E. →

Lorenz transformations:

$$t' = \gamma(t - \beta\, x/c), \;\; x' = \gamma(x - vt)$$

with $\beta = v/c$, $\gamma = (1 - \beta^2)^{-1/2}$

$$(ct')^2 - x'^2 = (ct)^2 - x^2$$

$$\Downarrow$$

★ Einstein

Special Relativity

where... Space and Time "mix" in a 4-vector

$$x^\mu = (x^0, x^1, x^2, x^3) \equiv (ct, \vec{x})$$

4-vectors in Maxwell's equations :

$$\begin{aligned} A^\mu &= (A^0, A^1, A^2, A^3) \equiv (\Phi, \vec{A}) \\ j^\mu &= (j^0, j^1, j^2, j^3) \equiv (c\rho, \vec{J}) \end{aligned}$$

and field strength : $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

▲ Relativistic formulation of Maxwell's Equs

$$\begin{aligned} \partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} &= 0 \\ \partial_\nu F^{\mu\nu} &= j^\mu / c \end{aligned}$$

★ Next Major Steps

1. General Relativity

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

2. Quantum Mechanics

$$i\hbar \frac{\partial}{\partial t}\psi(\vec{r},t) = \mathcal{H}\psi(\vec{r},t)$$

3. Quantum Electrodynamics

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Unification ... in a ... modern perspective...

The Standard Model

Unification of Strong, ElectroMagnetic and Weak forces

SM Ingredients: *3 copies (families) and 1 Higgs pair (SUSY)* :

1. Lepton and Higgs fields

$$SU(2)_{\downarrow} \; : \begin{pmatrix} \nu \\ e \end{pmatrix}, \begin{pmatrix} h_0 \\ h_+ \end{pmatrix} \begin{pmatrix} h_- \\ \bar{h}_0 \end{pmatrix}$$

2. Quarks in three varieties: Left-handed doublets

$$SU(3)_{\rightarrow} \begin{pmatrix} u & u & u \\ d & d & d \end{pmatrix}$$

and Right handed singlets

$$e^c, \; (u^c, u^c, u^c), \; (d^c, d^c, d^c)$$

$$\Downarrow$$

Standard Model Gauge Symmetry

$$SU(3) \times SU(2)_L \times U(1)_Y$$

Predictions-Discoveries

1. Gauge Bosons (*CERN 1983*)

$$W_+,\ W_-,\ Z,\ \gamma$$

2. Neutral currents (*CERN 1973*)

$$\nu_e\, e \to \nu_e\, e$$

3. Charm, Bottom, Top Quark... $m_t \sim 176$GeV (*CDF, D0*)
4. Higgs Boson $m_H \sim 126$ GeV (*CERN 2013*)

★ STANDARD MODEL: $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\mathcal{Q} = \begin{pmatrix} u \\ d \end{pmatrix}, \ \mathcal{L} = \begin{pmatrix} \nu \\ e \end{pmatrix} \qquad u^c, d^c, \ e^c$$

$$H = \begin{pmatrix} h^0 \\ h^- \end{pmatrix}, \ \bar{H} = \begin{pmatrix} h^+ \\ \bar{h}^0 \end{pmatrix}$$

Although successful, it is not the final theory.

Reasons:

▼ gravity not unified

▼ fermion mass hierarchy not explained, (21 arbitrary parameters)

▼ neutrino masses not incorporated (absence of ν_R), unless

$$\text{N.R. Operator} \rightarrow \frac{(\bar{H}\mathcal{L})^2}{\mathcal{M}} \ \rightarrow \ m^{\nu}_{Maj.}\, \nu\, \nu$$

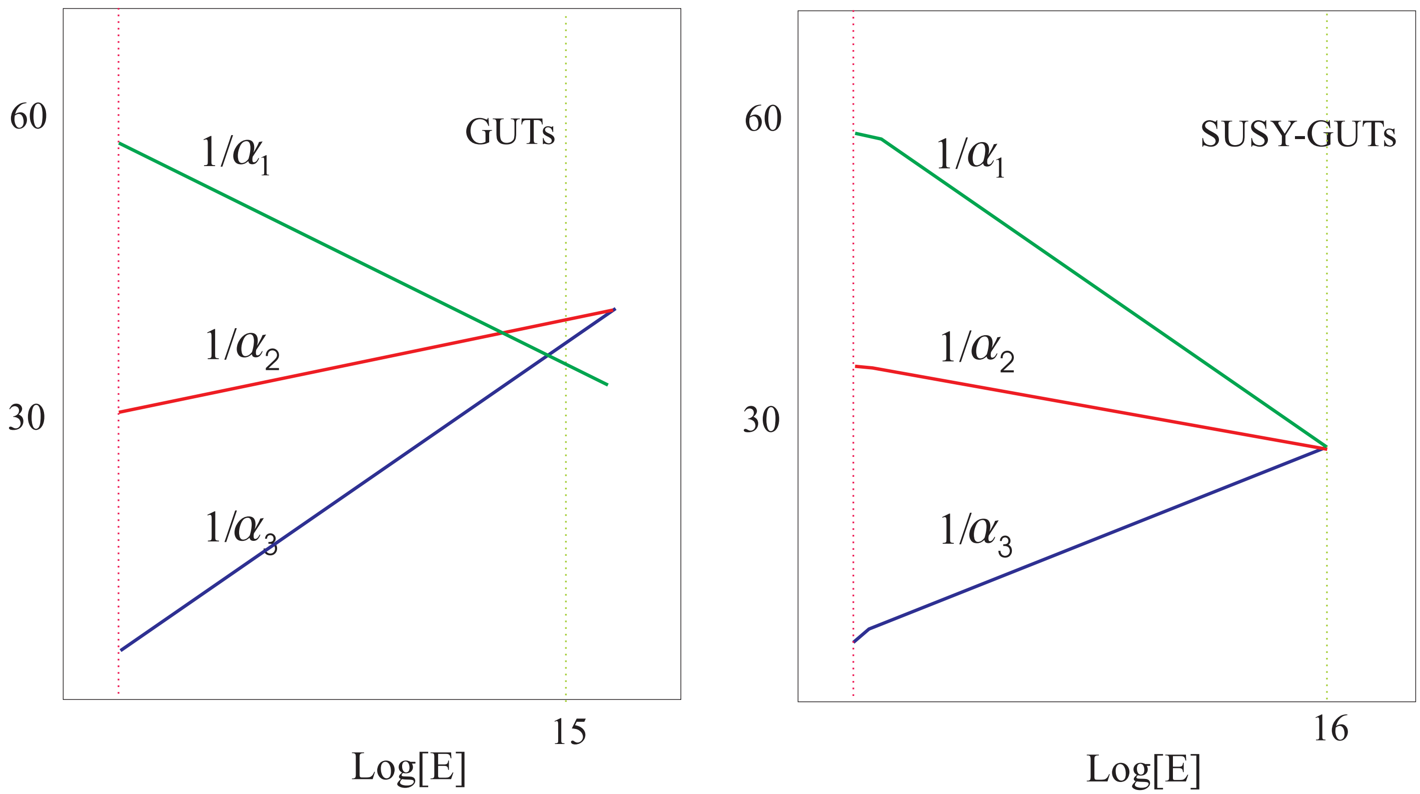
$\Longrightarrow$ New scale $\mathcal{M}$ and gauge coupling running naturally imply:

★Early extensions of SM: GUTs , SUSY GUTs, etc.

