RF noise and Crab Cavities

How much transverse emittance growth can we anticipate?

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Objective of the work

- Collaboration with Pr. Mastoridis (Cal Poly, San Luis Obispo) on the CC LLRF design
- First problem considered: Propose a design that keeps the transverse emittance growth caused by RF noise below 4-5%/h

Content of today's talk

- Derivations of emittance growth caused by CC RF noise
- Influence of power source
- Optimization of LLRF
- Conclusions and future work

Derivations

- "First principle" formulas for emittance growth caused by CC phase and amplitude noise

Derivations

- We follow the approach used by V. Ledeev in SSCL,1993 and applied to the effect of vibrations and magnetic field fluctuations, for the SSC [1]
- We have adapted it to the CC RF noise
- We use normalized transverse (x,p) coordinates

$$x = \frac{X}{\sqrt{\beta}} \quad \text{and} \quad p = \beta \frac{d \frac{X}{\sqrt{\beta}}}{ds}$$

- Consider a particle α of transverse tune $2\pi\mu_\alpha$, receiving a small momentum kick Δp_n at each turn n. In normalized coordinates, its position at turn n is given by

Unperturbed trajectory ↓ Cumulative effect of the kicks ↓

$$x_n = x_0 \cos(\mu_\alpha n + \theta_0) + \sum_{k=-\infty}^{n} \Delta p_k \sin(\mu_\alpha (n-k))$$

- We now consider the statistical properties of the noise process Δp_n (zero mean, $R_{\Delta P}(nT_{rev})$) and compute the variance of the position of particle α at turn n
- As the noise is zero mean, we have

$$\left\langle x_n{}^2 \right\rangle = x_0{}^2 \cos^2(\mu_\alpha n + \theta_0) + \sum_{k=-\infty}^{n} \sum_{l=-\infty}^{n} R_{\Delta p}\left[(k-l)T_{rev}\right] \sin(\mu_\alpha(n-k)) \sin(\mu_\alpha(n-l))$$

$$\left\langle x_n{}^2 \right\rangle = x_0{}^2 \cos^2(\mu_\alpha n + \theta_0) + \frac{1}{4} \sum_{k=-\infty}^{n} \sum_{l=-\infty}^{n} R_{\Delta p}\left[(k-l)T_{rev}\right] \left[e^{\pm j\mu_\alpha (k-l)} - e^{\pm j\mu_\alpha (2n-k-l)} \right]$$

- The emittance growth is due to the second term only

$$\left\langle \tilde{x}_n{}^2 \right\rangle = \frac{1}{4} \sum_{k=-\infty}^{n} \sum_{l=-\infty}^{n} R_{\Delta p}\left[(k-l)T_{rev}\right] \left[e^{\pm j\mu_\alpha (k-l)} - e^{\pm j\mu_\alpha (2n-k-l)} \right]$$

- Now using the relation between auto-correlation and Power Spectral Density

$$R_{\Delta p}(t) = \int_{-\infty}^{\infty} S_{\Delta p}(f)\, e^{j2\pi ft} df$$

- We get

$$\left\langle \tilde{x}_n^{\ 2} \right\rangle = \frac{1}{4} \sum_{k=-\infty}^{n} \sum_{l=-\infty}^{n} \int_{-\infty}^{\infty} S_{\Delta p}(f) e^{j2\pi f(k-l)T_{rev}} \left[e^{\pm j\mu_\alpha (k-l)} + e^{\pm j\mu_\alpha (2n-k-l)} \right] df$$

$$\left\langle \tilde{x}_n^{\ 2} \right\rangle = \frac{1}{4} \sum_{k=-\infty}^{n} \left[\int_{-\infty}^{\infty} S_{\Delta p}(f) \sum_{p=-\infty}^{n-k} \left[e^{-jp[2\pi fT_{rev} \pm \mu_\alpha]} + e^{-j[2\pi pfT_{rev} \pm \mu_\alpha (2n-2k-p)]} \right] df \right]$$

- The first term in the sum has an integrable discontinuity on the betatron bands. With *n* going to infinity, we get

$$\left\langle \tilde{x}_n^{\ 2} \right\rangle = \frac{f_{rev}}{4} \sum_{k=-\infty}^{n} \left[\int_{-\infty}^{\infty} S_{\Delta p}(f) \sum_{p=-\infty}^{\infty} \delta\left(f \pm \frac{\mu_\alpha}{2\pi} f_{rev} - p\, f_{rev} \right) df \right]$$

$$\left\langle \tilde{x}_n^{\ 2} \right\rangle = \frac{f_{rev}}{4} \sum_{k=-\infty}^{n} \sum_{p=-\infty}^{\infty} S_{\Delta p}\left(\pm f_{b,\alpha} - p\, f_{rev} \right)$$

- For particle α of betatron frequency $f_{b,\alpha}$, the rate of increase of its rms transverse motion is thus

$$\left\langle \tilde{x}_n^{\ 2} \right\rangle - \left\langle \tilde{x}_{n-1}^{\ 2} \right\rangle = \frac{f_{rev}}{4} \sum_{p=-\infty}^{\infty} S_{\Delta p}\left(\pm f_{b,\alpha} - p\, f_{rev} \right)$$

Growth is caused by the noise PSD on the betatron bands

- In our normalized coordinates

$$\varepsilon_{x,\alpha} = \left\langle \tilde{x}_n^{\,2} \right\rangle \rightarrow \frac{d\varepsilon_{x,\alpha}}{dt} = \frac{\left\langle \tilde{x}_n^{\,2} \right\rangle - \left\langle \tilde{x}_{n-1}^{\,2} \right\rangle}{T_{rev}} = \frac{f_{rev}^{\,2}}{4} \sum_{p=-\infty}^{\infty} S_{\Delta p}\left(\pm f_{b,\alpha} - p\, f_{rev}\right)$$

