Mass and color coherence in antennas

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M. R. Calvo, MRM and C. A. Salgado Phys. Lett. B **738C** (2014) [arXiv:1403.4892 [hep-ph]]
MRM and C. A. Salgado work in progress

Outline

1. Motivation
2. Introducing the Radiative Antenna
3. Mass Effect
4. Discussion
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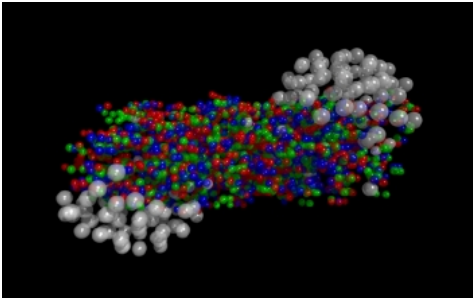
6. Summary

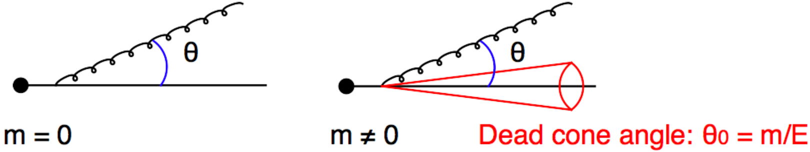
Motivation



- A state of deconfined coloured particles is formed in heavy ion collisions.
- One of the most striking effects of this coloured medium is the energy loss of high energy partons in heavy ion collisions.
- Is this energy loss the same for heavy quarks than for massless quarks?

Dead Cone Effect

- In vacuum, the presence of a mass leads to a suppression of gluon radiation inside the so-called dead cone.



Massive BDMPS Spectrum

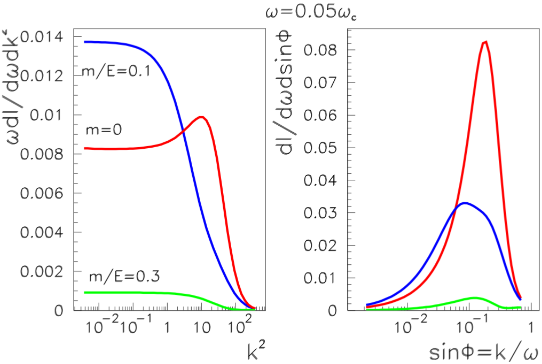
- In a medium, there is also a suppression due to mass effects.

$$k^+ \frac{\mathrm{d}N}{\mathrm{d}k^+\mathrm{d}^2\mathbf{k}} = \frac{\alpha_s C_F}{(2\pi)^2(k^+)^2} 2\mathrm{Re} \int_0^\infty \mathrm{d}y'^+ \int_0^{y'^+} \mathrm{d}y^+ \times \exp\left[i\frac{k^+}{2}\theta_{DC}^2(y^+ - y'^+)\right] \int \mathrm{d}^2\mathbf{z} \exp\Bigg[- i\mathbf{k}\cdot\mathbf{z} - -\frac{1}{2}\int_{y'^+}^\infty \mathrm{d}\xi\, n(\xi)\sigma(\mathbf{z})\Bigg] \boldsymbol{\partial}_y \cdot \boldsymbol{\partial}_z \mathcal{K}(y'^+, \mathbf{z}; y^+, \mathbf{y})\Big|_{\mathbf{y}=0}$$

(Armesto, Salgado, Wiedemann; Dokshitzer, Karzeev; Zhang, Wang, Wang; Djordjevic, Gyulassy, ...)

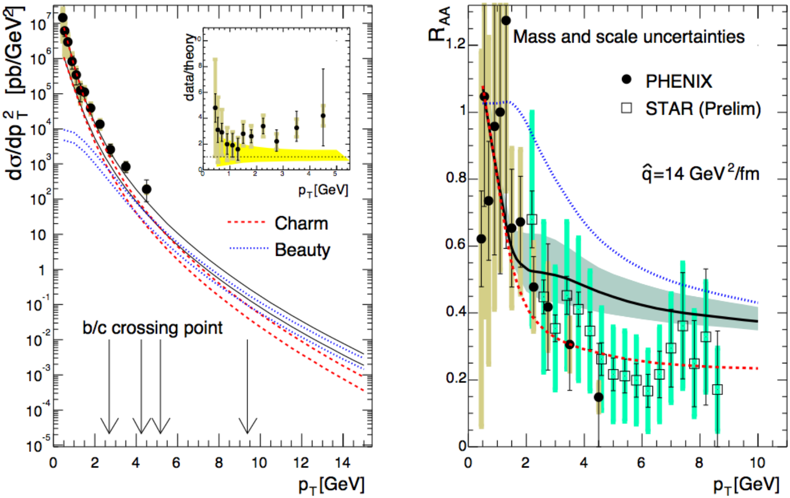
Massive BDMPS

- Smaller energy loss in the massive case (dead cone).



