Effect of crab cavity/damper noise on emittance evolution in the presence of beam-beam: summary of strong-strong simulations done at CERN/KEK/LBNL

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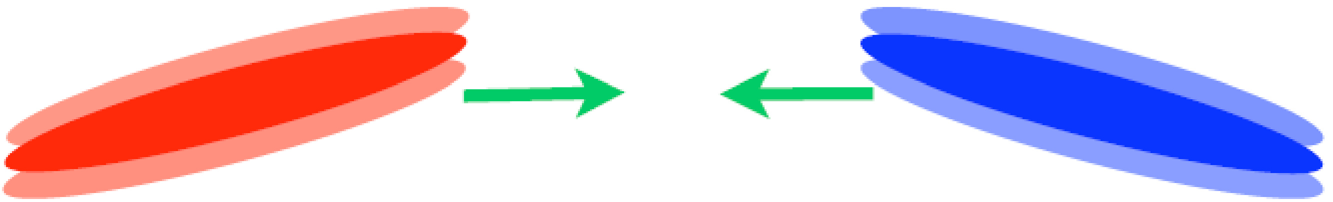
Thanks to K. Akai, Y. Morita, R. Calaga, R. Tomas

Introduction

- Beam-beam interaction in the presence of noise.
- Collision offset
- Tune
- Beam size



- Noise characteristics
  - Turn-by-turn white noise or time correlated noise in collision offset. Typical model
  - White noise in betatron motion with damping. Feedback noise
  - Sinusoidal noise in betatron motion with damping. Crab cavity noise

Physics of beam-beam and noise

- Noise of Collision offset

$$\Delta x_n = \left(1 - \frac{1}{\tau}\right)\Delta x_{n-1} + \delta x \hat{r}$$

δx:

Δx:

$$\Delta x^2 = \langle \Delta x^2_{n\to\infty} \rangle = \frac{\tau \delta x^2}{2}$$

$$\langle \Delta x_\ell \Delta x_{\ell+n} \rangle = K(n) = \Delta x^2 e^{-|n|/\tau}$$

$$\langle \Delta x_\ell \Delta x_{\ell+n} \rangle = \Delta x^2 \delta_{n0}$$ turn by turn noise

$$\Delta x_n = (1 - 1/\tau)(\Delta x_{n-1} \cos 2\pi\nu_x + \Delta p_{x,n-1} \sin 2\pi\nu_x) + \delta x \hat{r}$$

$$\Delta p_{x,n} = (1 - 1/\tau)(-\Delta x_{n-1} \sin 2\pi\nu_x + \Delta p_{x,n-1} \cos 2\pi\nu_x) + \delta x \hat{r}$$

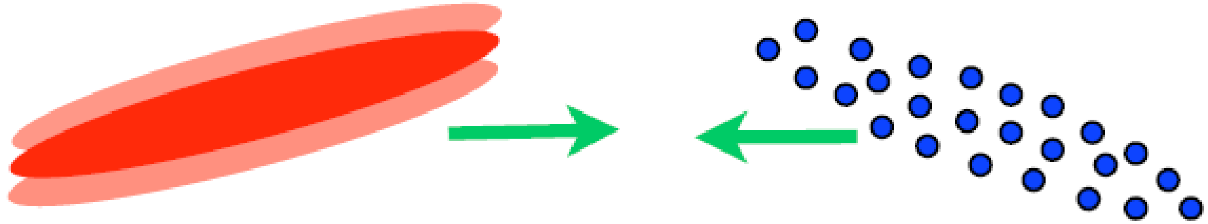
- Noise of collision offset

$$\Delta x^2 = \langle \Delta x^2_{n\to\infty} \rangle = \frac{\tau \delta x^2}{2}$$

$$\langle \Delta x_\ell \Delta x_{\ell+n} \rangle = K(n) = \frac{1}{2}\Delta x^2 \cos 2\pi n\nu e^{-|n|/\tau}$$

Weak-strong picture

G. Stupakov, SSC-560 (1991).
T. Sen and J. Ellison, PRL 77, 1051 (1996)



$$U = \delta_P(s)\frac{N r_p}{\gamma}\sum_{k=0}^{\infty} U_k(a)\cos 2k\psi$$

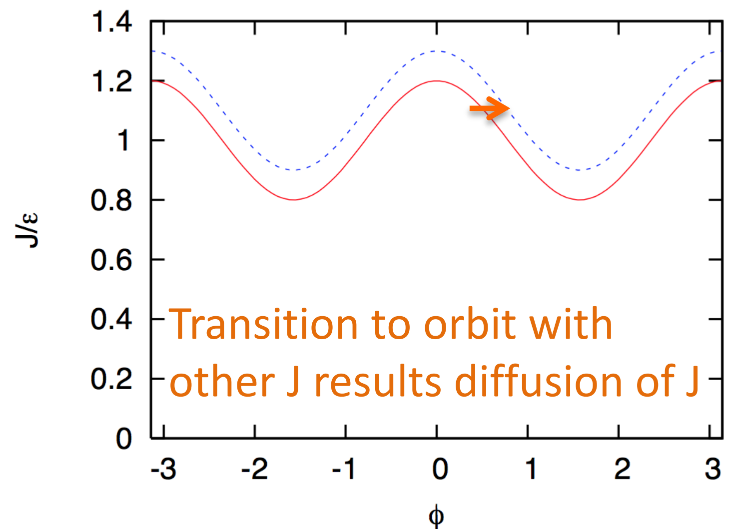
Requirement for the theory: near integrable system, far from resonances.

$$U_k = \int_0^a \left[\delta_{0k} - (2-\delta_{0k}(-1)^k e^{-w} I_k(w)\right]\frac{dw}{w},$$

$$a = \beta^* J/2\sigma_r^2, \quad \Theta = -\ln(1-1/\tau_c),$$

$$\Delta J = -\frac{\partial U}{\partial \psi} = \frac{N r_p}{\gamma}\sum 2k U_k \sin 2k\psi$$

ΔJ /turn



Weak-strong picture

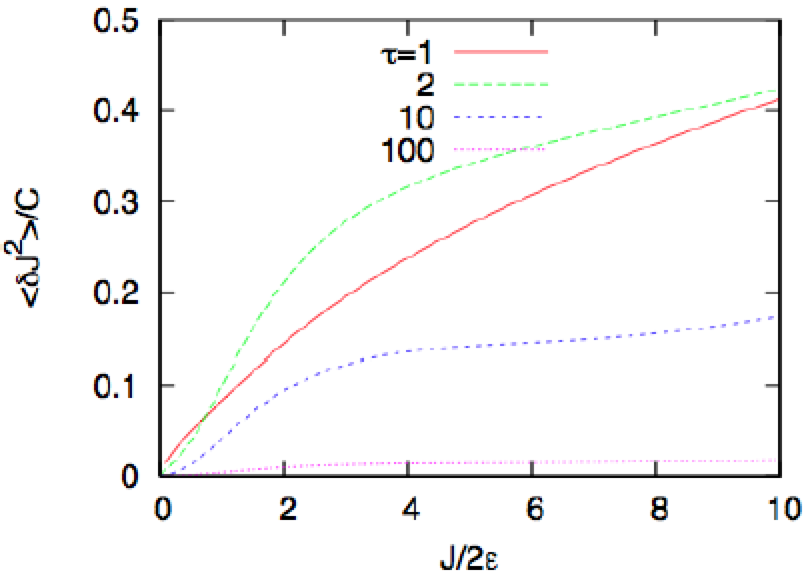
$$\frac{\langle \Delta J^2(N)\rangle}{N} = \left(\frac{N_p r_p}{\gamma}\Delta x\right)^2 \frac{1}{8}\sum_{k=0}^{\infty} \frac{(2k+1)^2 G_k(J)^2 \sinh\Theta}{\cosh\Theta - \cos(2\pi(2k+1)\nu_x)},$$

$$\Theta = -\ln(1-1/\tau)$$

$$= \left(\frac{N_p r_p}{\gamma}\Delta x\right)^2 \frac{1}{8}\sum_{k=0}^{\infty}(2k+1)^2 G_k(J)^2$$

for $K(n)=\delta_{0n}$

- τ: correlation time of the noise



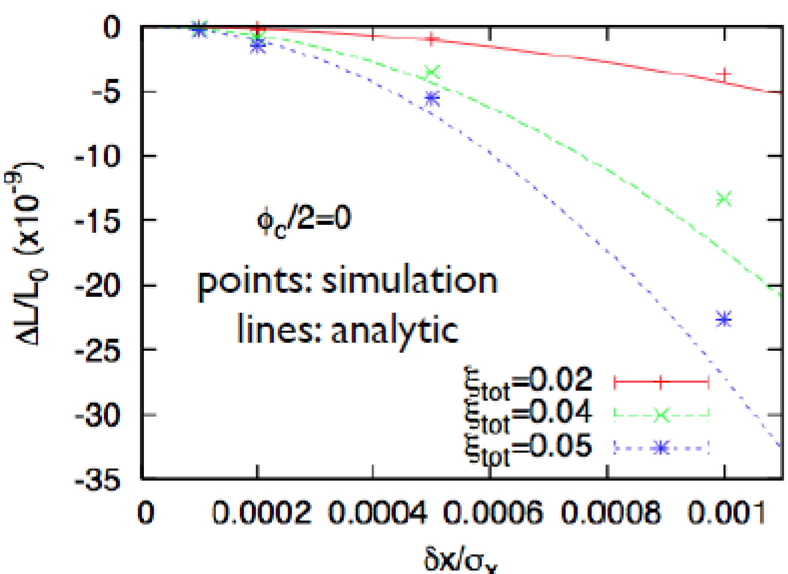
$$C = \frac{1}{8}\left(\frac{N_p r_p}{\gamma}\frac{\delta x}{\sigma_r}\right)^2$$

$$\frac{\langle \Delta J^2(N)\rangle}{N} \propto 1/\tau \qquad \tau \gg 1 \;\; \sinh\Theta \sim 1/\tau, \;\; \cosh\Theta = 1$$

