

IDENTIFICATION OF INTRA-BUNCH TRANSVERSE DYNAMICS FOR FEEDBACK CONTROL PURPOSES AT CERN SUPER PROTON SYNCHROTRON*

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Abstract

The design of feedback controllers to stabilize intra-bunch transverse instabilities requires of a reduced model to represent the bunch dynamics. These parametric models require the estimation of the parameters via measurements of the bunch motion in the machine. We present a method to identify a linear reduced model representing the vertical intra-bunch dynamics when the bunch is driven by appropriate signals. The effort is motivated by the plans to increase currents in the Super Proton Synchrotron as part of the HL-LHC upgrade where control techniques could be applied to stabilize the bunch dynamics, allowing greater freedom in the machine lattice parameters. Identification algorithms use subspace methods to compute a discrete linear multi-input multi-output (MIMO) representation of the nonlinear dynamics. Data from macro particle simulation codes (CMAD and HEADTAIL) and SPS machine measurements is used to identify the reduced model for the bunch dynamics. These models capture the essential dynamics of the bunch motion or instability at a particular operating point, and can then be used analytically to design model-based feedback controllers. The robustness of the model parameters against noise and external excitation signals is studied, as is the effect of the MIMO model order on the accuracy of the identification algorithms.

INTRODUCTION

Electron clouds and machine impedance can cause intra-bunch instabilities at CERN Super Proton Synchrotron (SPS). The high current operation of the SPS for LHC injection requires mitigation to these problems. Modern control techniques can be used to stabilize the bunch but they require reduced order models of intra-bunch dynamics to design optimal or robust controllers for wide-band feedback systems.

Nanosecond-scale bunch stabilization is challenging since it requires sufficient bandwidth to sense transverse motion of multiple locations along the bunch and apply correction signals to the corresponding parts of the bunch. Apart from these technological constraints, modeling intra-bunch dynamics is also more challenging compared to the case of modeling beam dynamics including bunch to bunch interactions.

The feedback system senses the vertical positions at multiple locations within nanosecond-scale bunch. Control filters use these measurements to calculate correction signals and apply them back onto the bunch using the kicker as actuator. A 4 Gs/Sec. digital feedback system has been developed to process the motion signals and generate the correction actions [1]. Due to very the fast intrinsic time characteristics of the system, parallelized control filters architecture has been developed. A very similar method had been used for bunch by bunch feedback control systems [2].

In this paper, we show the use of system identification techniques to estimate parameters of linear models representing single bunch dynamics. We define the form of the reduced order model. We pose the identification problem in least squares form [3] for given input-output data set. After a brief discussion of identification constraints, we show results of identification applied on data from SPS measurements and nonlinear macro particle simulation codes.

MODEL AND IDENTIFICATION

Reduced Order Model

Any linear dynamical system can be represented in state space matrix form. A discrete time system sampled at every revolution period k with p inputs and q outputs is represented by

$$\begin{aligned} X_{k+1} &= AX_k + BU_k \\ Y_k &= CX_k \end{aligned} \quad (1)$$

where $U \in R^p$ is the control variable (external excitation), $Y \in R^q$ is the vertical displacement measurement, $A \in R^{n \times n}$ is the system matrix, $B \in R^{n \times p}$ is the input matrix, and $C \in R^{q \times n}$ is the output matrix. For a MIMO system, the model order determines the complexity. In this study, we assumed time invariant dynamics which means having constant A, B and C matrices in state space model. When it comes to the interactions between the bunch with electron clouds or during energy ramping operations, time variant dynamics has to be accounted for tune shifts, changing beam parameters, etc...

Identification

System identification requires to drive the system with appropriated signals and measure its response to those excitation signals. A dynamical model is estimated based on the measured input-output data. In particular for the bunch dynamics identification, the bunch is driven by a wide-band

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kicker using a persistent excitation and measuring the vertical displacements along the $3.2ns$ bunch length. The signals used in the identification process are input $U(k)$, where the vector is represent the multiple momentum kicks applied to the different locations along the bunch and the output $Y(k)$ are the vertical displacements measured at those locations.

$$Y(z) = [D^{-1}(z)N(z)] U(z) \quad (2)$$

where $[\]$ represents the transfer function matrix ($\in R^{q \times p}$) for a system with p inputs and q outputs. $D(z)$ and $N(z)$ represents denominator and numerator of each discrete time transfer function between input-output couples.