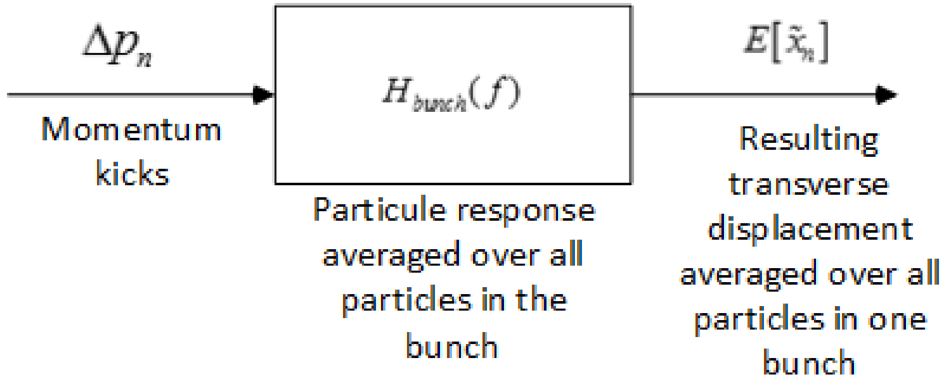
- So far we have treated one particle only. We can now generate another stochastic process that is the average of the particle positions in the bunch at turn *n*. We use *E[.]* when considering the in-bunch statistics (while we used *<.>* for the noise statistics). Let *g(f)* be the distribution of betatron frequency within the bunch. Non-zero for positive f only. Assuming that kicks are independent of particle (OK for phase noise, short bunches, not for AM noise), we get

$$E\left[\tilde{x}_n\right] = \Delta p_n * \mathrm{E}\left[\sin(\mu n)\right] = \Delta p_n * \int_{-\infty}^{\infty} g(f) \left[\frac{e^{j2\pi \frac{f}{f_{rev}} n} - e^{-j2\pi \frac{f}{f_{rev}} n}}{2j} \right] df = \Delta p_n * \int_{-\infty}^{\infty} \frac{g(|f|)}{2j} e^{j2\pi \frac{f}{f_{rev}} n} df$$

- The last term is the inverse Fourier Transform of $H_{bunch}(f)$ defined as

$$H_{bunch}(f) = \frac{f_{rev}}{2j} g(|f|)$$

- The new stochastic process $E[\tilde{x}_n]$ is the transverse damper input. It can be viewed as the momentum kick process filtered by $H_{bunch}(f)$



- The bunch emittance growth can also be related to $H_{bunch}(f)$

Growth is caused by the noise PSD on the betatron bands filtered by the bunch response

$$\frac{d\varepsilon}{dt} = \frac{f_{rev}^{\ 2}}{4} \sum_{p=-\infty}^{\infty} \int_{0}^{\infty} g(|f|) S_{\Delta p}(f - p\, f_{rev})\, df = \frac{f_{rev}}{2} \int_{0}^{\infty} jH_{bunch}(f) \sum_{p=-\infty}^{\infty} S_{\Delta p}(f - pf_{rev})\, df$$

- This formula is useful when the noise PSD varies within the betatron spread as it is the case with a damping system.

Crab Cavity case. Phase noise

- In our normalized coordinates, the crab cavity momentum kick on a particle is given by

$$\Delta p_x = \sqrt{\beta_{CC}}\,\frac{e\,\Delta V}{E_b}$$

- Assuming small noise level, the phase noise produces momentum kicks (independent of particle α)

$$\Delta p_n = \sqrt{\beta_{CC}}\,\frac{eV_o}{E_b}\,\Delta\varphi_n$$

- As a result, the emittance growth is given by

$$\frac{d\varepsilon}{dt} = \beta_{CC}\left(\frac{eV_o f_{rev}}{2E_b}\right)^2 \sum_{n=-\infty}^{\infty} S_{\Delta\phi}(\pm f_b - n f_{rev})$$

← Growth is caused by the phase noise PSD on the betatron bands

- We have assumed that the noise Spectrum was flat within the bunch betatron spread (bunch response). This is a reasonable assumption in the absence of damper (see below)..

Crab Cavity case. Amplitude noise

- Assuming small noise level, the amplitude noise produces momentum kicks on a particle α, dependent of its synchrotron motion $(\phi_\alpha, \mu_{s,\alpha}, \psi_\alpha)$

$$\Delta p_n = \sqrt{\beta_{CC}}\,\frac{eV_o}{E_b}\,\Delta A_n \phi_\alpha \sin\left(\mu_{s,\alpha} n + \psi_\alpha\right)$$

- The kick depends on the synchrotron frequency. Depending on the frequency of the amplitude noise, this will enhance or reduce its effects. Resonant behavior can be expected with noise on the synchro-betatron side-bands. It can be shown that the emittance growth is given by

$$\frac{d\varepsilon}{dt} = \beta_{CC}\left(\frac{eV_o\sigma_\varphi f_{rev}}{2E_b}\right)^2 \sum_{n=-\infty}^{\infty} S_{\Delta A}(\pm f_b \pm f_s - n f_{rev})$$

← Growth is caused by the amplitude noise PSD on the synchro-betatron bands

- Notice the importance of bunch length, as expected.

Crab Cavity. Total noise

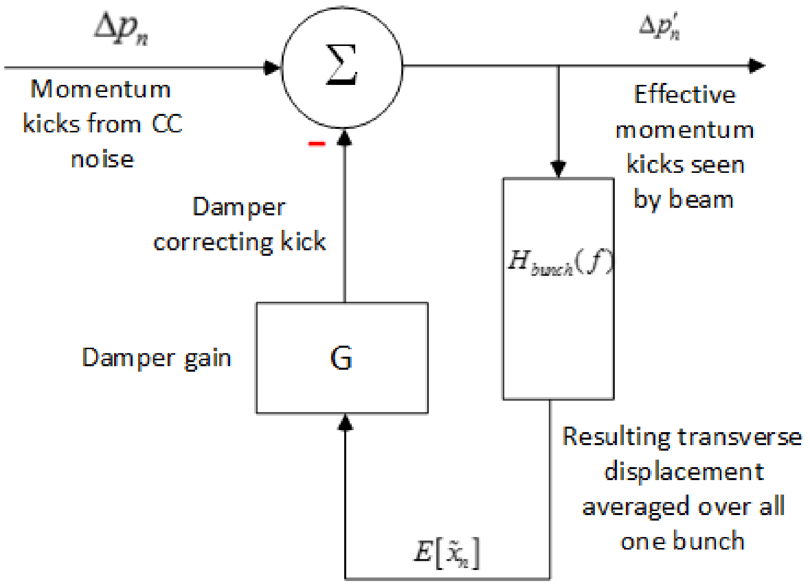
- The total CC emittance growth is given by

$$\frac{d\varepsilon}{dt} = \beta_{CC}\left(\frac{eV_o f_{rev}}{2E_b}\right)^2 \sum_{n=-\infty}^{\infty} S_{\Delta\phi}(\pm f_b - n f_{rev}) + \beta_{CC}\left(\frac{eV_o \sigma_\varphi f_{rev}}{2E_b}\right)^2 \sum_{n=-\infty}^{\infty} S_{\Delta A}(\pm f_b \pm f_s - n f_{rev})$$

- The phase noise spectrum is summed over all betatron bands while the amplitude spectrum is summed over all synchro-betatron bands (factor 2)
- The effect of the amplitude noise is scaled by the bunch length square. With the 1.00 ns long (4σ) LHC bunches, the scaling $2\sigma_\phi^2 \approx 0.8$
- The result is similar to the effect of RF noise in the longitudinal plane (G. Dome – D. Boussard [2], S.V. Ivanov [3],…)

$$\frac{d\sin^2\left(\frac{\hat{\phi}}{2}\right)}{dt} = (\pi f_s)^2\left[S_\phi(f_s) + 2\sin^2\left(\frac{\hat{\phi}}{2}\right) S_A(2 f_s)\right]$$

Transverse damper



- The damper receives as input the particle displacements averaged over one bunch and generates a correcting momentum kick
- We can use the property of filtering a stochastic process to derive the power spectrum of the effective kicks

$$S_{\Delta p'}(f) = \frac{S_{\Delta p}(f)}{\left|1 + GH_{bunch}(f)\right|^2}$$

- The damper reduces the effect of the noise in the core of the bunch particle distribution, where the average signal responds strongly to the noise
- The same effect is observed with the beam phase loop on the main cavities [4].

