Identification of Intra-Bunch Transverse Dynamics for Feedback Control Purposes at CERN Super Proton Synchrotron

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Abstract

A promising new approach for designing controllers to stabilize intra-bunch transverse instabilities is to use multi-input multi-output (MIMO) feedback design techniques. However, these techniques require a reduced model and estimation of model parameters based on measurements. We present a method to identify a linear reduced order MIMO model for the vertical intra-bunch dynamics. The effort is motivated by the plans to increase currents in the Super Proton Synchrotron as part of the HL-LHC upgrade where feedback control techniques could be applied to stabilize the bunch dynamics, allowing greater freedom in the machine lattice parameters. Identification algorithms use subspace methods to compute a discrete linear MIMO representation of the nonlinear bunch dynamics. Data from macro particle simulation codes (CMAD and HEADTAIL) and SPS machine measurements are used to identify the reduced model for the bunch dynamics. These models capture the essential dynamics of the bunch motion or instability at a particular operating point, and can then be used analytically to design model-based feedback controllers. The robustness of the model parameters against noise and external excitation signals is studied, as is the effect of the MIMO model order on the accuracy of the identification algorithms.

INTRODUCTION

- Electron clouds [1] and machine impedance can cause intra-bunch instabilities at the CERN Super Proton Synchrotron (SPS).
- Modern control techniques can be used to mitigate these problems but require reduced order models of intra-bunch dynamics to design optimal and robust controllers for a wideband feedback systems [2].
- We use system identification techniques to estimate parameters of linear models representing single bunch dynamics.
- Experimental data was collected from a single bunch with 1×10^{11} protons at 26 GeV with low chromaticity and Q26 optics configuration at CERN SPS.
- These studies uses 3.2 GS/s sampling rate allowing us to sample 16 different locations across 5 ns RF bucket [2]

REDUCED ORDER MODEL & IDENTIFICATION

- The physical system is a nanosecond scale SPS bunch.
- Control variable is momentum kick / driving signal and measured variables is vertical displacements. The control variables and measured variables are discretized to represent the physical system in a discrete-time MIMO system sampled at every revolution period k :

$$\begin{aligned} X_{k+1} &= AX_k + BU_k \\ Y_k &= CX_k \end{aligned} \qquad (1)$$

- where $U \in R^p$ is the control variable (external excitation), $Y \in R^q$ is the vertical displacement measurement, $A \in R^{n \times n}$ is the system matrix, $B \in R^{n \times p}$ is the input matrix, and $C \in R^{q \times n}$ is the output matrix.

$$Y(z) = \left[D^{-1}(z)N(z)\right] U(z) \qquad (2)$$

- [] represents the transfer function matrix ($\in R^{q \times p}$) for a system with p inputs and q outputs. $D(z)$ and $N(z)$ represent denominator and numerator of each discrete time transfer function between input-output couples.

$$N(z)U(z) - D(z)Y(z) = 0 \qquad (3)$$

$$U(z) = \sum_{i=0}^{T} U_i z^i, \quad Y(z) = \sum_{i=0}^{T} Y_i z^i \qquad (4)$$

$$D(z) = \sum_{i=0}^{T} D_i z^i, \quad N(z) = \sum_{i=0}^{T} N_i z^i \qquad (5)$$

$$\left[N_r \mid -D_r\right] \begin{bmatrix} U(k) \\ Y(k) \end{bmatrix} = 0 \qquad (6)$$

- Given the input and output signals, the estimation of the parameter matrices N_r and D_r is obtained by solving the last linear equation.
- Identify A, B, C matrices in discrete time observable canonical form. This minimizes the number of parameters to be identified [4].
- Figure 1 shows the impact of noise on estimation of system parameters. For identification algorithm to perform well, we need to have SNR $> \sim 8$.