(Armesto, Salgado, Wiedemann)

Trouble with RHIC data



Armesto, Cacciari, Dainese, Salgado, Wiedemann

- BDMPS does not fit energy loss of heavy quarks.

Radiative Antenna (massless case)

- The BDMPS spectrum does not deal with interference effects between emitters.
- We consider an antenna composed of two partons to study the role of these interferences.
- In vacuum, interference effects lead to *angular ordering*: emission angles within the partonic cascade decrease from one emission to the next one.

$$\theta_1 > \theta_2 > \ldots > \theta_n > \ldots$$

- Interference spectrum for massless partons inside a QCD medium:

$$\mathcal{J} = \mathrm{Re}\left\{ \int_0^\infty \mathrm{d}y'^+ \int_0^{y'^+} \mathrm{d}y^+ \, \left(1 - \Delta_{\mathrm{med}}(y^+, 0)\right) \, e^{i\frac{k^+}{2} y^+ \delta \mathbf{n}^2} \right.$$
$$\times \int \mathrm{d}^2\mathbf{z} \exp\left[-i\bar{\boldsymbol{\kappa}} \cdot \mathbf{z} - \frac{1}{2} \int_{y'^+}^\infty \mathrm{d}\xi \, n(\xi)\sigma(\mathbf{z}) \right]$$
$$\left. \times \left(\boldsymbol{\partial}_y - ik^+ \delta\mathbf{n}\right) \cdot \boldsymbol{\partial}_z \, \mathcal{K}(y'^+, \mathbf{z}; y^+, \mathbf{y}) \Big|_{\mathbf{y} = \delta\mathbf{n} y^+} \right\} + \text{sym.}$$

where $\mathcal{K}$ takes into account the rescattering of the emitted gluon with the medium.

$$\mathcal{K}(y'^+, \mathbf{z}; y^+, \mathbf{y} | k^+) = \int_{\mathbf{r}(y^+) = \mathbf{y}}^{\mathbf{r}(y'^+) = \mathbf{z}} \mathcal{D}\mathbf{r} \exp\left\{ \int_{y^+}^{y'^+} \mathrm{d}\xi \, \left(i\frac{k^+}{2} \dot{\mathbf{r}}^2(\xi) - \frac{1}{2} n(\xi)\sigma(\mathbf{r}) \right) \right\}$$

Antenna Parameters

- Opening angle:

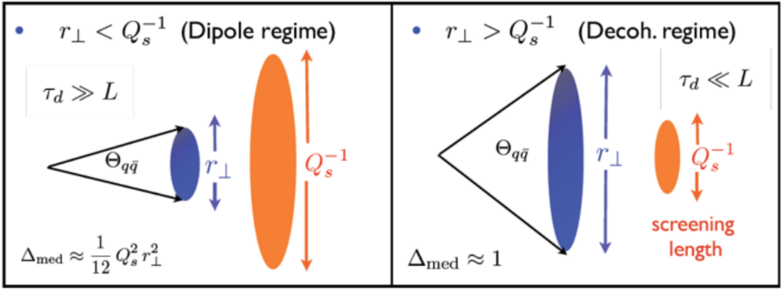
$$|\delta \mathbf{n}| \equiv \sin\theta_{qg} \sim \theta_{qg}$$

- Decoherence parameter:

$$\Delta_{\rm med}(y^+, 0) \equiv 1 - \exp\left[-\frac{1}{2}\int_0^{y^+} \mathrm{d}\xi\, n(\xi)\sigma(\delta\mathbf{n}\xi)\right] \simeq 1 - \exp\left[-\frac{1}{12}\hat{q}\delta\mathbf{n}^2 L^3\right]$$

$\sigma(\mathbf{r})$ is the dipole cross-section.
$\hat{q}$ is the medium transport coefficient/jet quenching parameter.
L is medium lenght.