Transverse damper (cont'd)

- The emittance growth rate, with a damper, becomes

$$\frac{d\varepsilon}{dt} = j\frac{f_{rev}}{2}\int_0^{\infty}\sum_{p=-\infty}^{\infty}\frac{1}{\left|1+GH_{bunch}(f)\right|^2}H_{bunch}(f)S_{\Delta p}(f-pf_{rev})\,df$$

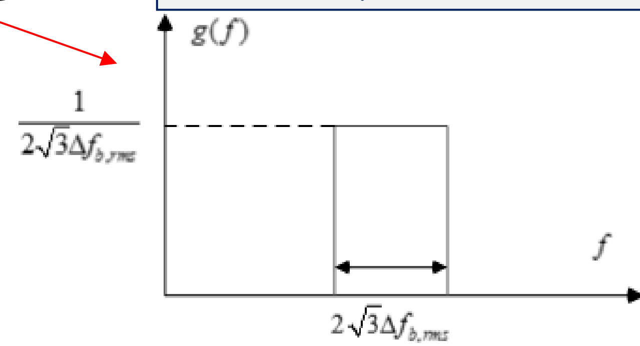
- As mentioned above, the reduction factor is not constant over the betatron tune, except for an (unrealistic) uniform distribution of betatron tune. In that case the reduction is easily computed we get

$$Factor = 48\left[\frac{\Delta f_b}{G}\right]^2$$

- With the design parameters we get a factor ~100

ν (tune)	64.31
Δν (rms tune spread)	0.0015
θ_c (μrad)	500
V_c (MV/cav)	3
N_{cav}	8
β* (cm)	20
β_{cc} (m)	3500
G_{ADT} (s^{-1})	1/(10 Trev)

Parameters used for the computations

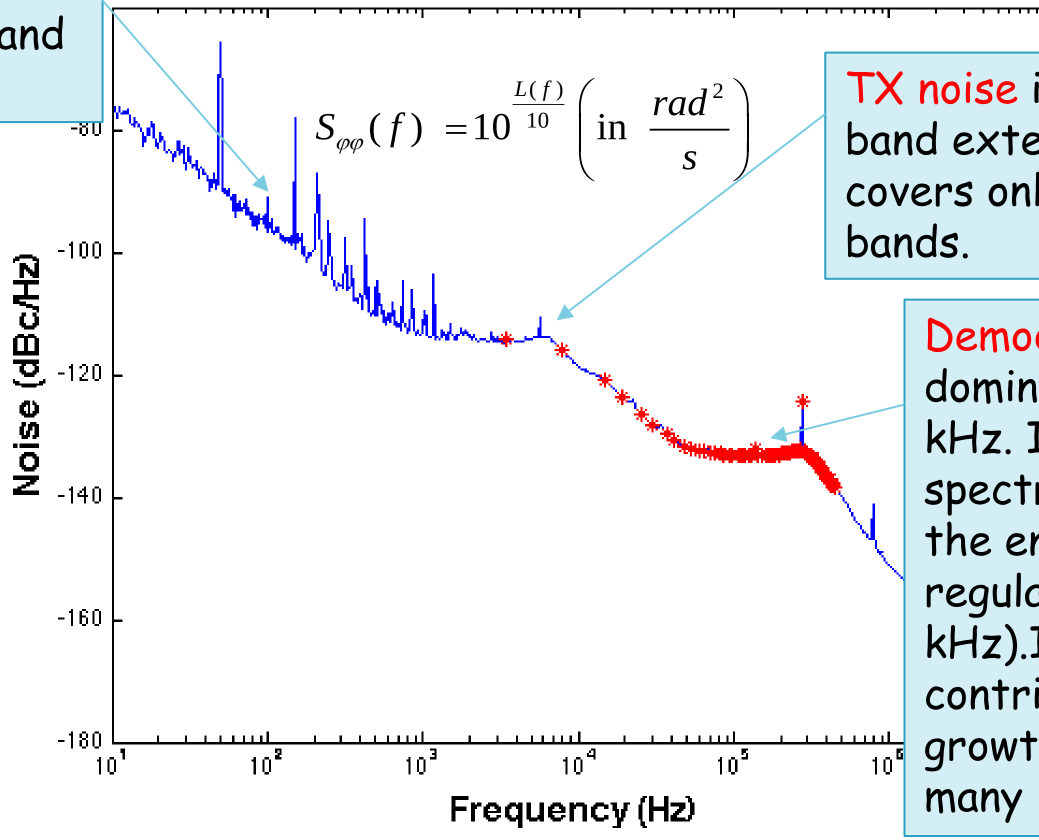


Uniform tune distribution

Scaling the ACS noise to CCs

Noise in the 10Hz-1kHz range is not an issue as the first betatron band is around 3 kHz

$$L(f) \text{ in } \frac{dBc}{Hz}$$



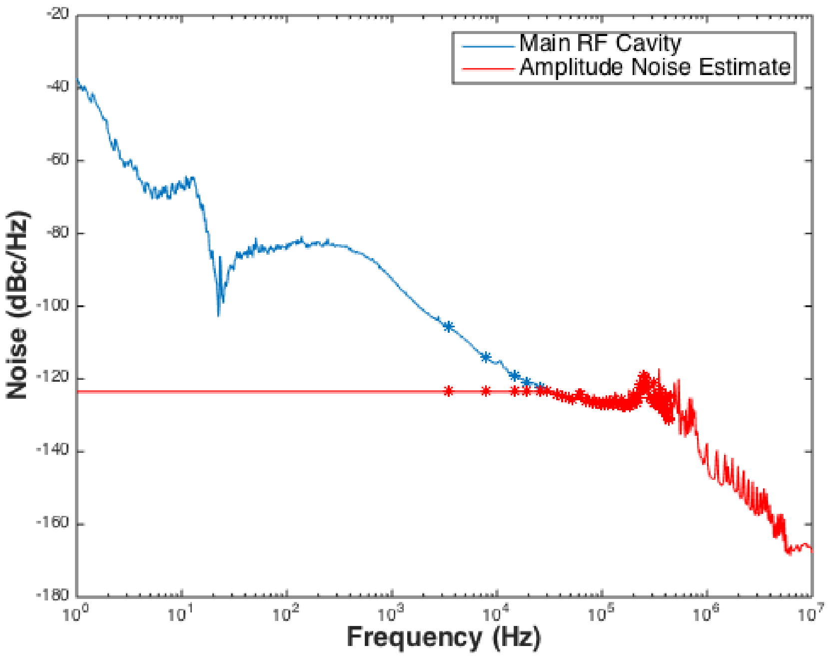
TX noise is dominant in the band extending to 20 kHz. It covers only 3-4 betatron bands.

