SPL RF Frequency Choice: Cavity Scaling Considerations

~~**Cryogenics, RF superconductivity, 2K-4.5K, $$$,…**~~

(Most) presented ideas developed in the framework of the LEP2 sc. cavity design studies (partly unpublished) …

–> acknowledgements to **Ernst Haebel** and **Philippe Bernard**

(both retired …..)

… but any errors in this talk are mine …

(<— still roaming …)

Brontosaurus superconductus altafrequencis

SPL-f-Review 30/4/08

J. Tückmantel, CERN AB-RF

Outline

1) Pure scaling of cavities/couplers/…

$f \longrightarrow 2 \cdot f$ (—> cavity length / 2)

1a) ramifications (only perfect cavities)

2) Increase cell number (same cavity/couplers/..)

$N \longrightarrow 2 \cdot N$ (—> recover 'old' cavity length)

2a) ramifications for perfect cavities

2b) ramifications for real cavities

– absolute 'calibration' with SNS simulations

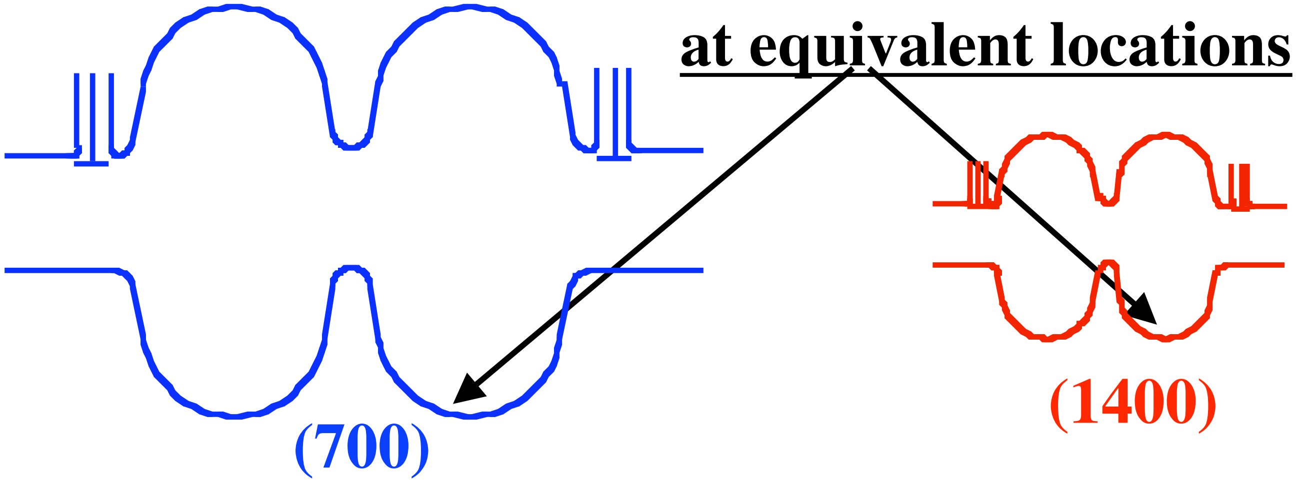
2c) powering up cavities

2d) RF vector feedback

Conclusions

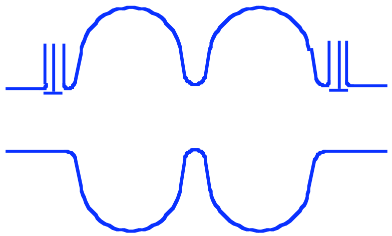
1a) Scaled – else identical – perfect cavities

Scaling of some RF/beam quantities for <u>same local fields at equivalent locations</u>

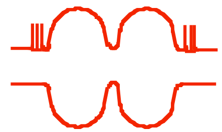


L	fundamental quantities	**L/2**
area		**Area/4**
volume		**Volume/8**
f		**2·f**
$V_{acc}(v_{part})$	–> trivially …	$V_{acc}/2$ (v_{part})
U_{st}		$U_{st}/8$

identical



derived quantities(1)



same local fields

$$\boxed{\left(\frac{R}{Q}\right)_{\|} = \frac{1}{2}\frac{V^2}{\omega \cdot U_{st}}}$$

Excitation independent

$$\left(\frac{R}{Q}\right)_{\|} = \frac{1}{2}\frac{(V/2)^2}{(2\cdot\omega)\cdot(U_{st}/8)} = \left(\frac{R}{Q}\right)_{\|}$$

antenna HOM coupler $P_{ext} = \frac{1}{2} I_{coup}^2 \cdot Z_{Line}$; $I_{coup} = A_{plate} \cdot i\omega \cdot E_{plate}$

loop HOM coupler $P_{ext} = \frac{1}{2} V_{coup}^2 / Z_{Line}$; $V_{coup} = A_{loop} \cdot i\omega B_{RF}$

resistive material $dP_{\text{int}} / dA \propto E^2 \propto B^2$; $P_{\text{int}} \propto A$

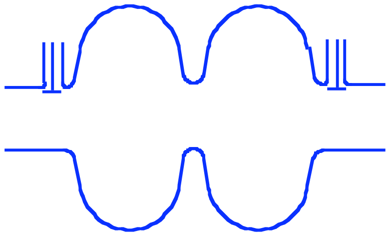
always $\boxed{P_{lost}/4} = \boxed{P_{lost}}$ $\Leftrightarrow$ for equivalent local fields!!

$$\boxed{Q_{ext} = \frac{\omega \ U_{st}}{P_{ext}}}$$

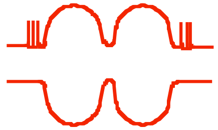
Excitation independent

$$Q_{ext} = \frac{(2\cdot\omega\cdot U_{st}/8)}{P_{ext}/4} = Q_{ext}$$

$\boxed{\text{sc. cav: } Q_0 >>> Q_{ext} \Rightarrow \quad Q_{tot} = Q_{ext}}$



derived quantities(2)



Monopole (longitudinal) wakes

beam induced fields

$$\Delta V_{ind}(cavity) = q \cdot \omega \cdot (R/Q)$$

$$\Delta V_{ind}(cavity) \cdot 2 = \Delta V_{ind}(cavity)$$

Longitudinal short range wakes/L scale as f²

$$(\Delta V_{ind}/L) \cdot 4 = (\Delta V_{ind}/L)$$

Longitudinal long range wakes (fields with memory)

$$Z_{\|}(cavity) = (R/Q) \cdot Q_{ext}$$

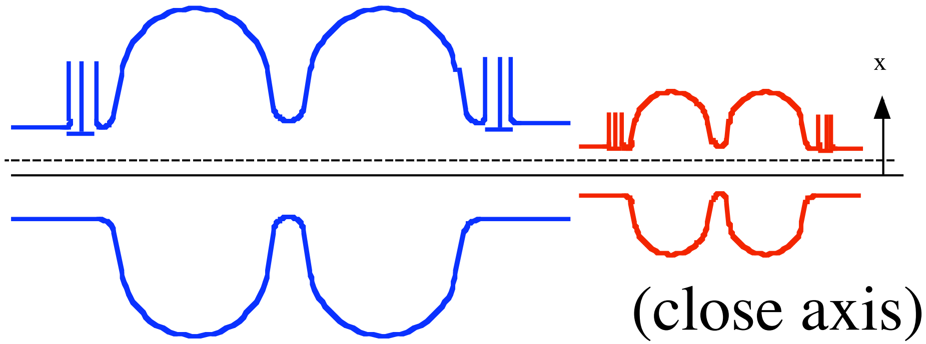
$$Z_{\|}(cavity) = Z_{\|}(cavity)$$

Excitation independent

Longitudinal long range wakes/L scale as f

$$(Z_{\|}/L) \cdot 2 = (Z_{\|}/L)$$

derived quantities(3)



Dipole wakes:

(close axis) $E_{\|}$ prop. to x (–>$E_{\|}(0)=0$)

For **same local fields** and the **SAME UNSCALED offset x**

$E_{local}(x)\cdot 2 = E_{local}(x)$ hence $V_{acc}(x) = V_{acc}(x)$

$\omega U_{st}/4 = \omega U_{st} \Rightarrow (R/Q)_{\|}(x)\cdot 4 = (R/Q)_{\|}(x)$ Excitation independent

---

beam induced $V_{\|,ind}(x) = I_{B,RF}\cdot(R/q)_{\|}(x)\cdot Q_{ext}$ $V_{\|}(x)\cdot 4 = V_{\|}(x)$

Induced voltage/L at (same) offset x: $(V_{\|}(x)/L_{cav})\cdot 8 = (V_{\|}(x)/L_{cav})$

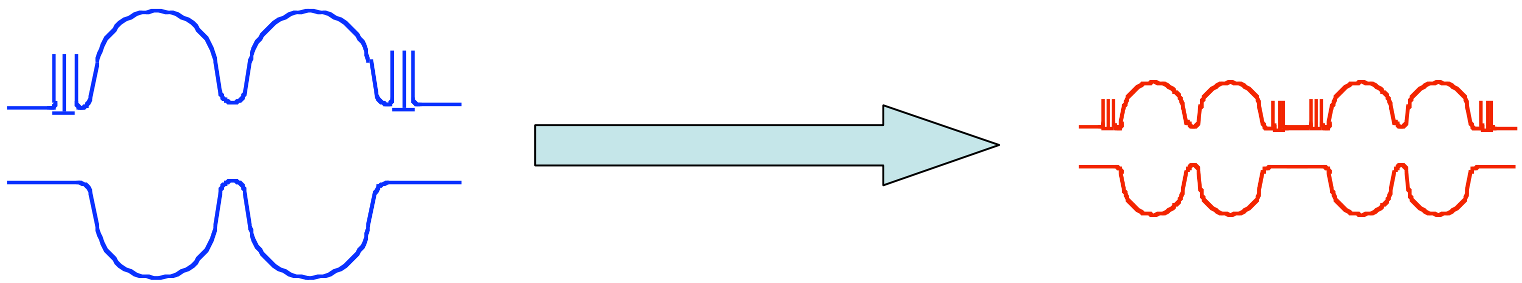
Panofsky-Wenzel: $\Delta p_x = -\dfrac{i\cdot e}{\omega}\cdot\dfrac{\partial V_{\|}}{\partial x}$

$\Delta p_x/L\cdot 4 = \Delta p_x/L$

$(Z_{\perp}/L)\cdot 4 = (Z_{\perp}/L)$

Excitation independent

Pure scaling of cavities/couplers/… by f —> 2·f



$$(Z_{\parallel}/L) \bullet 2 = (Z_{\parallel}/L)$$

$$(Z_{\perp}/L) \bullet 4 = (Z_{\perp}/L)$$

Transverse kicks/L (same offset) scale as f² (e.g. CEBAF design report)

-> beam break-up threshold current scales as 1/f²
becomes 1/4 = 25 %

Two aspects of the beam-cavity-interaction:

1) Beam Instabilities

No f_s nor f_β as in a circular machine:

an impedance at any frequency can excite the beam

–> creates its own 'line(s)' in modulating the beam (bunch position)

Also impedances far away from machine lines can be dangerous concerning instabilities

Mode frequency scatter along the linac may save the day ..