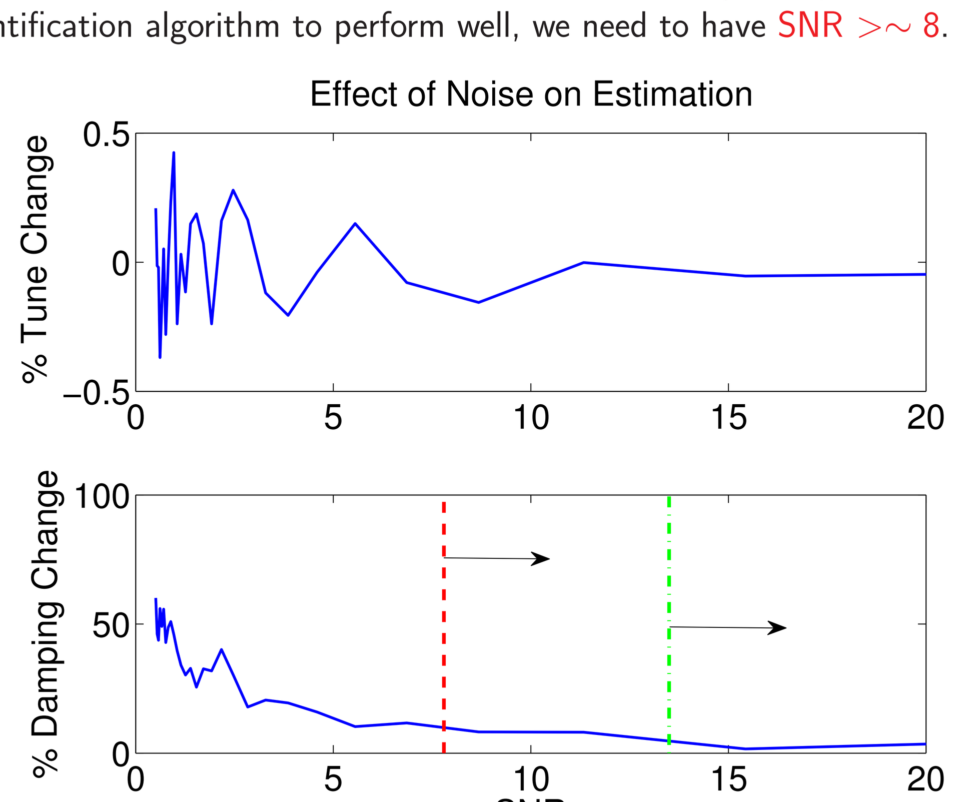

Figure 1 : Deviation of estimated natural tune and damping of the 1st mode from the true value for different SNR values. Red line shows min SNR to get errors less than 10%, green line is for errors less than 5%.

RESULTS OF MODELS PARAMETER ESTIMATION

- In our driven measurements we used both mode 0 and mode 1 excitations [2]
- Our data processing uses a time varying bandpass filter to improve SNR to $\sim$20 to overcome possible signal to noise problem showed in Fig. 1 .
- The existing limited bandwidth kicker [3] forces us to set our reduced model to detect low order modes corresponding to frequencies up to the second sideband ($2f_s$) around the betatron frequency (f_β).