$$N(z)U(z) - D(z)Y(z) = 0 \quad (3)$$

$$U(z) = \sum_{i=0}^N U_i z^i, \quad Y(z) = \sum_{i=0}^{\infty} Y_i z^i \quad (4)$$

$$D(z) = \sum_{i=0}^N D_i z^i, \quad N(z) = \sum_{i=0}^{\infty} N_i z^i \quad (5)$$

$$\begin{bmatrix} N_r & | & -D_r \end{bmatrix} \begin{bmatrix} U(k) \\ Y(k) \end{bmatrix} = 0 \quad (6)$$

Given the input and output signals, the estimation of the parameter matrices N_r and D_r is obtained by solving the last linear equation.. There are many different subspace based methods to solve this linear MIMO problem. These methods use projections or singular value decomposition (SVD) to cast the problem into linear least squares [4]. We follow [3] where the construction of data matrix from input and output signals, the solution algorithm for equation 6 and the relationship between the transfer function coefficients and the observable canonical state space form are shown. Assuming full observability of the system, we can represent our state space in discrete time observable canonical form. This will enable us to estimate the minimum number of parameters [3]. However, even for linear systems there are two well known limitations for identification. These are the effect of noise and lack of persistent excitation.

Persistence Input

Input signal design and persistent excitation are critical aspects of system identification. Given a quasi-stationary input of u with a dimension nu and with a spectrum $\phi_u(\omega)$, $\phi_u(\omega) > 0$ should hold for at least n distinct frequencies for u to be a persistent excitation [9]. Random noise would be ideal to excite all the modes in the system but requires high excitation power and bandwidth. The hardware used in these measurements puts constraints on both power and bandwidth. The design of an input signal for identification under given constraints becomes an important question for the future studies.

Noise Sensitivity

Noise affects the performance of the identification, and in certain cases can make identification impossible. We can quantify the effect of the noise by adding noise on a known

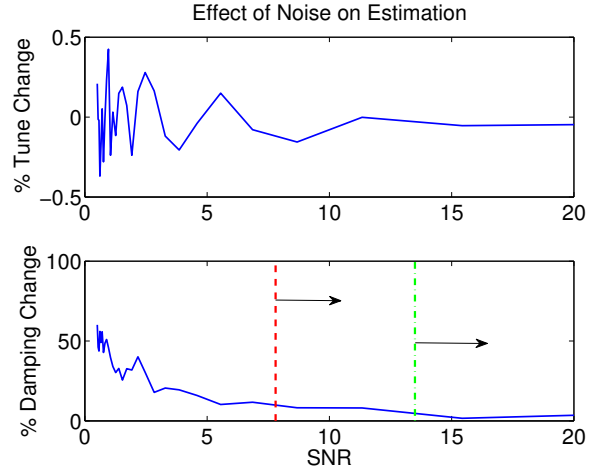


Figure 1: Deviation of estimated natural tune and damping of the 1st mode from the true value for different SNR values. Red line shows min SNR to get errors less than 10%, green line is for errors less than 5%.

system until the identification can no longer clearly estimate the known dynamics.

We drive a synthetic 2×2 coupled MIMO system using a frequency chirp signal with random noise added to the output signals. The effect of noise is tested by running the identification algorithm on input - output data as we increase the noise level. Parameters of the model and the corresponding modes of the identified model are estimated for different noise cases. We observe the effect of noise on the mode estimation by looking at the natural frequency and damping values of the first mode for different values of signal to noise ratio (SNR). Figure 1 shows the impact of noise on estimation of system parameters. For identification algorithm to perform well, we need to have $SNR \gg 8$.

APPLICATIONS

Multiples MDs have been conducted at the CERN SPS ring to drive the bunch with different excitations (open loop) and also test feedback controllers to stabilize the bunch (closed loop). Those measurements were conducted using a single bunch in the machine with intensities about $1 - 1.5 \times 10^{11}$ protons at the injection energy, 26 GeV and Q26 lattice configuration [6], [7]. The driven tests with different excitation signals have been designed such as the kicking signal is a persistent input for the system, and the collected data can be use to study the identification algorithms and quantify a reduced parametric model of the beam dynamics. Similarly, data obtained from macro-particle simulation codes (CMAD-HeadTail) has been used to test the identification algorithms and compare the dynamic model results with those obtained from machine measurements.