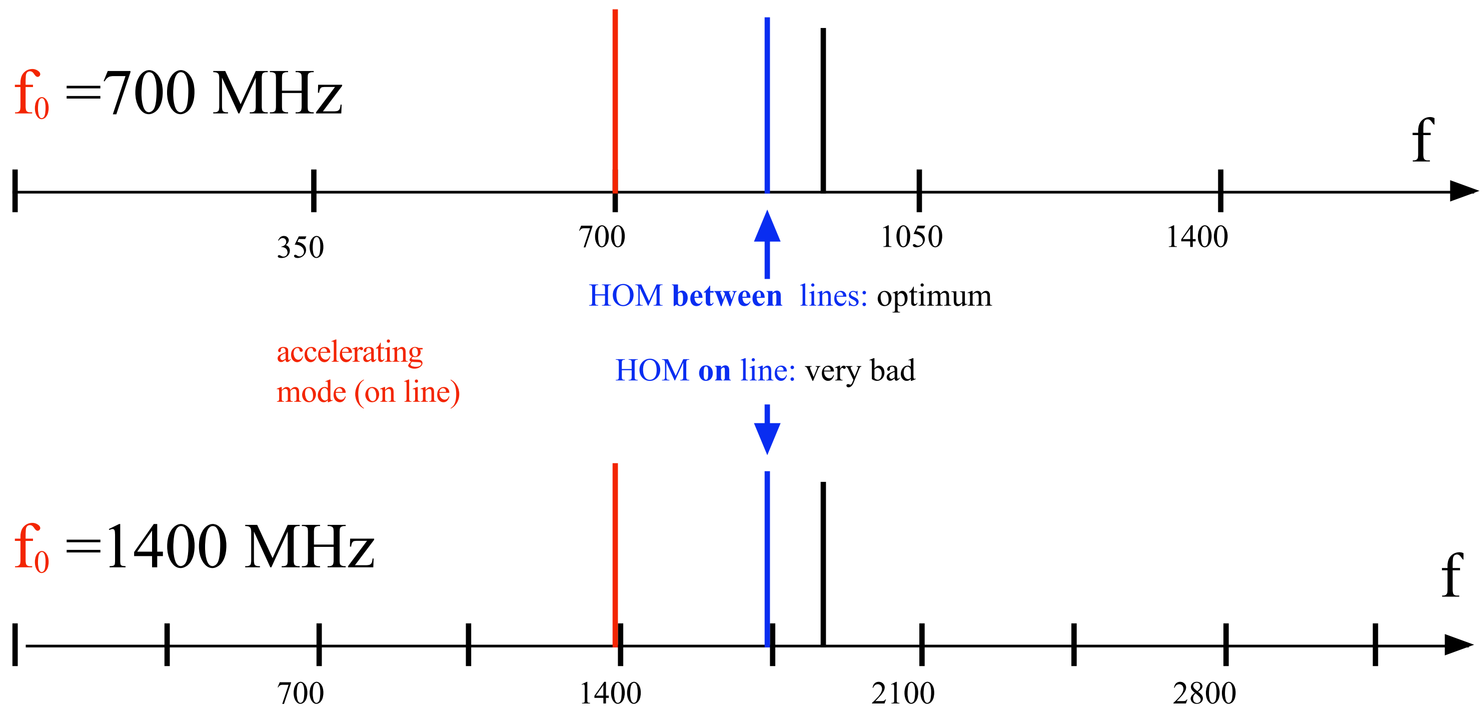
An **experienced** linac beam expert should investigate … train pattern, mode f, Q_{ext} … scatter ….

SNS @ 800 MHz, 6-cells: extensive studies (random !)

2) Power Extraction

(Derivations —> Appendix)

– principal Machine Lines (ML): multiples of 350 MHz
– weak (n/m ·350 MHz) lines if bunch trains have a m-pattern
– 50 Hz train rate 'invisible': decisive envelope = 350 MHz ML
– relat. form-fact. ≥0.85 up to 5 GHz (<— using info A. Lombardi)



Spectrum relatively more dense at 1400 MHz

$$P_{ext,cav}(\delta\omega) = \frac{2 \cdot (R/Q) \cdot Q_{ext} \quad I^2_{b,DC}}{1 + \left(2\delta\omega \;\; Q_{ext}/\omega_{ML}\right)^2}; \quad \delta\omega = \omega_{\mathrm{mod}\,e} - \omega_{ML}$$

$P_{ext,cav}$ scales **f-independent** ⇔ for the same exiting beam !!

—> power-density in coupler ***4**, local fields (arcing) ***2**

$P_{ext,cav}$ is **per cavity** ⇔ for the same exciting beam !!

—> total extracted HOM power ***2**

$P_{ext,cav}$ can become considerably, destroy coupler/load

(there is no principal limit for Q_{ext} …. as long as $\ll Q_0$)

Example on resonance:

$I_{b,DC}$ = 40 mA, (R/Q)=50Ω (e.g. TM_{011}), Q_{ext}=50000, δω=0

$P_{ext,cav}$ = 8 kW 'equilibr. on train' / 0.9 kV on 50Ω line

$\langle P_1 \rangle$ = 8 kW*duty-factor(5%) —> 400 W **(.. tolerable ..)**

$V_{HOM,cav}$ = –0.2 MV (in equilibrium): about 1% of V_{acc}

—> 1% voltage swing at start of train

{ LHC has 15*$I_{b,DC}$, duty-f.=1 —> $\langle P_1 \rangle$ = 1800 kW  **}**

No **coincidence** with a principal **machine line** (n·350 MHz)
—> no ‘over-power problem’ expected

<u>‘shifting’ of modes :</u> not easy for ‘all’ modes
- is a different problem at 700 and 1400 MHz
since beam spectrum does NOT scale !

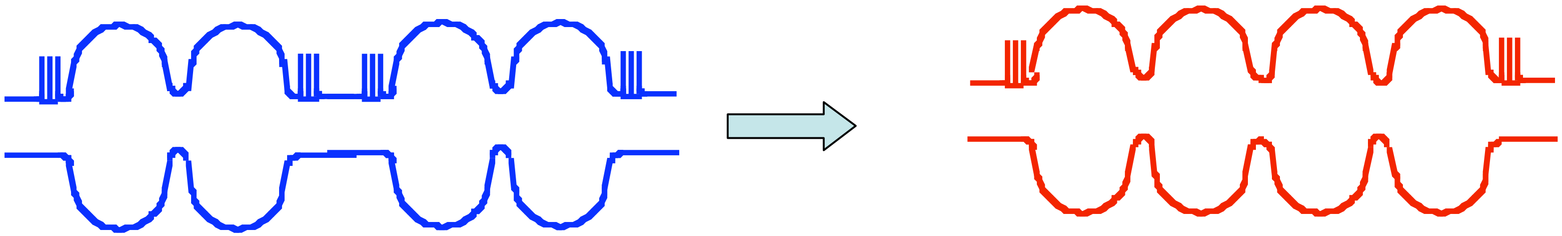
—> cannot simply scale TESLA/ILC case as is:
ILC/FLASH : 1 rare BIG bunch (long time between bunches)
SPL/SNS/X : rapid sequence of SMALL bunches (as CBI)

Only safe way: <u>guarantee damping : low Q_{ext}</u>
—> lower extracted power
—> lower (long range) impedances

Q_{ext} transparent under f-scaling.
How does it behave under cell-scaling ?

2*f —> double number of cavities, couplers, tuners, controllers, …. —> 2x $$ (*) ?

Avoid $$-increase: keep 'same' cavity length —> double number of cells N —> 2*N



same frequency (1400)

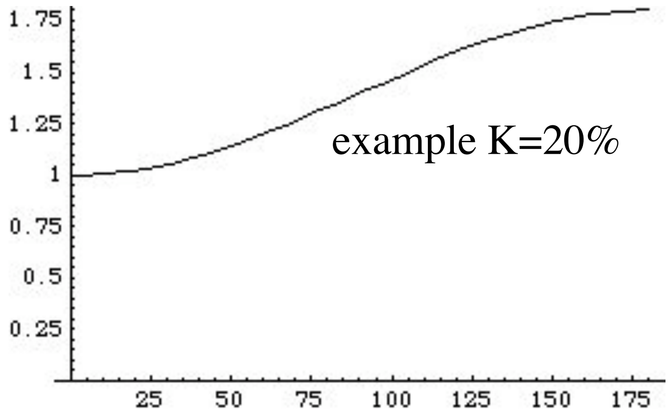
(*) see e.g. Ph. Bernard, E. Chiaveri, J.T. : "Technical and Financial Implications of the frequency choice for a sc. accelerator section", Jan. 1996 (unpublished)

2a) Perfect cavities (same 'cell frequency' for ALL cells)

… including end-cells ….

N-cell cavity, mode **m**: 1≤m≤N **(regular) passband** (see e.g. (*))

K=cell-to-cell coupling (K=0.85% LEP2) , ω_0 = cell basic frequency



$$\omega_m^2 = \omega_0^2 \cdot \left(1 + 2 \cdot K \cdot (1 - \cos(\pi \cdot m / N)\right)$$