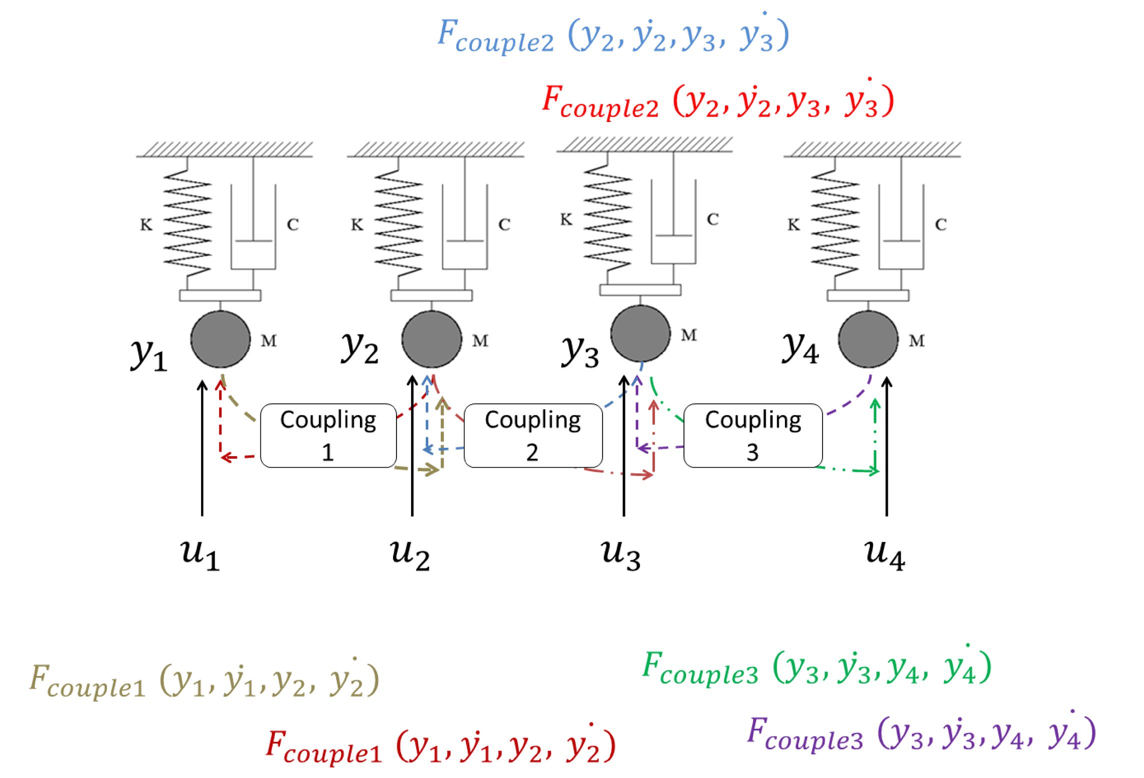

Figure 2 : Reduced model for intra-bunch dynamics.

- Dynamics, input-output relation of momentum kick and vertical displacement, is represented by 4 × 4 MIMO system with $p = 4$, $q = 4$ and $n = 8$.
- In Fig. 3 plots show the vertical displacement of 4 samples across the bunch. Measured data is represented by the blue trace and the response of the identified model is the red trace.