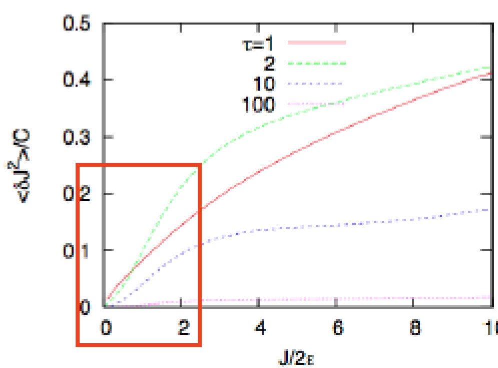
Weak-strong picture

For $\langle \Delta J^2 \rangle = \frac{1}{8}\left(\frac{N_p r_p}{\gamma}\frac{\Delta x}{\sigma}\right)^2 A\frac{J}{2\varepsilon},$

$$\frac{\delta\varepsilon}{\varepsilon} = \frac{1}{8}\left(\frac{N_p r_p}{\gamma\varepsilon}\frac{\Delta x}{\sigma}\right)^2 \frac{A}{4} = \left(\pi\xi\frac{\Delta x}{\sigma}\right)^2 \frac{A}{2} = 21.7 \times \left(\xi\frac{\Delta x}{\sigma}\right)^2$$



From BBWS



$$C = \frac{1}{8}\left(\frac{N_p r_p}{\gamma}\frac{\delta x}{\sigma_r}\right)^2$$

$$A = 4.4 \text{ for } \tau = 1$$

Noise in betatron motion

- Frequency range of feed back is fast.
- White noise in betatron motion with damping

$$\Delta x_n = (1 - 1/\tau)(\Delta x_{n-1}\cos 2\pi\nu_x + \Delta p_{x,n-1}\sin 2\pi\nu_x) + \delta x \hat{r}$$
$$\Delta p_{x,n} = (1 - 1/\tau)(-\Delta x_{n-1}\sin 2\pi\nu_x + \Delta p_{x,n-1}\cos 2\pi\nu_x) + \delta x \hat{r}$$

$$\Delta x^2 = \langle \Delta x^2_{n\to\infty} \rangle = \frac{\tau \delta x^2}{2}$$
$$\langle \Delta x_\ell \Delta x_{\ell+n} \rangle = K(n) = \frac{1}{2}\Delta x^2 \cos 2\pi n\nu e^{-|n|/\tau}$$

- Equivalent to

White noise spectrum

$$\ddot{x} + 2\lambda\dot{x} + \omega_\beta^2 x = \int f(\omega) e^{-i\omega t} d\omega \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

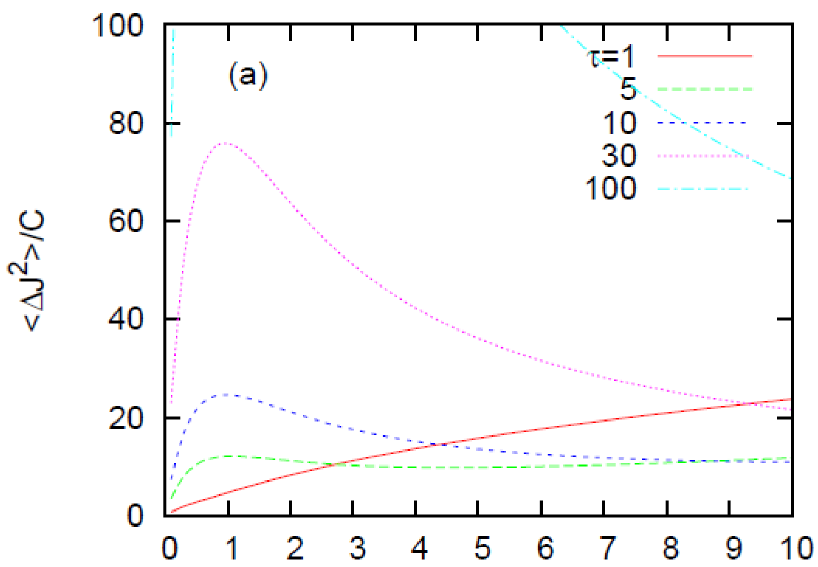
$\lambda = 1/\tau$

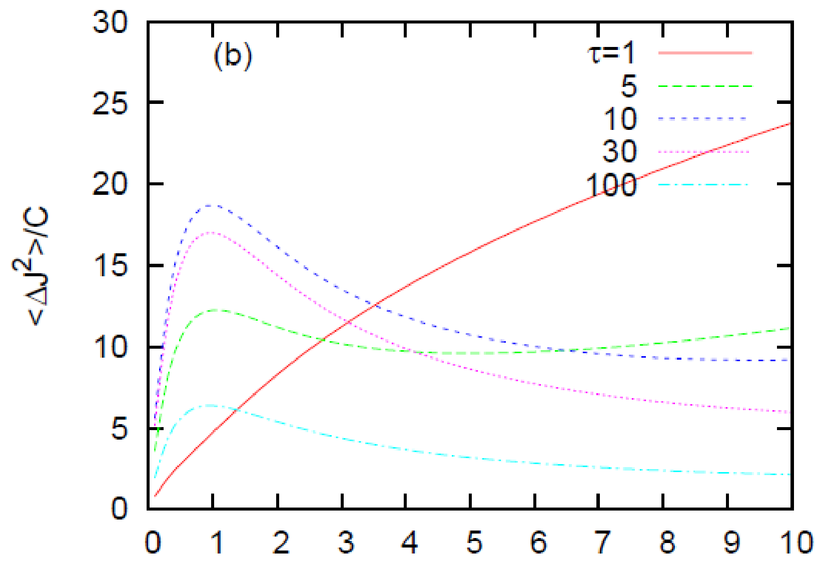
Diffusion rate for betatron noise

$$\langle \Delta J^2 \rangle \approx \frac{N_p^2 r_p^2 \Delta x^2}{16\gamma^2 \sigma_r^2} \sum_{k=0}^{\infty} (2k+1)^2 G_k(a)^2 \sinh 1/\tau$$

$$\left[\frac{1}{\cosh 1/\tau - \cos(2k\mu - \delta\mu)} + \qquad (31) \right.$$

$$\left. \frac{1}{\cosh 1/\tau - \cos(2(k+1)\mu + \delta\mu)} \right] ,$$





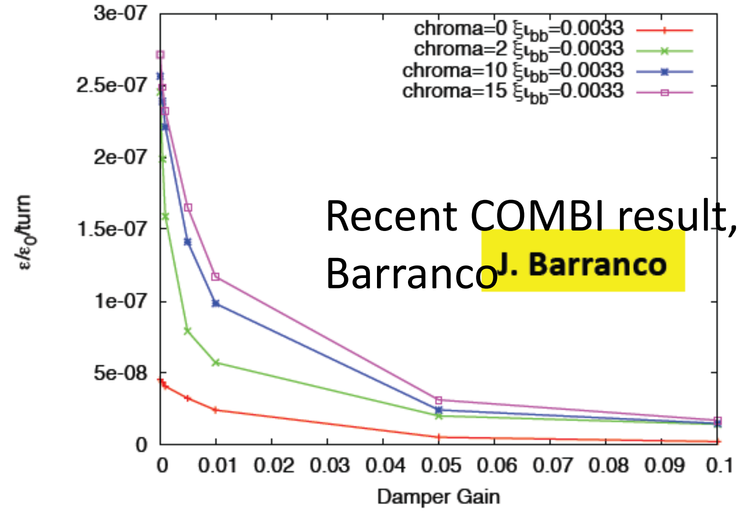
Beam-beam mode

- In weak-strong model, betatron excitation for $\delta\mu$=0 , resonant excitation
- Frequency of beam motion is ambiguous due to beam-beam tune shift. Decoherence is taken into account for $\delta\mu=2\pi\xi$.
- Diffusion rate $\Delta x<3\sigma$ is factor 2-3 enhanced for betatron nonlinear for dx.
- We expect the simple formula is useful even betatron noise for Δx (not dx).
- The formula is confirmed by strong-strong simulations.