$$\Theta_m = \pi \cdot \frac{m}{N}$$

TW cell-to-cell phase advance

For mode **m**

(rel.) amplitude in cell **n**

$$a_n^m = \sin\left(\pi \cdot \frac{m}{N} \cdot (n - 1/2)\right)$$

cell amplitudes
vector for mode **m**

$$A^{(m)} = \left\{a_1^m, a_2^m, \ \ldots. \ , a_{N-1}^m, a_N^m \right\}$$

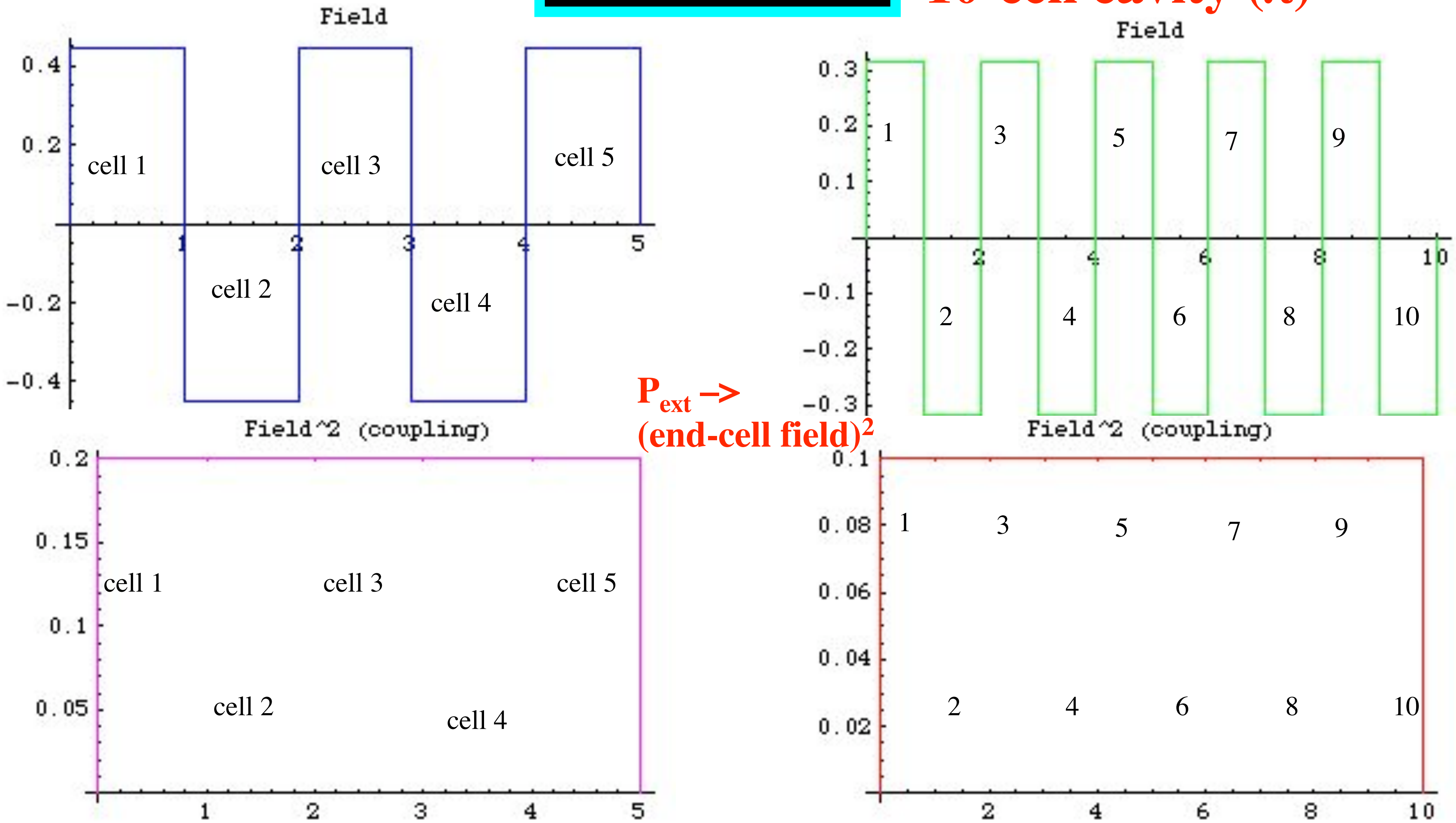
(*) E. Haebel & J.T., CERN/EF/RF **81**-5 "Tuning of a …", Part 1 (theory)

The highest passband mode(s): field amplitudes in cells

5-cell cavity (π)

norm: $U_{st} = 1$

10-cell cavity (π)



cell 1

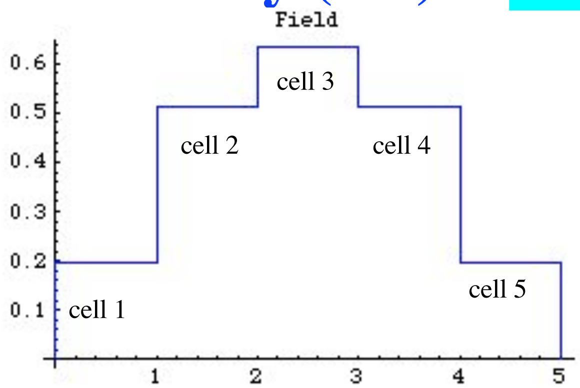
P_{ext}/U_{st} ratio : 2:1, same ω —> $Q_{ext} = U_{st}/\omega P_{ext}$ 1:2

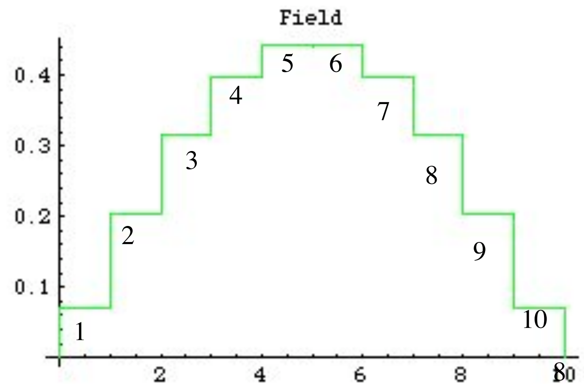
The lowest passband mode(s): field amplitudes in cells

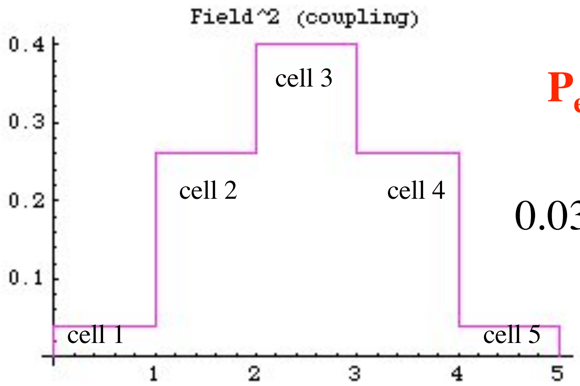
5-cell cavity ($\pi/5$)

norm: $U_{st} = 1$

10-cell cavity ($\pi/10$)



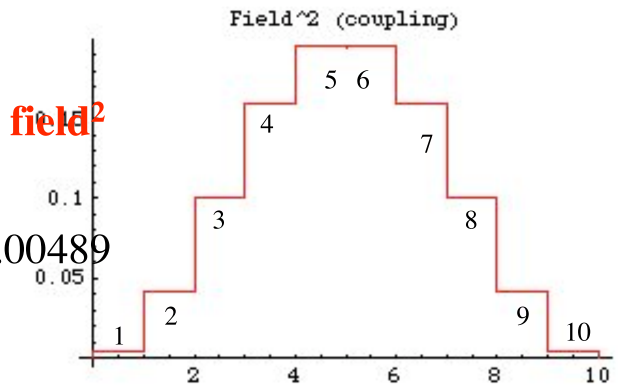




P_{ext} –> end-cell field^2

0.0382

0.00489

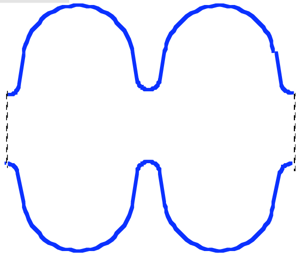


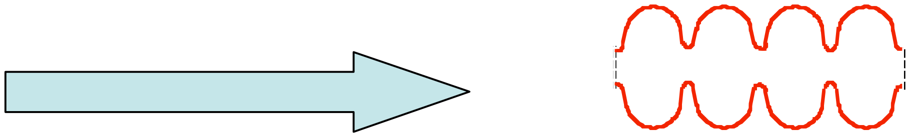
P_{ext}/U_{st} ratio : 8:1, same ω, —> $Q_{ext}=U_{st}/\omega P_{ext}$ 1:8

$Q_{ext,\pi} = Q_{ext,\pi/5} \approx$ 1:5

$Q_{ext,\pi} = Q_{ext,\pi/10} \approx$ 1:20

Scaling of cavities/couplers/… by f —> 2·f and cell number N —> 2 ·N





(perfect cavities, no end-cell-problem, no f-scatter)

$$Q_{ext} \bullet 2 \;....\; 8 = Q_{ext}$$

$$(Z_{\|} / L) \bullet 4 \;....\; 16 = \left(Z_{\|} / L\right)$$

$$(Z_{\perp} / L) \bullet 8 \;....\; 32 = (Z_{\perp} / L)$$

-> beam break-up threshold current becomes 1/8 = 12.5% …. 1/32 = 3%

2b) Before entering 'imperfection statistics', some facts:

∞ **number** of modes; only **a single bad one** can be sufficient to 'kill'

above cut off frequency: propagation into next cavity/'warm' damping

Let's find the 'bad guy' and do something about it .....

HOM couplers (but also **test antennas**) are on the cut-off tubes (*)

–> **coupling** depends **ONLY** on **end-cell fields** uniquely

- **Modes with low end-cell field are potentially dangerous**

  –> the **more dangerous** –> the more **'invisible'** in bench-meas.

- R/Q and end-cell field-levels 'not' correlated

  –> high peaks in bench measurement have high **or** low R/Q

  –> **no distinction** for high R/Q in transmission test

  (–> bead-pull: 'a mess' except lowest modes)

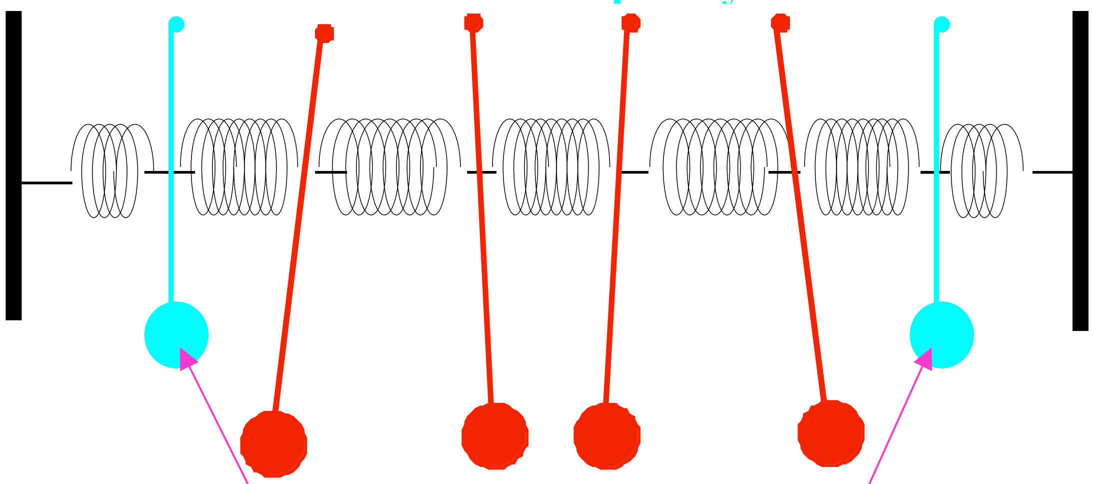
?

