Gravitational microlensing and the finite dimensions of the stars

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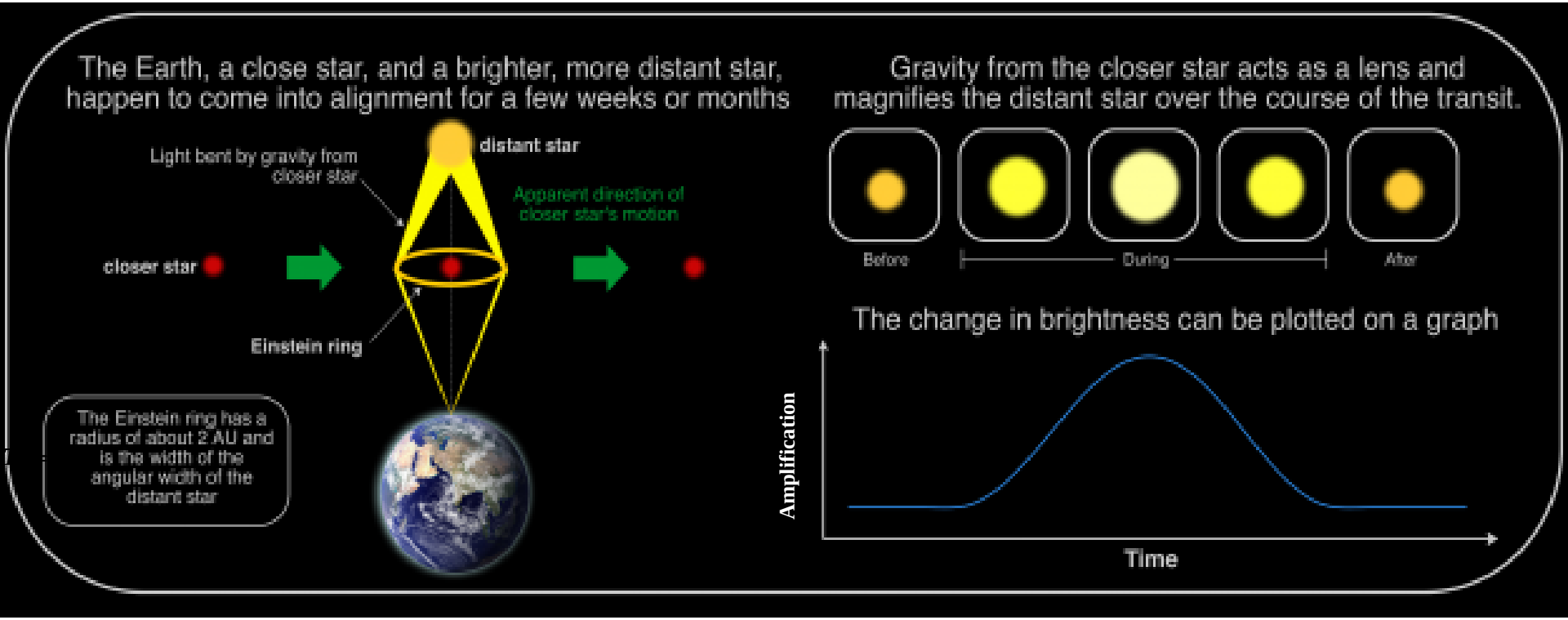
Gravitational microlensing



$$R_E(M, D_L) = D_L \sqrt{\frac{4GM}{c^2} \frac{D_{LS}}{D_L D_S}}$$

$$A(u) = \frac{u^2(t) + 2}{u(t)\sqrt{u^2(t) + 4}}$$

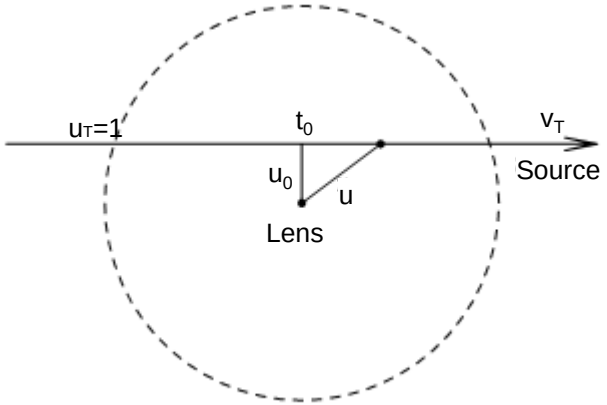
$$T_E = \frac{R_E(M, D_L)}{\mathrm{v}_T}$$