$SU(5)$, $SO(10)$, $SU(3)^3$, $SU(4) \times O(4)$

Predictions:

▲ unification of couplings at $M_S \sim 10^{15} - 10^{16}$GeV,



Pati-Salam Model

Treating leptons as a fourth colour

1. Left handed Quark-Lepton multiplet

$$SU(4)_{\rightarrow} \begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix} \downarrow SU(2)_L$$

2. and Right handed fields

$$SU(4)_{\rightarrow} \begin{pmatrix} u^c & u^c & u^c & \nu^c \\ d^c & d^c & d^c & e^c \end{pmatrix} \downarrow SU(2)_R$$

... many advantages (including RH-neutrino) but does not predict gauge unification

The simplest GUT : $\mathbf{SU(5)}$

- SM representations are accommodated as follows:
- $SU(5)$ Chiral and Higgs Representations:

$$
\begin{aligned}
10 &\rightarrow \mathcal{Q} + u^c + e^c \\
\bar{5} &\rightarrow d^c + \mathcal{L} \\
5 + \bar{5} &\rightarrow (T + h_u) + (\bar{T} + h_d)
\end{aligned}
$$

- Yukawa Couplings:

$$10 \cdot 10 \cdot 5 \rightarrow m_{top} \quad (3)$$

$$10 \cdot \bar{5} \cdot \bar{5} \rightarrow m_b \quad (4)$$

SU(5) predictions

1. ▲ $SU(5)$ Gauge Coupling Unification at M_{GUT}:

$$g_3 = g_2 = \sqrt{5/3}g_1$$

2. ▲ Electric Charge Quantization

$$\bar{5} = (d^c, d^c, d^c, e, \nu) \rightarrow 3Q_{d^c} + Q_e = 0$$

3. Proton Decay

   i) From new gauge bosons X, Y ($Q_X = 4/3$, $Q_Y = 1/3$)

   ii) From colour triplets T, $\bar{T}$ residing in Higgs multiplets

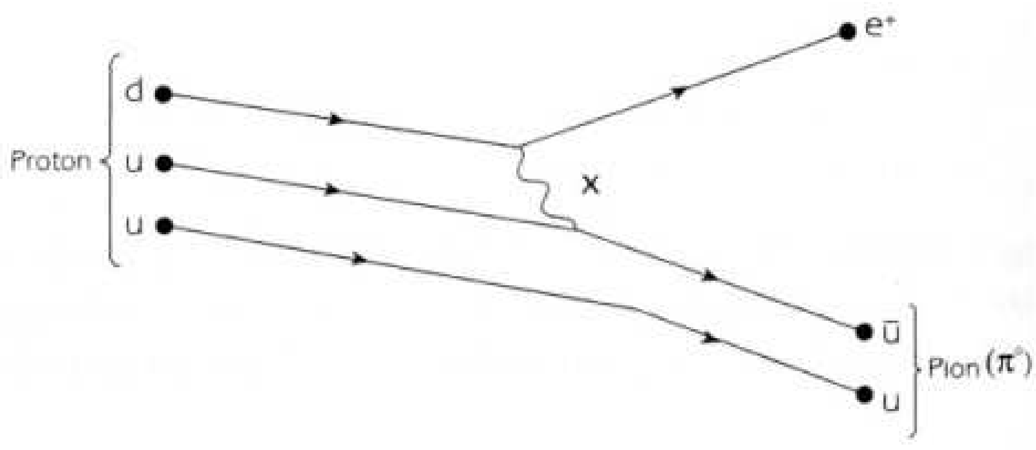
4. No room for ν^c unless...

A representative graph for proton decay

Proton consists of $(2 \times u + 1 \times d)$-**quarks**

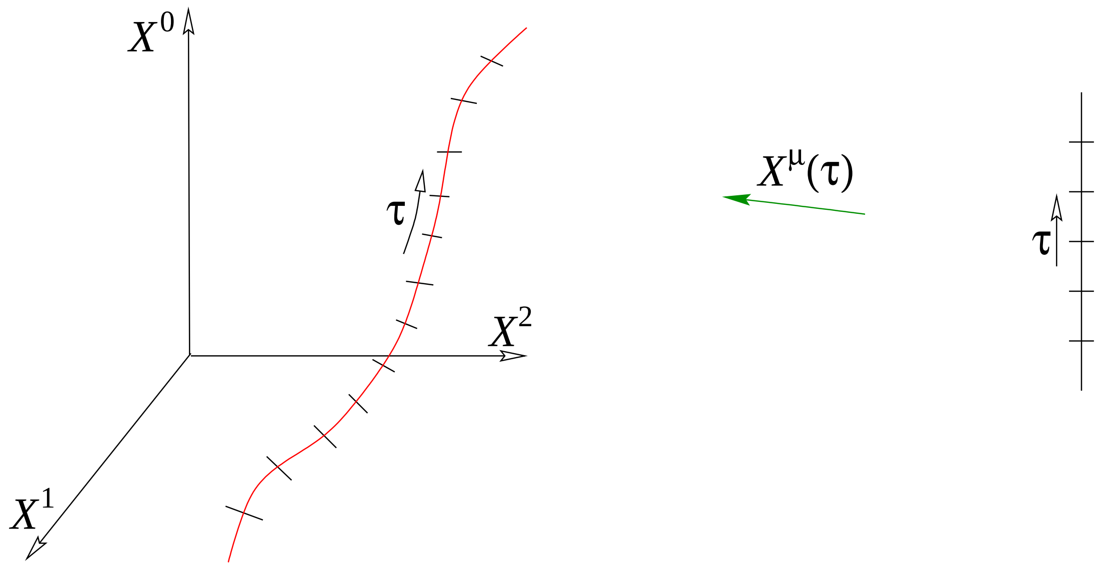
A d *and a* u *quark exchange a boson* X. *A* $\bar{u}$ *and an* e^+ *generated...*

$\bar{u}$ combines with u to a pion.



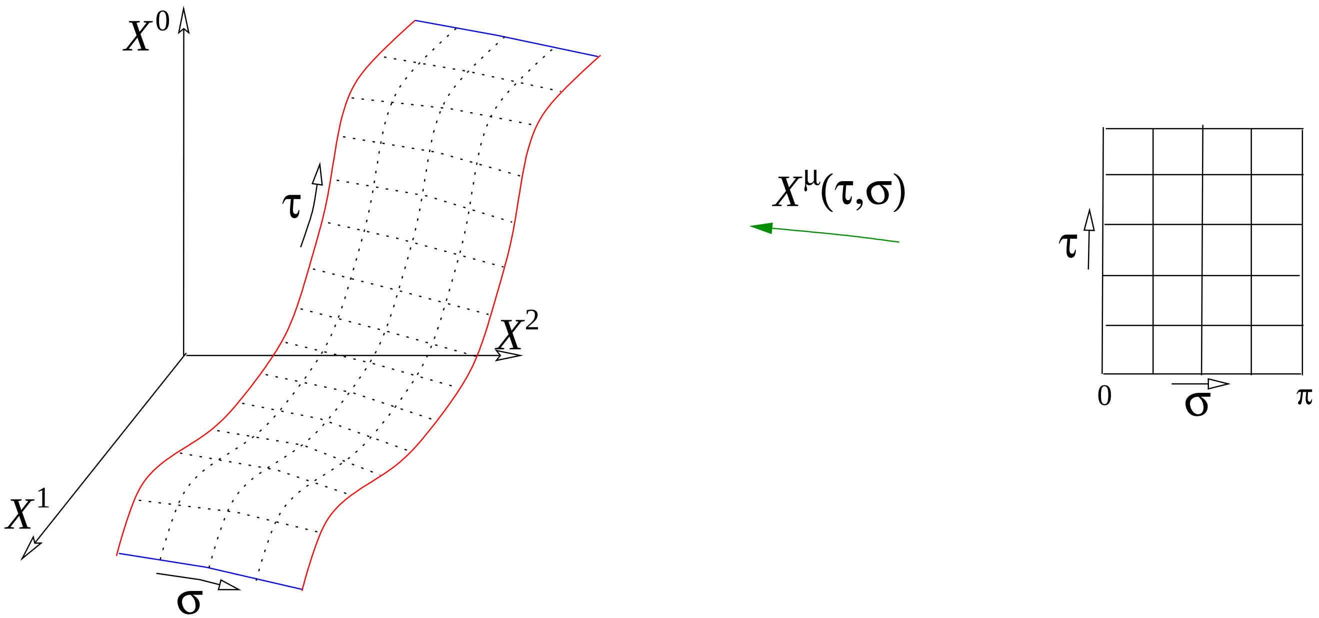
... worldline of a point particle in D-dimensions described by coordinates :