(*) experience from 500 MHz 5-cell cavity test —> never ports on cells

Imperfect cavities (each cell has 'its own' frequency)

End-cell correction (tube!) done for accelerating mode (not HOMs!)
Cell-f scatter is intrinsic property of manufacturing process!!
—> have to 'cheat' for accelerating mode by individual cell tuning
(include. f_0 tuning) after cavity fabrication of whole cavity.
HOMs have to accept 'what is' after the fundamental mode tuning

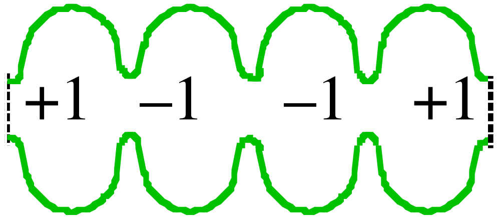
Trapped mode(s)



HOM coupler 'feel' (end-cell field)^2 only !!!

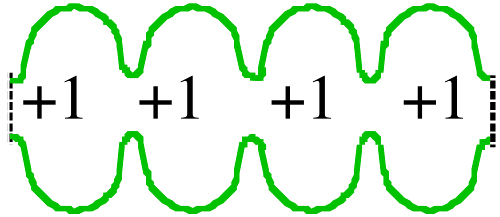
If '**strong**' coupling (K) end-cell **oscillate a little bit** —> high Q_{ext}
If '**weak**' coupling (K) end-cell **do 'not' oscillate** —> very high Q_{ext}

Trapped Modes <—> Mode Mixing

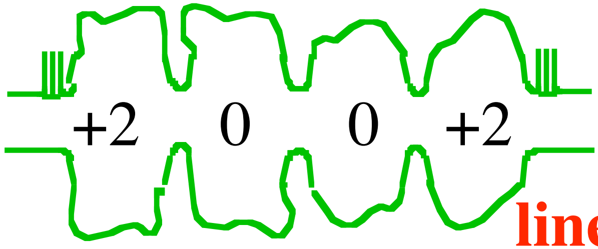


3π/4 like mode

ideal cavities:
‘ideal modes’

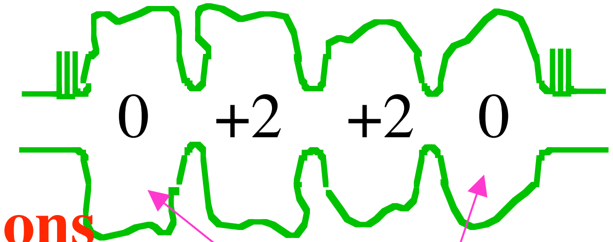


π/4 like mode



‘sum mode’

real cavities:
linear combinations
of ‘ideal modes’



‘difference mode’:

trapped

HEPL: differing end-cells: **trapped modes limited I_b far below specs**

(no external HOM damping at all)

LEP2: The ‘civilized’ TM_{012} mode (low K) had strong mode-mixing:
2 **opposingly inclined field profiles** (high at one, low at other end)
—> put one HOM coupler on BOTH sides (also for dipoles)
… if dipole modes (2 polarisations !) have such a pattern ???

‘Stolen’ from (Proc. LINAC06, Knoxville)

J. Sekutowicz, HOM Damping and …. sc. Cavities

(calculated examples)

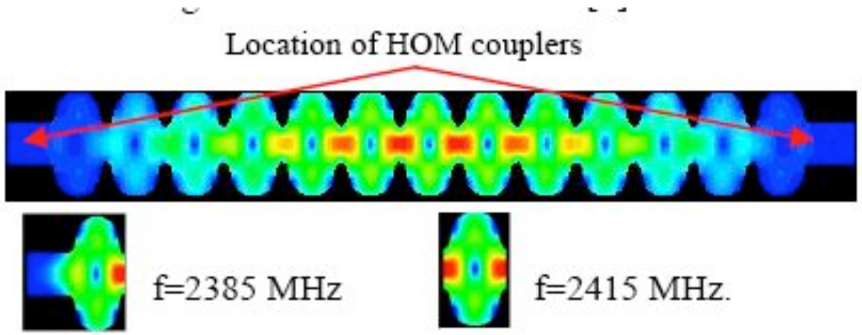


Figure 1: Example of mode trapping in a 13-cell cavity. End-cells and inner-cells have different frequencies for this resonant pattern.

frequency-difference
centre cells <–> end-cells

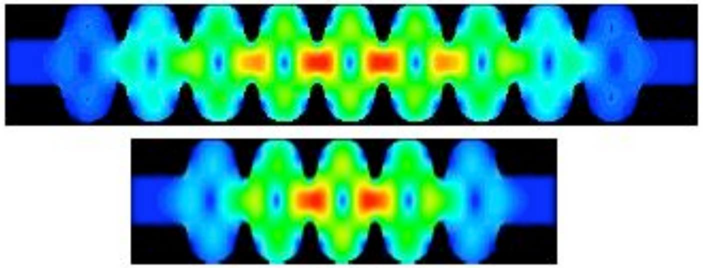


Figure 3: Shorter structures make trapping less probable.

less cells
makes it better …

Perturbation theory 'stolen' from **Quantum Mechanics!**

Diagonal perturbation operator **P**: relative cell frequency errors

using **cell** index

$$\delta(\omega_k^2)/\omega_0^2 \approx 2\cdot\delta\omega_k/\omega_0$$

$$P = 2\cdot\begin{pmatrix} \delta\omega_1/\omega_0 & 0 & & & & & 0 \\ 0 & \delta\omega_2/\omega_0 & 0 & & & & \\ & 0 & \dots & \dots & & & \\ & & \dots & \dots & 0 & & \\ & & & 0 & \delta\omega_{N-1}/\omega_0 & 0 \\ 0 & & & & 0 & \delta\omega_N/\omega_0 \end{pmatrix}$$

(H+P)(ψ+δψ)= (E+dE)(ψ+δψ)

Then the perturbations in the eigenvectors = **mode cell-field-levels** are determined in QM lingo as

$$\left|\delta\psi_m\right\rangle = \left|\delta a^{(m)}\right\rangle = \sum_{k\neq m}^{N} \frac{\left\langle a^{(k)}\middle|P\middle|a^{(m)}\right\rangle}{\left\langle a^{(k)}\middle|a^{(k)}\right\rangle} \frac{\omega_m^2}{\omega_m^2-\omega_k^2}\left|a^{(k)}\right\rangle$$

(change to 'new' $a^{(m)}$ by component of $a^{(k)}$)

Decisive factor for 'mode mixing'

—> 'trapped modes'

$$\boxed{\frac{\omega_m^2}{\omega_m^2-\omega_k^2} >> 1}$$

$$\frac{1}{E_m - E_k}$$

Details: E. Haebel & J.T., CERN/EF/RF **81**-5 "Tuning of a …", Part 1 (theory)

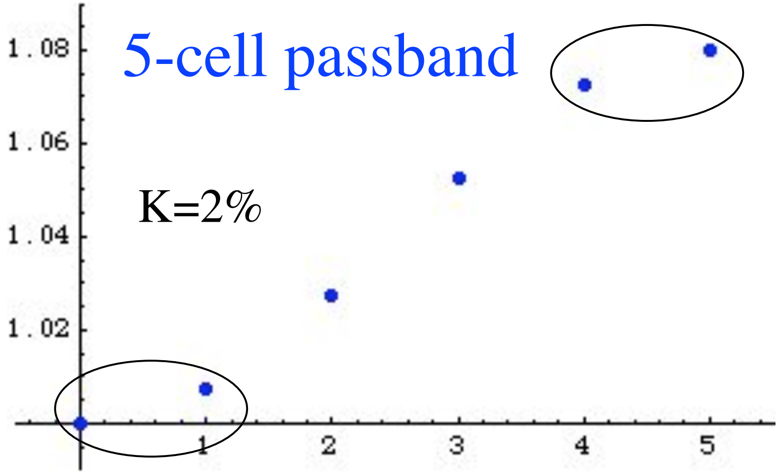
Mode frequencies in passband

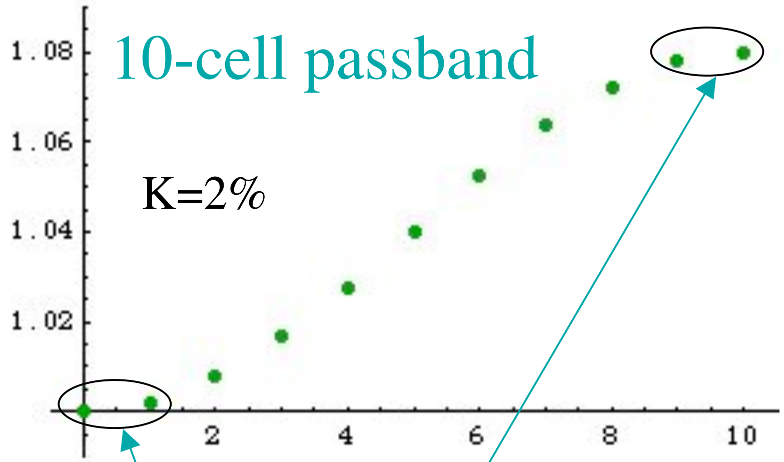
K=cell-to-cell coupling , ω_0 = cell basic frequency ('zero-mode')

$$\omega_n^2 = \omega_0^2 \cdot \left(1 + 2 \; K \cdot (1 - \cos(\pi \cdot n / N)\right) \Rightarrow \frac{1}{\omega_m^2 - \omega_k^2}$$

Distance of neighboring modes: smaller for more cells (larger N)

Most critical close to zero and π mode





$$\omega_m^2 - \omega_{m-1}^2 = 2 \; K \cdot \omega_0^2 \left(\cos(\pi \cdot (m-1)/N) - \cos(\pi \cdot m/N)\right) = 4 \; K \cdot \omega_0^2 \cdot \sin(\pi/2N) \cdot \sin(\pi \cdot (m-1/2)/N) \propto 1/N$$ *2

especially $\omega_N^2 - \omega_{N-1}^2 = \omega_1^2 - \omega_0^2 = 4 \; K \cdot \omega_0^2 \cdot \left[\sin(\pi/2N)\right]^2 \propto 1/N^2$ *4

Scaling of cavities/couplers/… by f —> 2·f

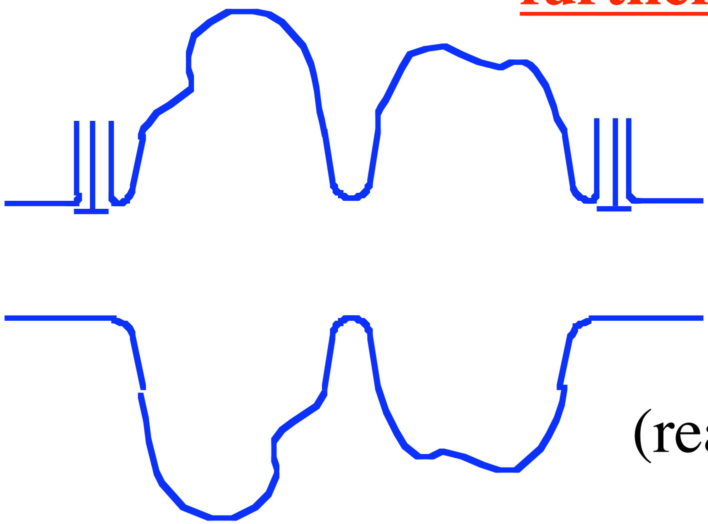
and cell number N —> 2 ·N

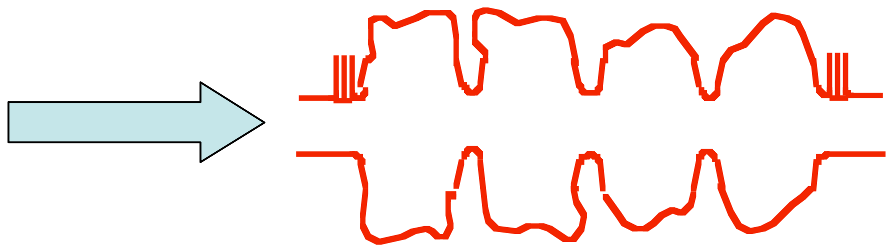
(—> Loss of I_{th} by factor 1/8 … 1/32)

and assuming real production scatter

Production scatter —> origin of field-flatness problem–> trapped mode

further degradation of $1/Q_{ext}$ –> I_{th}





(real cavities: end-cell, f-scatter)

Sensitivity per $\delta\omega/\omega_0$: ‘1’

Sensitivity per $\delta\omega/\omega_0$: ‘2’-’4’

For compensation:
increase cell-to-cell coupling K ?!?!