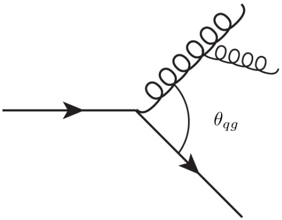
- What is the role of the parton mass in the picture below?

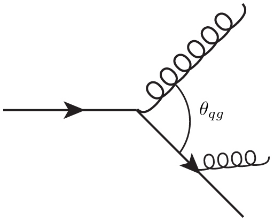


(Mehtar-Tani, Tywoniuk, Salgado, Casalderrey, Iancu)

Massive Antennas

- There are two types of massive antennas:
  - Heavy quark-antiquark antenna. (Armesto, Ma, Mehtar-Tani, Salgado, Tywoniuk)
  - Heavy quark-gluon antenna.





Framework Set-up

- BDMPS and interference spectra are calculated by solving the Classical Yang-Mills (CYM) equations

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

with the help of the continuity equation

$$[D_\mu, J^\mu] = 0$$

- Light-cone gauge ($A^+ = 0$) + linearized CYM equations.

$$\Box A^i - 2ig[A^-_{\text{med}}, \partial^+ A^i] = -\frac{\partial^i}{\partial^+} J^+ + J^i$$

Currents

- High energy partons are represented in the vacuum by the classical currents $J^{\mu}_{(0)}$

$$J^{\mu,a}_{(0)}(x) = g\frac{p^{\mu}}{E}\delta^{(3)}\left(\vec{x} - \frac{\vec{p}}{E}t\right)\theta(t)Q^{a}$$

- The classical currents get color rotated because of interactions with the medium.

$$J^{\mu}(x) = U_{p}(x^{+},0)J^{\mu}_{(0)}(x)$$

- Wilson lines are responsible for in-medium color rotation.

$$U_{p}(x^{+},0) \equiv \mathcal{P}\exp\left\{\int_{0}^{x^{+}} \mathrm{d}\xi\; T\cdot A^{-}_{\mathrm{med}}(\xi,\xi\mathbf{p}_{\perp}/p^{+})\right\}$$

Emitted Gluon

- The propagation in the medium of the emitted gluon is expressed by the following path integral, depicting its color rotation and Brownian motion in the transverse plane due to interactions with the background field.

$$\mathcal{G}(x^+, \mathbf{x}; y^+, \mathbf{y}|k^+) = \int_{\mathbf{r}(y^+)=\mathbf{y}}^{\mathbf{r}(x^+)=\mathbf{x}} \mathcal{D}\mathbf{r} \exp\left\{ \frac{ik^+}{2} \int_{y^+}^{x^+} \mathrm{d}\xi\, \dot{\mathbf{r}}^2(\xi) \right\} U(x^+, y^+; \mathbf{r})$$

Amplitude

- Applying the usual formulation to compute amplitudes, we get for the heavy quark-gluon antenna:

$$\mathcal{M}_\lambda(\vec{k}) = \frac{g}{k^+} \int\limits_{x^+=+\infty} \mathrm{d}^2\mathbf{x}\, e^{ik^- x^+} e^{-i\mathbf{k}\cdot\mathbf{x}} \int_0^{+\infty} \mathrm{d}y^+ \, e^{ik^+ \frac{p^-}{p^+} y^+}$$

$$\times \epsilon_\lambda(k) \cdot (i\partial_y + k^+\mathbf{n})\, \mathcal{G}(x^+, \mathbf{x}; y^+, \mathbf{y}|k^+)\Big|_{\mathbf{y}=\mathbf{n}y^+} U_p(y^+, 0) Q$$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Mass Effect

- The difference between the massive and massless cases comes from the dispersion relation:

$$2p^+p^- - \mathbf{p}^2 = M^2$$

- This leads to the appearence of a new phase:

$$\exp\left(ik^+\frac{p^-}{p^+}y^+\right) = \exp\left(i\frac{k^+}{2}\theta_{DC}^2 y^+\right)\exp\left(i\frac{k^+}{2}\mathbf{n}^2 y^+\right)$$

