Black holes as tools to understand gravity

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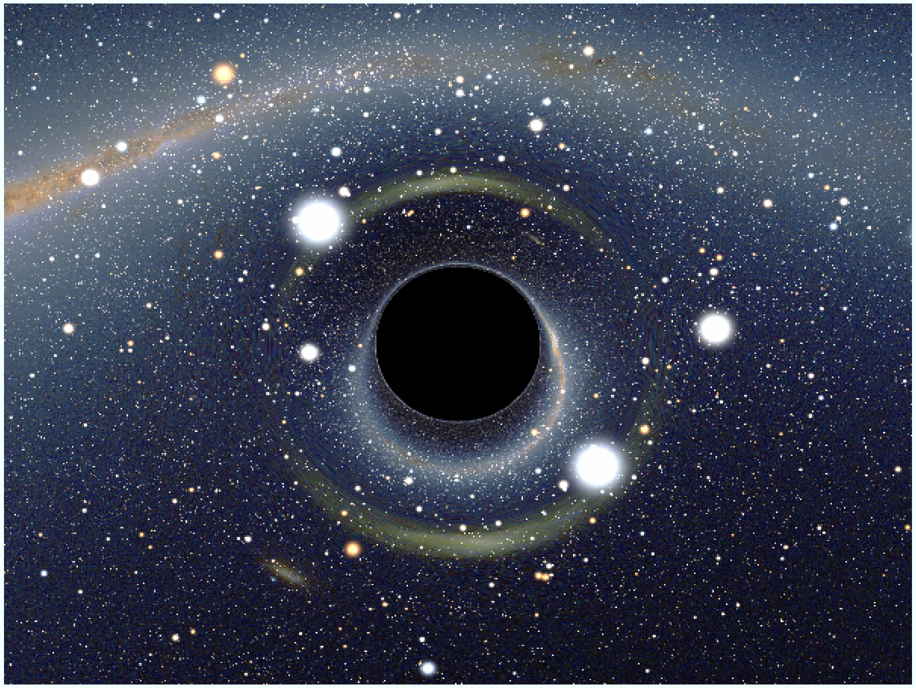
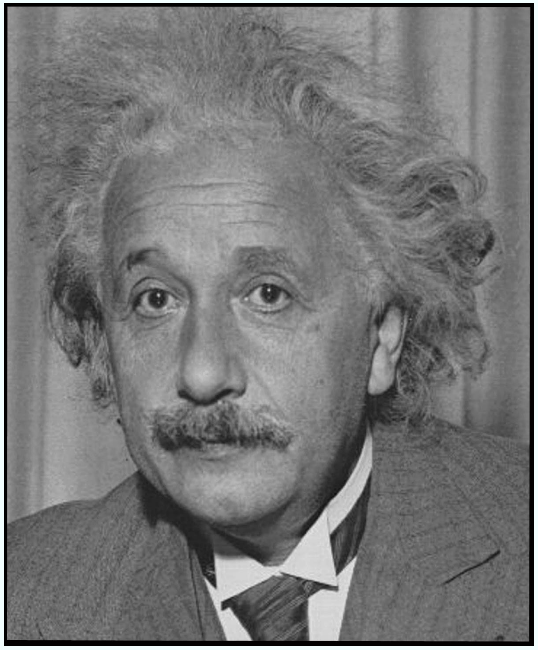


Numerical simulation by Alain Riazuelo (2007)

Shkodra, 7 Septembre 2014

1. Black holes are an outcome of general relativity



Albert Einstein (1879-1955)

David Hilbert (1862-1943)

Einstein's equations : $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ 25 November 1915

Matter "curves" spacetime

The Einstein equations, $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$,
are differential equations for 10 functions $g_{\mu\nu}(t, x, y, z)$