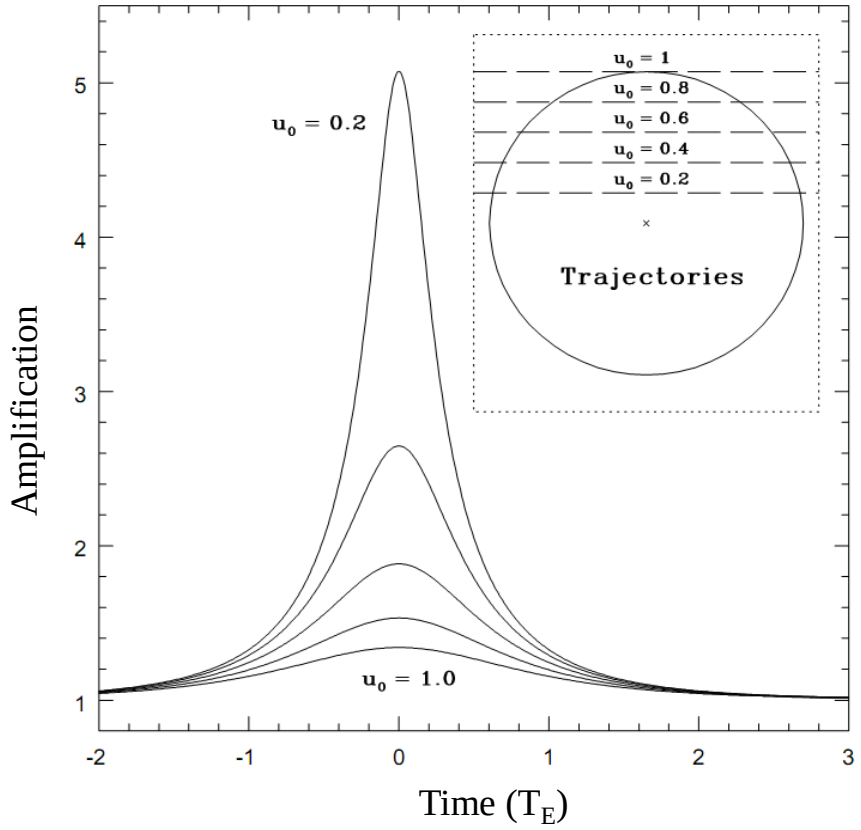
Gravitational microlensing

Measurable parameters are: u_0, t_0, T_E

$$u(A) = \sqrt{2}\left[\left(\frac{A^2}{A^2-1}\right)^{1/2} - 1\right]^{1/2}$$

$$u(t) = \sqrt{u_0^2 + \left(\frac{t-t_0}{T_E}\right)^2}$$





Gravitational microlensing

Anomalies in the light curves

- Parallax effect
- Binary lens
- Finite source effects

The limb-darkening coefficients (LDC) laws by the stellar atmosphere models

- Linear $I(\mu) = I(1)[1 - U(1-\mu)]$
- Quadratic $I(\mu) = I(1)[1 - a(1-\mu) - b(1-\mu)^2]$
- Square root $I(\mu) = I(1)[1 - c(1-\mu) - d(1-\sqrt{\mu})]$
- Logarithmic $I(\mu) = I(1)[1 - e(1-\mu) - f\,\mu \ln \mu]$
- New non-linear $I(\mu) = I(1)[1 - a_1(1-\mu^{1/2}) - a_2(1-\mu) - a_3(1-\mu^{3/2}) - a_4(1-\mu^2)]$

$\mu = \cos(\theta)$, where θ is the angle between the line of sight and the emergent intensity.

The last law describe the specific intensity at any part of the disk, better than the others

Free-floating planets (FFPs)

- **Population of objects with M<0.01Ms**

  Direct observations in Sigma Orionis and Taurus
  Gravitational microlensing towards Galactic Bulge

- **Unbound to a host star or very distant (over 100 AU)**

- **Their origin is uncertain**

  Believed that these objects may have formed in proto-planetary disks and subsequently scattered into unbound or very distant orbits, becoming FFPs.