SPS Measurements

The hardware installed in the CERN SPS allows us to drive the bunch with limited bandwidth. The kicker used for these test has a maximum bandwidth of 160 MHz, limiting the spectrum of the momentum applied to the bunch. This limit conditions the number of modes that can be identified in the dynamic model because the final momentum is not a persistent input to detect high order transverse modes in the bunch. New kicker designs, fabrication and installation in the CERN SPS ring are in progress to be able to drive the bunch with higher frequency momentums [5].

The existing limited bandwidth kicker forces us to set our reduced model to detect low order modes corresponding to frequencies up to the second sideband ($2f_s$) around the betatron frequency (f_β). We use both mode 0 (barycentric shape along longitudinal axis - amplitude modulation with chirp in turns) and mode 1 (200 MHz, head - tail shape along longitudinal axis - amplitude modulation with chirp in turns) excitations [1]. Chirp frequency covers $f_\beta \pm 2f_s$ in ~ 15000 turns. Time alignment between excitation signal and bunch gives us the flexibility to excite a specific mode more dominantly.

To improve the SNR used in the identification, the vertical motion signal is processed using a time-varying band-pass filter whose central frequency follows the frequency variation of the excitation signal applied.

Tailoring the reduced model to the low-order modes, it is possible to use 4 coupled 2^{nd} order differential equations to capture the low-order dynamics of the bunch (mode 0 - barycentric motion and mode ± 1 - head-tail motion). The input-output relationship (momentum kick to vertical displacement) of the bunch is defined by a 4×4 MIMO system with $p = 4$, $q = 4$ and $n = 8$. This MIMO model sets the input and output vectors dimension (equation 1) to 4 for each sampling instance k . Measurement set-up acquires either 16 or 32 samples long momentum kick or vertical displacement signals across the bunch at each sampling time k . To do identification with the corresponding MIMO model, each sample in vectors U_k and Y_k ($\in R^{4 \times 1}$) is calculated averaging either 4 or 8 consecutive non overlapping samples of the 16 or 32 sample long original data (e.g $U(1, k), Y(1, k)$ is the average of samples 1-4, $U(2, k), Y(2, k)$, the average of samples 5-8 ... etc).

Using these input-output signals, the identification algorithm is evaluated and some results are analyzed. Fig. 2 shows the time evolution of the 4 components of the vertical displacement vector Y_k for about 10000 turns. Measured data is represented by blue trace and the response of the identified model is red trace. It is important to notice that the reduced order model is linear time invariant and cannot capture external perturbations or parameter variations in the bunch. Still, the envelope of the amplitude of the intra-bunch vertical motion is captured in the time domain. The plots on the right show measurements and response of model in frequency domain.

Figure 3 shows another data with strong excitation of both mode 0 and 1^{st} sideband (mode 1). We also see some motion around 2^{nd} sideband. On the left, we see the RMS spectrogram of the driven measurement with clear mode 0, mode 1, and mode 2 excitation around turns ~ 7000 , ~ 11500 and ~ 15500 . On the right side, we show the RMS spectrogram of bunch's vertical motion predicted by reduced model. It is important to notice from the measured data (Fig. 3 - left) the effect of nonlinearities in the driving systems. The spectrogram analysis of the measured signal shows that the 2^{nd} sideband ($f_\beta + 2f_s = 0.189$) is excited before the chirp excitation drives the bunch motion at that particular frequency. As expected, our linear model is able to capture dominant characteristics and linear dynamics such as motions at mode 0, mode 1 and mode 2 tunes, but not that effect attributed to the non-linearity in the driver.

Nonlinear Simulations

Similar techniques were applied to data obtained by non-linear macro particle simulations such as HEADTAIL and CMAD. These tools are especially very useful and helpful because they are accessible to study beam dynamics without beam time in the machine. As soon as simulations are benchmarked with machine measurements, the intra-bunch dynamics can also be identified using data from these simulations. As opposed to machine conditions and experiments, simulations have control over noise, impurities, disturbances, etc ... This gives more flexibility and control to check the performance of the identification algorithm.