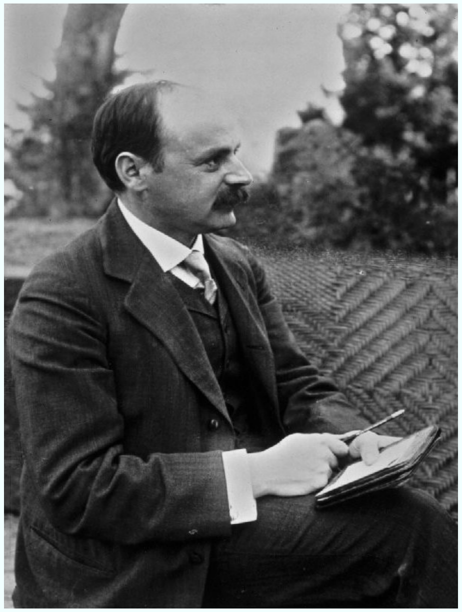
- The source of the gravitational field is the mass-energy of matter represented by a tensor, e.g. $T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu + p g_{\mu\nu}$
  ϵ and p are its energy density and pressure, u_μ its velocity.

- $g_{\mu\nu}$ is the metric of spacetime. It depends on time and location.
  It measures distance between points and duration between events
  $ds^2 = g_{xx}(t, x, y, z)dx^2 + g_{yy}(t, x, y, z)dy^2 + \cdots$
  (to be contrasted to Pythagoras theorem : $ds^2 = dx^2 + dy^2 + \cdots$)

- $G_{\mu\nu}$ is the Einstein tensor, a function of $g_{\mu\nu}$ and its derivatives,
  It is related to the Riemann tensor
  which describes the curvature of spacetime.

Schwarzschild's solution, december 1915 :

$G_{\mu\nu} = 0$, static, spherically symmetric case



Karl Schwarzschild
(1873-1916)

- $ds^2 = -\left(1 - 2m/r\right) dt^2 + \frac{1}{1-2m/r} dr^2 + r^2 d\Omega^2$

$m \equiv GM/c^2$ where M is the mass of the star

- at $r = 2m$ the metric / gravitational appears to be singular :

Eddington's "magic circle",
Hadamard's "catastrophy"

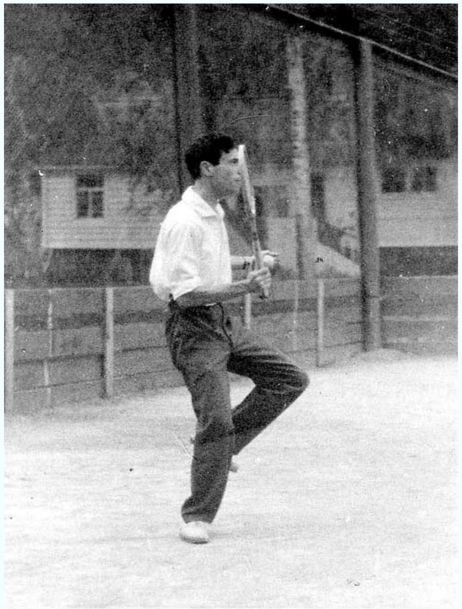
- However, for "normal stars", e.g. the Sun :

$R_\odot = 7 \times 10^5 kms$; $m_\odot = 1.5\, kms$; hence $R_{\mathrm{star}} \leq 2\, m_{\mathrm{star}}$ is "unrealistic".

$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$: Input from quantum physics

Not only compact stars can exist
but sufficiently dense and compact stars, must collapse

Chandrasekhar
(1910-1995)



Lev Landau
(1908-1968)

Robert Oppenheimer
(1904-1967)

Gravitational collapse :
Oppenheimer Snyder (Phys Rev, September 1st, 1939)

"When all thermonuclear sources of energy are exhausted a sufficiently heavy star will collapse...

Light from the surface of the star is progressively reddened ...