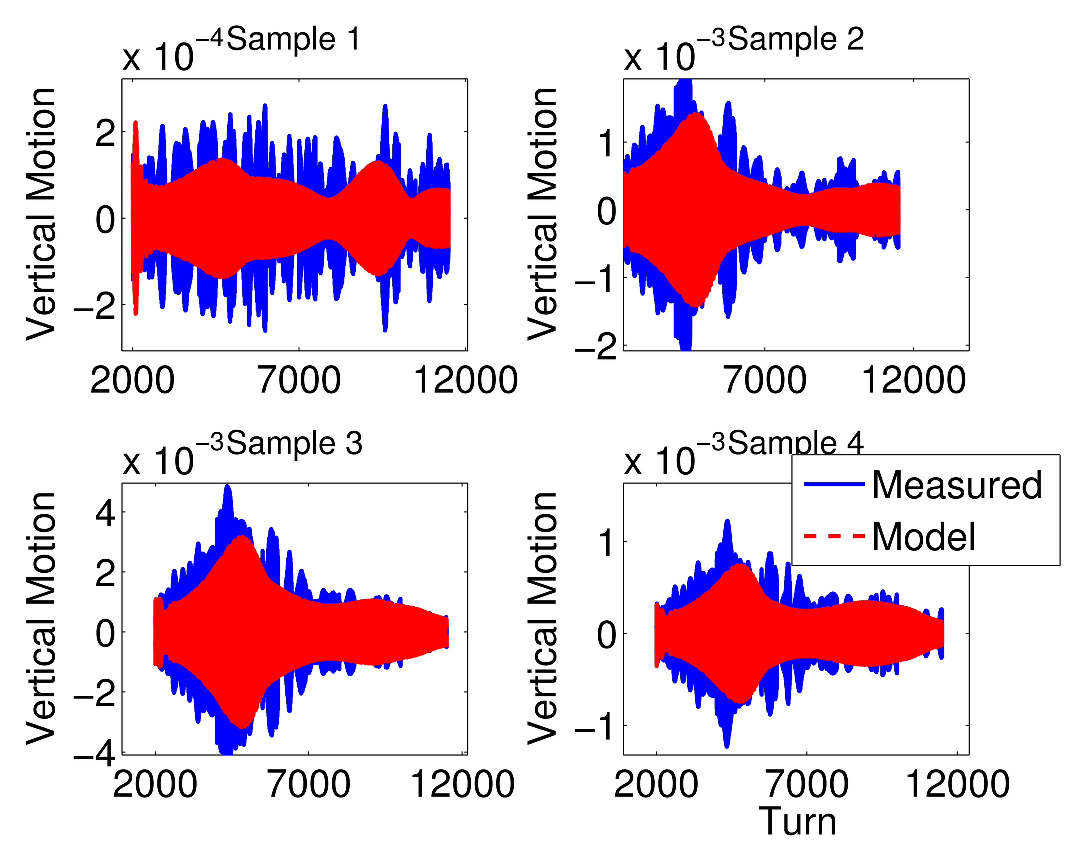

Figure 3 : The vertical displacements at multiple locations across 3.2ns SPS bunch with measurements in blue and the response of the reduced order model in red.

- Reduced order model is linear time invariant. It can't capture external perturbations or parameter variations in the bunch.
- The envelope of the amplitude of the centroid motions (each sample is calculated averaging 4 consecutive non overlapping samples of the 16 samples long original data) is captured in time domain.
- Figure 4 shows measurements and response of model in frequency domain.