Demodulator noise is the dominant factor above 20 kHz. It gives a flat noise spectrum extending to the end of the LLRF regulation band (300 kHz).It has a large contribution to the growth rate as it contains many betatron bands

- ACS (without beam) SSB phase noise Power Spectral Density measured in the cavity, in dBc/Hz. The red marks indicate the betatron bands.

Using the ACS design for CC

- Phase Noise
  - A conservative estimate using the main RF Cavity phase noise PSD, leading to an emittance growth by a factor of 86 per hour. Yes that is 8600 %/h !
  - This would be reduced to 93% with ADT
- Amplitude Noise
  - The main RF cavity amplitude noise is similar to the phase noise except at frequencies below 30 kHz (TX phase noise). Using this design we would have a factor of 32 emittance growth per hour.
  - The transverse damper can do nothing (with its 20 MHz BW). Amplitude noise will kick head and tail of the bunches in opposite direction so that the average will always be zero.

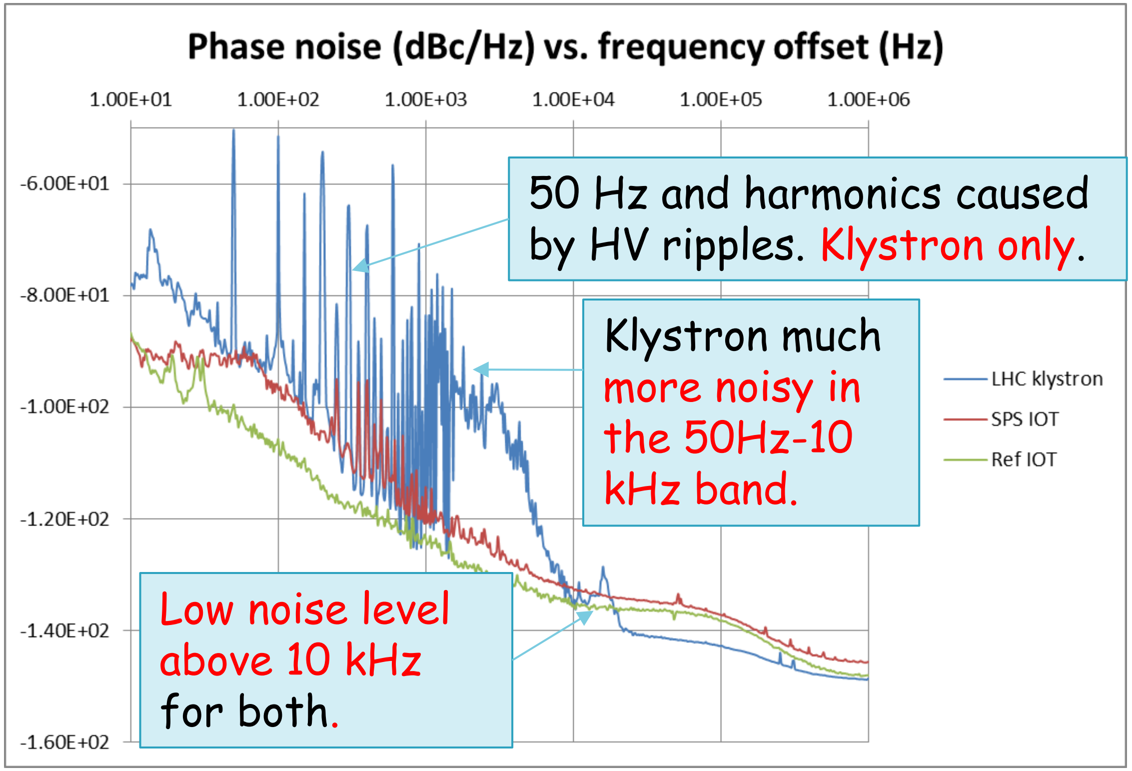


- ACS (with beam)
- Blue: Phase noise Power Spectral Density
- Red: Amplitude noise
- The red marks indicate the betatron bands.

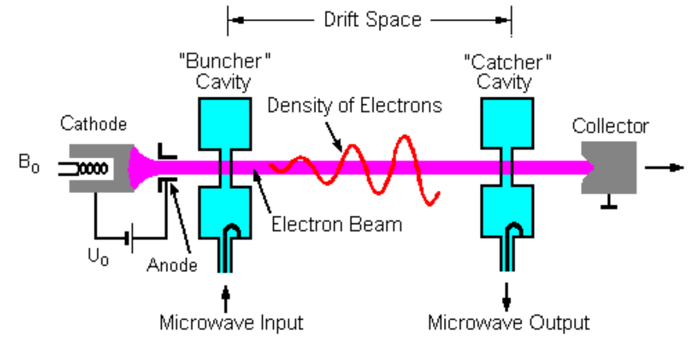
Choice of power source

- Objective: reduce noise PSD coming from the TX

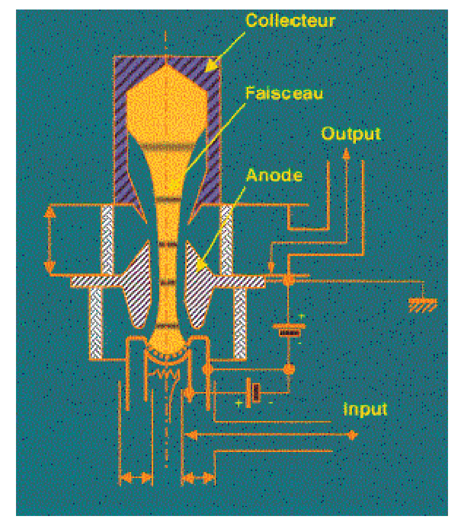
Klystron vs. IOT (phase noise)



- LHC 400 MHz 300 kW klystron vs. SPS 800 MHz 240 kW IOT SSB phase noise.