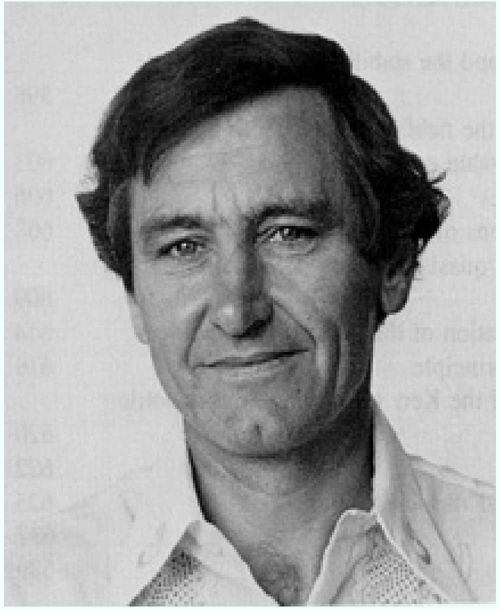
An external observer sees the star asymptotically shrinking to its gravitational radius."

The total time of collapse for an observer comoving with the stellar matter is finite...

Birth date of "black hole" concept

Today the "gravitational radius" $r = 2m$ is called an "horizon"

$G_{\mu\nu} = 0$: The Kerr Black Hole, 1963 :



In Kerr-Schild coordinates :

$$ds^2 = d\bar{s}^2 + f(h_\mu dx^\mu)^2$$

$d\bar{s}^2$ is flat ST in spheroidal coordinates :

$$d\bar{s}^2 = -dt^2 + \frac{\rho^2}{(r^2+a^2)} dr^2 + \rho^2 d\theta^2 + (r^2+a^2)\sin^2\theta \, d\phi^2$$

with $\rho^2 = r^2 + a^2 \cos^2\theta$

h_μ is the null and geodesic vector :

$$h_\mu dx^\mu = dt + \frac{\rho^2}{(r^2+a^2)} dr + a \sin^2\theta d\phi$$

$G_{\mu\nu} = 0$ (linear in f !) gives $f = \frac{2mr}{\rho^2}$

This solution describes a rotating black hole (horizon at $g^{rr} = 0$)
of mass m and angular momentum $J = ma$
(When $J = 0$ the solution reduces to Schwarzschild's)

Some properties of Roy Kerr's solution

- **It is "unique"**. (From Carter, 1971, to Bunting, 1983)

The Kerr metrics (parametrised by their mass and angular momentum) are the only solutions which fulfill all the following conditions :

They solve Einstein's vacuum equations (in 4 dimensions)
They are asymptotically flat, axisymmetric and stationary
They possess an event horizon which is smooth and convex

- **It is an enormous reservoir of energy** :

Up to 29 % of its mass can in principle be extracted.
(Penrose, Wheeler, Christodoulou, Ruffini...)

A first lesson :

- Finding black holes solutions...

involves elaborate mathematical techniques in

– riemannian and differential geometry ; group theory and topology
– non-linear partial differential equations theory ; stability analysis

- Finding critical masses of heavy, neutron- stars...

involves a detailed knowledge of their equations of state, that is, nuclear physics

- Finding how to extract their energy...

involves a detailed analysis of astrophysical processes in their neighbourhood

Still a lot to understand and develop !

2. The evidence for stellar mass black holes

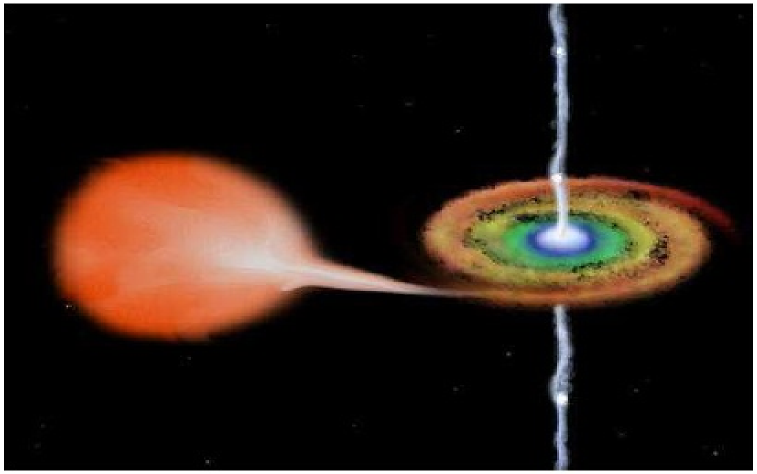


Fig. 6.4 – Vue d'artiste d'une binaire X de faible masse.