Problems:

Needs <u>wider iris opening</u> (for elliptical cavity: ‘sc. holy cow’)

—> lower R/Q (more cold He / MV)

—> higher E_{peak}/E_{acc} (field emission !!)

—> passbands get deformed ($d(\omega^2)/d\Theta=0$ –> mode mixing)

magnetic and electric coupling may cancel –> K=0
(which happens sometimes for ‘higher HOMs’ anyway)

Not a ‘saves all’ solution,
can even become worse …

‘Calibration’ with SNS simulations (R. Sundelin et al. PAC 91)

Optimists live easier; here assume always worst case (*) …

(6-cell cavities @ 806 MHz, I_{train} = 20 mA)

Transverse instabilities OK, error magnifications acceptable ….

Longitudinal instabilities OK ….

…. if the loaded cavity Q for **each** (*) **HOM** is less than **10^8**

Beam current SPL *2 -> $Q_{ext}/2$

Limit $5\cdot10^7$ **all** modes SPL @ 40 mA

f_{SPL} *2 —> $Z_\perp$ *4 need $Q_{ext}/4$:

Limit $1.25\cdot10^7$ **all** modes SPL @1408 MHz, 5-cells

5-cell -> 10 cell: $Q_{ext}/2$ … **$Q_{ext}/8$** perfect cavities

Limit $1.6\cdot10^6$ **on 5-cell SPL (each HOM** (*))

End-cell ‘problem’, fabrication tolerances, say worst factor 4

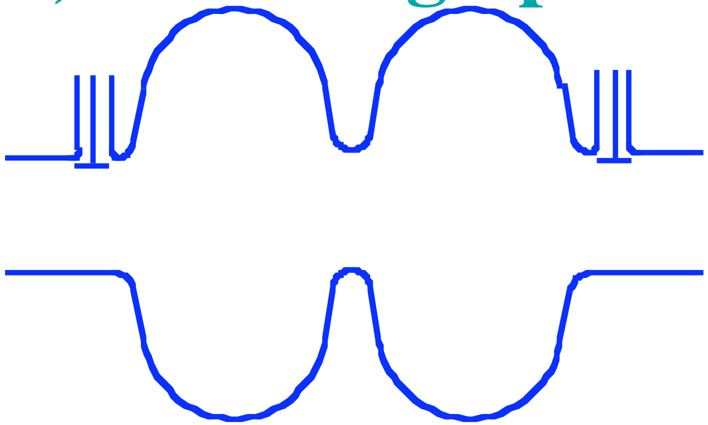
Limit $4\cdot10^5$ **on 5-cell SPL (each HOM** (*))

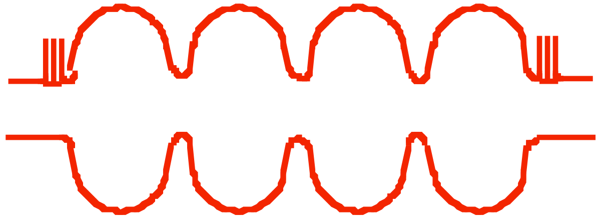
SPL is 2x as long as SNS —> factor 2….

Limit **$2\cdot10^5$** **on 5-cell SPL(each HOM** (*)) **$Q_{MC}\approx10^6$**

(*) Terrorist to FBI: You have to be always successful, we only once !

2c) Powering up Cavities (before beam)





$$\left(\frac{R}{Q}\right)_{\|} \cdot 2 = \left(\frac{R}{Q}\right)_{\|}$$

$$Q_{ext,opt} = \frac{V}{2\ (R/Q)\ \ I_{b,DC}\, f_b \sin(\phi)}$$

$$Q_{ext,opt}/2 = Q_{ext,opt}$$

$$\tau_{fill} = Q_{ext}/\omega$$

$$\tau_{fill}/4 = \tau_{fill}$$

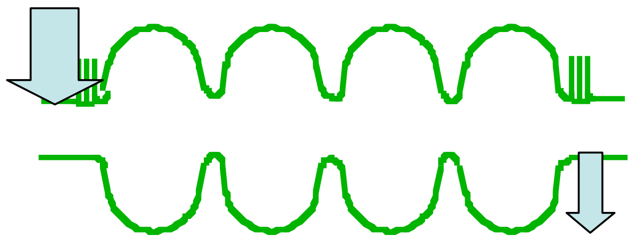
$$P_{pump} = \frac{V_{equil}^2}{2\ (R/Q)\ \ Q_{ext}}$$

$$P_{pump} = P_{pump}$$

$$U_{fill,waste} = \int_0^{\eta \cdot \tau_{fill}} P_{pump}\, dt \quad \Rightarrow \quad U_{fill,waste}/4 = U_{fill,waste}$$

f_b = <u>relative</u> bunch form factor; f_b =1 for point-bunches

2d) Fast RF vector feedback considerations



- The probe antenna (PA) should be on the cavity end **opposing** the main coupler (MC) (avoid cross-talk !!)
- The **polarity** between MC and PA **alternates** along the (fundamental) passband modes (m) + – + – + – ….

  —> Modes with **inverted** (wrsp to acc. mode) **polarity** **without special filters** or …. the loop **auto-oscillates** on these f_m

In LEP times a few sc. LEP2-type 4-cell sc. cavities were also used in the SPS injector but had to be made invisible during the proton cycle by a **high gain RF vector feedback** (120 dB !!).

Main problem for feedback:
separate 4 modes of fund. passband to prevent auto-oscillation and still act on these modes ('wide band' tetrode amplifier)

'Large box' full of (low power) RF components ($$$), …. , watchmakeres's work, setting up was time intensive.

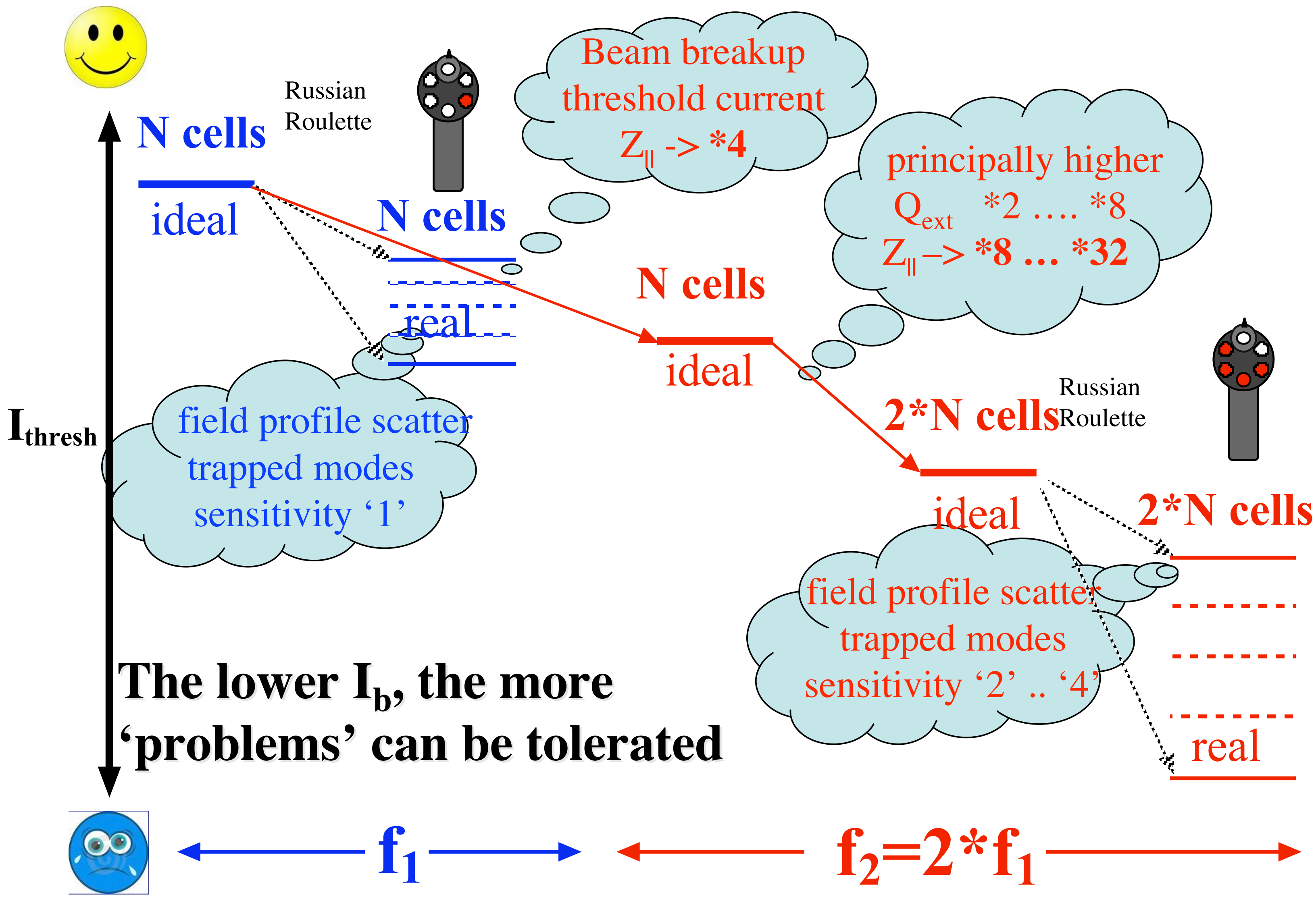
But still not possible to separate the accelerating π-mode (352.2 MHz) and the 3π/4 mode at about 351.2 MHz (Δf= 1 MHz) by 'classical' means to sufficient attenuation.