$$X^\mu(\tau),\ \mu = 0, 1, ..., D - 1$$



... a string in D-dimensions sweeps a worldsheet

$$X^\mu(\tau,\sigma),\ \mu = 0, 1, ..., D-1$$



Strings have lengths

$$0 \leq \sigma \leq 2\pi$$

Boundary Conditions

- Closed Strings:

$$X^\mu(\tau, \sigma + 2\pi) = X^\mu(\tau, \sigma)$$

For Open Strings:

i) Neuman b.c. : *free end-points of the string:*

$$\frac{\partial}{\partial \sigma} X^\mu(\tau, \sigma)\Big|_{\sigma=0} = \frac{\partial}{\partial \sigma} X^\mu(\tau, \sigma)\Big|_{\sigma=2\pi} = 0$$

ii) Dirichlet b.c. : *fixed end-points of the string:*

$$X^\mu(\tau, \sigma)|_{\sigma=0,2\pi} = c^\mu$$

String Spectrum

Equation of Motion of X^μ on the worldsheet

$$(\partial_\tau^2 - \partial_\sigma^2) X^\mu = 0$$

General solution

$$X^\mu(\tau, \sigma) = f^\mu(\tau - \sigma) + \tilde{f}^\mu(\tau + \sigma)$$

Standing Waves (open string)

$$f^\mu(\tau - \sigma) = \frac{1}{2} x_0^\mu + \frac{\alpha'}{2} p^\mu (\tau - \sigma) + i\sqrt{\frac{\alpha'}{2}} \Sigma \frac{1}{n} a_n^\mu e^{i(\tau - \sigma)}$$

and analogously for $\tilde{f}^\mu(\tau + \sigma)$

Quantization

continuous → *discrete spectrum*

center of mass:

$$[x^\mu, p^\mu] = \eta^{\mu\nu}$$

Fourier coefficients → *operators*

$$(a_n^\mu)^\dagger, (\tilde{a}_n^\mu)^\dagger \to \text{creation operators}$$

String Spectrum generated by action of creation operators on ground states

$$(a_{n_r}^{k_r})^\dagger \cdots (\tilde{a}_{m_s}^{\ell_s})^\dagger \cdots |\mathbf{p}\rangle$$

Massless Spectrum

$$m^2 = \frac{\beta}{a'^2}\left(N - \frac{D-2}{24}\right), \; N = \sum n_r$$

(Open string $\beta = 1$, closed string $\beta = 4$, massless spectrum $N = 1$)

1. open string: Photon: ${a_1}^{k\dagger}|p>$

$$m^2 = \frac{1}{a'^2}\left(1 - \frac{D-2}{24}\right) = 0, \rightarrow D = 26$$

2. closed string: Graviton, Axion, Dilaton: ${a_1}^{k\dagger}\tilde{a}_1^{l\dagger}|p>$

$$m^2 = \frac{4}{a'^2}\left(1 - \frac{D-2}{24}\right) = 0, \rightarrow D = 26$$

3. bad news! $N = 0 \rightarrow$ **tachyon!** $m^2 < 0$

... therefore:

Consistency of the theory requires number of Dimensions to be:

$$D = 26$$

$$\Downarrow$$

Bosonic String Theory

... when fermionic degrees are introduced...

$$D = 10$$

$$\Downarrow$$

SuperString Theory

1. Classical bosonic string: $X^\mu \to$ coordinate description
2. Quantisation: $X^\mu \to$ Quantum commuting operators
3. For superstring introduce world-sheet fermions $\psi_1^\mu,\ \psi_2^\mu$ (anticommuting). Equivalently:

$$\psi^I(\tau,\sigma) = \begin{cases} \psi_1(\tau,\sigma) & \sigma \in [0,\pi] \\ \psi_2(\tau,\sigma) & \sigma \in [-\pi,0] \end{cases}$$

4. Boundary Conditions

$$\psi^I(\tau,\pi) = +\psi^I(\tau,-\pi) \quad : \quad (\mathrm{Ramond}\ \mathrm{R})$$

$$\psi^I(\tau,\pi) = -\psi^I(\tau,-\pi) \quad : \quad (\mathrm{Neveu-Schwarz}\ (\mathrm{NS}))$$

5. **good news: No tachyon!**

Closed String Spectrum

…emerges from the combination of L*-moving and* R*-moving open strings…*

Closed String Sectors:

$$(NS, NS),\ (NS, R),\ (R, NS),\ (R, R)$$

bosons: (NS, NS) and (R, R)
fermions: (NS, R) and (R, NS)

Example: Type **II-B**:
truncation of spectrum to preserve supersymmetry: $(-)^{F_{L,R}}$

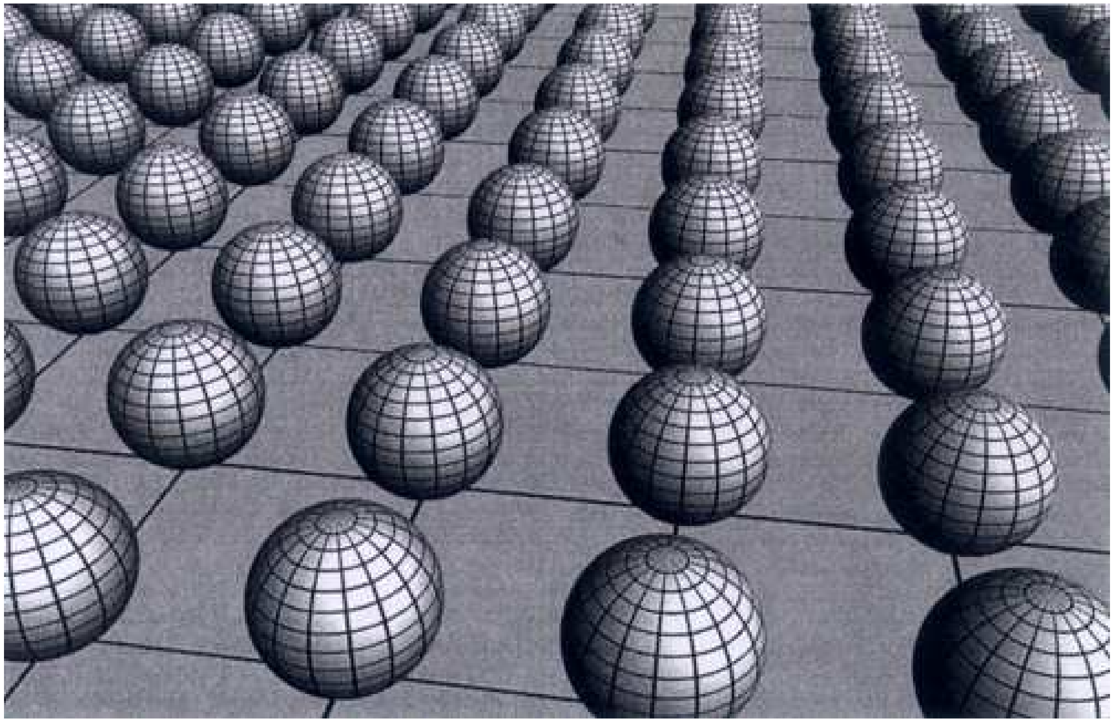
$$Left = \left\{ \begin{array}{c} NS_+ \\ R_- \end{array} \right\},\quad Right = \left\{ \begin{array}{c} NS_+ \\ R_- \end{array} \right\}$$