Riccardo Giacconi, Nobel 2002
(from 1961, first X-ray rocket, with H. Gursky ; to 1970, launch of UHURU satellite ; 1972 : Cygnus X1, first black hole candidate)

About 20 stellar mass BH strong candidates today

The evidence for super massive black holes

An “overwhelming” evidence for a SMBH in the Milky Way

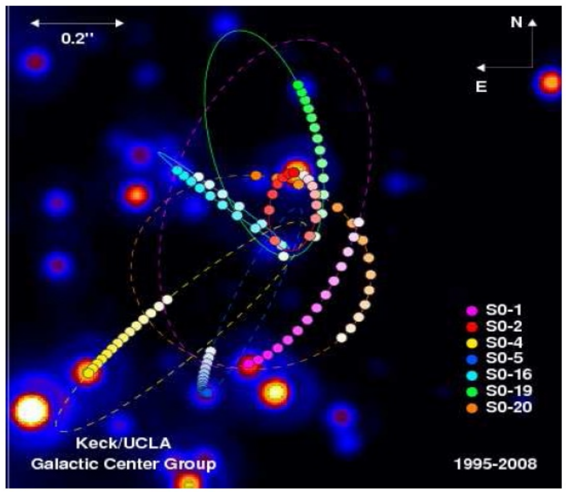


Fig. 7. Stars within the 0.02 parsecs of the Galactic center orbiting an unseen mass. Yearly positions of seven stars are indicated with filled colored circles. Both curved paths and accelerations (note the non-uniform spacings between yearly points) are evident. Partial and complete elliptical orbital fits for these stars are indicated with lines. All orbital fits require the same central mass of $\approx 4 \times 10^6$ $M_\odot$ and a common focus at the center of the image, the position of the radio source Sgr A*. (Image courtesy A. Ghez.)