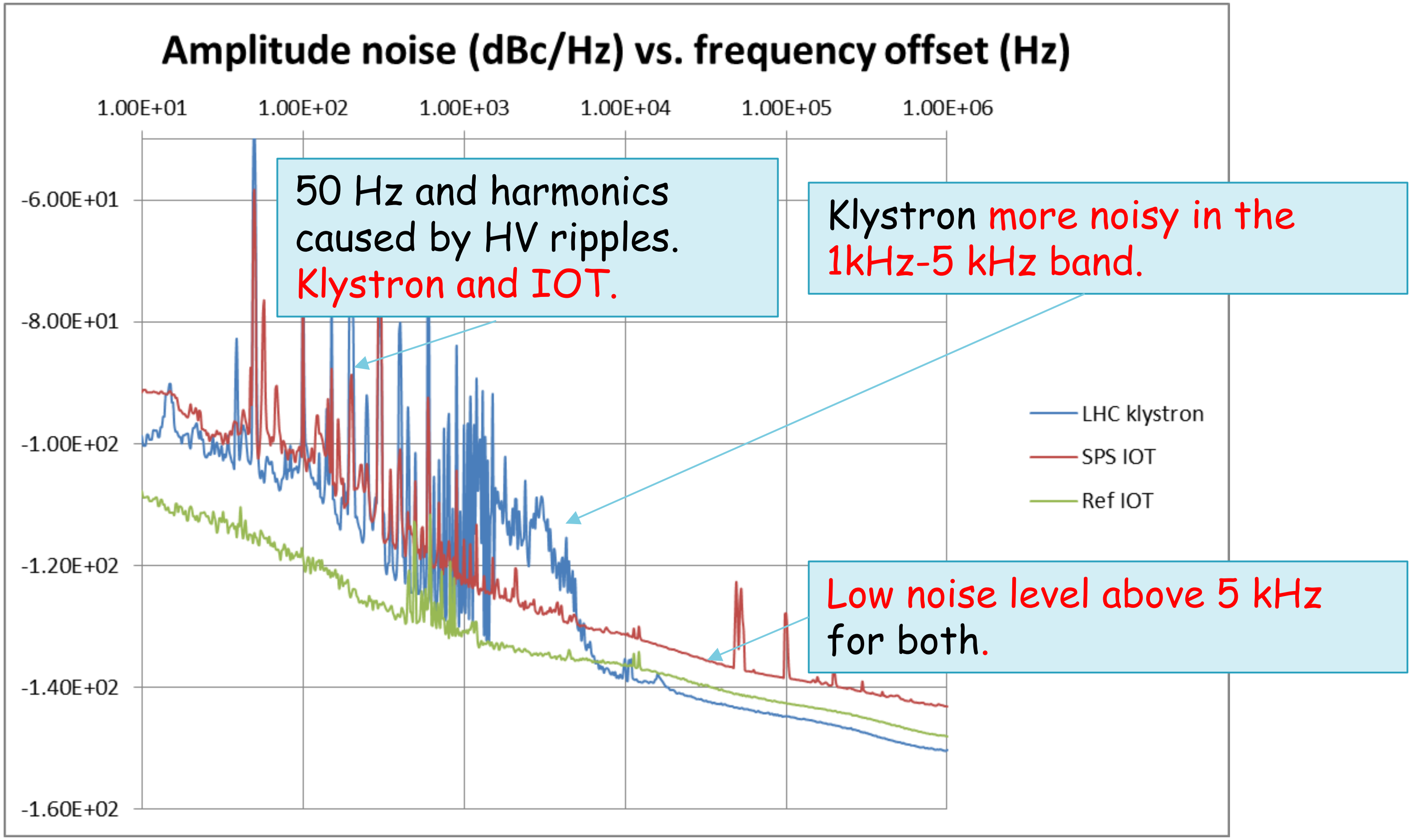


- In klystrons, the HV ripples change the electron velocity. Given the long drift space we get ~10 RF dg/1% HV ripple.



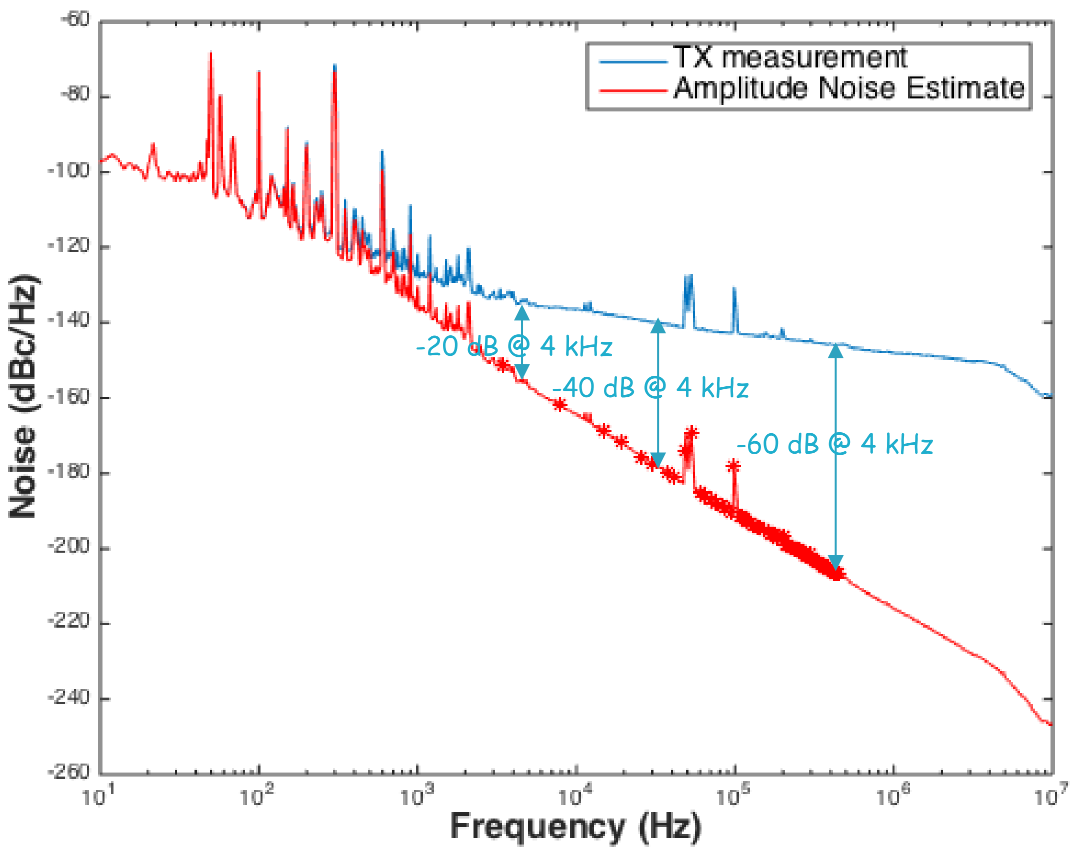
- In IOT, the distance between cathode and output cavity is small (few cms). HV ripples have small effect on RF phase.