Compactification

We see only 4 dimensions! $\rightarrow$ six must be invisible! *We "compactify" them so they look like circles with a tiny radius* R

$$x^i \sim x^i + 2\pi R$$

Compact Dimensions



IMPLICATIONS

1. *A scalar field* ϕ *look like :*

$$\phi(x^\mu) \sim \sum_{n_i} \phi_i(x^0, \vec{x}) \prod_{i=4}^{9} \cos(n_i x^i / R_i)$$

   (*because of* $x_{4\ldots9}$ *periodicities*)

2. Equation of motion $\partial_\mu \partial^\mu \phi = 0$ implies masses

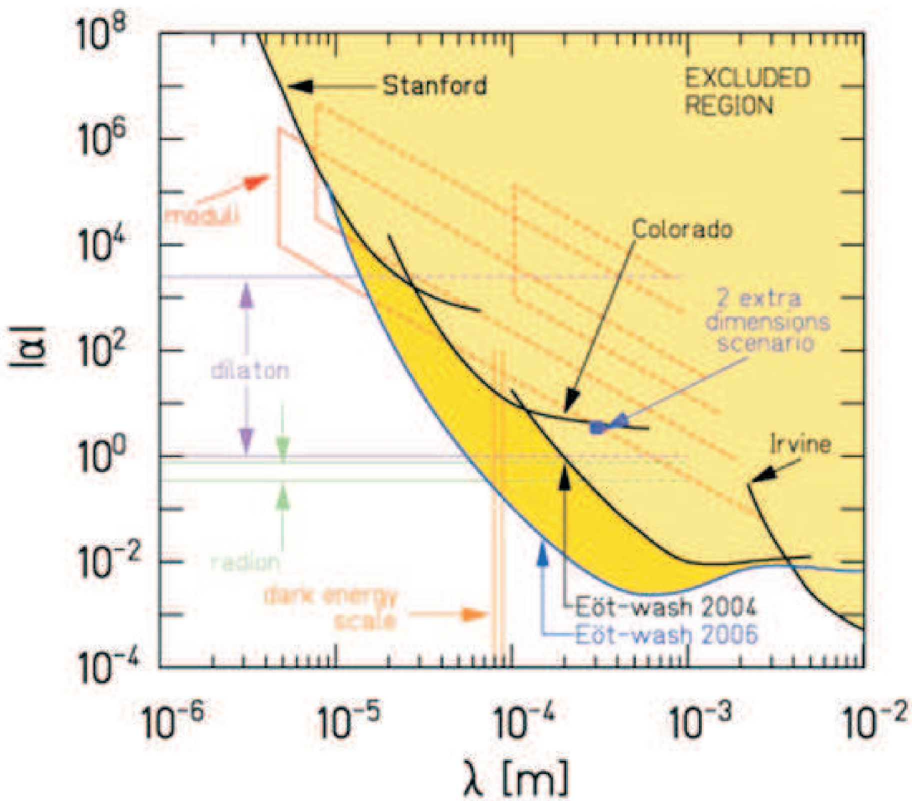
$$m_n^2 = \sum_{i=4}^{9} \frac{n_i^2}{R_i^2}$$

3. Some compact dimensions may not be so small. Then, we get measurable deviations:

$$F = G \frac{mm'}{r^2} \left(1 + \alpha\, e^{-r/\lambda}\right), \;\; \alpha = 2n, \lambda = R_{compact}$$

Newton's Law modifications have attracted the interest of physicists and have been investigated long before Strings

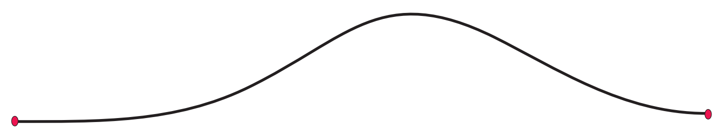
Bounds from various experiments



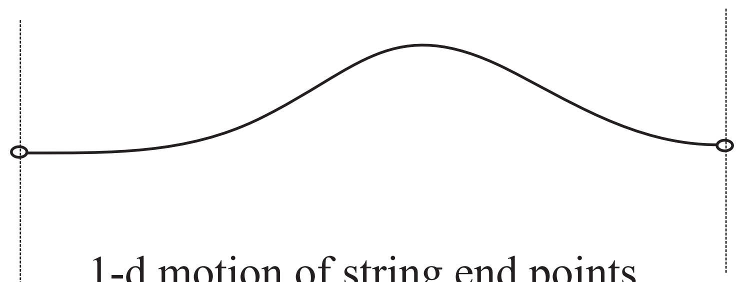
D-BRANES

Fixed end-points of open string define "objects" with zero dimensions →

$$D0 - \text{branes}$$



String with fixed end points



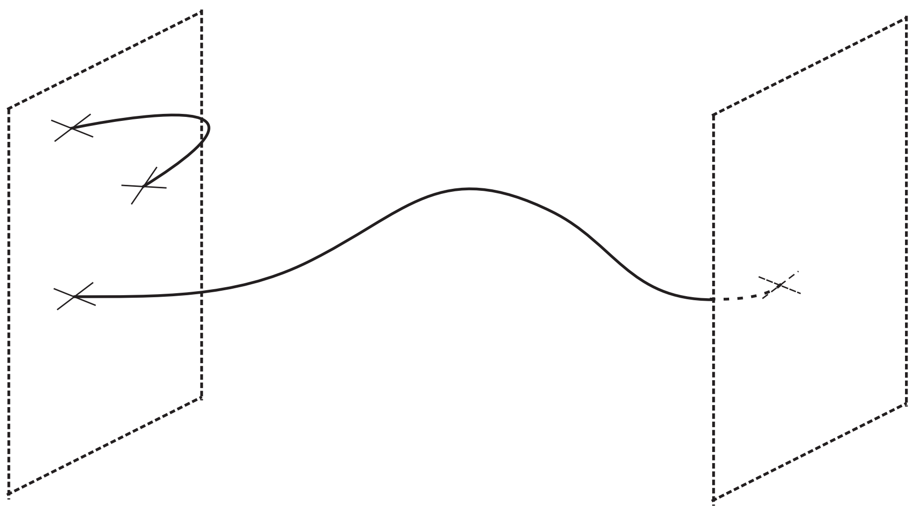
1-d motion of string end points

String End - points allowed to slide along lines define one-dimensional objects →