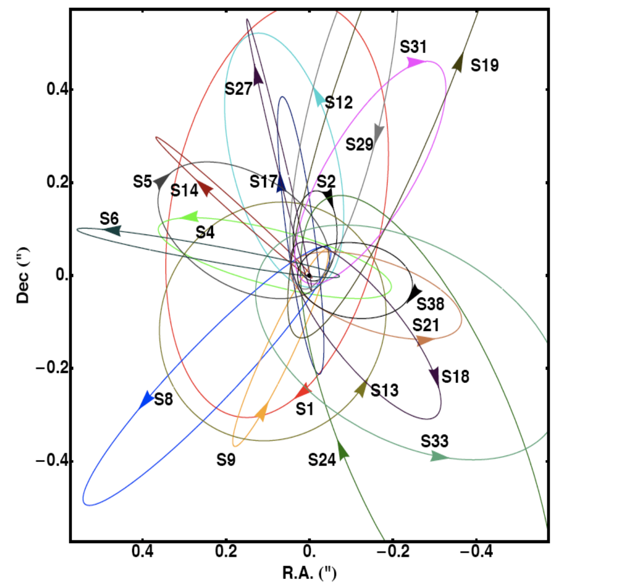


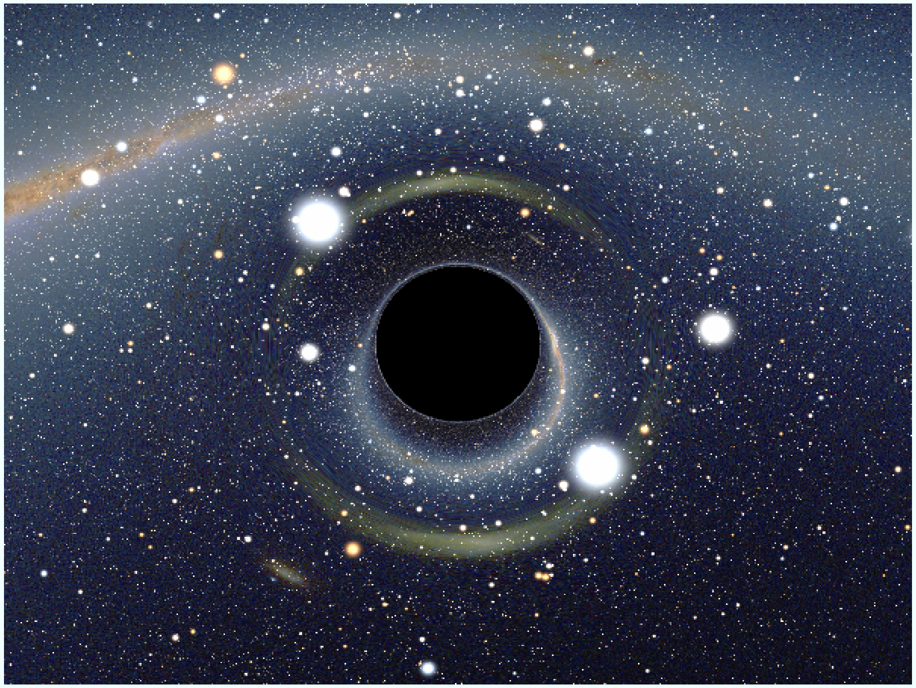
Fig. 16.— The stellar orbits of the stars in the central arcsecond for which we were able to determine orbits. In this illustrative figure,

Stars orbit an “immense unseen mass concentration”.

$M = 4.31 \times 10^6 M_\odot$, $r < 12M$. See e.g., Gillessen, Mark J. Reid.

A second lesson :

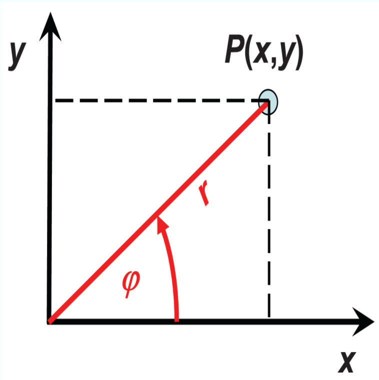
Black holes... are observed in the universe



Many ongoing efforts (observational and theoretical) to know more !

3. Black holes are "exotic" objects

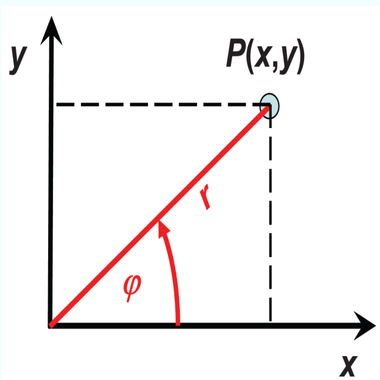
• Consider the line element : $ds^2 = dr^2 + r^2 d\phi^2$, with $\phi \in [0, 2\pi[$ and $r \in]0, \infty]$; $r = 0$ is a priori excluded because the metric is ill defined there.