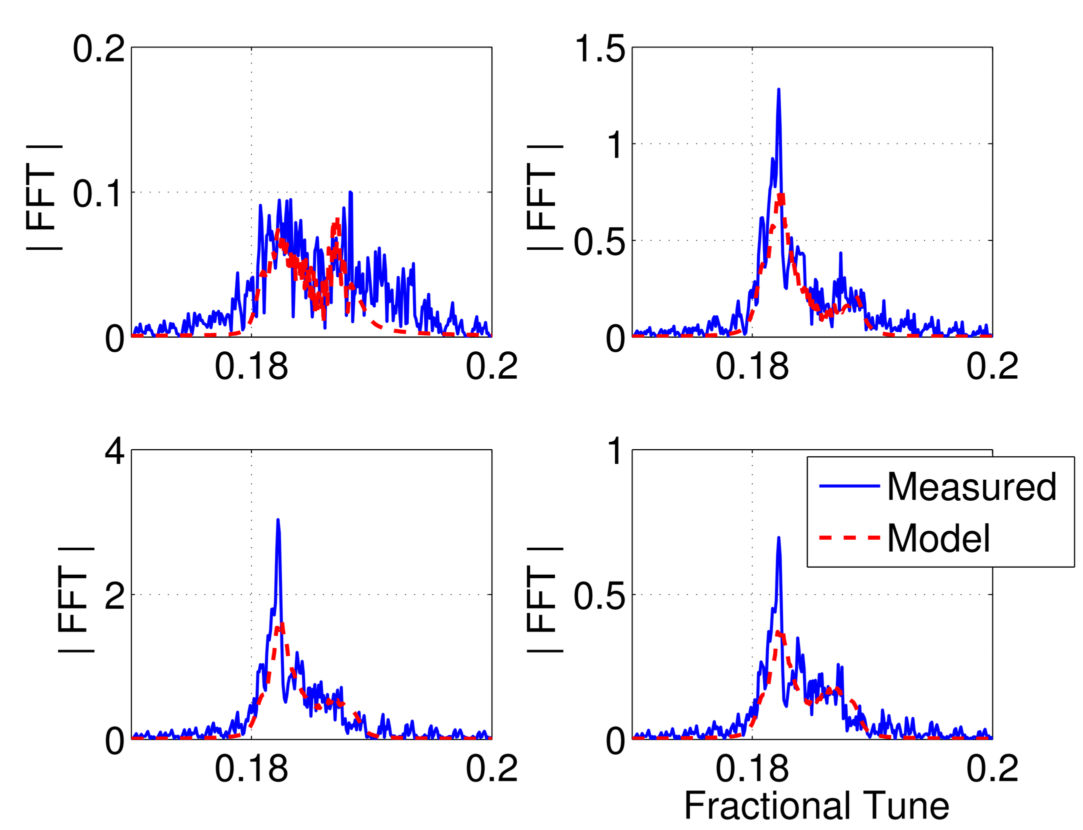

Figure 4 : FFT of vertical displacement. Mode 0 tune is around 0.182 and mode 1 tune is around 0.187.

- Estimation of state space matrices enables us to calculate eigenvectors of the system.
- Another data set where the bunch has 3 modes excited.
- On the left, RMS spectrogram of the driven measurement with clear mode 0, mode 1 and mode 2 excitation around turns $\sim$ 7000, $\sim$ 12000 and $\sim$ 17000.
- On the right side, RMS spectrogram of bunch's vertical motion predicted by reduced model.
- As expected, our linear model is able to capture dominant characteristics and linear dynamics such as motions at mode 0, mode 1 and mode 2 tunes, but not the effect attributed to the non-linearity in the driver.

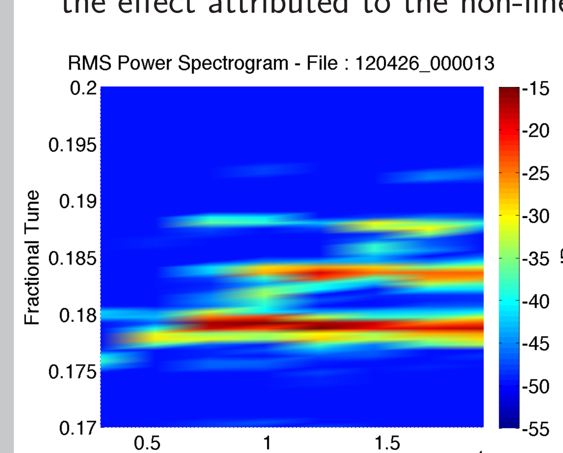

Figure 5 : Spectrogram of physical measurement.

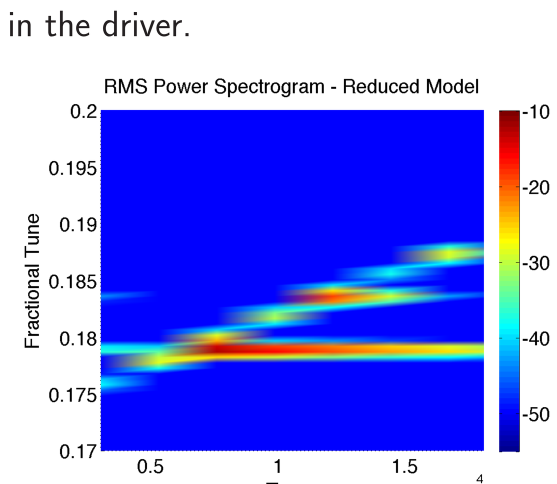

Figure 6 : Spectrogram of the model with same excitation and analysis applied.