- **The gravitational microlensing, is the only way, to detect these objects at distances larger than a few tens of parsecs.**

EUCLID Mission (2018)

- Gravitational microlensing observations for 10 months

- Euclid will observe towards the Galactic center (l =1.1^0 , b=-1.7^0)

- Field of view is 0.54 deg^2

- Cadence is 20 min

- The threshold amplification A_{th} = 1.001.

- The photometric error, 0.1%

Mass function of FFPs

Sumi et al. 2011 by the detailed analysis of the MOA-II 2006-2007 data and from the statistical investigation of the event timescale distribution revealed a significant excess of events with t < 2 days, compared to prediction.

For stars and brown dwarfs they consider a power-law mass function continuous in overall interval of masses.

$$\frac{dN}{dM} \propto M^{-\alpha} = \begin{cases} \alpha = 2 & for \quad 0.7M_S < M < M_S \\ \alpha = 1.3 & for \quad 0.08M_S < M < 0.7M_S \\ \alpha_{BD} = 0.49^{+0.24}_{-0.27} & for \quad 0.01M_S < M < 0.08M_S \end{cases}$$

They proposed with a high confidence level the power-law mass function for FFPs

$$\frac{dN}{dM} = k_{PL} M^{-\alpha_{PL}} \qquad \alpha_{PL} = 1.3^{+0.3}_{-0.4} \qquad 10^{-5} M_S < M < 10^{-2} M_S$$

Spatial distribution of FFPs

For free-floating planets we considerate two models:

a) triaxial bulge with mass density given by

$$\rho(M,x,y,z)=\rho_0(M)e^{-s^2/2} \qquad s^4=(\frac{x^2}{a^2}+\frac{y^2}{b^2})^2+\frac{z^4}{c^4}$$

$$a=1.49kpc$$
$$b=0.58kpc$$
$$c=0.40kpc$$

b) double exponential disk with mass density given by

$$\rho(M,R,z)=\rho_0(M)e^{-|z|/H}e^{-(R-R_0)/h}$$

with parameters for thin components

$R_0=8.5kpc$ $h=3.5kpc$ $H=300\,pc$

with parameters for thick components

$R_0=8.5kpc$ $h=3.5kpc$ $H=1000\,pc$

Velocity distribution of FFPs

We assume that the velocity distribution follow a Maxwellian form (Han & Gould)

$$f(v_i) \propto \exp[-\frac{(v_i - \overline{v_i})^2}{2\sigma_i^2}] \qquad i \in \{x, y, z\}$$

Bulge $\overline{v_z} = \overline{v_y} = 0$ $\sigma_z = \sigma_y = 100 km/s$

Thin disk $\overline{v_z} = 0$ $\overline{v_y} = 220 km/s$ $\sigma_z = \sigma_y = 30 km/s$

Thick disk $\overline{v_z} = 0$ $\overline{v_y} = 220 km/s$ $\sigma_z = \sigma_y = 50 km/s$

Finite source effects

In microlensing events, in which, $u_0 \lesssim \rho_*$, the point-source approximation is not valid anymore. In these events, the lens may pass over a source disk, providing a chance to measure:

- **The limb-darkening profile of a remote source**,
- **The angular Einstein radius of the lens** $\theta_E = \frac{R_E}{D_L}$

Finite source effects

When the lens pass over the source disk, at the moment of the entrance (exit) of the lens into (from) the source disk, in the light curve exhibits inflections. The duration between them is