Strong-strong simulation

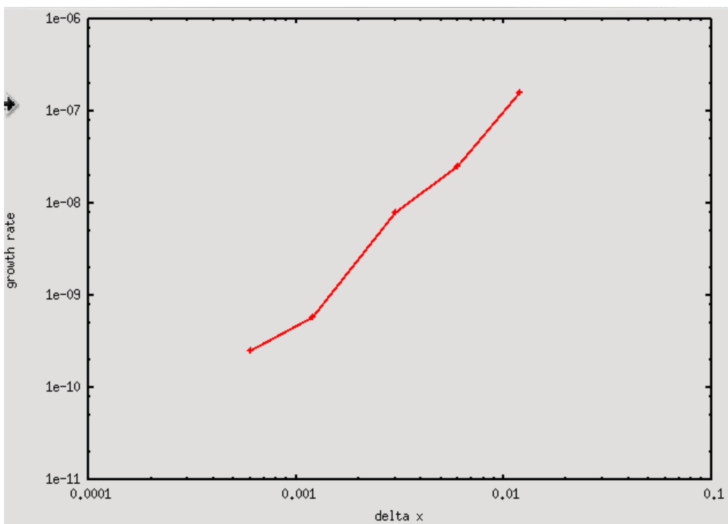
- Beambeam3D (J.Qiang) and BBSS(K.Ohmi)
- Both are Particle in Cell based simulations.
- Solver is Green function method.
- 3D symplectic using z-interpolated potential.

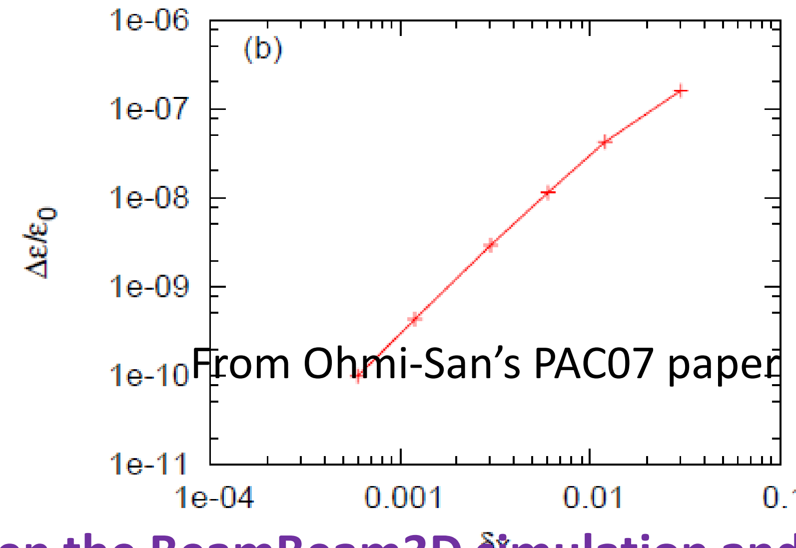
Emittance Growth vs. Normalized Noise Amplitude



Recent COMBI result, J. Barranco **J. Barranco**

From BeamBeam3D

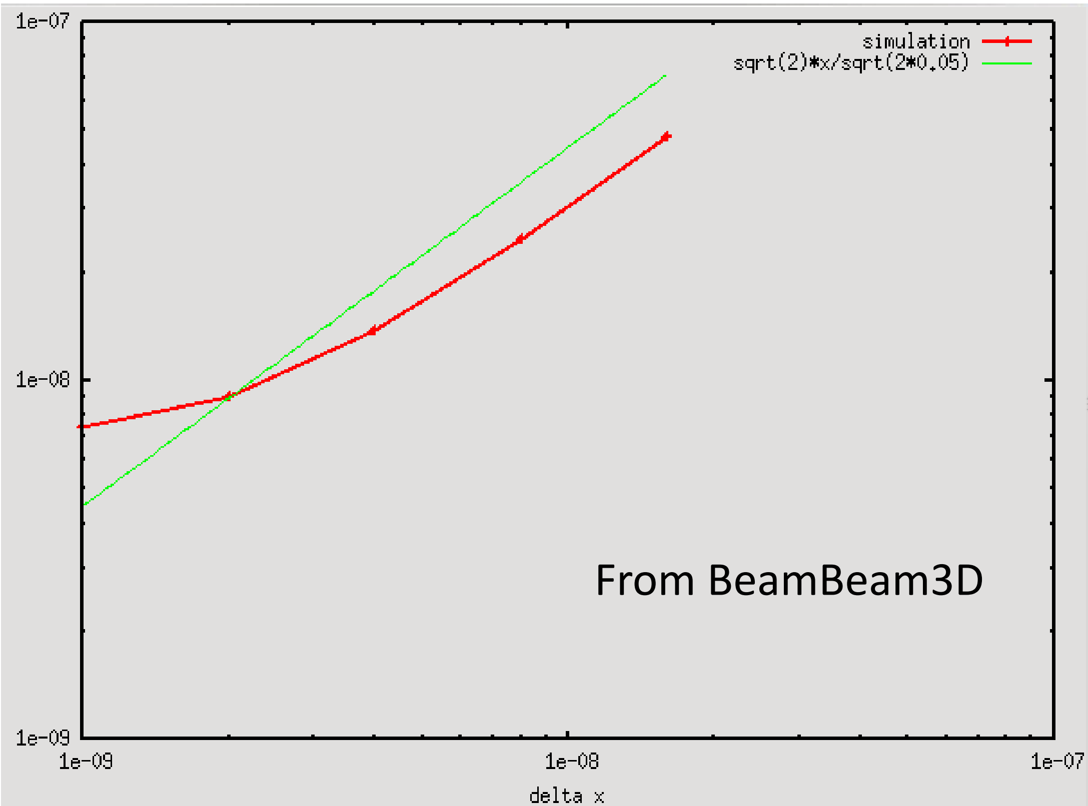




From Ohmi-San's PAC07 paper

Good agreement is obtained between the BeamBeam3D simulation and Ohmi-San's PAC07 paper for the LHC parameters with 1 IP and 1 error.

Beam RMS <x> fluctuation vs. Normalized Noise Amplitude



δx (m)