In the simulation, the bunch is represented by 64 slices. The bunch is excited by a chirp signal similar to the one use in the experimental test at CERN SPS. The shape of the excitation signal in longitudinal axis is a 200 MHz sine wave (single cycle in a $5ns$ RF bucket) at a given turn and the amplitude of this head-tail shaped signal is modulated by a frequency sweep between 0.144 and 0.22 corresponding to the fractional tune in turns. The frequency range covers from the 2^{nd} lower sideband to 2^{nd} upper sideband where betatron tune is 0.18 and synchrotron separation is 0.017 as given in Q-20 optics.

Fig. 4 shows on the left side the vertical displacement data across the bunch for 1000 turns. Due to the shape of excitation signal across the bunch in this specific case, most of excitation energy is coupled to ± 1 lateral bands and we see very small amount barycentric oscillations due to residual kick. With an appropriate excitation signals we can study many multi-modal dynamics with the help of these simulations.

Fig. 4 shows on the right side the response of the reduced order model to the same excitation in time domain. The relative error between these results is less than $\sim \%10$. The reduced order model can capture the dominant dynamics of the bunch successfully. Similarly, results can also be seen in frequency domain as show in Fig. 5 where on the left we see the spectrogram of the vertical displacement data from the simulation and on the right we have the spectrogram of the vertical displacement obtained by the reduced order model.

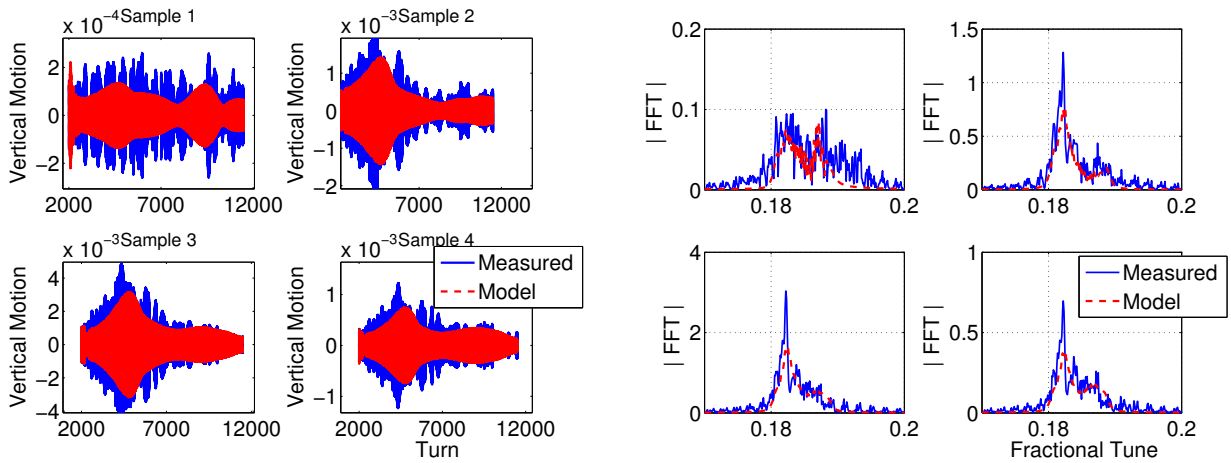


Figure 2: Comparison of the reduced order models response with machine measurements in time and frequency domain.