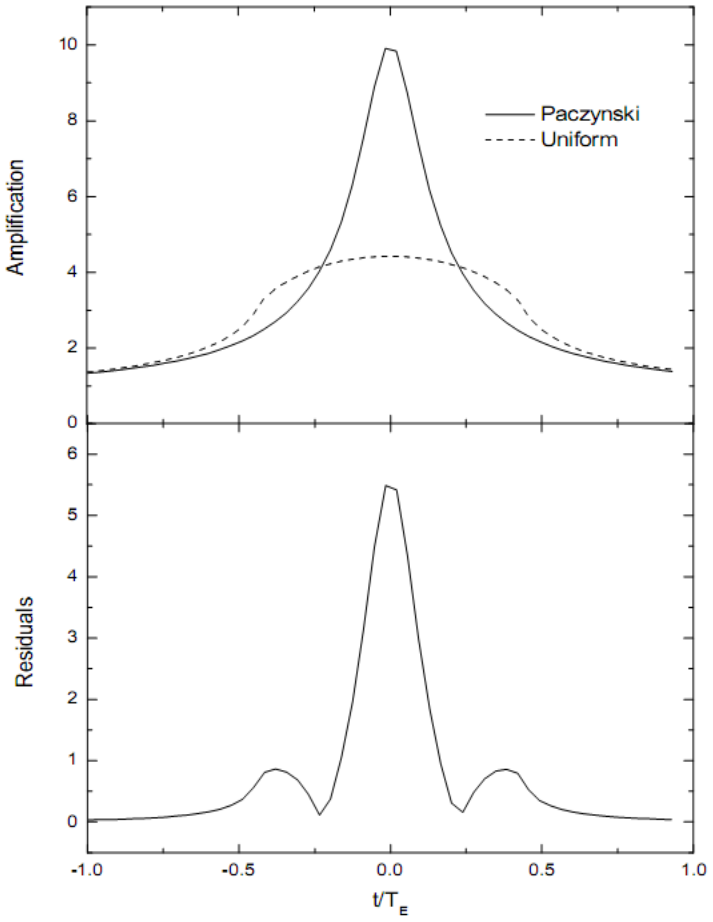
$$T = 2\sqrt{\rho_*^2 - u_0^2}\,T_E$$

By this relation can be defined ρ_*

With the additional information about the angular source size θ_* , can be define the angular Einstein radius by

$$\theta_E = \frac{\theta_*}{\rho_*}$$

Since θ_E does not depend on v_T , the physical parameters of the lens can then be better constrained.



Up. The amplification of a microlensing event. The event parameters are: $u_0 = 0.1$, $\rho_* = 0.46$, $T_E = 9.2$h. The duration of the lens transit over the source disk turns out to be $T \simeq 0.9T_E \simeq 8.2$ hours,

Down. The residuals between the two curves.

The amplification of finite source effects

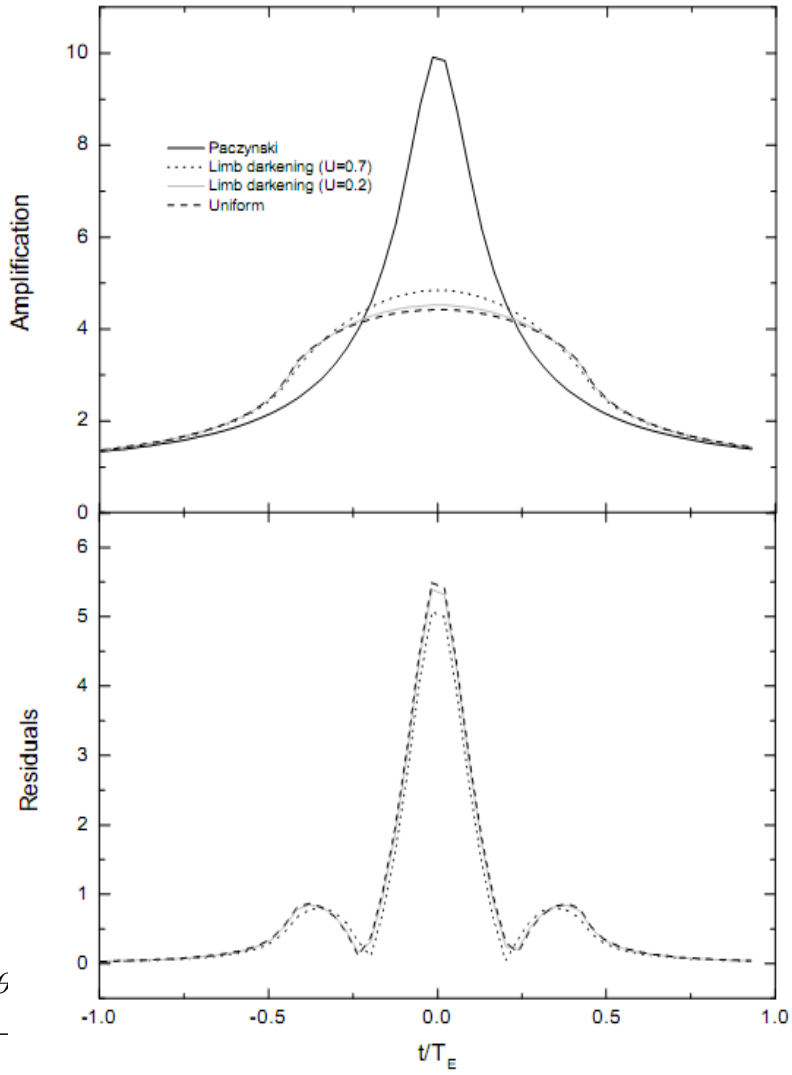
The amplification in light curve when the source brightness is uniform.