δx (nm)	Δx (m)
2	9.0e-9
4	1.4e-8
8	2.5e-8
16	4.8e-8

$$\Delta x = \sqrt{2}\,\frac{\delta x}{\sqrt{2G}}$$

G = 0.05

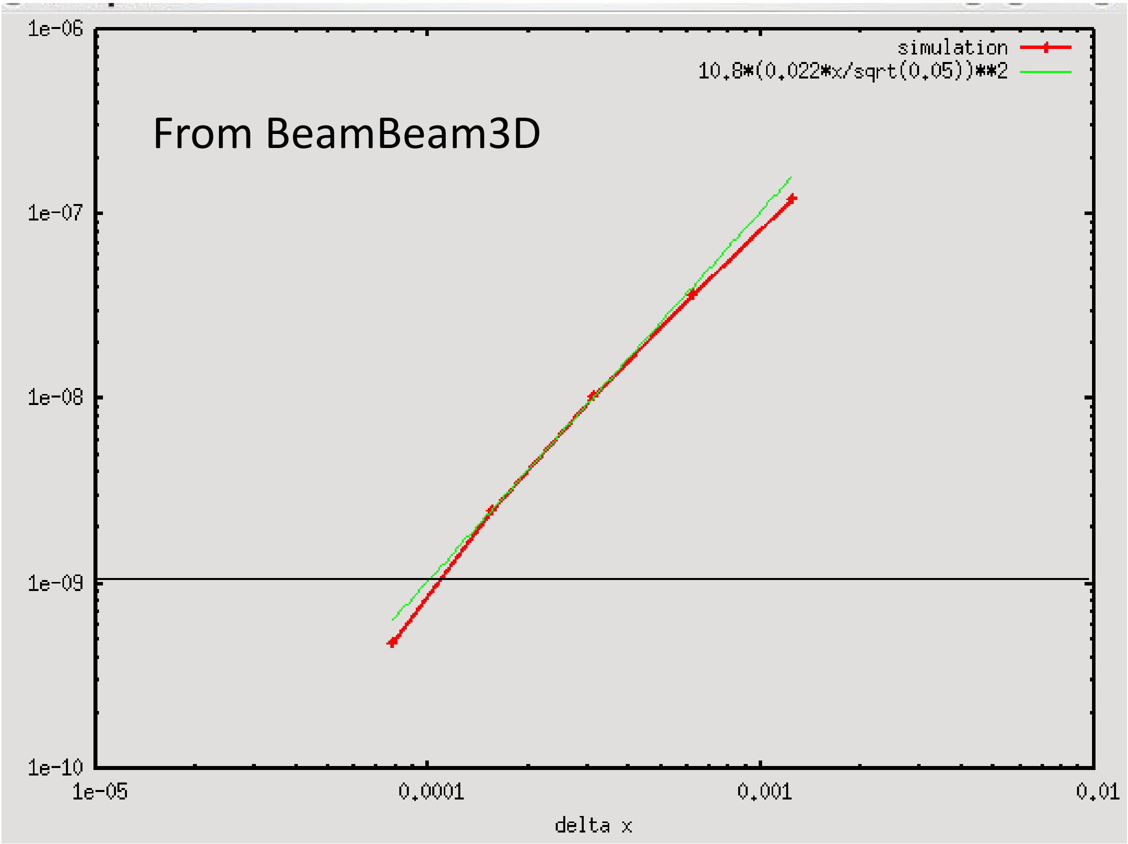
two CC errors in each direction

Numerical rms <x> fluctuation is in reasonable agreement with the model

K. Ohmi, "Beam-beam effect s under the influence of external noise ," in BeamBeam 2013 workshop.

Luminoisty Decrease from CC Noise with Beta* Leveling

-dL/L/turn vs. δx/σ



δx/σ

$$\xi = 2 \times 0.011$$

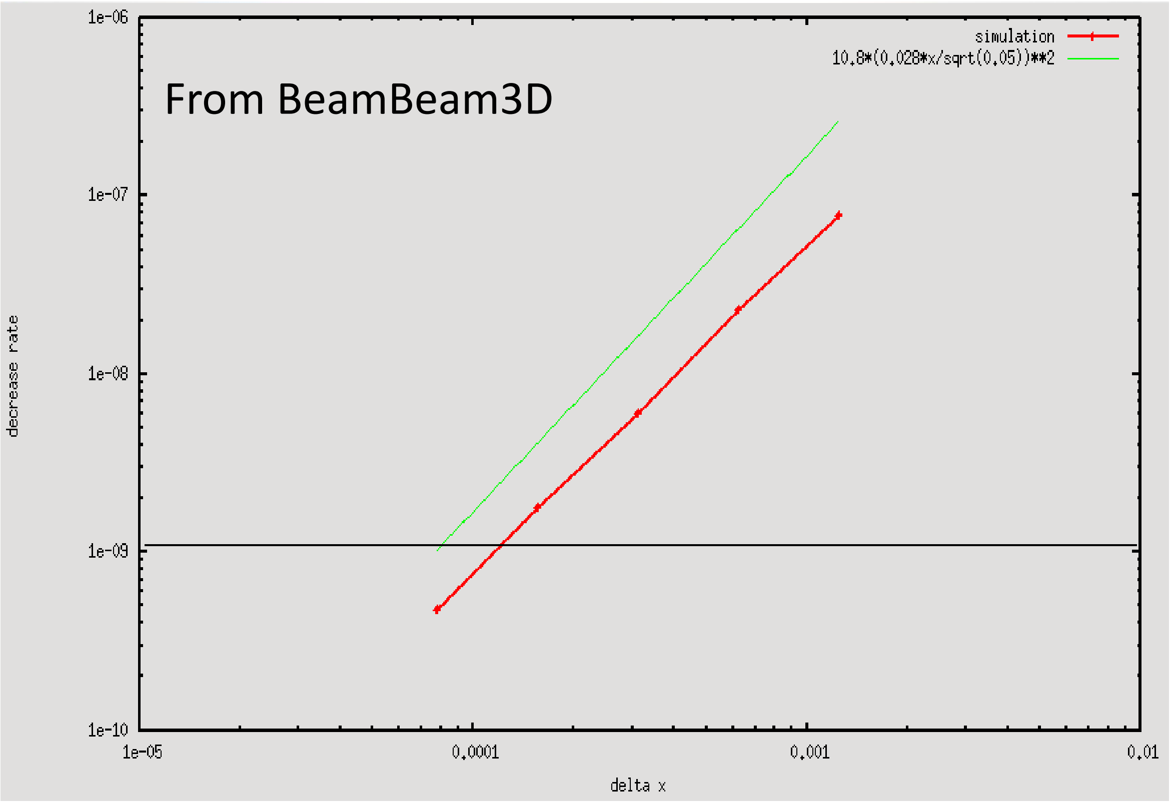
$$\frac{\Delta L}{L} = 10.8\left(\xi \frac{\Delta x}{\sigma}\right)^2$$

Luminosity decrease rate is in good agreement with the model

Luminoisty Decrease from CC Noise with Beta* Leveling

$\xi = 2 \times 0.014$

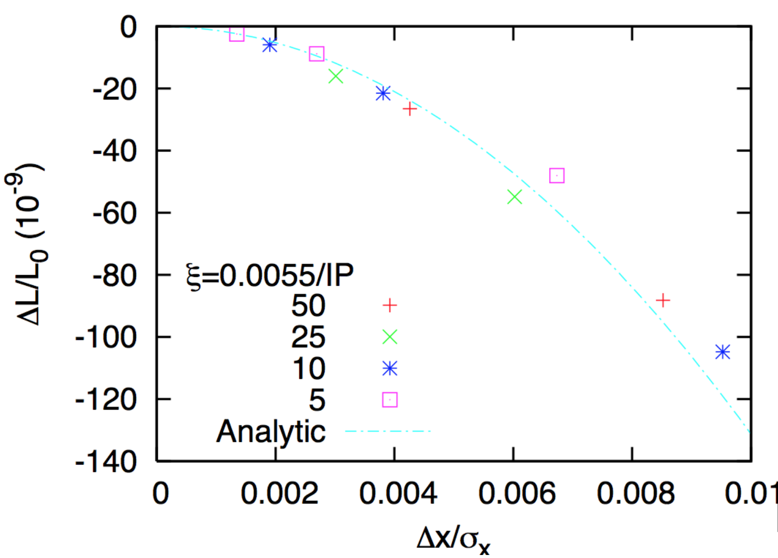
-dL/L/turn vs. δx/σ



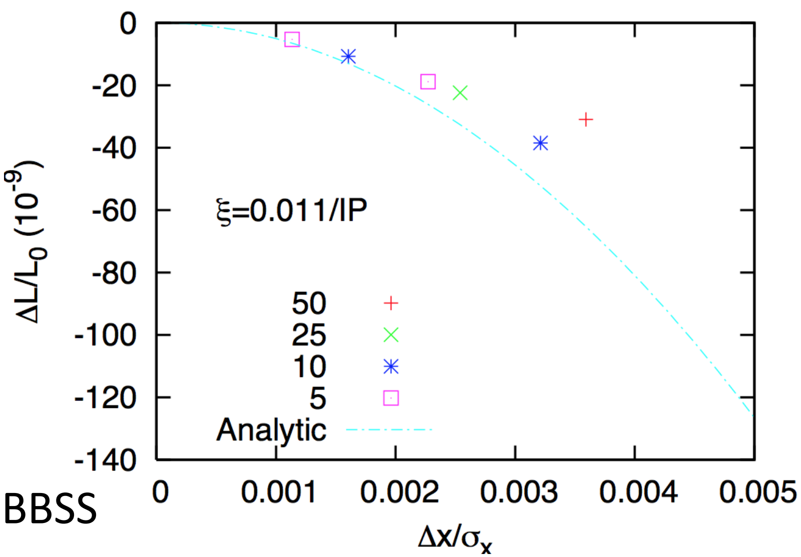
δx/σ

Luminosity decrease rate is in reasonable agreement with the model

- Strong-strong simulation showed luminosity degradation scale to Δx not δx.



From BBSS



Crab cavity noise

- Betatrpn oscillation is induce by crab cavity phase error randomly, or a frequency range.
- Feedback suppresses the betatron oscillation.

$$\delta x_{crab} = \frac{c \tan \theta_{crab}}{\omega_{crab}} \delta \varphi_{crab}$$

θ_{crab}: Crab angle at IP

$$\Delta x = \sqrt{\frac{N_{crab} \delta x_{crab}^2 + \delta x_{FB}^2}{G}}$$

- Δx tolerance for 2IP

$$\frac{\Delta L}{L_0} = 10.8 \left(\xi \frac{\Delta x}{\sigma} \right)^2$$

- Phase error tolerance

$$\delta \varphi_{crab} = \sqrt{\frac{G}{N_{crab}}} \frac{0.3\, \omega_{crab} \sigma}{\xi c \tan \theta_{crab}} \sqrt{\frac{\Delta L}{L_0}}$$

HL-LHC with β* leveling

25 ns

Physical parameters	
ε	0.335 nm
pick-up gain	0.05
pick-up noise	720 nm hor. 500 nm ver.
β^* σ θ	49 cm 12.8 um 0.59 mrad
ξ N	0.022 2.2×10^{11}

50 ns spacing

Physical parameters	
ε	0.40 nm
pick-up gain	0.05
pick-up noise	720 nm hor. 500 nm ver.
β^* σ θ	102 cm 20.25 um 0.59 mrad
ξ N	0.028 3.5×10^{11}