- In the simulation, the bunch is represented by 64 slices. All the individual samples across the bunch were taken into account to set N × N MIMO system with N inputs, N outputs and 2N states. Identification is performed based on an N × N MIMO model.
- Similar techniques are also applicable to the non-linear macro particle codes like HEADTAIL or CMAD data.
- As opposed to machine conditions and experiments, simulations have control over noise, disturbances, etc. This gives more flexibility and control to check the performance of the identification algorithm.

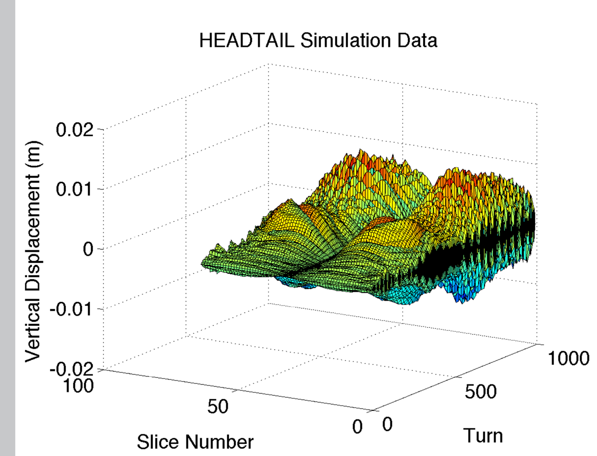

Figure 7 : HeadTail simulation data. Input chirp is between 0.144 and 0.22 fractional tunes covering 2 synchrotron side bands around betatron tune.

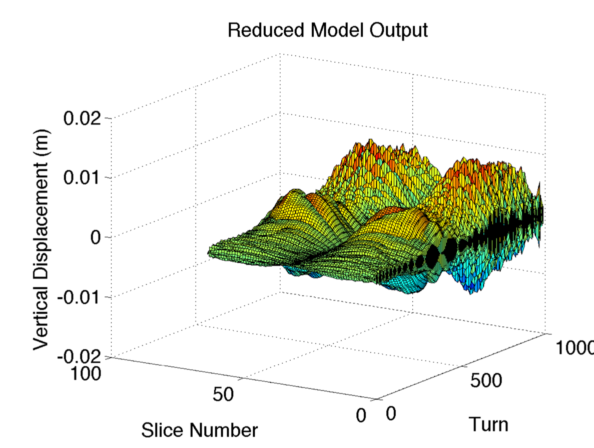

Figure 8 : Reduced order model response to same input signal (0.144 - 0.22 chirp). Reduced order parameters are estimated based on HeadTail simulation data.

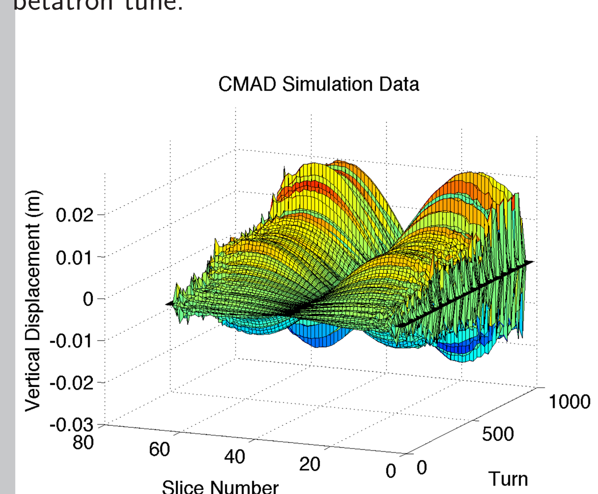

Figure 9 : CMAD simulation data. Input chirp is between 0.144 and 0.22 fractional tunes covering 2 synchrotron side bands around betatron tune.