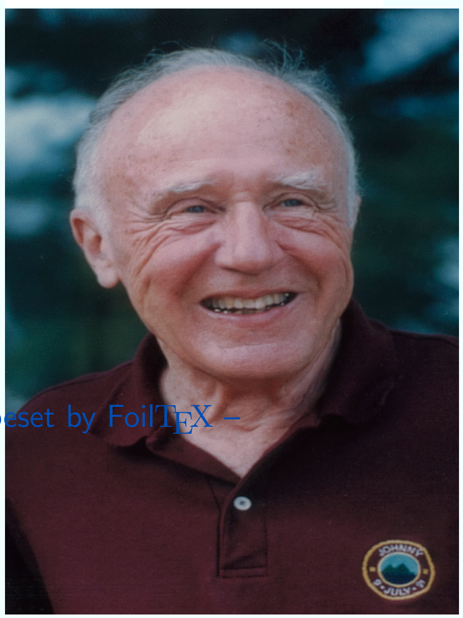


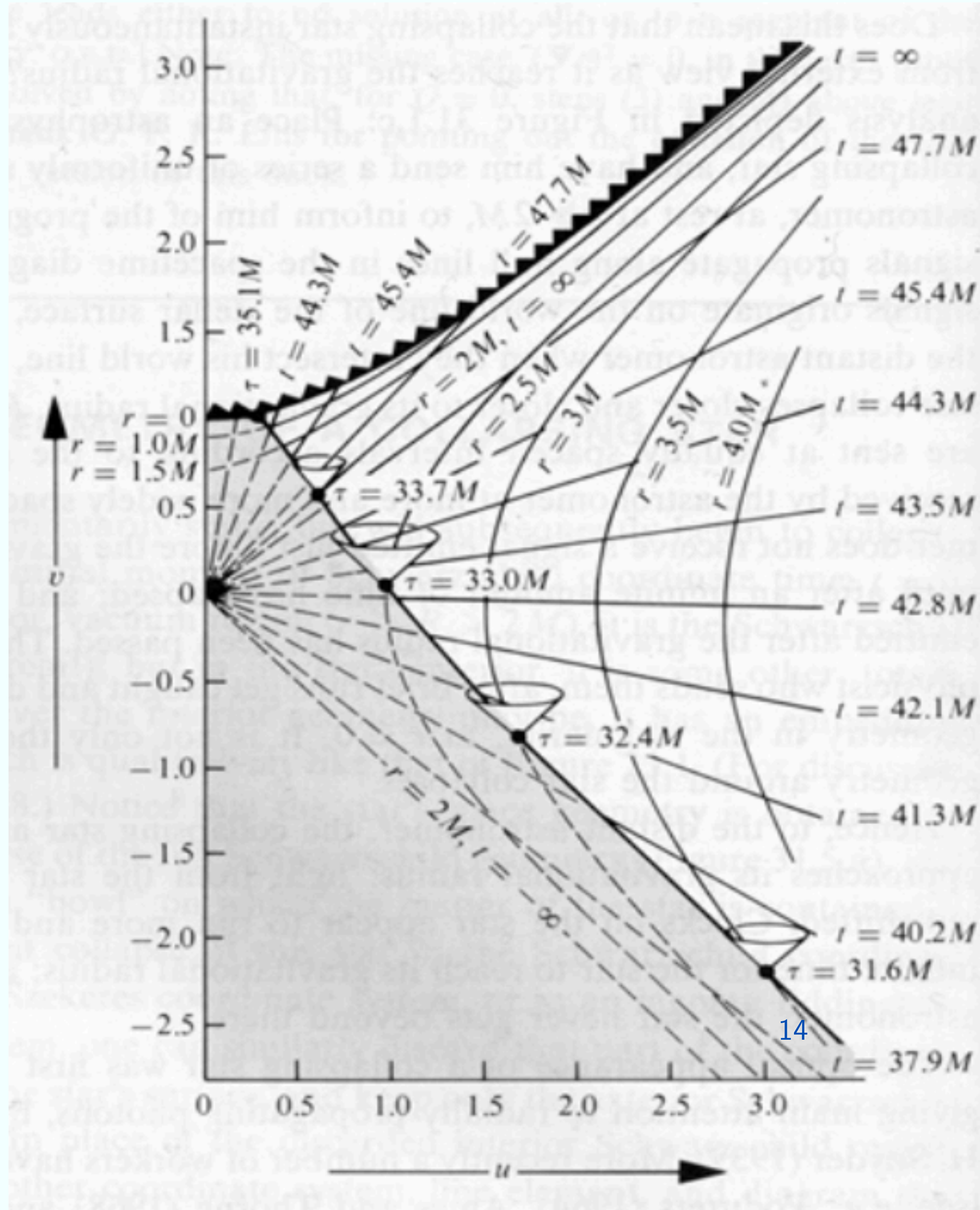
• Introduce new coordinates : $x = r\cos\phi$, $y = r\sin\phi$. The line element then becomes $ds^2 = dx^2 + dy^2$ which describes a flat, euclidean, plane in cartesian coordinates. The metric is now well defined everywhere including $x = y = 0$, that is $r = 0$.