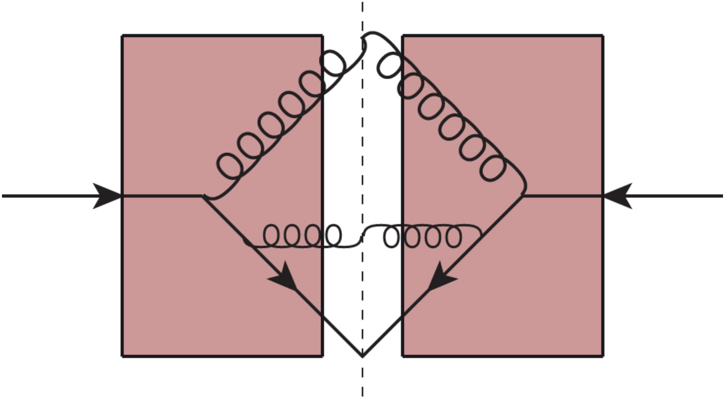
where

$$\boxed{\theta_{DC} \equiv \frac{M}{p^+}}$$

BDMPS $\mathcal{R}_q$

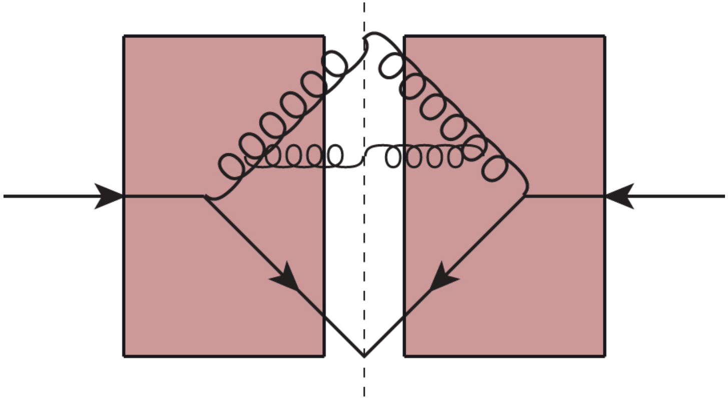
$$(k^+)^3 \frac{dN}{d^3k} = \frac{\alpha_s}{(2\pi)^2} \left[C_F \mathcal{R}_q + C_A \mathcal{R}_g - C_A \mathcal{J} \right]$$

$$C_F \mathcal{R}_q = (k^+)^2 \langle |\mathcal{M}_q|^2 \rangle$$



$$(k^+)^3 \frac{dN}{d^3k} = \frac{\alpha_s}{(2\pi)^2}\left[C_F \mathcal{R}_q + C_A \mathcal{R}_g - C_A \mathcal{J}\right]$$

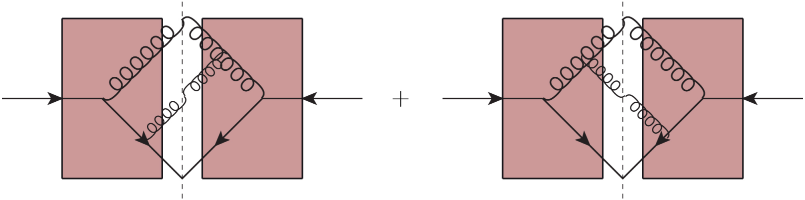
$$C_A \mathcal{R}_g = (k^+)^2 \langle |\mathcal{M}_g|^2 \rangle$$



Interference $\mathcal{J}$

$$(k^+)^3 \frac{dN}{d^3k} = \frac{\alpha_s}{(2\pi)^2}\left[C_F \mathcal{R}_q + C_A \mathcal{R}_g - C_A \mathcal{J}\right]$$

$$-C_A \mathcal{J} = (k^+)^2 \, \mathrm{Re}\langle \mathcal{M}_q \mathcal{M}_g^\dagger \rangle$$



Interference Spectrum

- The interference spectrum for the heavy quark-gluon antenna:

$$\mathcal{J} = \mathrm{Re}\Bigg\{ \int_0^\infty \mathrm{d}y'^+ \int_0^{y'^+} \mathrm{d}y^+ \, \left(1 - \Delta_{\mathrm{med}}(y^+, 0)\right) \, e^{i\frac{k^+}{2} y^+ (\theta_{DC}^2 + \delta \mathbf{n}^2)}$$
$$\times \int \mathrm{d}^2\mathbf{z} \exp\left[-i\bar{\boldsymbol{\kappa}} \cdot \mathbf{z} - \frac{1}{2} \int_{y'^+}^\infty \mathrm{d}\xi \, n(\xi)\sigma(\mathbf{z}) \right]$$
$$\times \left(\boldsymbol{\partial}_y - ik^+ \delta\mathbf{n}\right) \cdot \boldsymbol{\partial}_z \, \mathcal{K}(y'^+, \mathbf{z}; y^+, \mathbf{y})\Big|_{\mathbf{y} = \delta\mathbf{n} y^+} \Bigg\} + \mathrm{sym}.$$

- Same result as in the massless case... except for the mass phase!