Phase noise tolerance

$$\delta\varphi_{crab} = \sqrt{\frac{G}{N_{crab}}} \frac{0.3\,\omega_{crab}\sigma}{\xi c \tan\theta_{crab}} \sqrt{\frac{\Delta L}{L_0}}$$

- $\Delta L/L = 10^{-9}$, $N_{crab}=4$, $f_{crab}=400$MHz, G=0.05, $\theta_{crab}=590$mrad/2
- $\xi=0.011$x2, σ= 13μm $\delta\varphi_{crab} = 1.8\text{x}10^{-5}$ rad
- $\xi=0.014$x2, σ= 20μm $\delta\varphi_{crab} = 2.2\text{x}10^{-5}$ rad

Luminosity Decrease Rate vs. Phase Noise Amplitude
(Linear Model vs. Non-Linear Model)

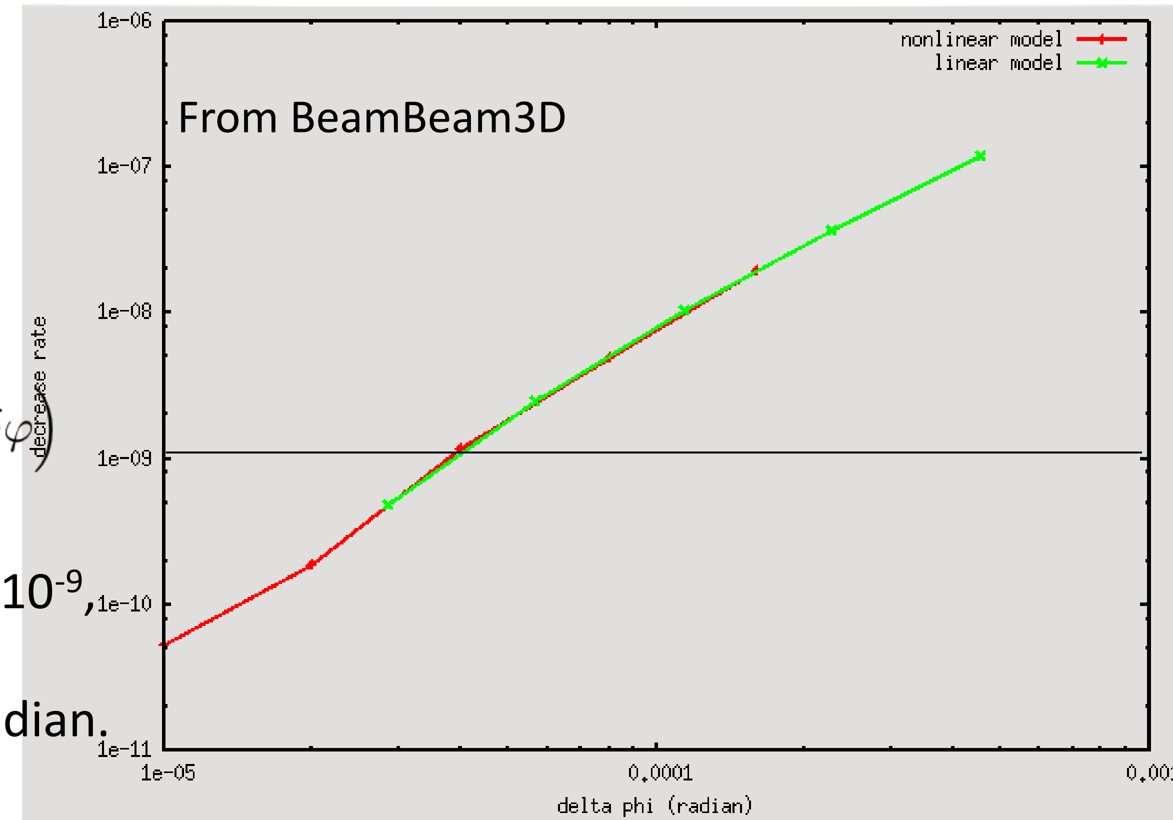
Linear model:

$$\delta X = -\frac{c}{\omega_{cc}} \tan\left(\frac{\theta}{2}\right) \delta\varphi$$

Non-Linear model:

$$x = -\frac{c}{\omega_{cc}} \tan\left(\frac{\theta}{2}\right) \sin\left(\frac{\omega_{cc} z}{c} + \delta\varphi\right)$$

For luminosity decrease rate 10^{-9},
i.e. 24 hours lifetime,
this corresponds to ~4×10^{-5} radian.



$\delta\phi$

Both models agree with each other very well!

Beam-Beam Simulation of HL-LHC with 10% Crab Cavity leveling and Crab Cavity White Noise for 25 ns scenario

Physical parameters	
ε	0.335 nm
pick-up gain	0.05
pick-up noise	720 nm hor. 500 nm ver.
β^* σ θ	15 cm 7.1 um 0.59 mrad
ξ N	0.022 2.2×10^{11}

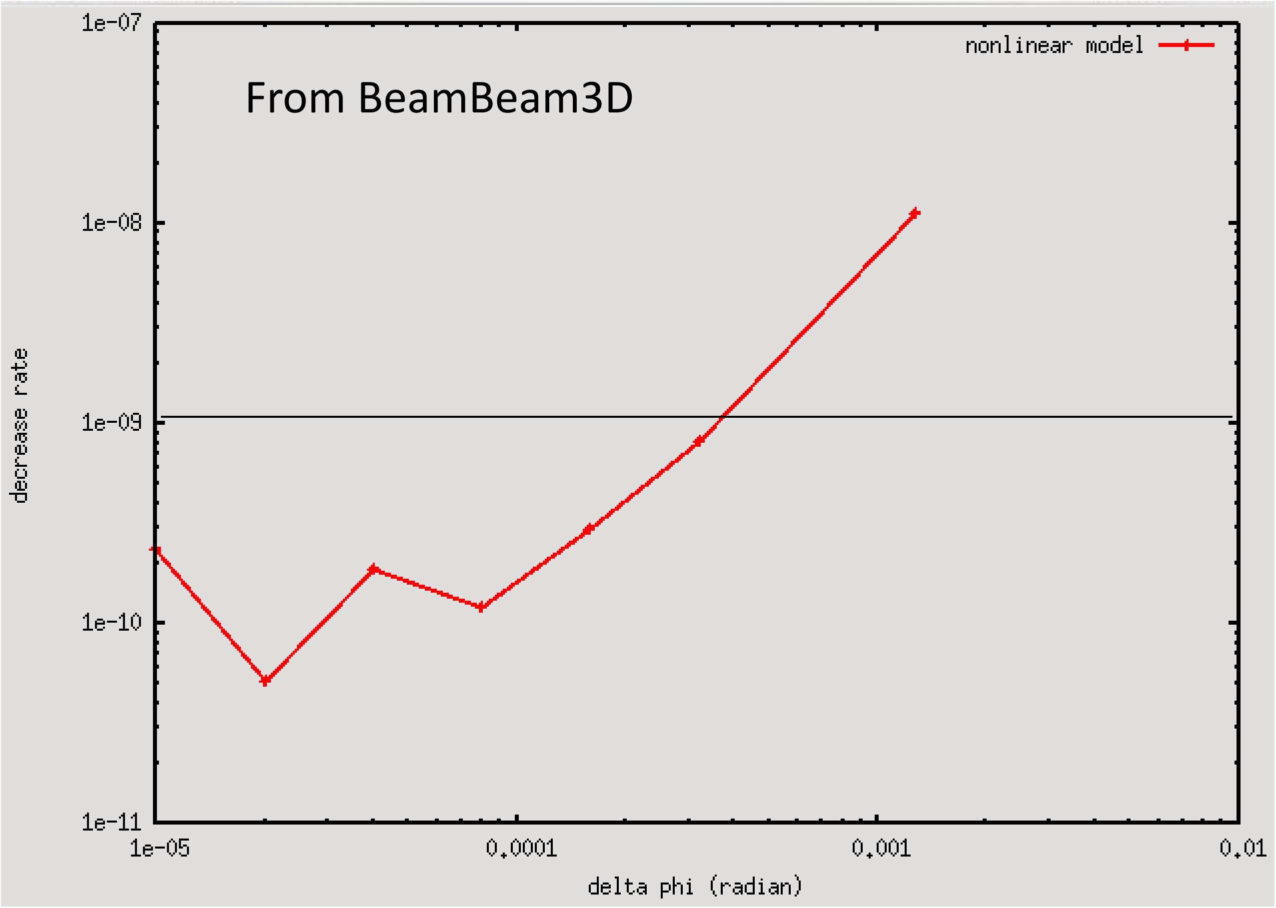
Phase noise tolerance

$$\delta\varphi_{crab} = \sqrt{\frac{G}{N_{crab}} \frac{0.3\,\omega_{crab}\sigma}{\xi c \tan\theta_{crab}}} \sqrt{\frac{\Delta L}{L_0}}$$