The same happens with the Schwarzschild metric written with the (t, r, θ, ϕ) coordinates. One can go to new coordinates, which make the metric well-behaved at the horizon $r = 2m$ and extend spacetime, not by a single point, but by a whole new region.

Kruskal diagram
of a collapsing star
(MTW p 848)



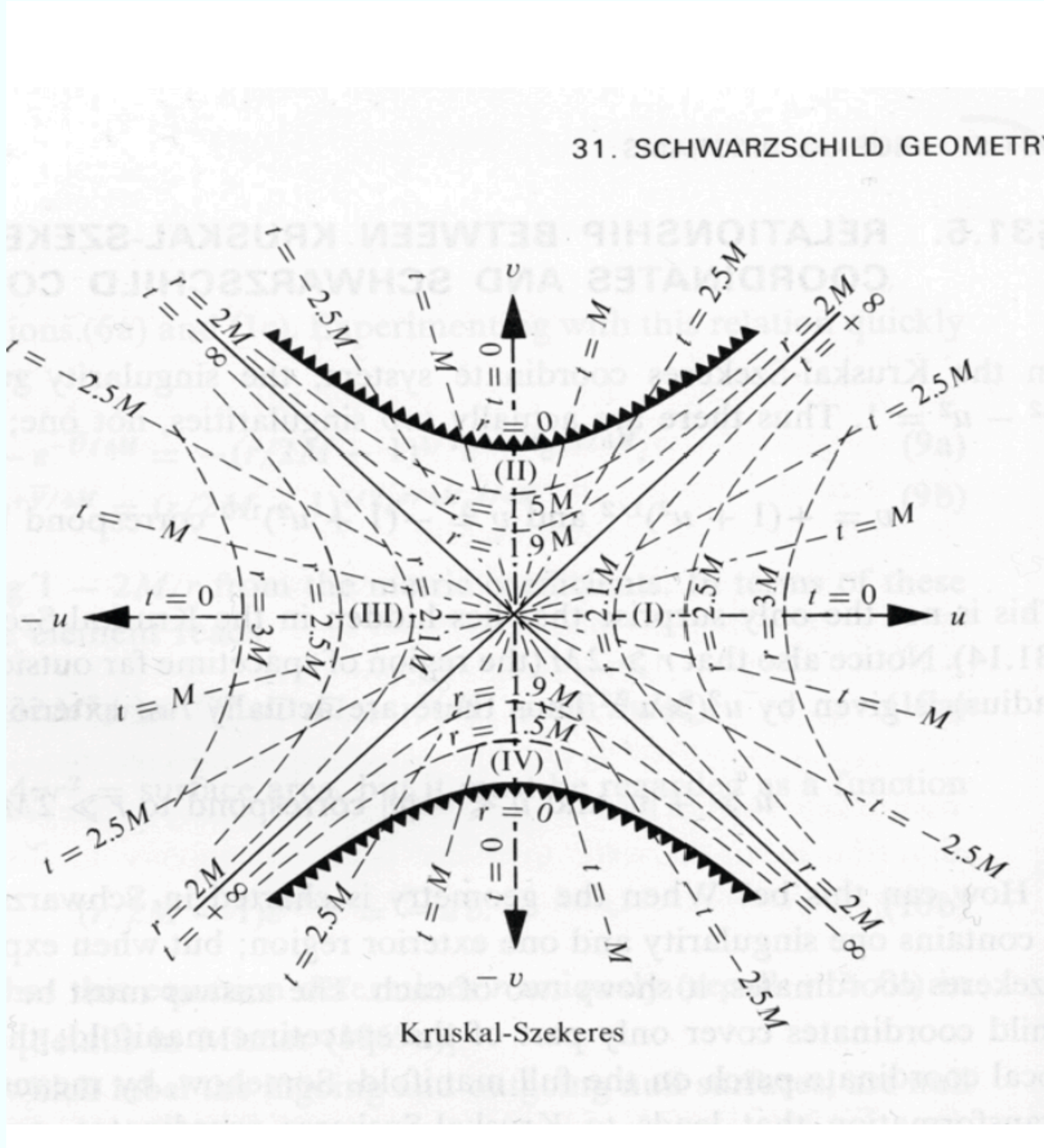




Kruskal diagram

of a black hole

(MTW p 852)



A third lesson :

Understanding the internal structure of black holes ...

involves many challenges :

- How "primordial black holes" (not formed by star collapse) can form ?
- Is their "exotic" geometry structure stable ?
- If it is, what does the "mirror universe" tell us about gravity ??
- Can it be ever observed ??

A speculative but potentially ground breaking field in the physics of gravity

4. Black holes as gateways to quantizing gravity ?

Stephen Hawking 1974

The definition of what is vacuum in the gravitational field of a black hole

is not the same as in flat, gravitation free, Minkowski spacetime.

As a result an observer far away from the black hole detects a flux of particles with a Planck spectrum.

Because matter IS quantized, black holes radiate like black bodies

Searching for (truly) exotic black holes...

Why ?... Because :

- Black holes evaporate, hence loose mass, “shrink” and MUST eventually be described as fully quantum objects

- Unfortunately there is no quantum theory of gravity “on the market”

- However there are indications (from string theory mainly) that

– spacetime may have $4+6$ or $5+6$ dimensions,
– Einstein’s equations may have corrections in $Riemann^2$,
– gravity may be described by a metric plus various fields, etc

Hence the search for black holes solutions