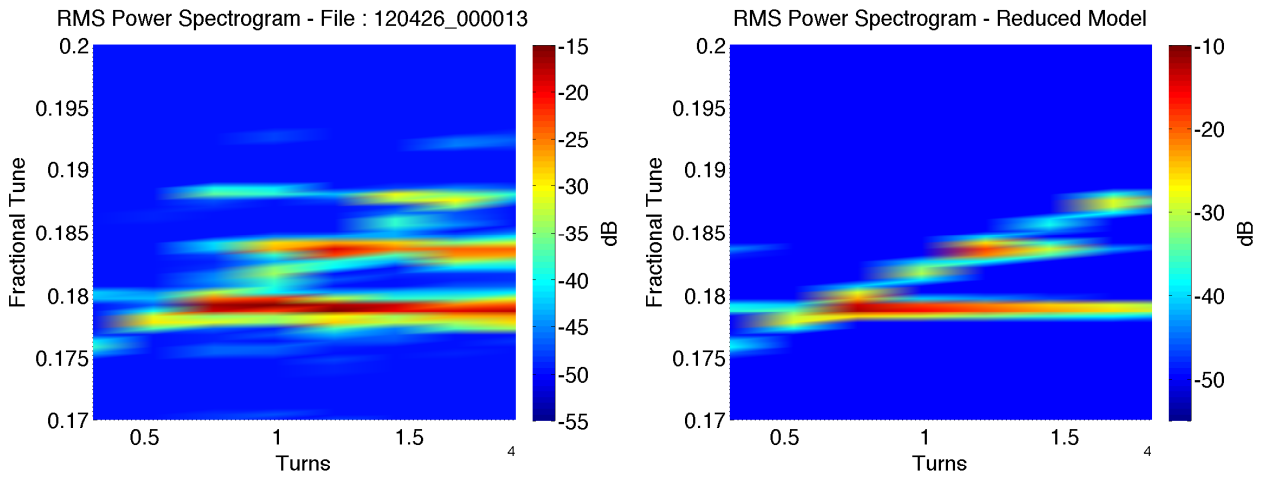


Figure 3: On the left we see the spectrogram of physical measurement showing both chirp excitation where we excite mode 0, mode 1, and mode 2 excitation around turns ~ 7000 , ~ 11500 and ~ 15500 , respectively. On the right, we see the same excitation and analysis applied to the reduced order model.

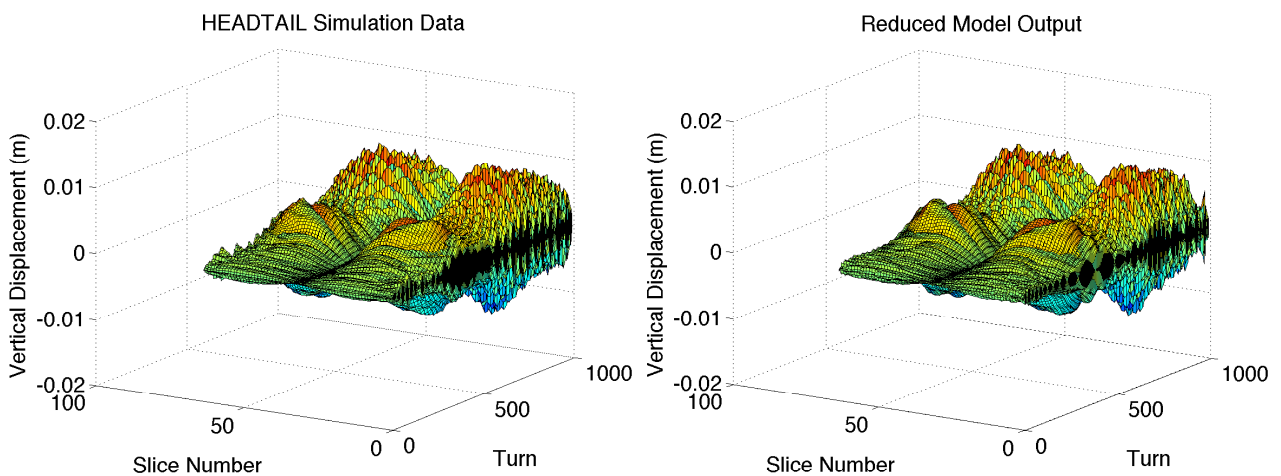


Figure 4: Comparison of the reduced order models response with HEADTAIL simulation data in time domain. Relative error between these two results are less than $\sim 10\%$. Both reduced model and HEADTAIL simulations are driven by same excitation signal.