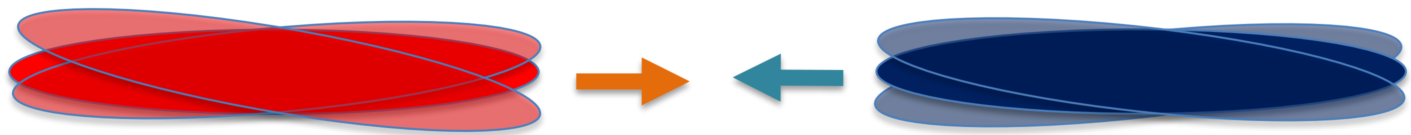
- $\Delta L/L=10^{-9}$, $N_{crab}=4$, $f_{crab}=400MHz$, $G=0.05$, $\theta_{crab}=59mrad/2$
- $\xi=0.011x2$, $\sigma= 7\mu m$ $\delta\varphi_{crab} = 1x10^{-4}$ rad

Luminosity Decreasing Rate vs. Phase Noise Amplitude with 10% Crab Cavity Leveling (nonlinear model)

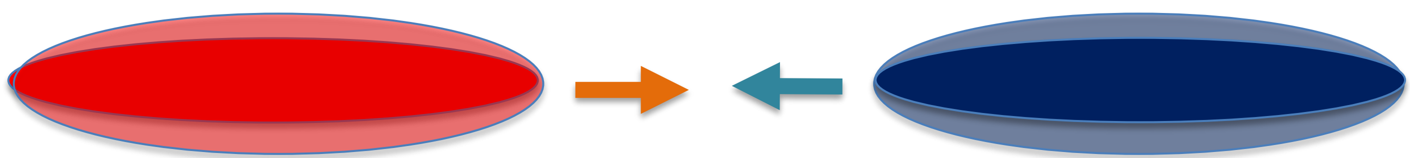
-dL/L/turn vs. $\delta x/\sigma$



Noise in crossing angle



- Equivalent to beam size fluctuation, when disruption parameter is small. Only projected size is important.



$$\Delta\sigma = \sigma_z \theta_{crab} \frac{\Delta V}{V} \qquad \frac{\Delta\sigma}{\sigma} = \frac{\sigma_z \theta_{crab}}{\sigma} \frac{\Delta V}{V}$$

Diffusion due to crab voltage

$$U(a,\psi;\sigma+\Delta\sigma) = U(a,\psi;\sigma) + \frac{dU(a,\psi;\sigma)}{d\sigma}\Delta\sigma$$

$$\frac{dU(a,\psi;\sigma)}{d\sigma} = \frac{\partial U(a,\psi)}{\partial a}\frac{da}{d\sigma} + \frac{\partial U(a,\psi)}{\partial \psi}\frac{d\psi}{d\sigma}$$

$$U(a,\psi) = \sum_{k=-\infty}^{\infty} U_k(a)\cos 2k\psi$$

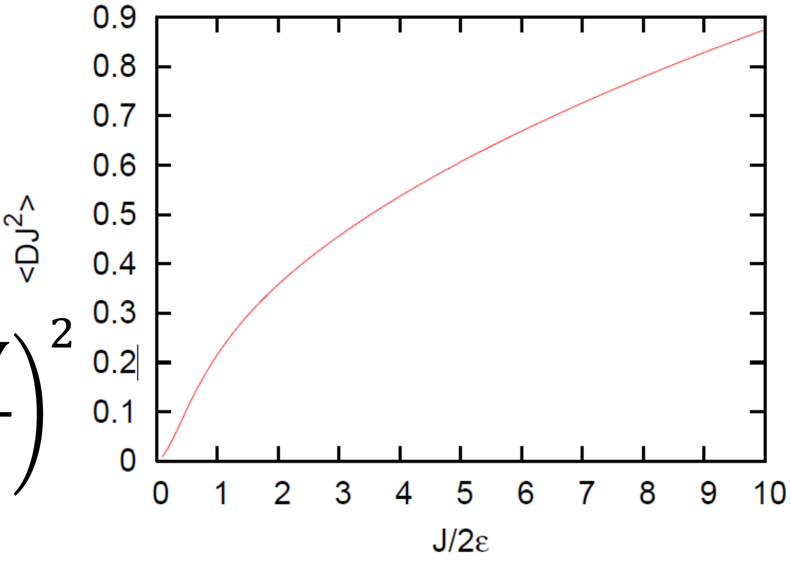
$$a = \frac{\beta J}{2\sigma^2} \qquad \frac{da}{d\sigma} = -\frac{2a}{\sigma}$$

$$\langle \Delta J^2 \rangle = \frac{4a^2}{\sigma^2}\frac{\partial U(a,\psi)}{\partial a}\frac{\partial U(a,\psi)}{\partial a}\langle \Delta\sigma^2 \rangle$$

$$= 8\left(\frac{Nr_p}{\gamma}\frac{\Delta\sigma}{\sigma}a\right)\sum_{k=-\infty}^{\infty}\frac{k^2 U'_k(a)^2 \sinh G}{\cosh G - \cos 4\pi k\nu}$$

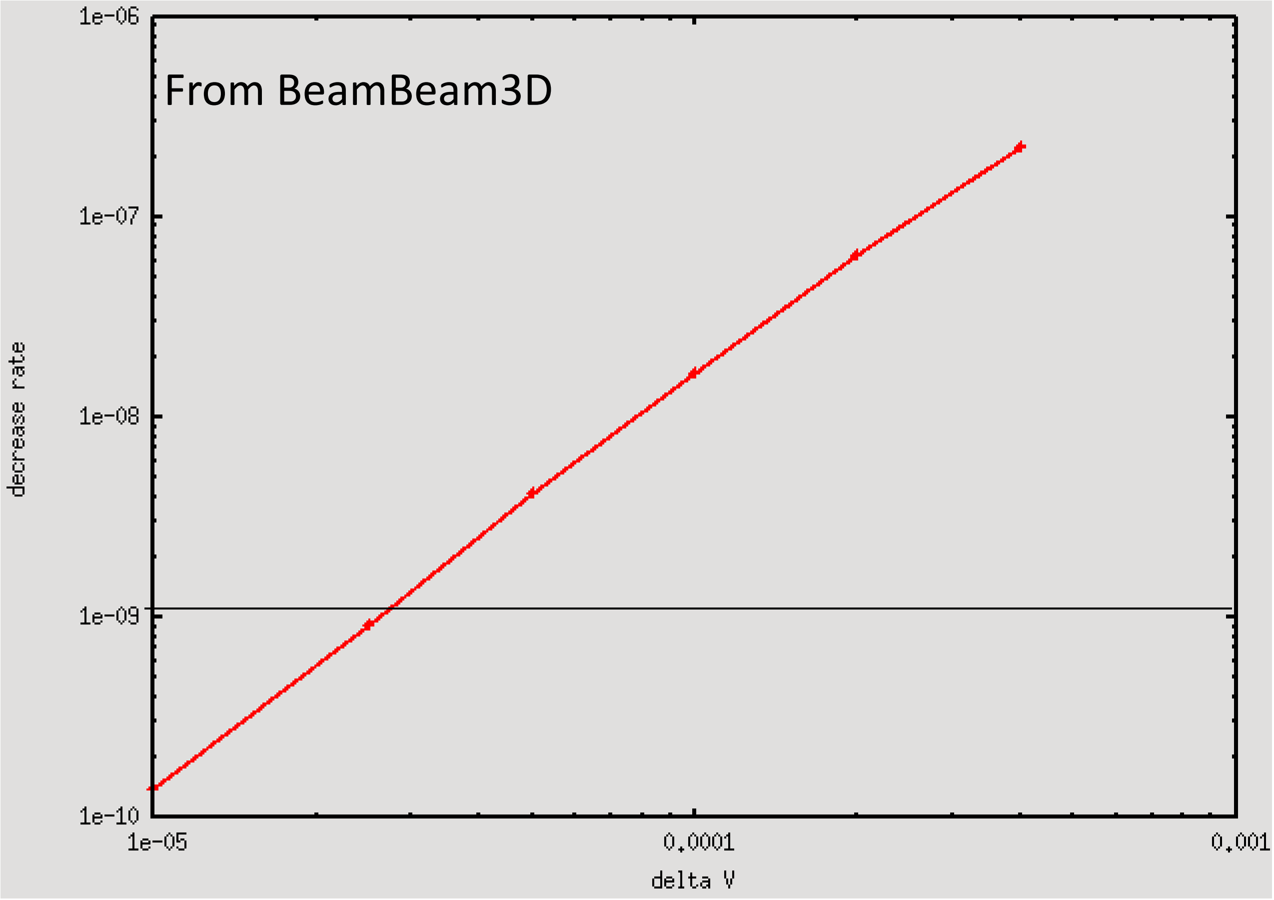
- Similar sensitivity as phase

$$\frac{\Delta L}{L_0} = 0.4\left(\xi\frac{\Delta\sigma}{\sigma}\right)^2 = 0.4\left(\xi\frac{\sigma_z\theta_{crab}}{\sigma}\frac{\Delta V}{V}\right)^2$$



CC Voltage White Noise Tolerance with Beta* Leveling

-dL/L/turn vs. δv/v



δv/v

For luminosity decrease rate below 10^{-9}/turn,
the voltage noise amplitude should be controlled below $3x10^{-5}$

Noise characteristics

- Beam has frequency component of external excitation, crab phase modulation frequency.