Use trick: cable roll that was $M*\lambda_{\pi}$ long for the π-mode and $(M-1/2)*\lambda_{3\pi/4}$ long for the 3π/4 mode, creating another factor (-1)

Worked well but demanding ….

—> **if possible keep fundamental passband modes as far as possible apart in f and only few of them —> low cell number**

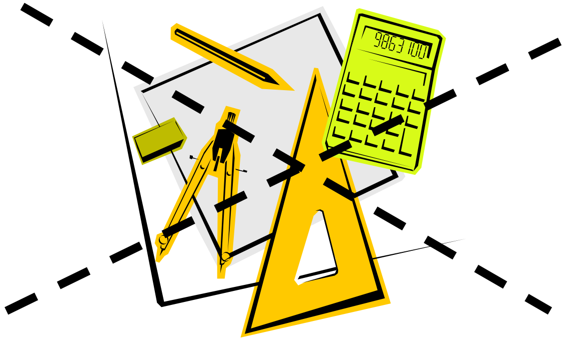
Conclusion(1) : Threshold Current



Conclusion(2) : Other Aspects

- **stiffer** bare cavity at higher f (same Nb sheet thickness)
- (possibility to) **cool** (hook type) HOM couplers **by conduction**
- **1/4 wasted energy** to charge up cavity before beam
- **complicating** the fast RF vector feedback design/prod./setting

The decision 700/1400 MHz – conc. HOMs, impedances, .. – is **NOT a clear-cut engineering decision** but has aspects of **a stock-market type decision: risk against benefit**

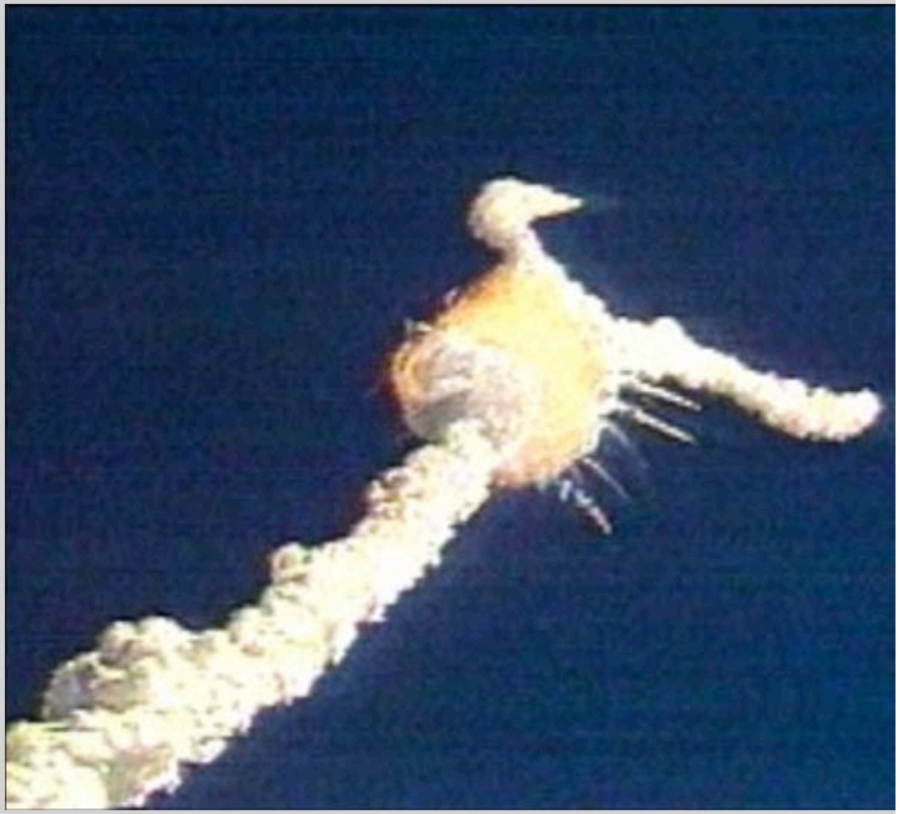


Hedge Fund	Blue Chip	T-Bond
	intel Leap ahead	
14.08%	**7.04 %**	**3.52%**

?

the % numbers are purely accidental and any resemblance to ….

Challenger Accident <u>28 Jan. 1986</u>



On January 28, 1986 America was shocked by the destruction of the space shuttle Challenger, and the death

National Aeronautics and Space Administration

NASA

George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama
35812

EP25 (79-13) January 19, 1979

TO: EE51/Mr. Eudy

FROM: EP25/Mr. Miller

SUBJECT: Evaluation of SRM Clevis Joint Behavior

As requested by your memorandum, EE51 (79-10), Thiokol documents TWR-12019 and letter 7000/ED-78-484 have been revaluated. We find the Thiokol position regarding design adequacy of the clevis joint to be completely unacceptable for the following reasons:

a. The large sealing surface gap created by excessive tang/clevis relative movement causes the primary O-ring seal to extrude into the gap, forcing the seal to function in a way which violates industry and Government O-ring application practices.

b. Excessive tang/clevis movement as explained above also allows the secondary O-ring seal to become completely disengaged from its sealing surface on the tang.

c. Contract End Item Specification, CPW1-2500D, page I-28, paragraph 3.2.1.2 requires that the integrity of all high pressure case seals be verifiable; the clevis joint secondary O-ring seal has been verified by tests to be unsatisfactory.

Questions or comments concerning this memorandum should be referred to Mr. William L. Ray, 3-0459.

John Q. Miller

John Q. Miller
Chief, Solid Motor Branch

First warning on deficient seal: <u>Jan. 1979</u>

when politics, ‘bean counting’, ….
collides with ‘too conservative’ engineers …

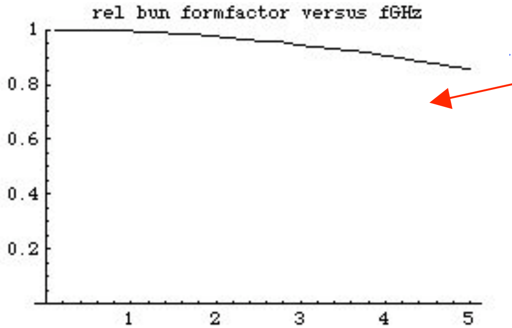
Thank you for your attention!

Appendix:

Induced voltage, impedance extracted power

Appendix: Induced voltage, impedance, extracted power

Induced voltage by single bunch train of M bunches: regular inter-bunch time T_B ; mode frequency ω; FIELD damping time $\tau_F = 2\omega / Q_{tot}$



Relative bunch form factor [0, 1] versus f in [0, 5 GHz]

A. Lombardi : 4σ-BL ±4.5° @ 352 MHz. No relief: $f_B \approx 1$

—> **short bunches**

$\Delta V = q\,\omega\,(R/Q)$ per bunch (scales with f !!)

$$V_M = \Delta V \cdot \sum_{m=0}^{m=M-1} \exp\left[(i\omega - 1/\tau_F) \cdot T_B \cdot m \right] = \Delta V \cdot \sum_{m=0}^{m=M-1} \rho^m = \Delta V \cdot \frac{1-\rho^M}{1-\rho}$$

$$\rho = \exp\left[(i\omega - 1/\tau_F) \cdot T_B \right] \qquad (geometrical\ \ series\ \ |\rho| < 1\ )$$

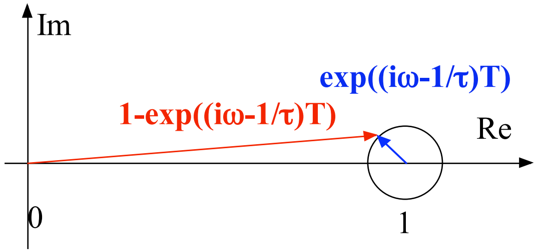
$$if\ \ train-length\ \ M \cdot T_B >> \ \tau_F : \ \ \rho^M \to 0 \ \ V_M = V_\infty = \frac{\Delta V}{1-\rho}$$

Example: f=2GHz, Q_{ext}=**100** –> **τ_F= 16 ns >> T_B=2.8 ns** (352MHz)

bunches are 'always' coupled —> **(only) 352 MHz multiples are true 'machine lines'**

1) 'strong' damping = **field mainly decays during T_B : T_B/τ_F 'large'**

$$\boxed{q = \exp\left[(i\omega - 1/\tau_F)\cdot T_B\right] \;=\; \varepsilon \cdot \exp(i\omega \cdot T_B); \quad \varepsilon = \exp(T_B/\tau_F) << 1}$$



(1-q) does not get close to zero:
<u>no resonant effect</u>

2) 'week' damping: **field mainly 'survives' during T_B : T_B/τ 'small'**

i.e. $\boxed{\exp(T_B/\tau_F) \;\approx\; 1}$

If **also** f close to multiple of $1/T_B$ = 352 MHz: $(\omega-\delta\omega)T_B = 2\pi\cdot n$

—> use $\exp(x) \approx 1 + x$ for small $|x|$

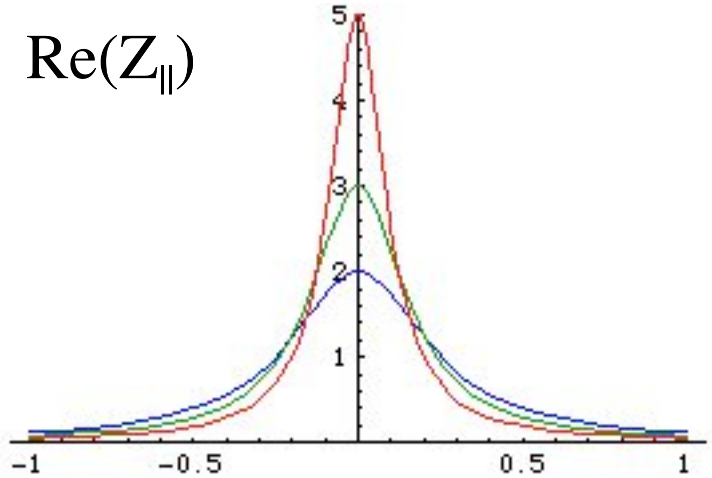
$$\boxed{V_\infty = \frac{\Delta V}{1-\rho} = \frac{q\cdot\omega\cdot(R/Q)}{1-\exp\left[(i\omega - 1/\tau_F)\cdot T_B\right]} \approx \frac{q}{T_B}\,\frac{\omega\cdot(R/Q)}{i\delta\omega\cdot -1/\tau_F}}$$