Klystron vs. IOT (AM noise)



- LHC 400 MHz 300 kW klystron vs. SPS 800 MHz 240 kW IOT SSB amplitude noise.

AM noise seen by the beam



- The TX noise seen by the beam is very small with high Q_L cavities, as it is filtered by the cavity response.
- The first betatron line is at −150 dBc/Hz
- The resulting emittance growth would be 0.1 %/hour…
- …but NO regulation.

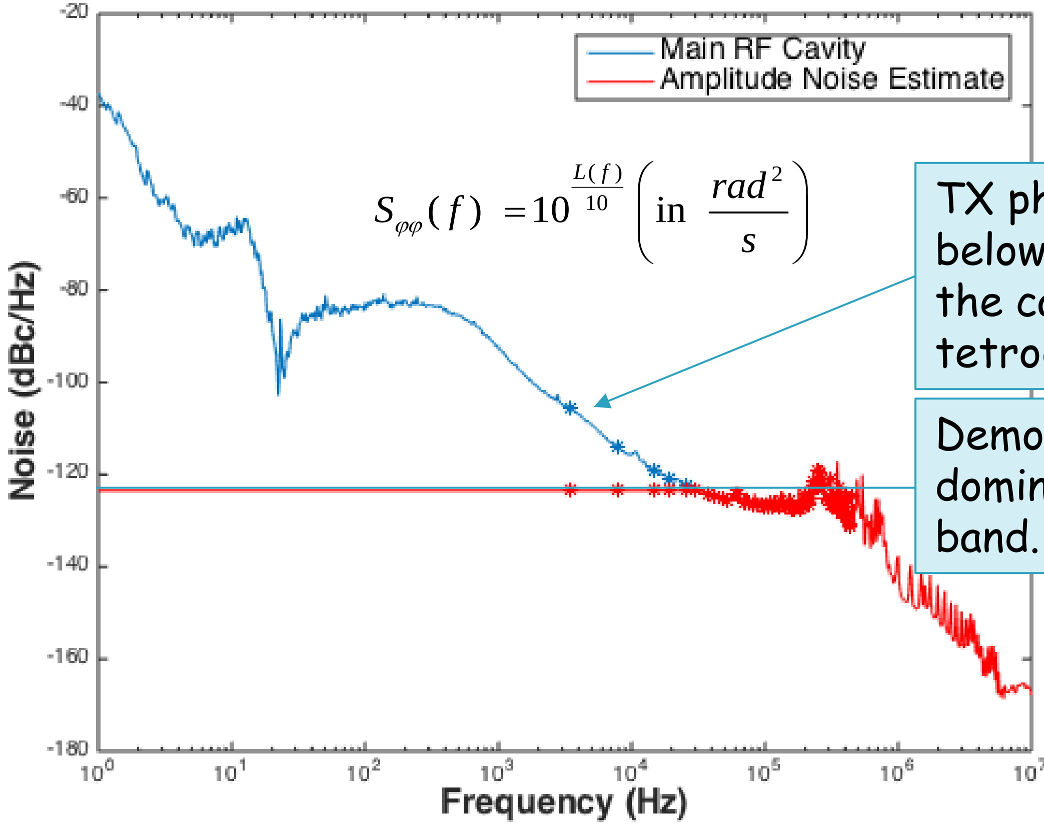
- *SPS 800 MHz 240 kW IOT SSB amplitude noise filtered by a cavity with Q_L=500000.*

LLRF optimization

- With fast regulation, the noise level is set by the demodulation used in the LLRF

RF system with LL regulation (LHC ACS)

$$L(f) \text{ in } \frac{dBc}{Hz}$$



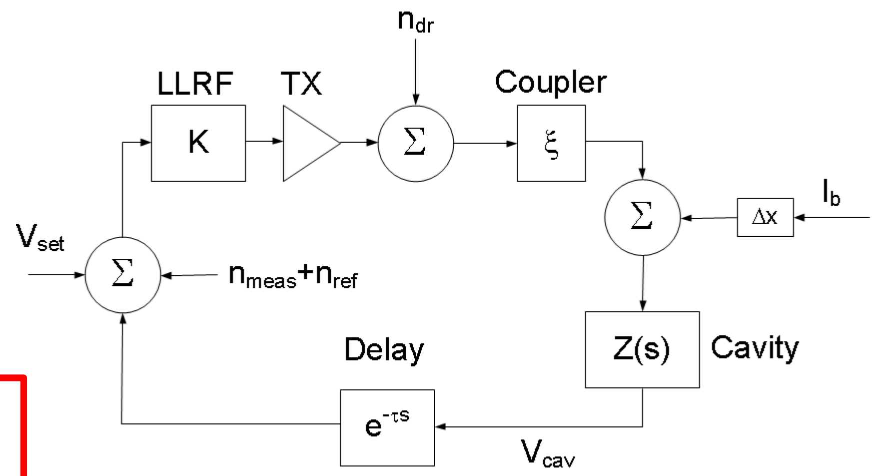
$$S_{\varphi\varphi}(f) = 10^{\frac{L(f)}{10}} \left(\text{in } \frac{rad^2}{s} \right)$$

TX phase noise dominant below 30 kHz. Will not be the case with IOT or tetrode

Demodulator noise dominant in the regulation band.

- The cavity response does not show-up
- Instead we have a “plateau” extending to the regulation BW and set by the measurement noise in the feedback chain
- In the LHC LLRF it is around -126 dBc/Hz with beam

Cavity RF Noise



- RF feedback noise sources:
  - The RF reference noise n_{ref}
  - The demodulator noise (measurement noise) n_{meas}
  - The TX (driver) noise n_{dr} . It includes also the LLRF noise not related to the demodulator
  - The Beam Loading $I_b\ \Delta x$
- We get

$$Z(s)=\frac{\frac{R}{Q}Q_L}{1+2Q_L\frac{s}{\omega_0}} \quad \text{with } s=j\,\Delta\omega$$

Main coupler

$$V_{cav}=\frac{K\ G\ e^{-\tau s}Z(s)}{1+K\ G\ e^{-\tau s}Z(s)}\left[V_{set}+n_{ref}+n_{meas}\right]+\frac{Z(s)}{1+K\ G\ e^{-\tau s}Z(s)}\left[\Delta x\ I_b+\sqrt{\frac{Z_0}{\frac{R}{Q}Q_L}}\,n_{dr}\right]$$

Closed Loop response CL(s)
- Equal to ~1 in the CL BW
- Increase of K increases the BW
- Within the BW, reference noise and measurement noise are reproduced in the cavity field