Discussion: heavy quark-gluon antenna

- Spectrum:

$$(k^+)^3 \frac{dN}{d^3k} = \frac{\alpha_s}{(2\pi)^2} \left[C_F \mathcal{R}_q + C_A \mathcal{R}_g - C_A \mathcal{J} \right]$$

- Mass phase:

$$\Delta_{DC} = \exp\left[i \frac{k^+}{2} \theta_{DC}^2 y^+ \right]$$

- $\mathcal{J} \to 0$ if $\theta_{DC} \gg 1/\sqrt{\omega L}$
- Loss of color coherence implies enhanced energy loss.

Discussion: heavy quark-antiquark antenna

- Spectrum:

$$(k^+)^3 \frac{dN}{d^3k} = \frac{\alpha_s}{(2\pi)^2} \left[C_F \mathcal{R}_q + C_F \mathcal{R}_{\bar{q}} + (C_A - 2C_F)\mathcal{J} \right]$$

- Mass phase:

$$\Delta_{DC} = \exp \left[i \frac{k^+}{2} \left(\theta_{DC}^2 y^+ - \bar{\theta}_{DC}^2 y'^+ \right) \right]$$

  - Asymmetric case ($\theta_{DC} \gg \bar{\theta}_{DC}$, for example). Similar to the qg case.

$$\mathcal{J} \to 0 \text{ if } \theta_{DC} \gg 1/\sqrt{\omega L}$$

  - Symmetric case ($\theta_{DC} \simeq \bar{\theta}_{DC}$)

$$\mathcal{J} \to 0 \text{ if } \theta_{DC} \gg 1/\sqrt{\omega t_{form}}$$

The symmetric case loses coherence slower than the asymmetric one.

- Loss of color coherence implies less energy loss ($C_A > 2C_F$).

Discussion: symmetric heavy quark-antiquark antenna

$$(k^+)^3 \frac{dN}{d^3k} = \frac{\alpha_s}{(2\pi)^2}\left[C_F \mathcal{R}_q + C_F \mathcal{R}_{\bar{q}} + (C_A - 2C_F)\mathcal{J}\right]$$

- Decoherence regime ($\Delta_{\text{med}} \sim 1$):

$$(k^+)^3 \frac{dN}{d^3k} = \frac{\alpha_s}{(2\pi)^2} C_F \left[\mathcal{R}_q + \mathcal{R}_{\bar{q}}\right]$$

- Total coherence regime ($\Delta_{\text{med}} \sim 0$):

$$(k^+)^3 \frac{dN}{d^3k} = \frac{\alpha_s}{(2\pi)^2} C_A \mathcal{R}_q$$

Radiation off a gluon but with the dead-cone suppression factor!

Quenching Weights

- Medium-induced energy loss distribution. For independent gluon emissions carrying away a total energy ΔE:

$$P(\Delta E) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int \mathrm{d}\omega_i \frac{\mathrm{d}N(\omega_i)}{\mathrm{d}\omega_i} \right] \delta\left(\Delta E - \sum_{i=1}^{n} \omega_i \right) \exp\left[- \int_0^{\infty} \mathrm{d}\omega \frac{\mathrm{d}N(\omega)}{\mathrm{d}\omega} \right]$$

$$= \int_C \frac{\mathrm{d}\nu}{2\pi i} \exp\left[- \int_0^{\infty} \mathrm{d}\omega \, \frac{\mathrm{d}N(\omega)}{\mathrm{d}\omega} \left(1 - e^{-\nu\omega}\right) \right] e^{\nu \Delta E}$$

- A quenching weight $P(\Delta E)$ for each antenna spectrum.

Quenching Weights Results

Summary

- Energy loss of heavy quarks is a remaining puzzle.
- The effect of the mass will suppress the interferences, thus losing coherence more easily than in the massless antenna.
- The loss of coherence implies:
  - larger energy loss (quark-gluon antenna).
  - smaller energy loss (quark-antiquark antenna).
- Interference in the heavy-qg/asymmetric heavy-$q\bar{q}$ antenna will dissappear faster than in the symmetric heavy-$q\bar{q}$ one.
- Comparison via quenching weights is being done.