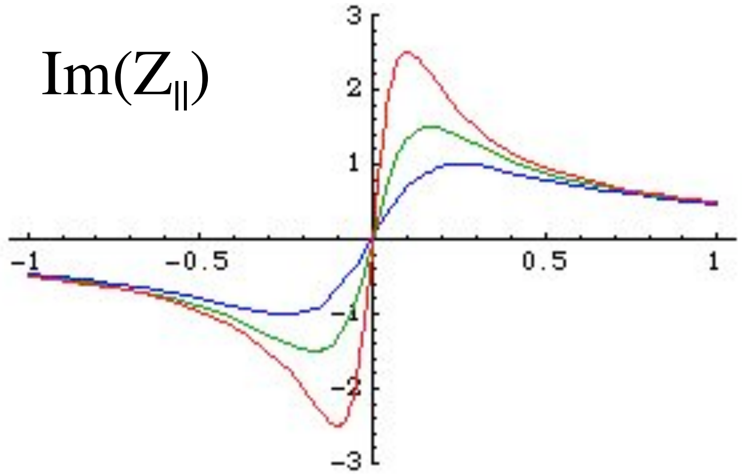
'stable field' (no large 'ripple')

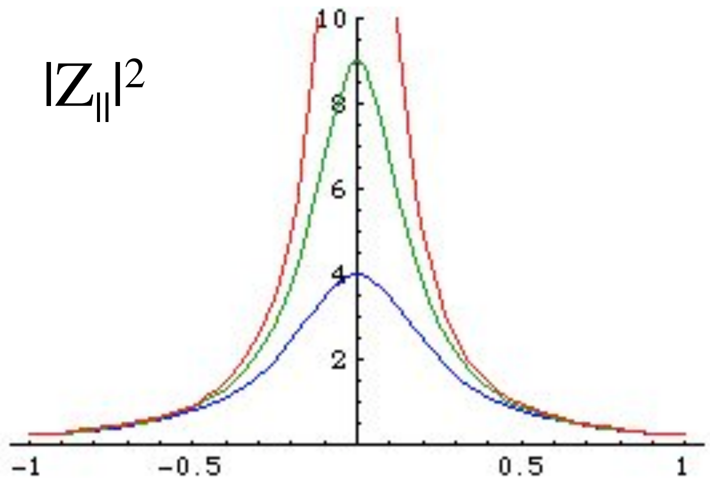
(1-q) gets close to zero at any ML ($f = n/T_B$) : **<u>resonant effect</u>**

express τ by Q_{tot} = Q_{ext}, and $q/T_B = I_{b,DC}$ $\quad I_{RF}=2*I_{b,DC}$

$$V_\infty \approx 2 \cdot I_{b,DC} \frac{(R/Q) \cdot Q_{ext}}{i \cdot (2 \cdot \delta\omega \cdot Q_{ext}/\omega) - 1} \Rightarrow Z_{\|}(\omega) = \frac{(R/Q) \cdot Q_{ext}}{i \cdot (2 \cdot \delta\omega \cdot Q_{ext}/\omega) - 1}$$







high/medium/low Q_{ext}

Extracted power:

to be transported by the coupler and digested by the load

and replaced by the accelerating field

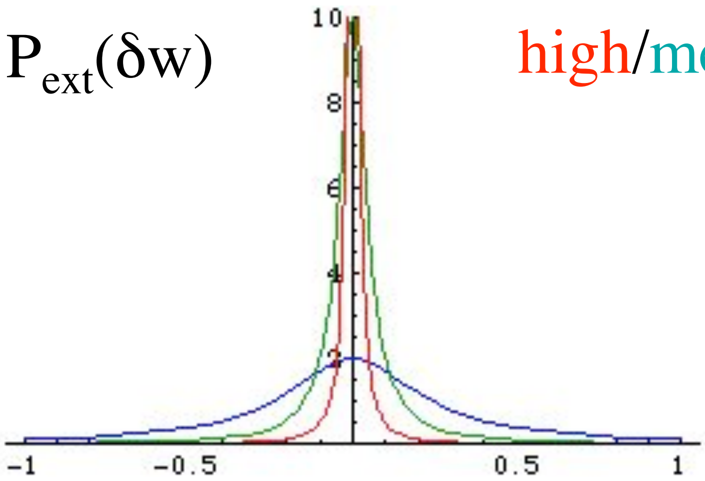
For 'stable field'

$$P_{ext}(\delta\omega) = \frac{|V_\infty|^2}{2\cdot(R/Q)\cdot Q_{ext}} = \frac{2\cdot(R/Q)\cdot Q_{ext}\quad I_{b,DC}^2}{1 + \left(2\delta\omega\ \ Q_{ext}/\omega\right)^2}$$

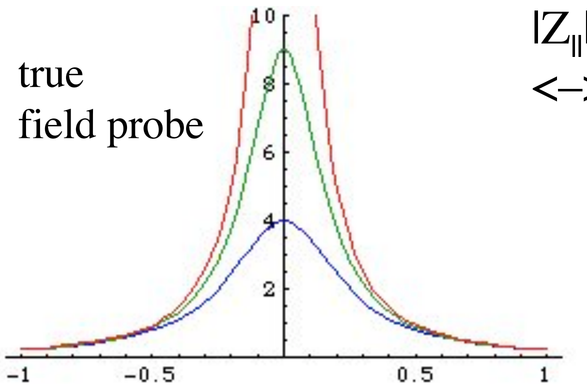
P_{ext} is ω-independent: coupler P-density ***4**, fields (arcing) ***2** !!

Total power extracted from beam **≥*2** (to be replaced by main RF)

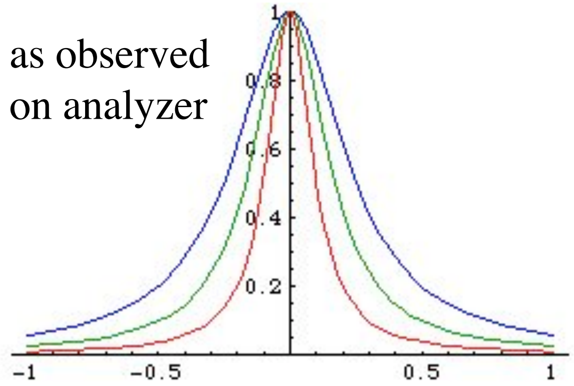
$P_{ext}(\delta w)$



high/medium/low Q_{ext}



$|Z_{\parallel}|^2 \Longleftrightarrow P_{probe}$
$\Longleftrightarrow$ (cavity field)^2



high/medium/low Q_{ext}

high/medium/low Q_{ext}

Reality

… and when people adapt the **‘amplitude reference level’ of their spectrum analyzer**

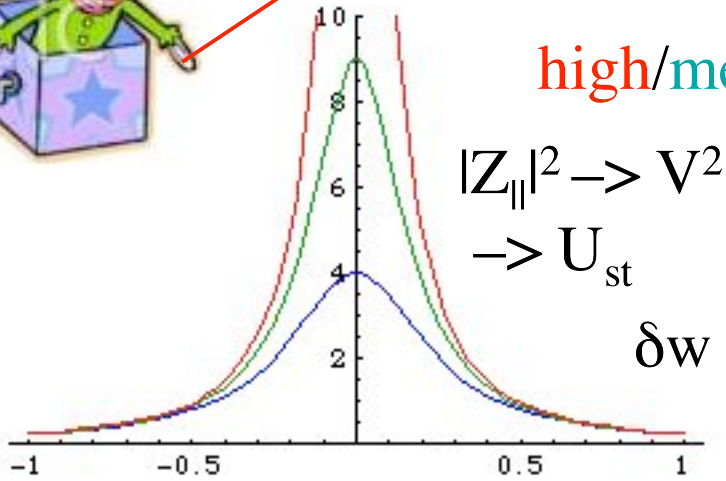
“You ‘never’ hit the **sharp line** of a **high-Q mode**”|[1]:

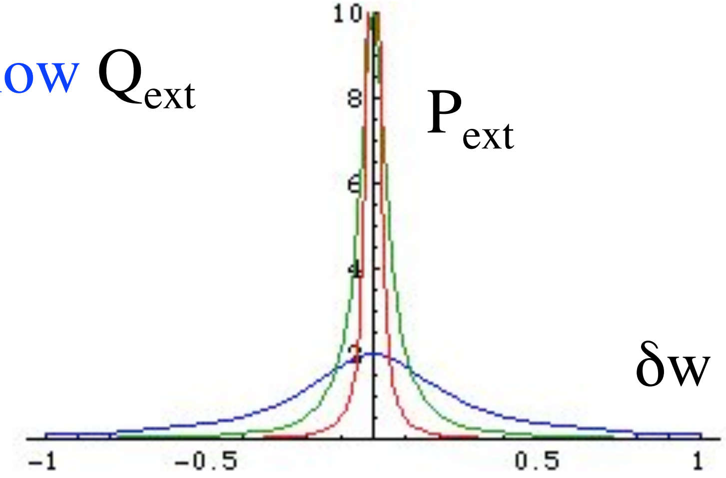
Nonsense !!!

higher Q_{ext} —> higher induced field at ANY frequency

|[1] Ernst Haebel got ‘ballistic’ each time that this ‘fact’ was ‘re-discovered’ about all 2 years by new people (joining the field) ….

Why HOM couplers at all ?





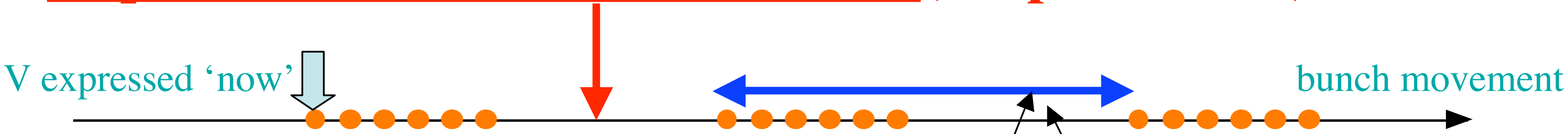
<u>Extracted power can be smaller</u> for higher Q_{ext} for 'larger' $\delta\omega$
BUT: <u>induced cavity field, U_{st}</u> <u>always larger</u> for higher Q_{ext}

Energy conservation, NO power conservation: higher Q_{ext} confines stripped beam energy longer in cavity; possible coupling train to train

This field (energy)

– may decelerate coming particles more: more stripped beam energy
– changes $V_{acc,tot}$ unpredictable (RF feedback only on main mode)
– makes additional cryo losses
– sneaks over MC and circulator (built for f_0) to klystron 

Sequence of L trains of M bunches (coupled trains)



Example: f=2 GHz, Q_{ext}=**10’000’000** –> **τ_F= 1.6ms << T_T=20ms** (50 Hz)

trains ‘always’ uncoupled —> **‘50 Hz multiples’ exist ONLY ON PAPER: NO PROBLEM**

do same ‘trick’ as before …. based on single train voltage V_M :
(train-repetition time T_T)

$$V_{M,L} = V_M \cdot \sum_{l=0}^{l=L-1} \exp\left[(i\omega - 1/\tau)\cdot T_T \cdot l\right] = V_M \cdot \sum_{l=0}^{l=L-1} \hat{\rho}^l$$

$$\hat{\rho} = \exp\left[(i\omega - 1/\tau)\cdot T_T\right] \qquad (geometrical \quad series \quad |\hat{\rho}|<1\ )$$