$$A_{fs}(u) = \frac{1}{\pi\rho_*^2}\int_0^{\rho_*}\int_0^{2\pi} A(u')rdrd\theta =$$

$$= \frac{1}{\pi\rho_*^2}\int_0^{\rho_*}\int_0^{2\pi} \frac{u^2+r^2-2ur\cos\theta+2}{\sqrt{u^2+r^2-2ur\cos\theta}\sqrt{u^2+r^2-2ur\cos\theta+4}} rdrd\theta$$

The amplification in light curve when the source brightness is given by a linear limb-darkening profile, $I(\mu) = I(1)[1-U(1-\mu)]$

$$A_{fs}(u) = \frac{\int_0^{\rho_*}\int_0^{2\pi} \frac{u^2+r^2-2ur\cos\theta+2}{\sqrt{u^2+r^2-2ur\cos\theta}\sqrt{u^2+r^2-2ur\cos\theta+4}}[1-U(1-\sqrt{\frac{\rho_*^2-r^2}{\rho_*^2}})]rdrd\theta}{2\pi\int_0^{\rho_*}[1-U(1-\sqrt{\frac{\rho_*^2-r^2}{\rho_*^2}})]rdr}$$



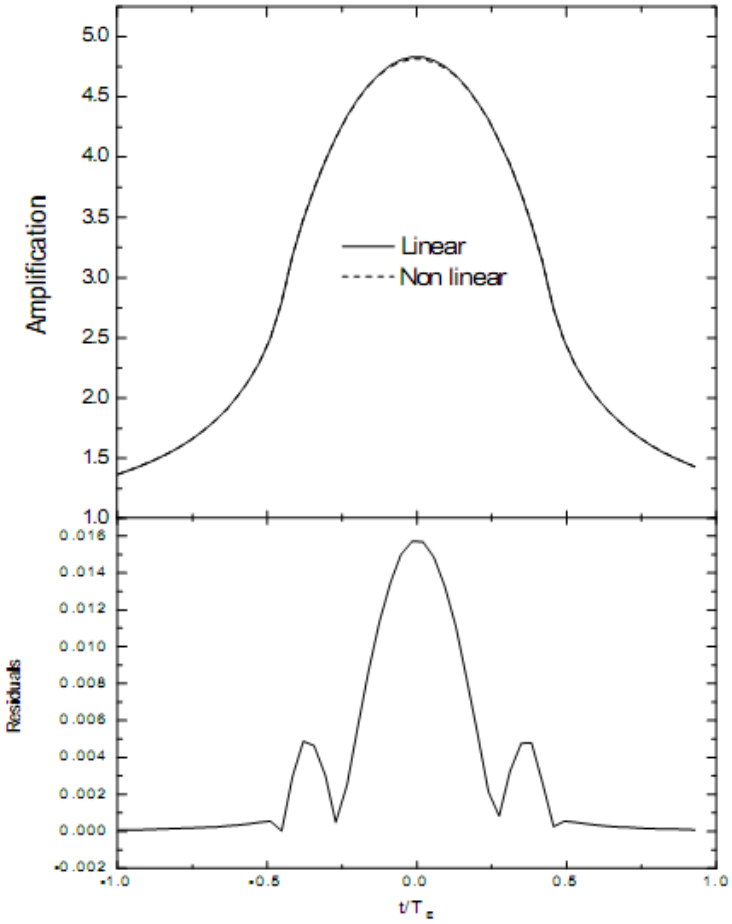
Conditions of observing the finite source effects

A microlensing event will be detected if there are at least eight consecutive points with amplification bigger than A_{th}=1.001. Since the cadency is 20 min, the event have to a duration longer than 2.67 h to be detectable.

Since the photometric error of Euclid is 0.1% we find that the residuals between the standard and the finite source curve have to be $Res=|A_s-A_{fs}|=0.001$.

The finite source effects are detected when there are at least 8 points around the event peak in which Res >0.001

The residuals at the peak is up to 0.32%, so the different limb darkening profiles are distinguishable by Euclid.