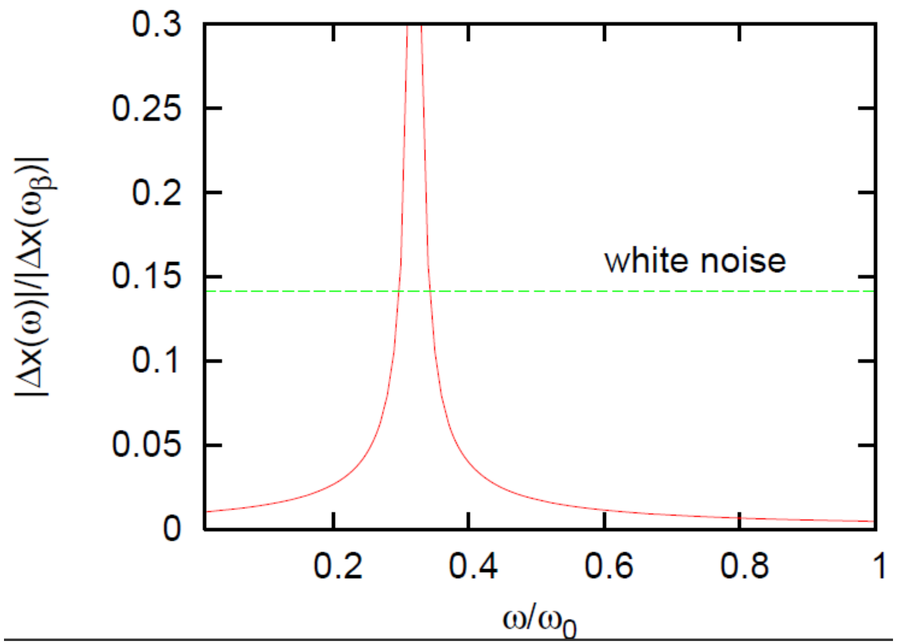
$$\ddot{x} + 2\lambda\dot{x} + \omega_\beta^2 x = \int f(\omega) e^{-i\omega t} d\omega \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

$$\lambda = \frac{\omega_0}{2\pi} G$$

$$x = \frac{\omega_0}{2\pi} \sum_{p=-\infty}^{\infty} \int d\omega \frac{f(\omega) e^{-i\omega_p t - i\varphi}}{\sqrt{\left(\omega_\beta^2 - \omega_p^2\right)^2 + \left(2\lambda\omega_p\right)^2}}$$

$$\omega_p = \omega + p\omega_0$$

$$\varphi = \tan^{-1} \frac{2\lambda\omega_p}{\omega_\beta^2 - \omega_p^2}$$

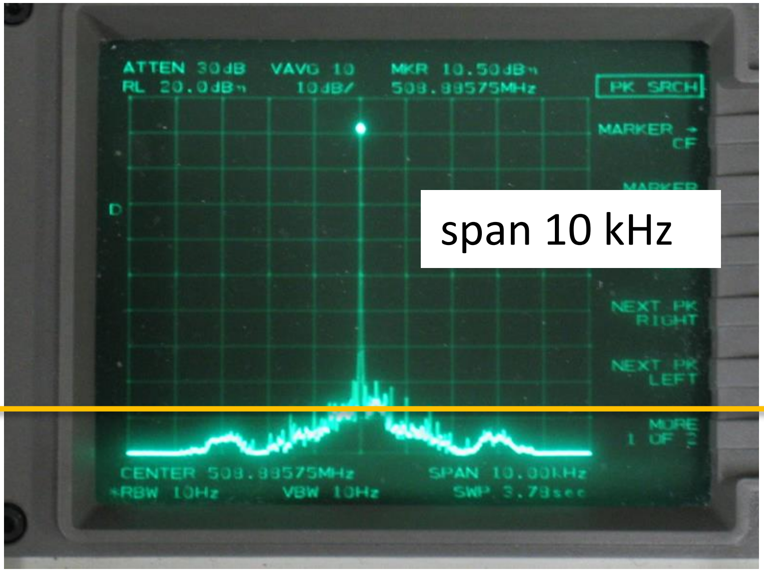


Beam response for sinusoidal and white noise

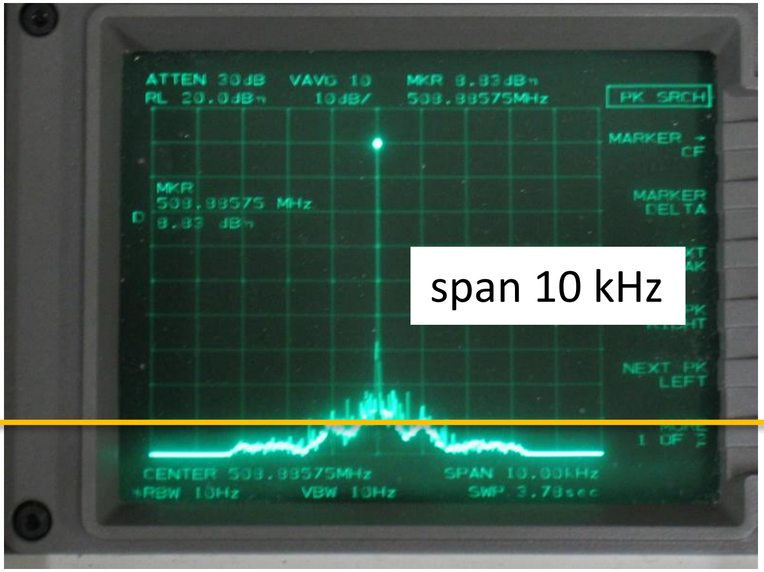
Measurement of noise level in KEKB

Jul 24, 2007

LER



HER

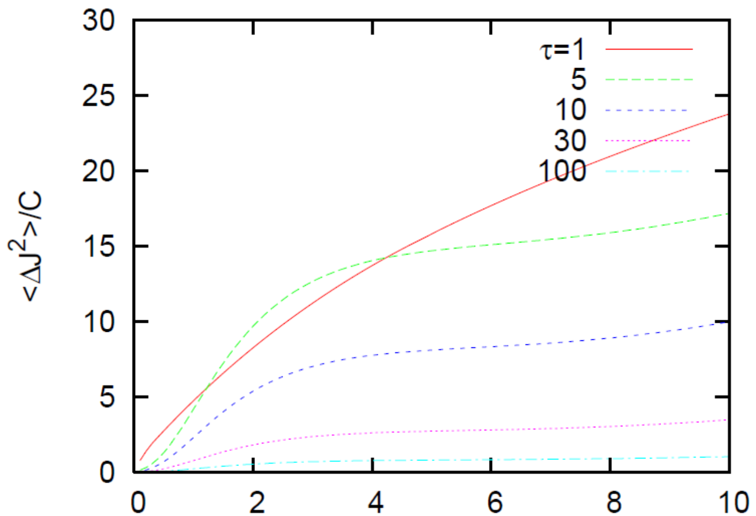


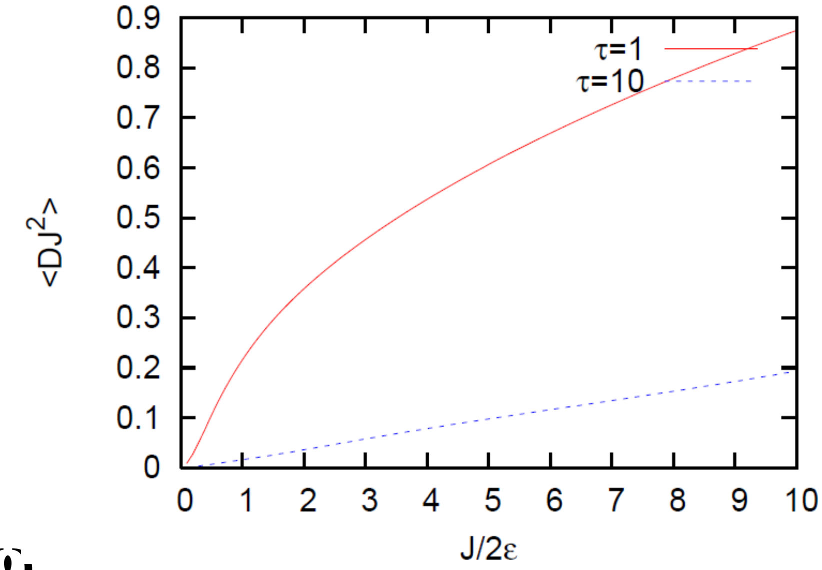
- Tune, beam-beam mode ~3kHz, f_0=11kHz
- 2 unit=20dB for power, 1/10 for amplitude.

by K. Akai, Y. Morita

Correlation time dependence

phase noise (coll. offset)   voltage noise (beam size)





Correlation time is ~ω_{crab}/ω_0.