$$D1 - \text{brane}$$

$\Downarrow$

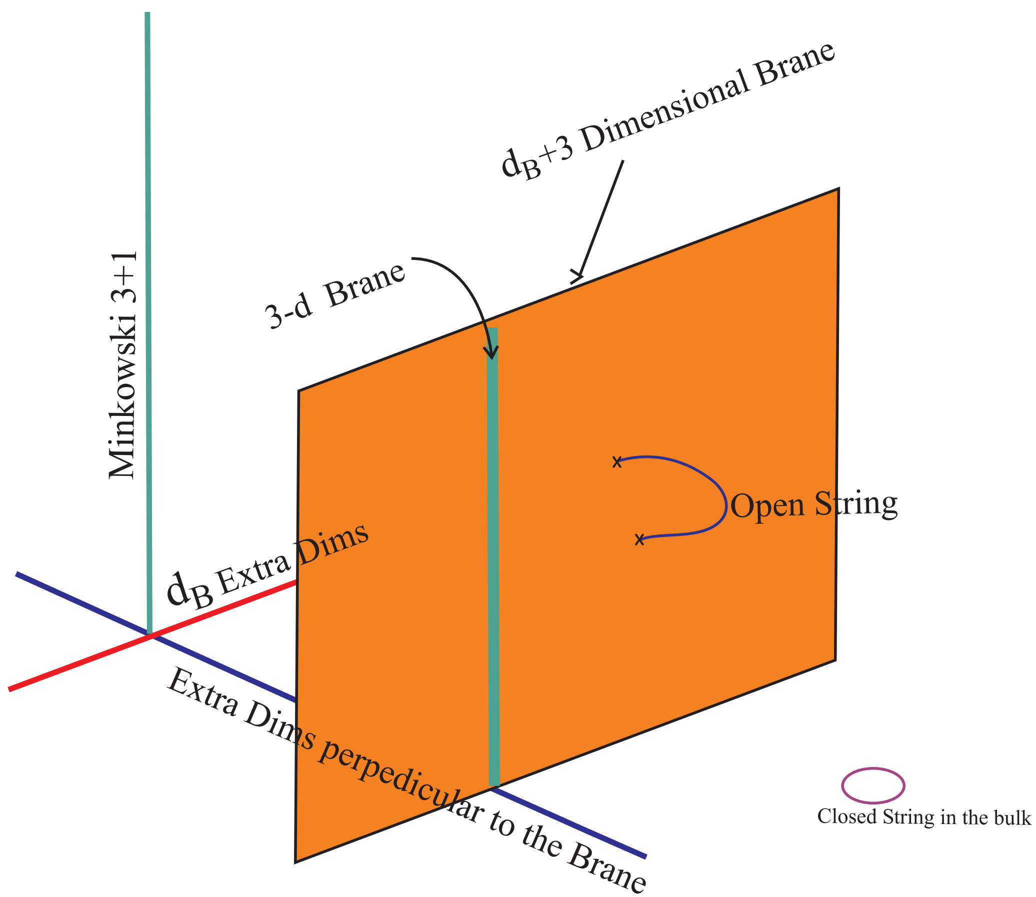
A *Dp*-brane is an object with p-spatial dimensions



Open string endpoints on D-branes

STRING-BRANE Unification Scenario...

Gravity (closed strings) and QFT (particles ∼ open strings)



Models from Intersecting Branes

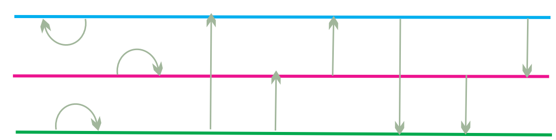
Stack of parallel D-branes generate $U(n) \to SU(n) \times U(1)$ *gauge symmetries*

Gauge bosons represented by string connecting parallel D-branes $(n^2 - 1 + 1)$

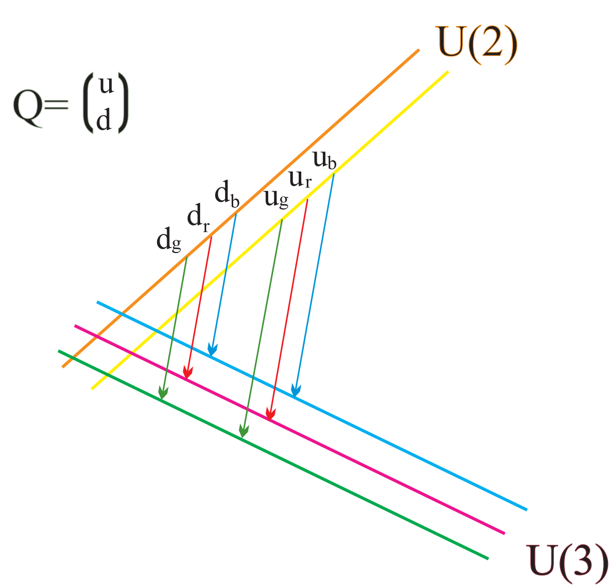
SM states are represented by strings stretched near intersections

The Standard Model in intersecting branes

U(3) gauge bosons



Quark doublets



Toroidal Compactification

internal space is a compact manifold with definite topological properties

The simplest ones are those of the sphere $\mathcal{S}^2$ *and torus* $\mathcal{T}^2$.

1. We can think of the internal 6d-space as three factorised torii

$$\mathcal{T}^2 \times \mathcal{T}^2 \times \mathcal{T}^2$$

2. We can cut the torus along the two radii and stretch it so it looks like a rectangle with opposite sides identified.
3. Now, branes wrapped around $\mathcal{T}^2$ can be depicted as lines circling the two radii of $\mathcal{T}^2$

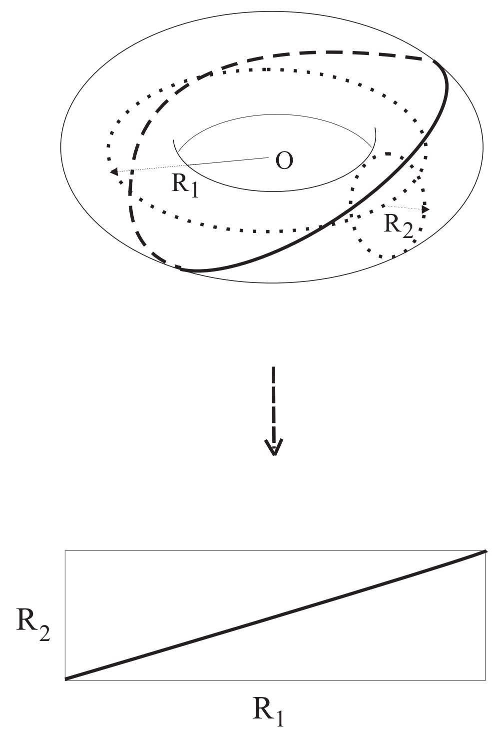


Figure 1: Representation of a $(1, 1)$ D-brane wrapping on a T^2 torus.