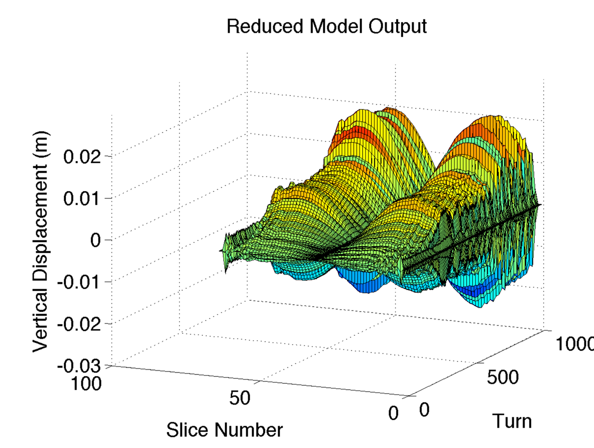

Figure 10 : Reduced order model response to same input signal (0.144 - 0.22 chirp). Reduced order parameters are estimated based on CMAD simulation data.

- A model reduction technique is applied to the result based on Henkel Singular Value (HSVD) analysis to get a minimum order balanced realization of the model [5]. HSVD analysis indicates that relative contributions of the dominant mode representing states are noticeable higher than the contributions of the remaining 128 states ($N = 64$ case).

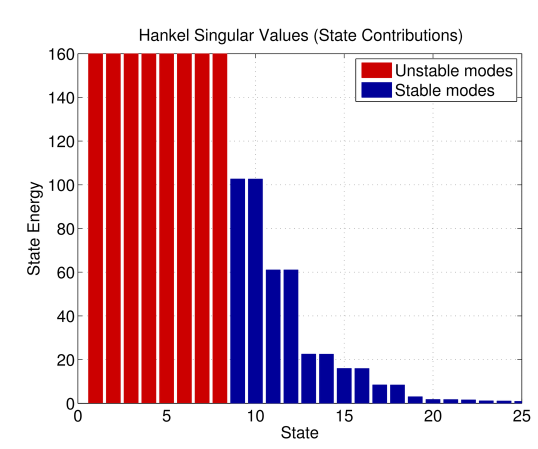

Figure 11 : HSVD Analysis suggests the model order for balanced realization of HEADTAIL simulation based identified model.

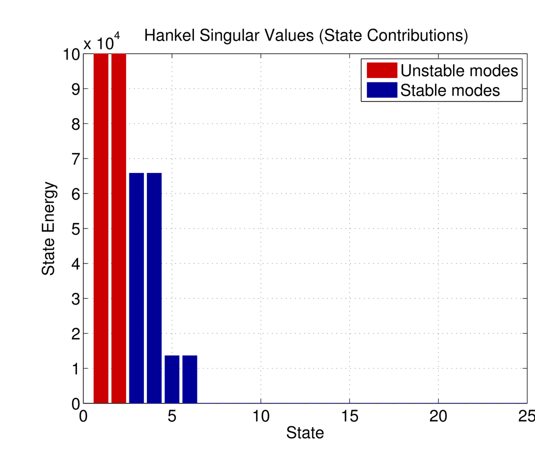

Figure 12 : HSVD Analysis suggests the model order for balanced realization of CMAD simulation based identified model.

CONCLUSION and FUTURE WORK

Model-based control design techniques for intra-bunch instabilities requires a reduced model of the intra-bunch dynamics. We proposed reduced order models and show initial results of the identification of those models. We identified parameters of a reduced order model that captures mode 0, mode 1 and mode 2 dynamics from the CERN SPS machine measurements. The natural tunes, damping values and the separation of modes associated with the motion seen in measurements are estimated correctly using a linear model. We also show similar results using macro particle simulation codes data. Dominant dynamics is captured with a reduced order model and simulation data is regenerated successfully in time domain. Future work is aimed at estimating more internal modes as the wideband kicker will be available early 2015. Availability of the new wideband kicker also requires careful analysis of persistency and optimality of the new excitation signals for the estimation of higher order internal modes. Optimal and robust controllers will be designed using identified reduced order models. These new model based control architectures will be compared with the existing parallelized control filter architecture in terms of performance, processing power and complexity requirements. We plan to evaluate new controllers using macro particle simulations and test in the SPS with single bunch mid 2015.

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