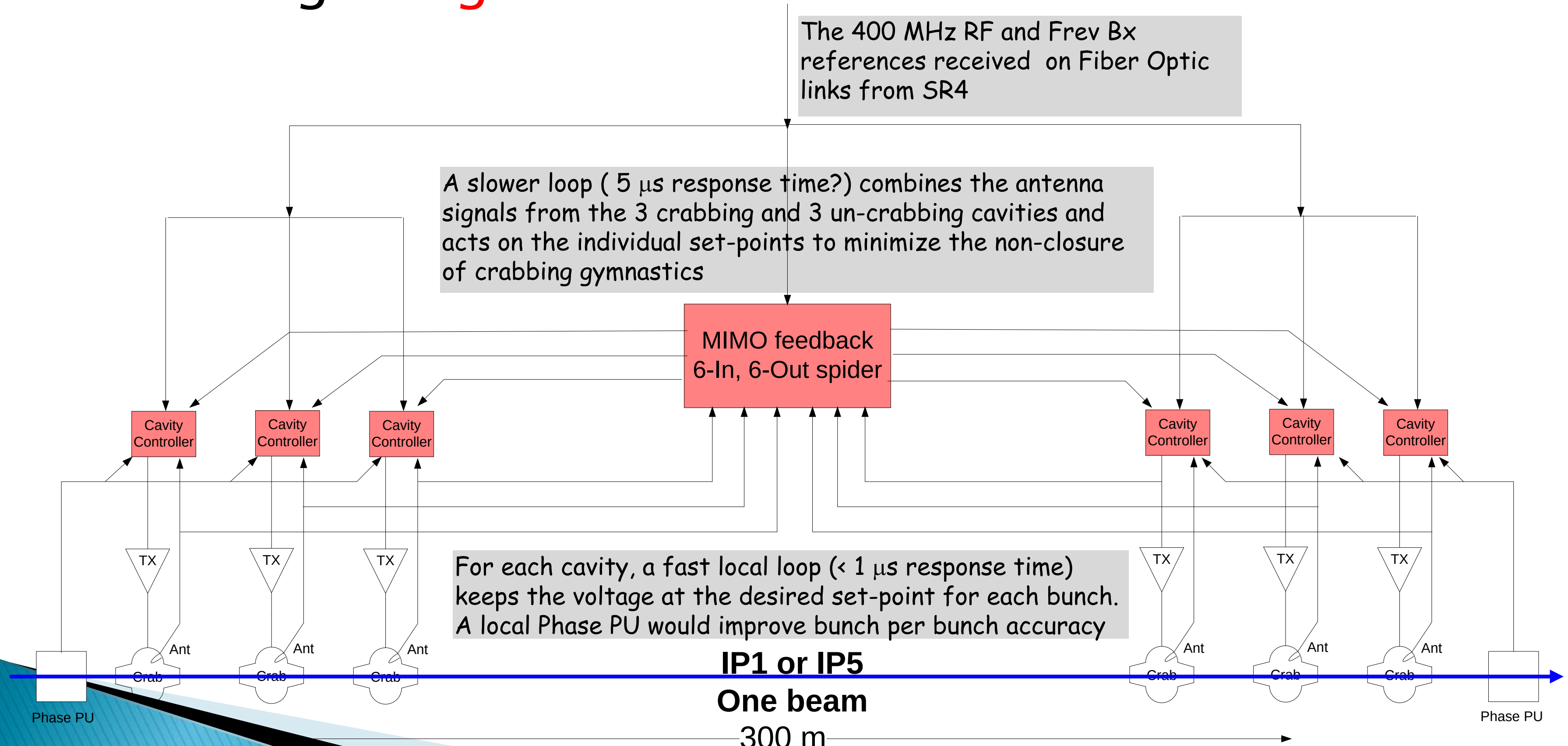
Beam Loading response = effective cavity impedance Zeff(s)
- Equal to ~1/KG in the CL BW
- Increase of K decreases Zeff within the CL BW
- Within the CL BW, TX noise and beam loading are reduced by the Open Loop gain KG

Do we need fast regulation?

- In the LHC main cavity, we wanted a broadband RF feedback to reduce transient beam loading and cavity impedance at the fundamental in order to increase the longitudinal stability threshold (coupled-bunch oscillations). Thanks to this, the cavity fundamental impedance will be no problem with HiLumi intensity (1.1 A DC). But this results in a wideband (300 kHz) regulation and … large noise bandwidth. Acceptable in the longitudinal plane (below IBS).
- For the CC, transient beam loading and cavity impedance are less of an issue as the beam will be fairly well centered, and the cavity will be on-tune.

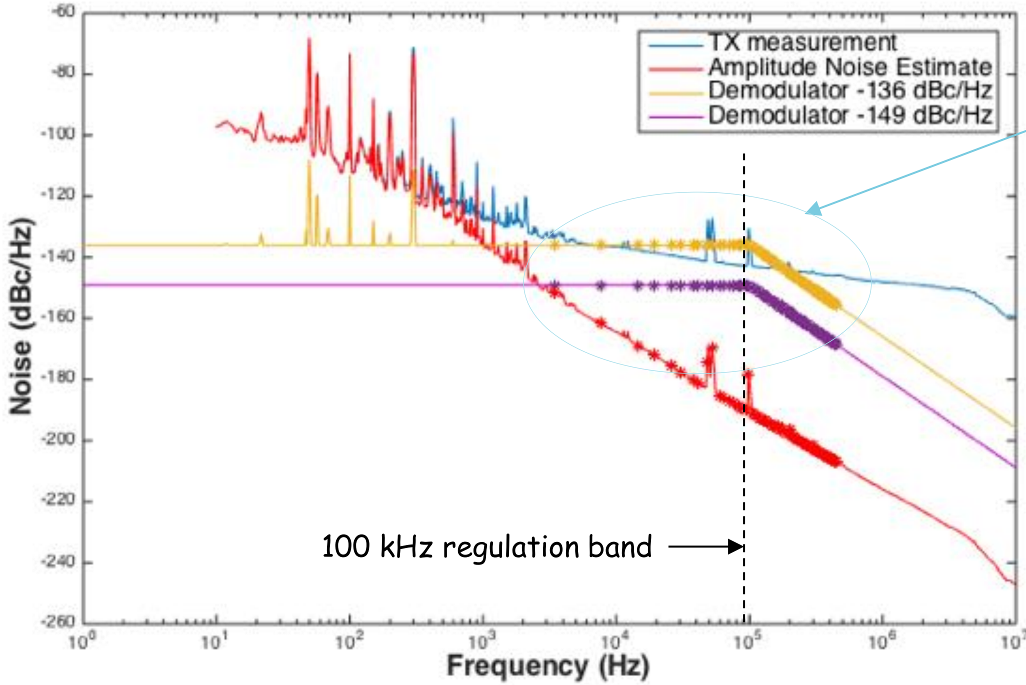
Do we need fast regulation? (cont'd)

- But we still want to react strongly and quickly for accurate field control and to track the fast changes in one cavity following a klystron trip or quench. So we set the target regulation BW to 100 kHz.