Up. The amplification of a microlensing event (u_0 = 0.1, ρ_* = 0.46) for linear LDC (U = 0.6866) and for four parameter non-linear power law (a1 = 0.7181, a2 = −0.5941, a3 = 1.1698 and a4 = −0.4558.
Down. The residuals between the two curves.

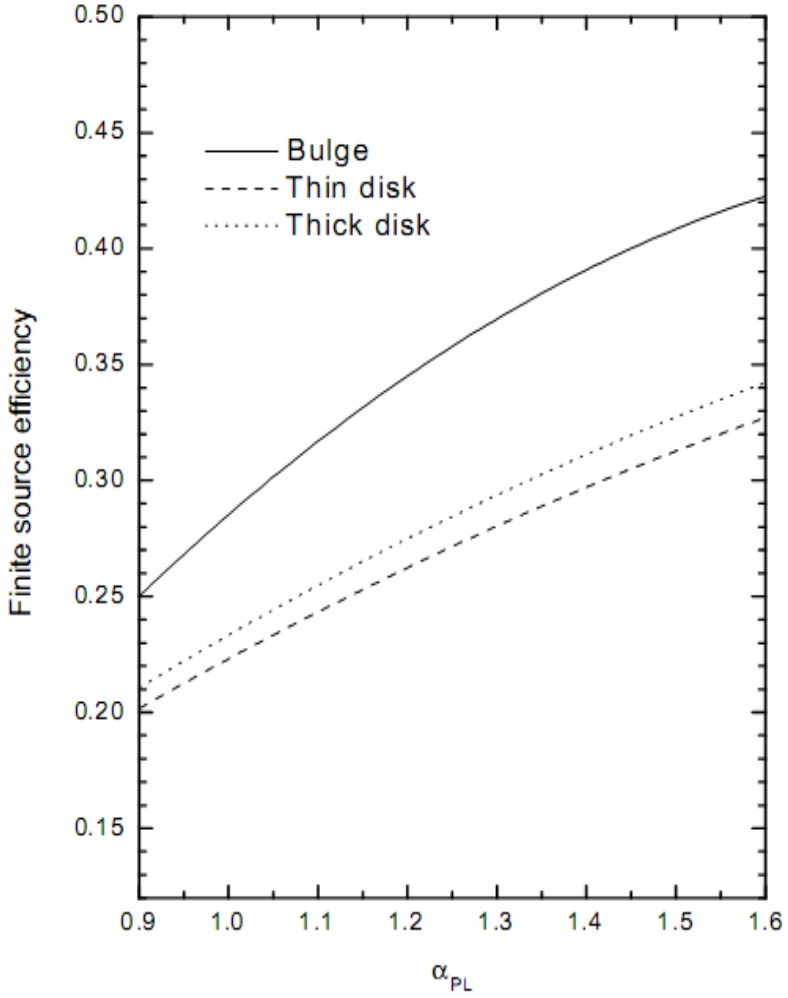
Monte Carlo simulations

We begin by drawing:

- The FFP distance from Earth D_L is extracted based on the disk and bulge, FFP spatial distributions along the Euclid line of sight ($b = 1.7^\circ$, $l = 1.1^\circ$).
- The event impact parameter u_0 is assumed to be uniformly distributed in the interval [0, 6.54].
- The FFP relative transverse velocity is extracted from the velocity distribution
- The lens mass is assumed to follow the mass function.
- By the free code available in the web site *http://iac-star.iac.es* we generate a synthetic stellar population.
- We use the site *http://vizier.u-strasbg.fr/viz-bin/VizieR-3?source=J/A%2bA/363/1081/phoenix* to find the four LDCs: a_1, a_2, a_3, a_4 of non-linear law.
- The source distances D_S, based on the above written bulge spatial distributions, result to be in the range 7 ÷ 10 kpc.

Efficiency of finite source effects

The efficiency for finite source effects is the ratio of the event's number with finite source effects with the total number of detectable events.