Chiral Matter

1. Imagine now that two ("non-equivalent") branes wrap the same torus.
2. Because space is compact intersections are unavoidable
3. But strings (*representing particles*) are jostling at the intersections
4. Therefore, the number of intersections 'counts' multiplicities of states we have. In particular if:

(n_a^i, m_a^i): wrapping numbers of the D_a brane-stack around the i^{th} torus, (n_b^i, m_b^i) those of the D_b-stack,
$\Rightarrow$ # of fermion generations = # of intersections:

$$I_{ab} \quad = \quad \prod_{i=1}^{3}(n_a^i m_b^i - n_b^i m_a^i) \qquad (5)$$

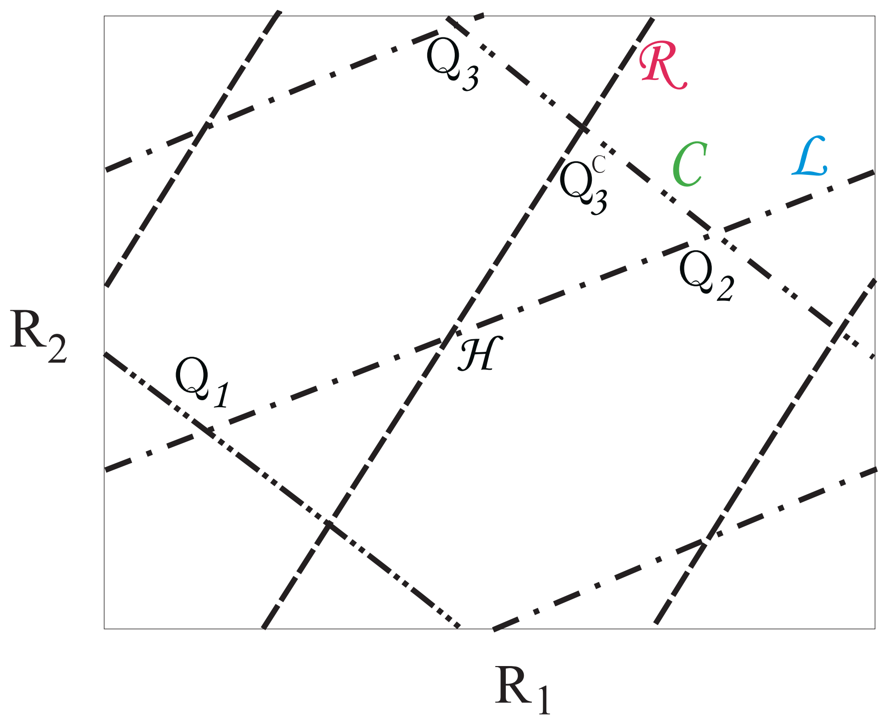


Figure 2: Intersecting D-branes wrapping on a T^2 torus, with fermion fields localized at the intersections with $\mathcal{C} = (-1, 1)$, $\mathcal{R} = (2, 1)$, $\mathcal{L} = (2, 1)$ wrappings along R_1, R_2