- Reproduced from 6th CC workshop, 2013

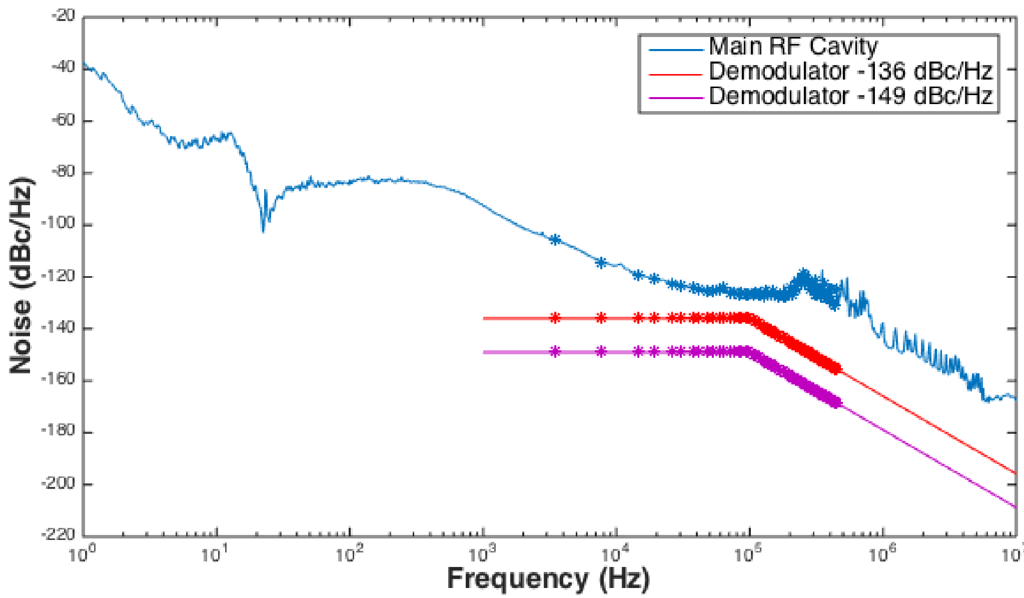
Target demodulator noise level



For all betatron band, the demodulator noise dominates the TX noise (filtered by cavity).

- Amplitude noise using an IOT type TX spectrum
  - The red curve (no regulation) gives 0.1 %/hour emittance growth
  - The yellow curve: regulation with 100 kHz BW and -136 dBc/Hz demodulator noise gives 80 %/hour
  - The violet curve (100 kHz BW, -149 dBc/Hz) gives **3.9 %/hour**. **Acceptable.**

Resulting Phase Noise Estimate



- Phase noise with factor 100 reduction by ADT
  - The blue curve (LHC main RF estimate with beam) gives 93%/hour emittance growth
  - The red curve: regulation with 100 kHz BW and −136 dBc/Hz demodulator noise gives 1.1%/hour
  - The violet curve (100 kHz BW, −149 dBc/Hz) gives **0.05 %/hour**.

Conclusions…

- …and future work

- Formulas have been presented for the effect of phase and amplitude noise, and the effect of transverse damper
- Although the phase noise has a marginally bigger effect, it is much attenuated by the transverse damper. Amplitude noise appears as the main problem
- Use of IOT or tetrode will help, compared to a scaling of the ACS case but does not make big difference
- Our target is to keep growth rate below 4 %/h with a 100 kHz regulation BW
- The dominant noise comes from the LLRF demodulator that must be improved: target=-149 dBc/Hz @ 3 kHz offset. 23 dB reduction of Noise Figure compared to ACS.
- Future work
  - Validation of the derivations using PyHeadTail simulations and later the SPS test bench
  - Design of a low-noise demodulator. Synergy with LCLS-II developments @ 1.3 GHz (-160 dBc/Hz @ 10 kHz offset, B. Chase FNAL).

Challenging but challenges were encountered before....

8. CONTRIBUTION OF RF NOISE TO THE FINITE BEAM LIFE-TIME IN THE SPS COLLIDER

This is a short historical survey of the improvements achieved by D. Boussard and the RF group on the life time of proton (and later anti-proton) bunches in the SPS when operated as a storage ring for $p\,\bar{p}$ collisions.

In the following, τ represents that contribution to the finite beam life-time which is due to RF noise only.

- With original RF system (June 1978): $\tau \approx 1$ minute.
- Insertion of a low pass filter (cut-off 10 Hz) in radial loop (December 1978): $\tau \approx 10$ minutes.
- Sampling of the beam-phase pick-up during the passage of the reference (proton) bunch (October 1979): $\tau \approx 10$ hours.

  This sampling is the change which produced the most dramatic improvement in τ.
  With an ideal phase loop having infinite gain and no phase noise, the particles in the reference bunch would see no phase noise.
- Replacement of the radial loop by a frequency programme (January 1980): $\tau \approx 40$ hours.
- New low-noise master oscillator (August 1981): $\tau = 100$ hours for the reference bunch, and almost the same for the other bunches.

Reproduced G. Dome, CAS 1985

Thank you for your attention…

- … questions?

References

[1] V. Lebedev et al., Emittance Growth Due to Noise and its Suppression with the Feedback System in Large Hadron Colliders, SSCL-Preprint-188, 1993

[2] G. Dome, Diffusion due to RF noise, CAS 1985. Also listed in A. Chao Handbook of Accelerator Physics and Engineering.

[3] S.V. Ivanov, Longitudinal Diffusion of a Proton Bunch under External Noise, IHEP 92-43, Protvina 1992

[4] P. Baudrenghien, T. Mastoridis, Longitudinal Emittance Blow-up in the Large Hadron Collider, NIM A, Vol 726, 2013

Bach-up slides

Comparison with Longitudinal Plane (Main RF)

- Why is effect so much smaller in longitudinal plane?

$$\frac{d\epsilon_z}{dt} = \frac{\pi f_s^2}{2}\sum_{n=-\infty}^{\infty}\left(S_{\Delta\phi}(f \pm f_s - nf_{rev}) + xS_{\Delta A}(f \pm f_s - nf_{rev})\right)$$

  - Leading coefficient 4 order of magnitude higher for longitudinal plane, BUT, the fractional rate is 5 orders of magnitude smaller (before damper action)

$$\frac{1}{\epsilon_z}\frac{d\epsilon_z}{dt}$$

- Why is amplitude noise effect insignificant in longitudinal plane?
  - 98% of the phase noise power in the longitudinal plane comes from the first sideband at f_s, due to the RF reference. It is reduced by the main phase loop. This noise does not affect the voltage amplitude. As a result, the phase noise power is at least a factor of 50 higher than amplitude noise power.