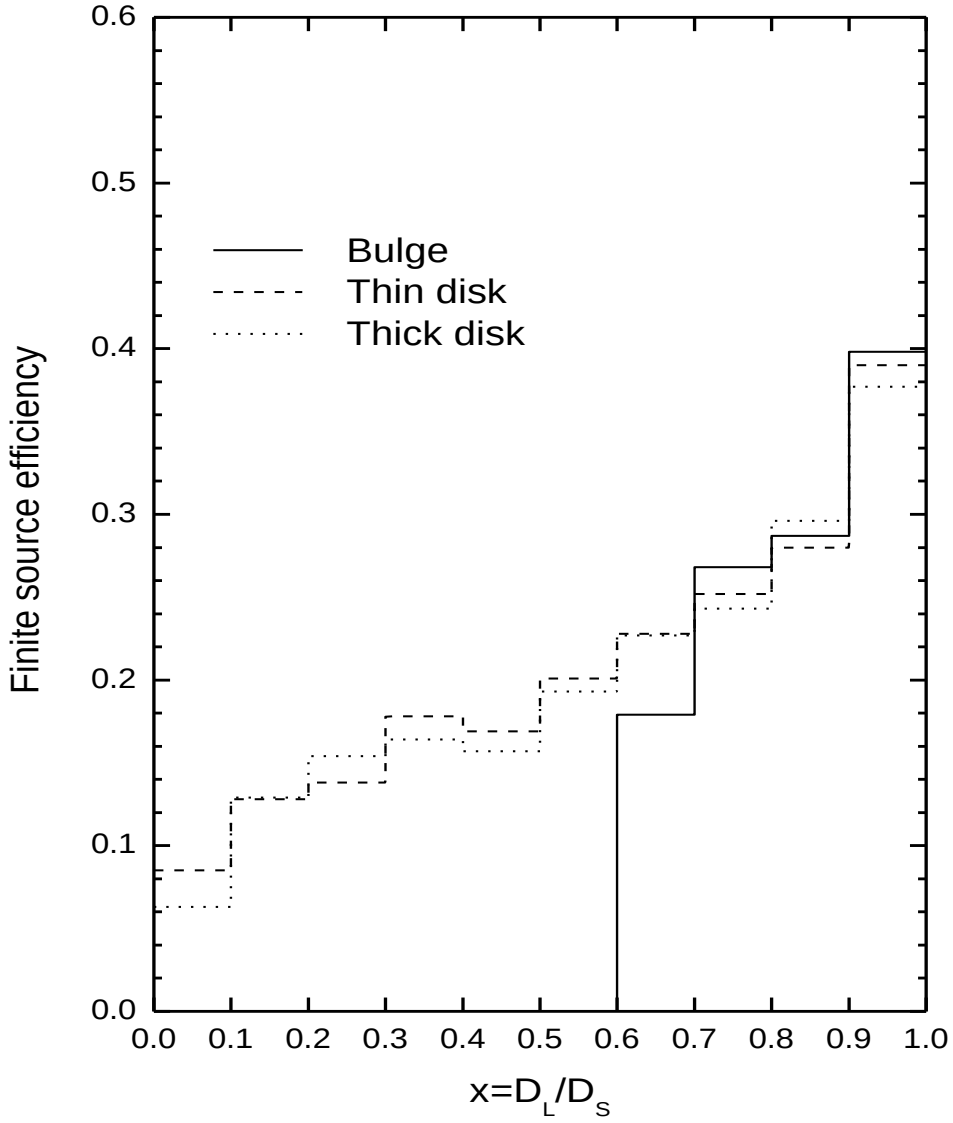


Finite source efficiency versus x

Simulating 1000 microlensing events for each defined bin range [0; 0.1]–[0.9; 1] and for $\alpha_{PL} = 1.3$, we calculate the finite source efficiency as defined above.

• The finite source efficiency increases with the x value.

•For FFPs in the thin and thick disks we note that the finite source efficiency slightly diminishes in the [0.4, 0.5] bin, because the R_E gets its maximum value at x = 0.5.



Conclusions

- We emphasize that the Euclid program will offer the unique opportunity to detect the finite source effects in microlensing events caused by FFPs.
- Using Monte Carlo numerical simulations, we investigate that 1/3 of observable events will have finite source effect detected.
- Since Euclid will detect a considerably large number of events caused by FFPs, the events with detectable finite source effects will allow to constrain the FFP mass and distances, which is a fundamental information necessary to investigate for FFP populations through out the Milky Way.
- Also the high-magnification microlensing events caused by FFPs are the unique possibility to investigate the Galactic bulge star atmospheres, and to define the source limb-darkening profile.

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