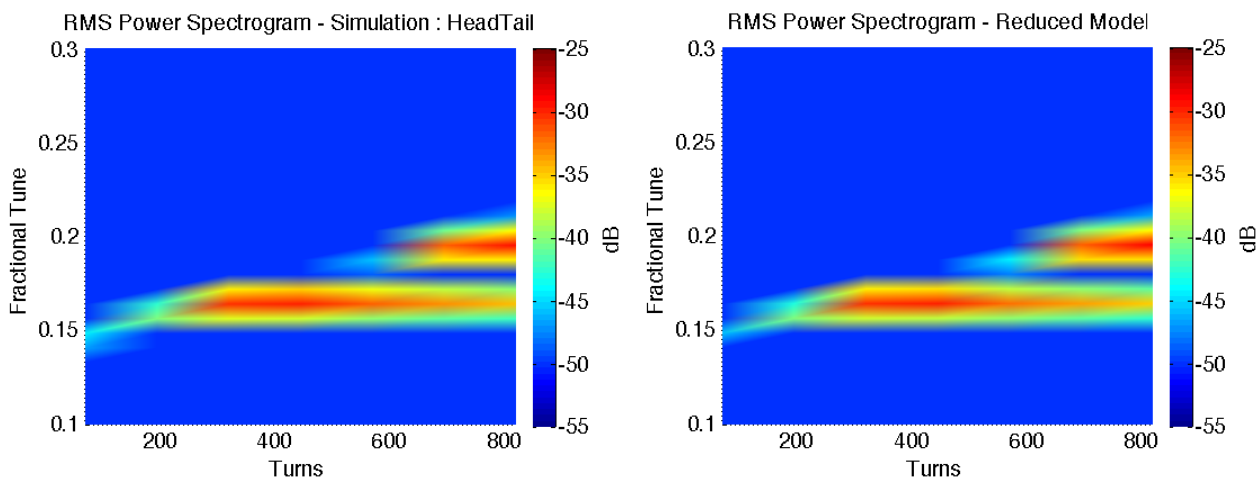


Figure 5: Comparison of the reduced order models response with HEADTAIL simulation data in frequency domain.

One of the important identification parameters that we can study is the estimation of the MIMO model order. Two different approaches could be used. In the first approach, as briefly explained in "SPS Measurement" section we proposed the order based on the persistence input signal. The model order was estimated based on the modes that the excitation signal is able to drive in the bunch and we constrained the input-output signals to identify that reduced model (e.g. we averaged the measured samples in order to reduce the number of inputs and outputs to identify a 4×4 MIMO system with $p = 4$, $q = 4$ and $n = 8$.) With the simulation data, we used the second approach in the analysis. All the individual samples across the bunch were taken into account to set $N \times N$ MIMO system with N inputs, N outputs and $2N$ states. Identification is performed based on $N \times N$ MIMO model and then model reduction is applied to the result based on Henkel Singular Value analysis to get a minimum order balanced realization of the model [4]. As an example, Fig. 6 shows relative contribution of first 25 states of all 128 states ($N = 64$ case). Therefore, Henkel Singular Values clearly indicate that there are 6 states - 3 modes as dominant dynamics in the system and a reduced order model with 6 states (order of 6) is enough to capture dominant dynamics driven by this excitation signal.

CONCLUSION AND FUTURE WORK

Control of intra-bunch instabilities via a wideband feedback control system with optimal control algorithms requires a model of the intra-bunch dynamics. We show initial promising system identification results. We identify parameters of a reduced order model that captures mode 0, mode 1 and mode 2 dynamics from CERN SPS machine measurements. The natural tunes, damping values and the separation of modes associated with the motion seen in measurements are estimated correctly using a linear model. We also show similar results using macro particle simulation codes data. Dominant dynamics is captured with a reduced order model and simulation data is regenerated successfully in time do-

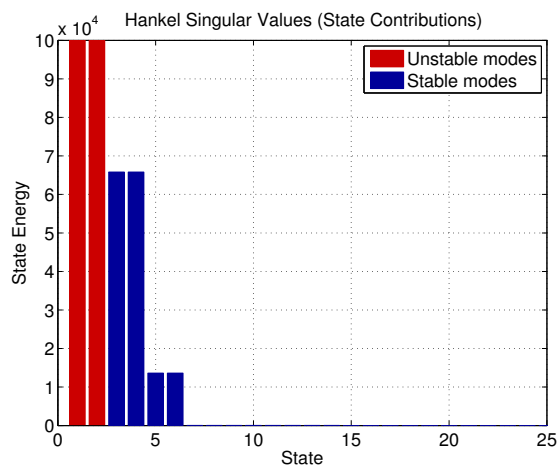


Figure 6: Henkel singular value analysis help us to get a minimum order balanced realization. Note that marginally stable modes are also labeled as "Unstable modes" in MATLAB.

main. Future work is aimed at estimating more internal modes as the wideband kicker is available late 2014 and early 2015. Optimal and robust controllers will be designed using identified reduced order models. These new model based control architectures will be compared with the existing parallelized control filter architecture in terms of performance, processing power and complexity requirements. We plan to evaluate new controllers using macro particle simulations and test in SPS for single bunch mid 2015.

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