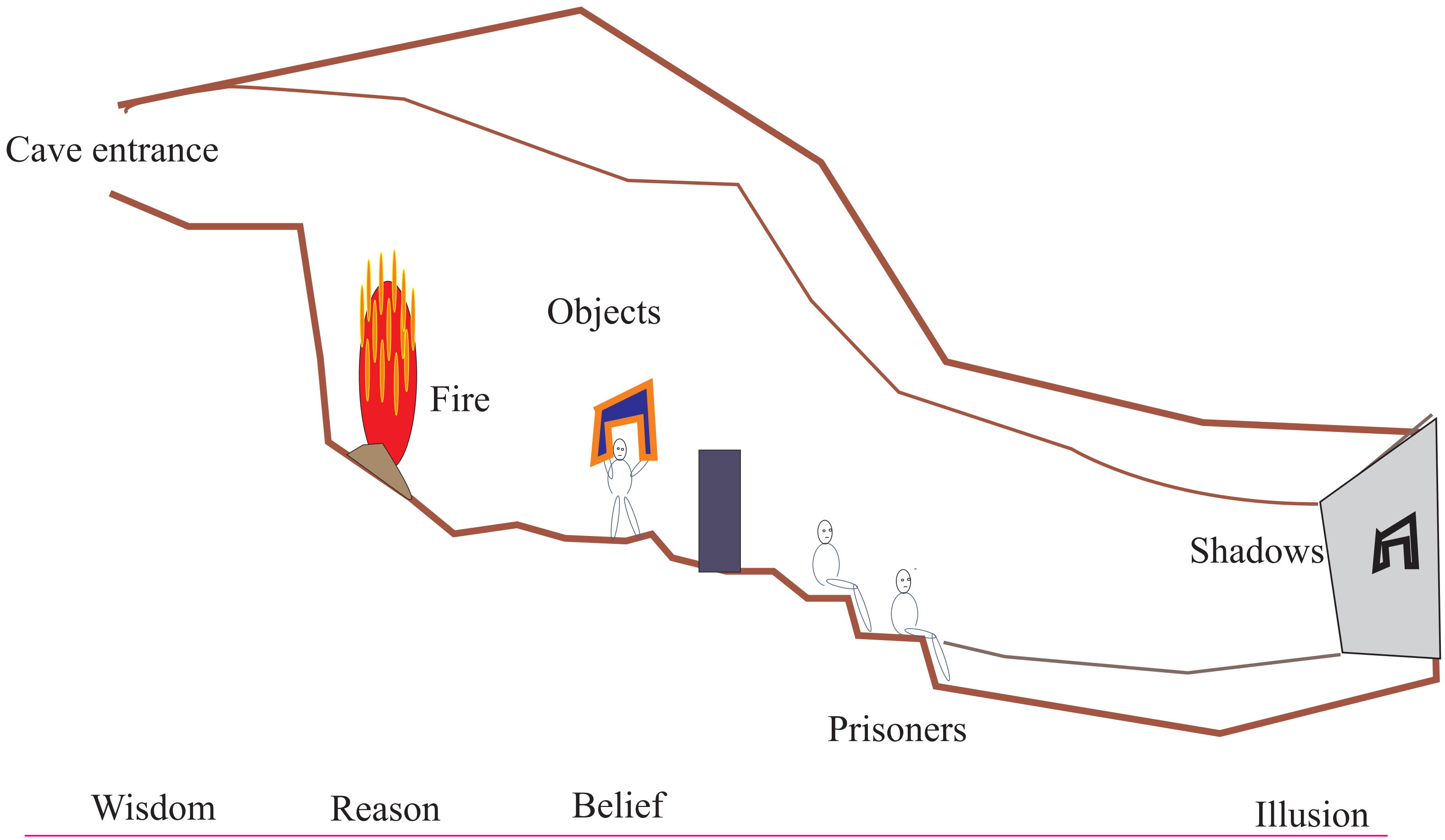
,

Plato's allegory of the cave

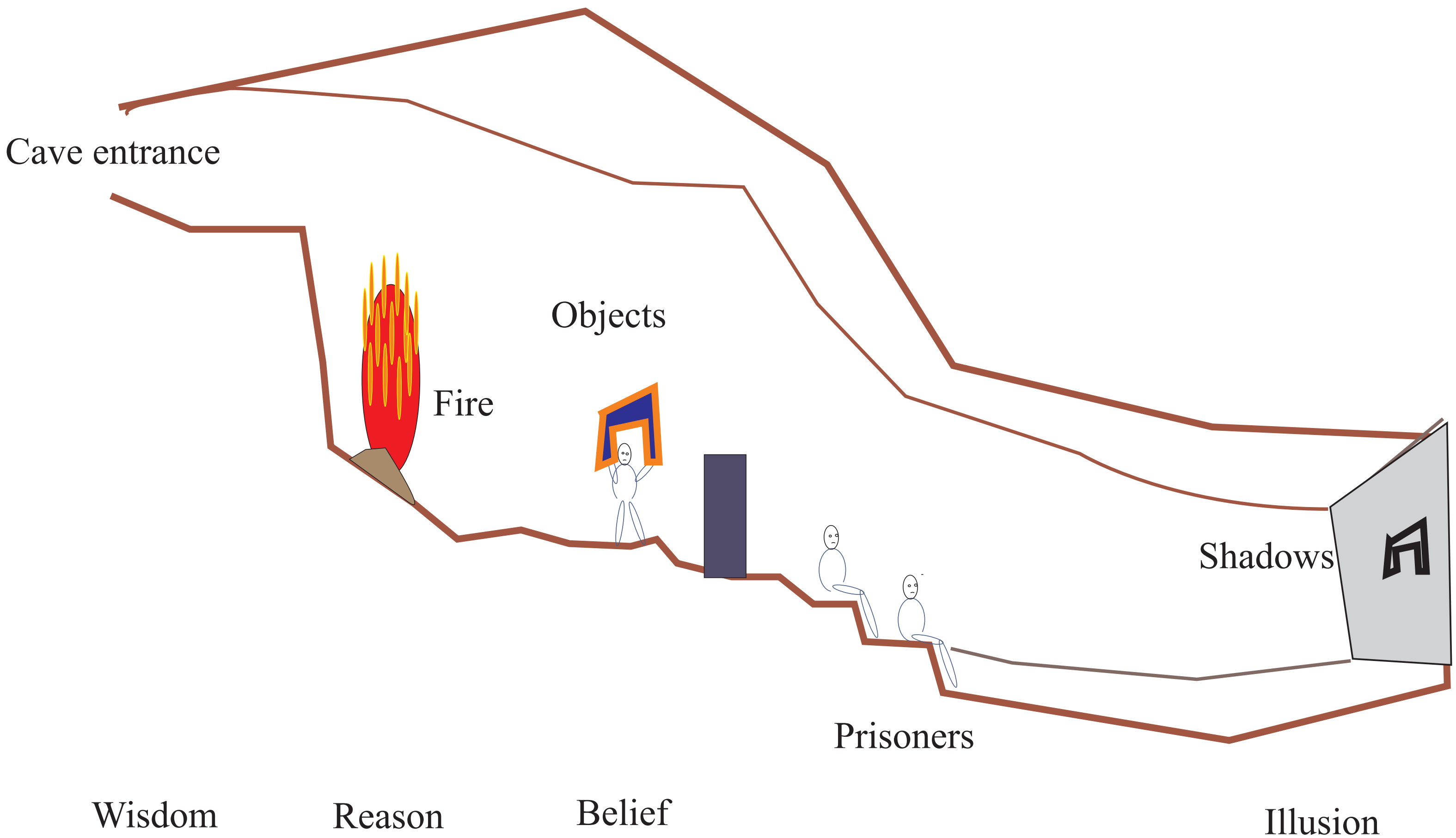
... and ...

extra dimensions

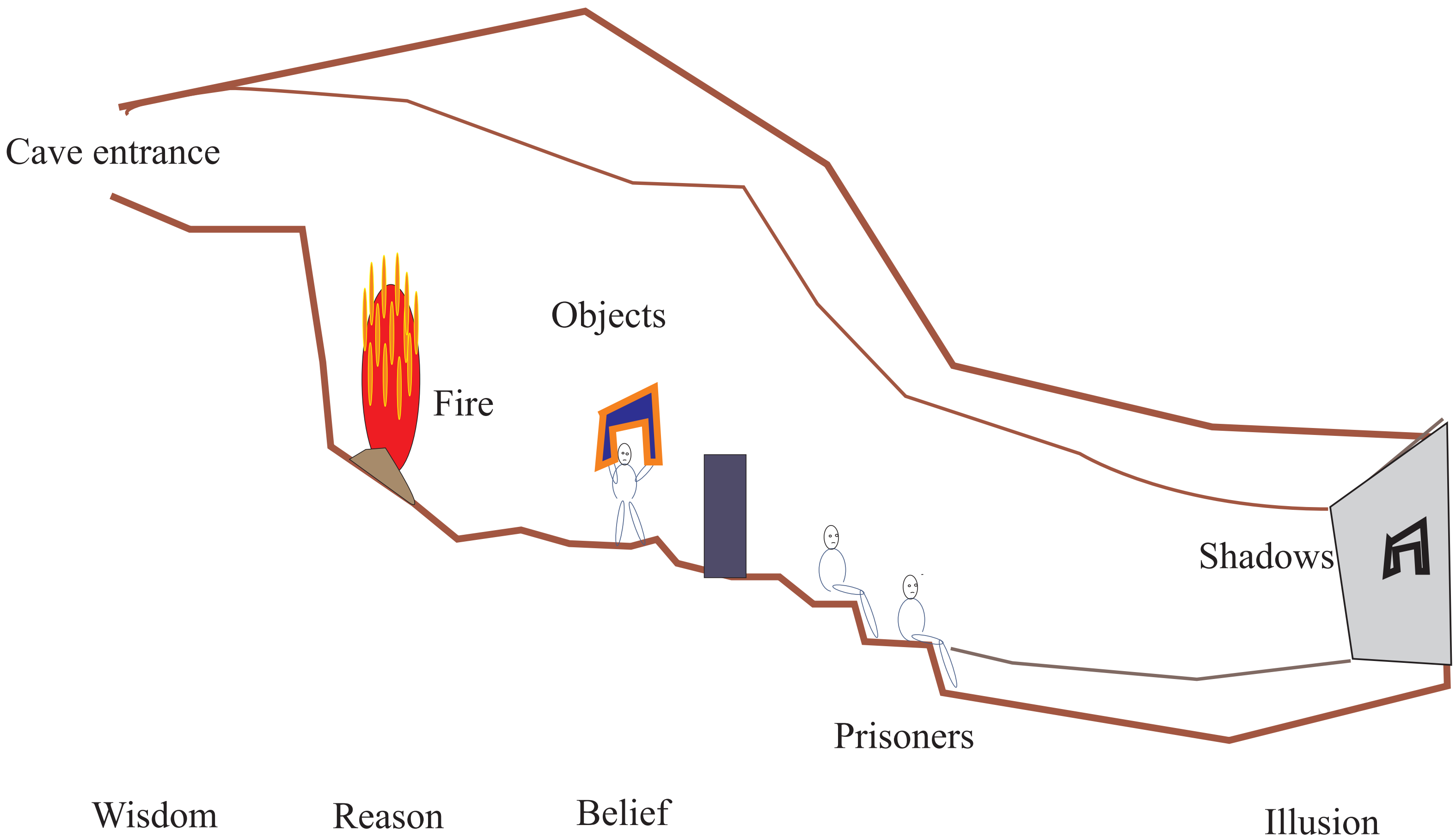
http://upload.wikimedia.org/wikipedia/commons/4/4a/Plato-raphael.jpg