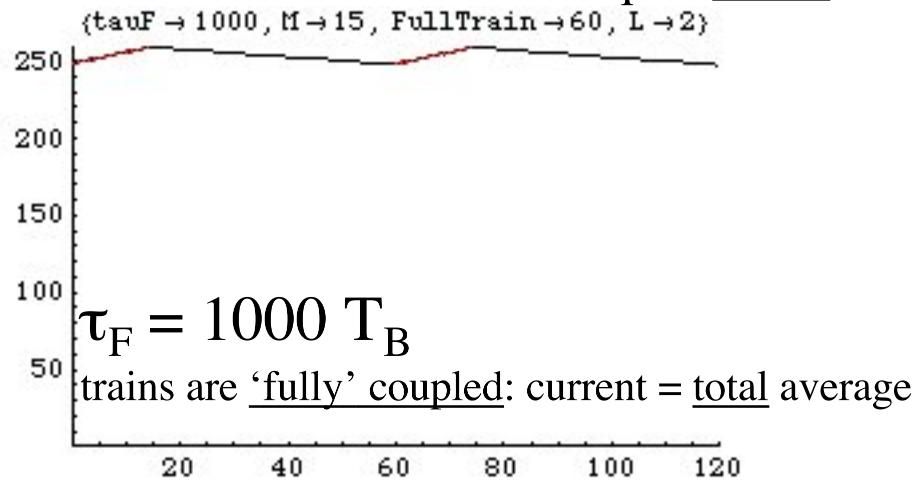
$$V_{M,L} = V_M \cdot \frac{1-\hat{\rho}^L}{1-\hat{\rho}} = \Delta V \frac{1-\rho^M}{1-\rho}\frac{1-\hat{\rho}^L}{1-\hat{\rho}} \underset{L\to\infty}{\Rightarrow} V_{M,\infty} = \Delta V \frac{1-\rho^M}{1-\rho}\frac{1}{1-\hat{\rho}}$$

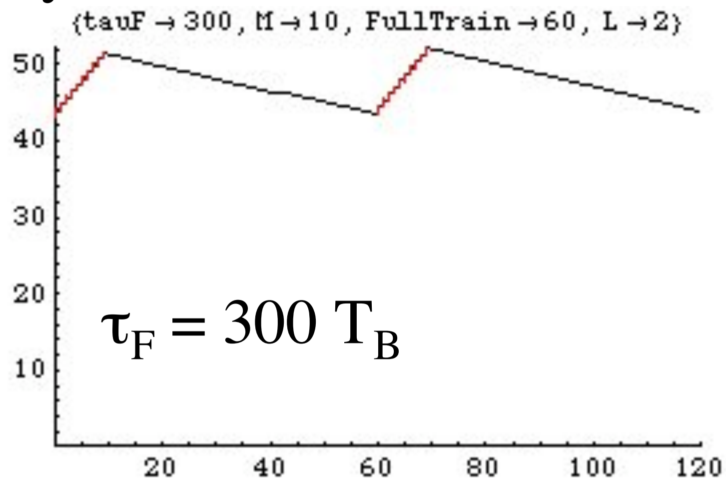
$$if \quad train-length \quad M\cdot T_B \quad >> \quad \tau: \quad \rho^M \to 0 \quad \Rightarrow \quad V_{M,\infty} = \frac{\Delta V}{(1-\hat{\rho})(1-\rho)}$$

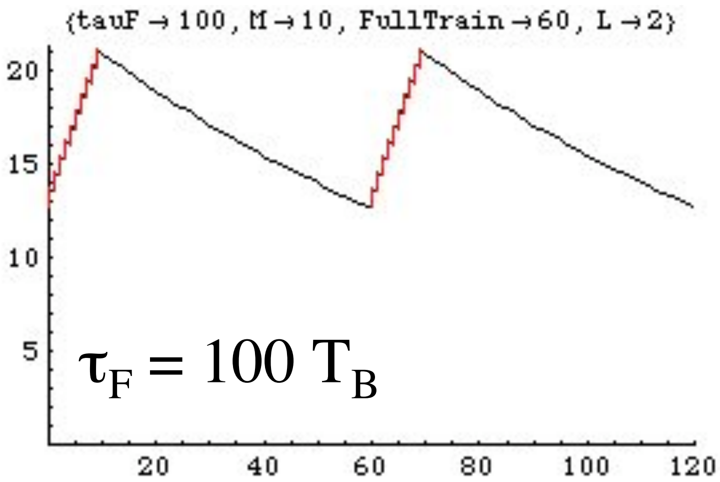
Result = product of ‘envelope functions’

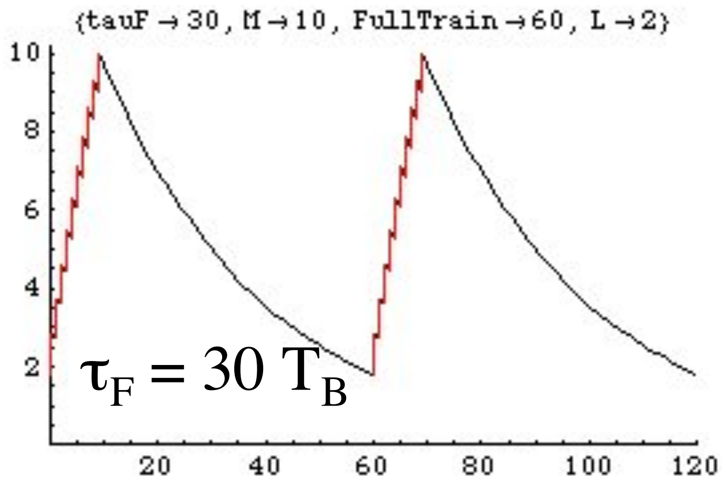
Example: Bunch-trains with **60 places :** {**15 bunches** , **45 voids**}

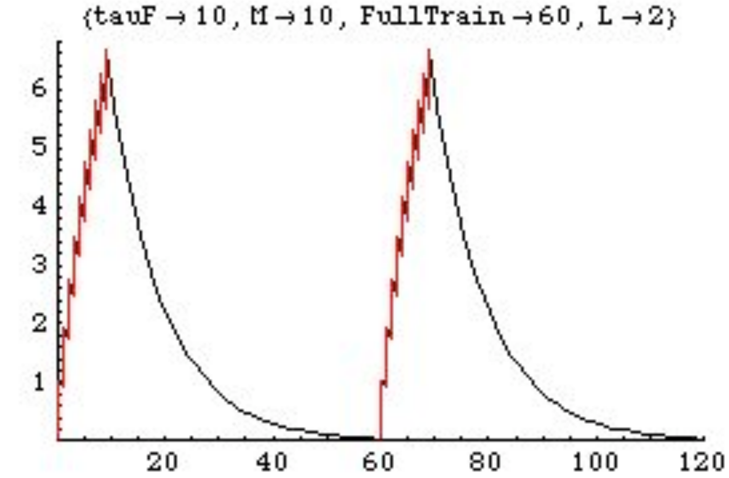
τ_F = field decay time



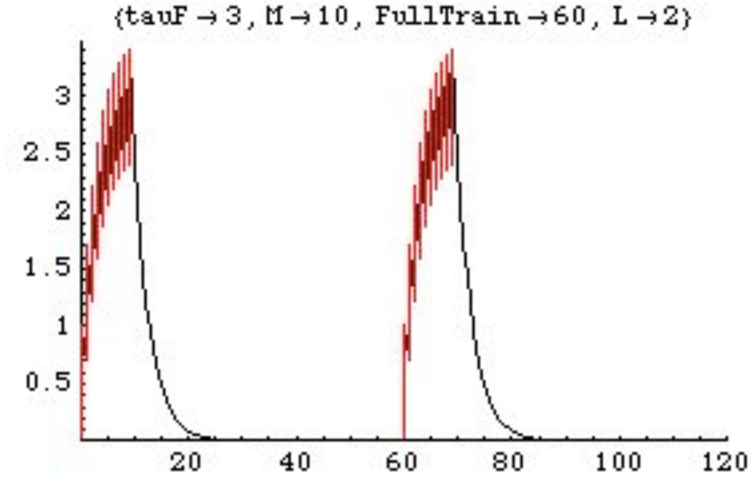




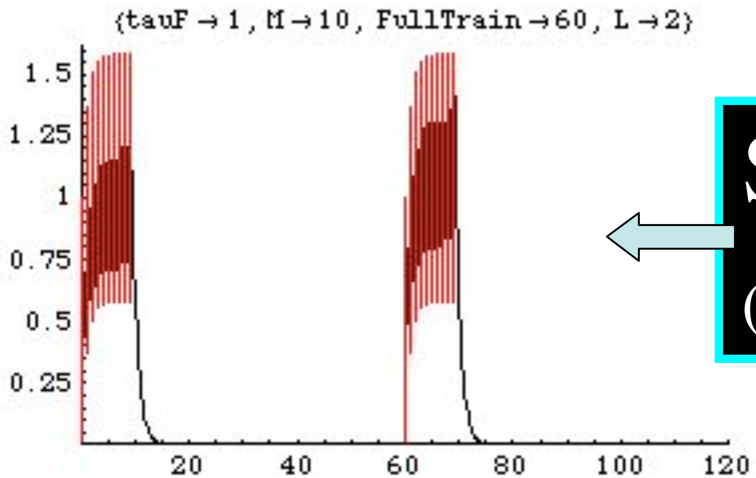




$\tau_F = 10\ T_B$



$\tau_F = 3\ T_B$



$\tau_F = 1\ T_B$

trains are <u>'fully' decoupled</u>:
current = <u>train average</u>

SPL case (T_B= 2.9 ns, T_T=50ms)

(except for extremely high Q_{ext})

If field is not stable **('ripple')** average power (over repetition period T_R) has to be expressed as

$$\boxed{\langle P_{ext} \rangle = \frac{1}{T_R} \int_0^{T_R} dt \cdot P_{ext}(t) = \frac{1}{2 \cdot (R/Q) \cdot Q_{ext} \cdot T_R} \int_0^{T_R} dt \cdot |V(t)|^2}$$

For SPL (except very high Q_{ext} modes) bunch-trains are separated ($T_R = T_T$) and about 'rectangular power profile'

$$\boxed{\langle P_{ext} \rangle \approx d \, \frac{2 \cdot (R/Q) \cdot Q_{ext} \quad I^2_{b,on \;\; train}}{1 + \left(2\delta\omega \;\; Q_{ext} / \omega\right)^2}}$$

d=duty-cycle (5%)

$I_{b,\text{on train}} = q/T_B$ the current during the pulse (40 mA… 64 mA)