in higher dimensional / modified gravity theories

The opening of a Pandora box

(because there is no uniqueness theorem)

A few examples :

- Rotating solutions of $G_{\mu\nu} = 0$ in 5D

The 5D generalisation of the Kerr black hole (Myers-Perry, 1985) **is not unique**

Indeed there exist "black ring" solutions with horizons having topologies different from a 3-sphere (Emparan-Reall 2001) etc

- Rotating solutions in "Gauss-Bonnet" gravity $(G_{\mu\nu} + \alpha H_{\mu\nu} = 0)$

An analytical solution is known (ND et al., 2008) : is it a black hole ?
Numerical solutions exist (Brihaye et al, Kunz et al, 2008- onwards)

- adding an electromagnetic field : work in progress

Summary

Even if the first black hole was invented almost 100 years ago...

- **Lesson 1** : there is still a lot to understand about

– their formation (e.g. when they form from inspiralling neutron stars)

– the astrophysical mechanisms to extract their rotational energy

- **Lesson 2** : there is still a lot to do to on the observational side

(measure their angular momentum, probe whether their gravitational field is the Kerr's)

- **Lesson 3** : there is still a lot to do to understand about

their internal structure, its stability, the meaning of the "mirror universes" they appear to hide within their horizons

- **Lesson 4** :

– New mathematical techniques and/or numerical algorithms are needed to solve Einstein's equations in 4D and/or their generalizations to higher dimensions

– Black holes have been thought for 40 years now to be the clue to understanding quantum gravity but the goal still seems to be far ahead.

Black holes as indeed tools to understand gravity !

THANK YOU FOR YOUR ATTENTION