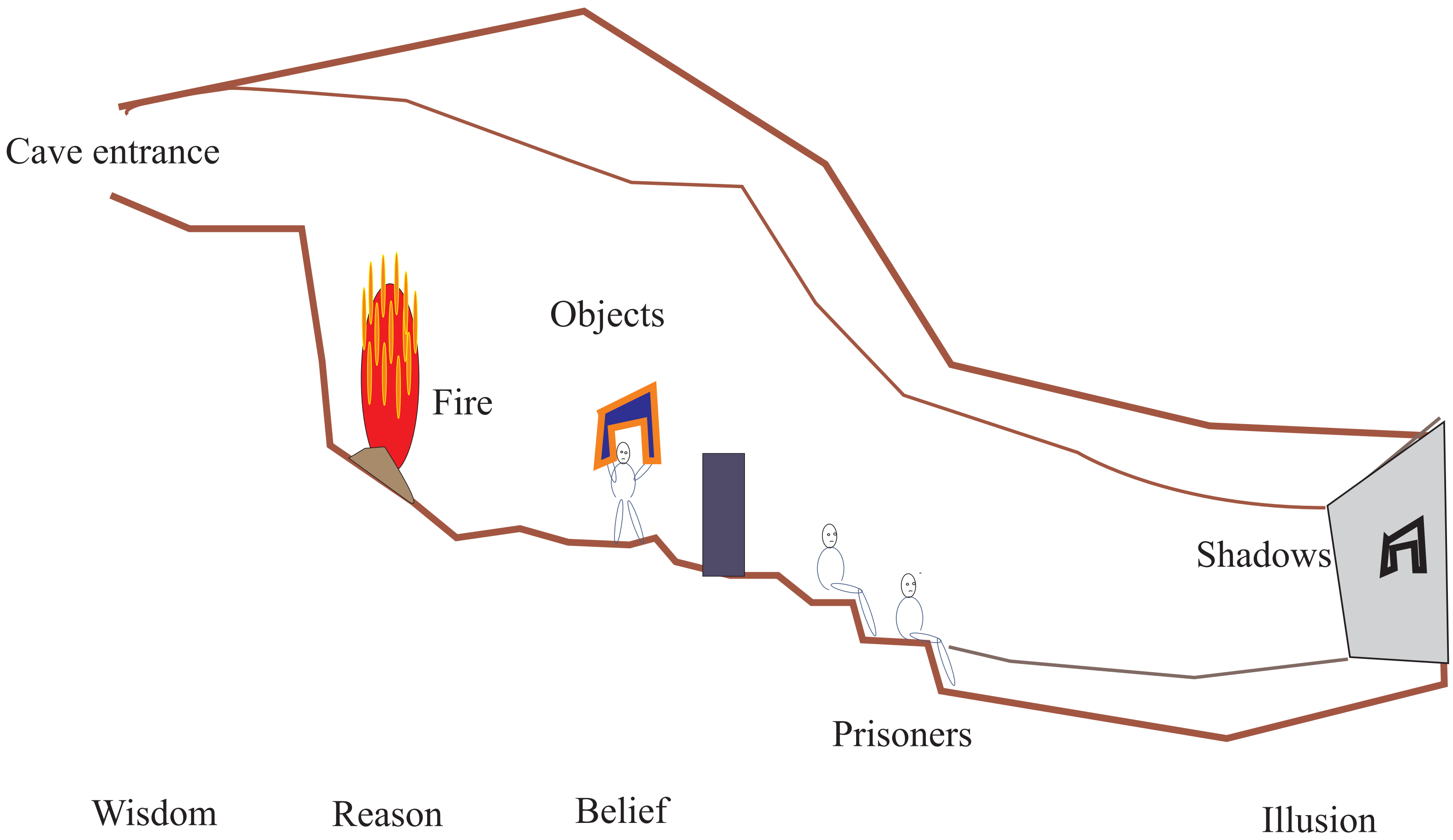
Trying to understand the higher dimensional reality from the 4-dimensional “shadows” ...



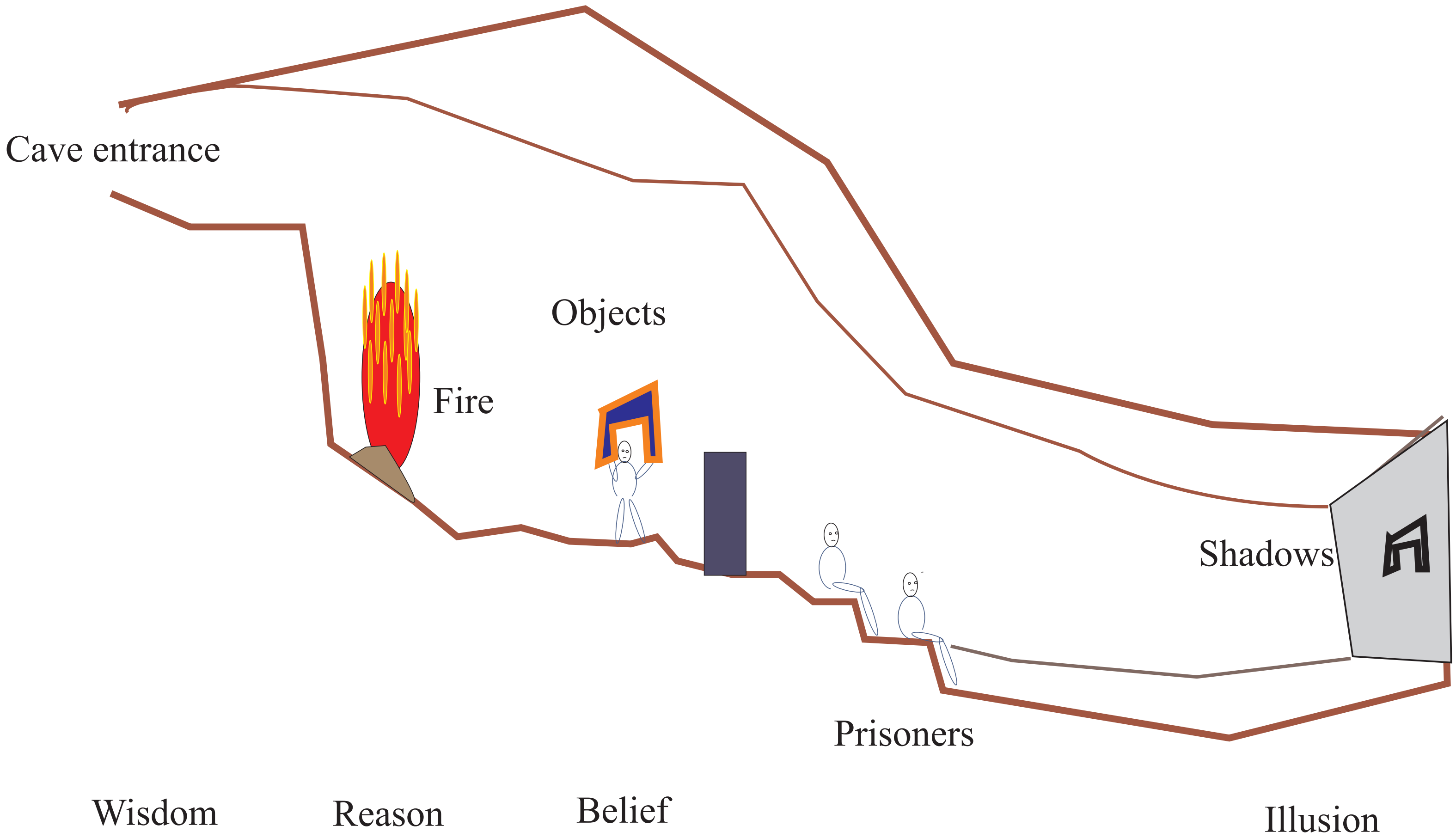
Imagine prisoners who have been chained since childhood, deep inside a cave. Their heads are immobilized, so that their gaze is fixed on a wall. ...



Behind the prisoners there is an enormous fire and between the prisoners and the fire there is a raised walkway along which animals, plants and other things are carried. ...



Their shapes cast shadows on the wall which occupy prisoners attention. These shadows is the only reality they know, even though they are seeing merely shadows of higher dimensional objects



(*adapted from* **Plato's** "*Republic*", *5th Century BC*)

{*Imagine prisoners who have been chained since childhood, deep inside a cave. Their heads are immobilized, so that their gaze is fixed on a wall. Behind the prisoners there is an enormous fire and between the prisoners and the fire there is a raised walkway along which animals, plants and other things are carried. Their shapes cast shadows on the wall which occupy prisoners attention. ... These shadows is the only reality they know, even though they are seeing merely shadows of higher dimensional objects*.}
(*adapted from Plato's "Republic", 5th Century BC*)

★ **Thank You**★