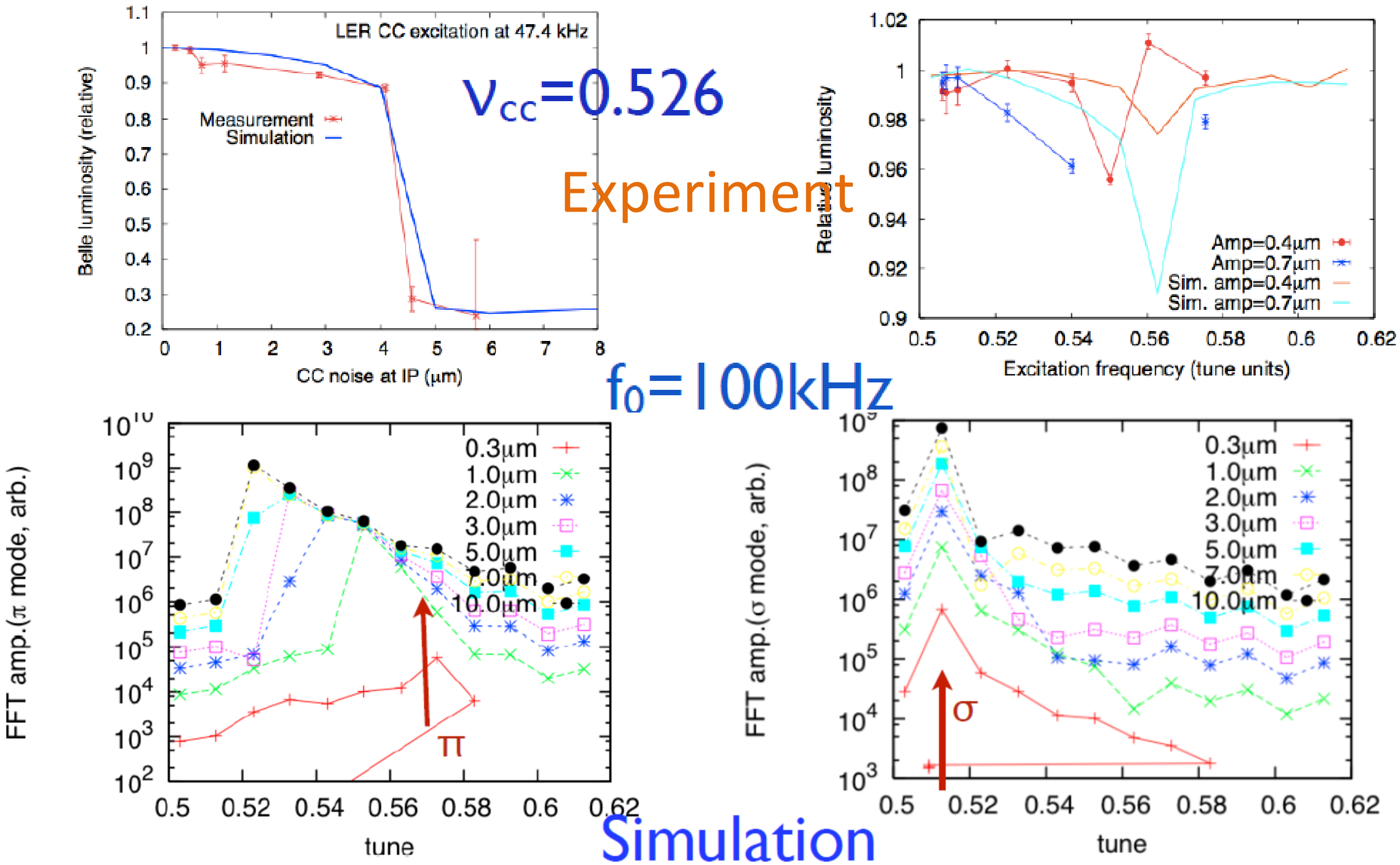
ω_0=2πx11kHz

ω_{crab}=2πx1kHz in typical. Diffusion is suppressed 1/2 (offset) and 1/10(size).

ω_{crab}=2πx3kHz may couple beam-beam mode.

Complex beam response seen in KEKB

- When the noise frequency range overlaps with beam-beam mode (3kHz for LHC), complex phenomena may appear.
- Strong and weak beam losses are seen near σ and π mode tunes, respectively.



Beam response near beam-beam mode

Summary

- Luminosity degradation due to the beam-beam effect in the presence of external noise has been studied.
- Studied noise characteristics are
  1. White noise of collision offset
  2. Time correlation noise of collision offset
  3. White noise in betatron motion with damping
  4. Sinusoidal noise in betatron motion with damping
- Feedback noise corresponds to 3rd. The luminosity degradation is similarly expressed as 1 by

$$\frac{\Delta L}{L} = 10.8 \times \left(\xi_{tot} \frac{\Delta x}{\sigma}\right)^2 \qquad \Delta x = \frac{\delta x}{\sqrt{G}}$$

$$\frac{\Delta x}{\sigma} = 5 \times 10^{-4} \quad \text{for} \quad \frac{\Delta L}{L} = 10^{-9} \quad \xi_{tot} = 0.02$$

Summary II

- Crab cavity noise corresponds to 4 which is similar as 2. Phase error tolerance is 10^{-5}-10^{-4} for white noise. Beam does not respond the crab cavity noise f<2kHz.

  1/10 of white noise for f < 2kHz, Phase error tolerance 10^{-4}-10^{-3}

- For crab voltage noise, beam size fluctuation, luminosity degradation is expressed by

$$\frac{\Delta L}{L_0} = 0.4\left(\xi\frac{\Delta\sigma}{\sigma}\right)^2 = 0.4\left(\xi\frac{\sigma_z\theta_{crab}}{\sigma}\frac{\Delta V}{V}\right)^2$$

  Tolerance is 10^{-5}-10^{-4} for white noise. Since frequency range is 1kHz, the tolerance may increase 10 times easier, 10^{-4}-10^{-3}.

Noise characteristics

- Equation of motion.

$$\ddot{x} + 2\lambda\dot{x} + \omega_\beta^2 x = \int f(\omega) e^{-i\omega t} d\omega \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

Fourier expansion $x = \int \hat{x}(\omega) e^{-i\omega t} d\omega$

$$\int (-\omega^2 - 2i\lambda\omega + \omega_\beta^2)\hat{x}(\omega) e^{-i\omega t} d\omega = \int f(\omega) e^{-i\omega t} d\omega \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

operate $\frac{1}{2\pi}\int e^{i\omega\prime t} dt$

$$(-\omega^2 - 2i\lambda\omega + \omega_\beta^2)\hat{x}(\omega) = \frac{1}{2\pi}\int d\omega' f(\omega') \int dt e^{-i(\omega' - \omega)t} \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

$$= \frac{1}{2\pi}\int d\omega' f(\omega') \sum_{n=-\infty}^{\infty} e^{-i(\omega' - \omega)nT_0} = \frac{1}{2\pi}\int d\omega' f(\omega')\, \omega_0 \sum_{p=-\infty}^{\infty} \delta(\omega' - \omega - p\omega_0)$$

$$= \frac{\omega_0}{2\pi} \sum_{p=-\infty}^{\infty} f(\omega + p\omega_0)$$

$$\hat{x}(\omega) = \frac{\omega_0}{2\pi} \sum_{p=-\infty}^{\infty} \frac{f(\omega + p\omega_0)}{-\omega^2 - 2i\lambda\omega + \omega_\beta^2}$$

$$x = \int \hat{x}(\omega) e^{-i\omega t} d\omega = \frac{\omega_0}{2\pi} \sum_{p=-\infty}^{\infty} \int d\omega \frac{f(\omega + p\omega_0) e^{-i\omega t}}{-\omega^2 - 2i\lambda\omega + \omega_\beta^2}$$

$$= \frac{\omega_0}{2\pi} \sum_{p=-\infty}^{\infty} \int d\omega \frac{f(\omega) e^{-i\omega_p t}}{\omega_\beta^2 - \omega_p^2 - 2i\lambda\omega_p} \qquad \omega_p = \omega + p\omega_0$$

$$= \frac{\omega_0}{2\pi} \sum_{p=-\infty}^{\infty} \int d\omega \frac{f(\omega) e^{-i\omega_p t - i\varphi}}{\sqrt{\left(\omega_\beta^2 - \omega_p^2\right)^2 + \left(2\lambda\omega_p\right)^2}} \qquad \varphi = \tan^{-1} \frac{2\lambda\omega_p}{\omega_\beta^2 - \omega_p^2}$$